Mortality Uncertainty
How to get a distribution around the Best Estimate Mortality

Henk van Broekhoven
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What are we going to do?

- This workshop contains 3 parts
  - Definition of mortality risk
  - Uncertainty in projections
  - Solvency II
Definition Life Risk

• Life Risk relates to the deviations in timing and amount of cash flows due to incidence or non-incidence of death
  – Deviations relative to the Best Estimate Assumptions

• Overall mortality can be described by a probability function in which the Expected Value is the Best Estimate.
Definition Life Risk

• This probability function is not just a single distribution

• Following the IAA Blue Book EVERY risk type should be analysed in 3 parts:
  – Volatility
  – Uncertainty (model + parameter)
  – Extreme events
Distribution around the BE mortality

- Best Estimate Mortality rates can be analysed into two parts:
  - Level
  - Trend

- The distribution is defined by the following sub-risks:
  - Volatility
  - Uncertainty Level
  - Uncertainty Trend
  - Extreme event risk (Calamity)
Question / Discussion

• Stochastic?

OR

• Analytic?
• Volatility
• Extreme Event (calamity)
  • Uncertainty Level
  • Uncertainty Trend
Volatility

• Risk of random fluctuations
  – frequency
  – important: variation of sum-at-risk across policies
  – also additional volatility because of external causes
    • like cold winters
    • influenza epidemics
    • severe accidents

<Mortality is not a full independent process>
Volatility

• Because of the (small) dependency between the several lifes the Compound Poisson distribution (instead of Compound Binomial) is used.
Volatility

- Via an analytical way this CP distribution can be estimated using the Normal Power Approach:

\[ P\{[S - E(S)]/\sigma_s \leq s + \frac{1}{6} \gamma (s^2 - 1)\} \approx \Phi(s) = \alpha \%
\]

- The NP approach is based on the Cornish Fisher expansion only using the first 3 or 4 moments.
Volatility

Standard deviation total claim level \((c(j))\) follows (for Compound Poisson):

– **\(X\) is capital at risk\) (=face value \(-\) reserve)

\[
\sigma = \sqrt{\sum q_i(x)X_i^2}
\]

• And Skewness:

\[
\gamma = \frac{\sum q_i(x)X_i^3}{\sigma^3}
\]

\(\gamma\) can be fine-tuned based on observed volatility in the past (Maximum likelihood)
• Volatility
• Extreme Event (calamity)
  • Uncertainty Level
  • Uncertainty Trend
Extreme event risk

• Worldwide the worst thing related to mortality that can happen is a Pandemic.
  – Some events are simple too extreme to model like an impact of a large asteroid.

• Still every insurance company should analyse concentration risk within their portfolio’s. An incident can, in case of high concentrations, have a larger impact than a pandemic.
Extreme Events in mortality

deads / 1000 age group 15-45
The Netherlands, source CBS

deads per 1000

year
Extreme event risk

• Although often a model for extreme events is connected to some confidence level, like 1 in 200 for Solvency II, in my opinion this is wrong.
• A pandemic is a conditional event.
  – The 1 in 200 connected to the 0.15% is valid in case we are in a WHO phase 1 situation.
  • We never were!
# Pandemic phases

<table>
<thead>
<tr>
<th>Inter-pandemic phase</th>
<th>Low risk of human cases</th>
<th>Higher risk of human cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>New virus in animals, no human cases</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Pandemic alert</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>New virus causes human cases</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Pandemic</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
Extreme event risk

• Better to talk about a scenario, like the Spanish Flu: combined with the 1 in 200 VAR of the other risks you should be able to survive a certain scenario.
  – Beside this “conditional” nature of a pandemic it is also a fact that nobody really knows the real pandemic distribution.
  – This also counts for other types of extreme events
Question / Discussion

• Now we start calculating the uncertainty.
• It should at the end result in an uncertainty around the BE liabilities.

• Should it be a one-year approach or a multi-year approach?
  – Solvency II Standard Model is based on a one year approach…
• Volatility
• Extreme Event (calamity)
  • Uncertainty Level
  • Uncertainty Trend
Uncertainty Level

• The best estimate is based on a factor times the population mortality (Basis Best Estimate)

• The factor in formula:

\[
\text{factor} = \frac{\mu_{\text{obs}}}{\mu_{\text{pop}}}
\]
Uncertainty Level

• Because the $\mu_{\text{obs}}$ is the result of a random process (=volatility in the past!) We are not sure that this number represent the real expected claim level.
  – It could have been an observation in the (wrong) tail.

• To be sure for $\alpha\%$ that the liabilities are sufficient we need to add a capital.
Uncertainty Level

• Recalculate the liabilities based on mortality rates on “α% level”, this is

\[ q_{EC}(x; t) = \text{factor}_{EC} \times q_{pop}(x; t) \]

\[ \text{factor}_{EC} = \frac{\mu_{obs} + (-)unc_{\alpha\%}}{\mu_{pop}} \]
Uncertainty Level

• To find the uncertainty (unc) around the Best estimate we use like in Volatility the Compound Poisson distribution
  – Poisson instead of binomialial because there is some dependency in mortality
  – Compound because of the spread of the insured sums
  – Like in volatility a correction can be made to the skewness based on historical volatility
• The Compound Poisson distribution can be calculated in a rather simple way using the normal power approach similar to the volatility calculations

• The distribution is translated into a normal distribution using the first three moments
  \[ \rightarrow \text{NP}(3) \]
The economic capital “level” follows:

\[ EC_{level} = LIAB_{EC} - LIAB_{BE} \]

Liabilities based on \( q_{EC}(x;t) \)

Liabilities based on \( q_{BE}(x;t) \)
• Volatility
• Extreme Event (calamity)
  • Uncertainty Level
  • Uncertainty Trend
Uncertainty Trend

• It is impossible to predict a future trend.
• Medical development, environment, new diseases and resistance against a medical cure can change trend (drift).
• Also volatility in the observations will cause uncertainty (random walk).
• Are we using the right model?
• We must try to say something about the uncertainty
Uncertainty Trend

• In Lee Carter the stochastic part is based on
  – The random walk around the drift
  – Mistake in estimation of the drift because of volatility in used data
  – The drift is rather linear (in standard model)
  – This means that changes in drift are modeled as random walk
  – Often the Normal Distribution is used.
Uncertainty Trend

• The real uncertainty is the uncertainty in the drift. But exactly this part is hard to model.
• Other than assumed under Lee Carter is the drift not a straight line and for sure not Normal Distributed
• It is influenced by several factors like medical developments, resistance against medicines, climate change.
• Even extreme events can occur like a cure against important causes of death (cancer)
Uncertainty Trend

Lee Carter model

\[ c = \frac{1}{T} \sum_{t=1}^{T} [k(t) - k(t-1)] = \frac{k(T) - k(0)}{T} \]

Standard error \( e(t) \sigma \):

\[ se(e) = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} [k(t) - k(t-1) - c]^2} \]

Error in \( c \):

\[ se(c) = \sqrt{\frac{\sigma^2}{T}} \approx \frac{se(e)}{\sqrt{T}} \]
Uncertainty Trend

• In my opinion this method is underestimating the risk around trends.

• The random walk part of a trend analysis is better separate modelled under volatility.
  – It contains for example impacts of cold winters, flu epidemics etc.
Uncertainty Trend

• The uncertainty is not only based on a statistical error but also:
  – Model choice
  – Used data
  – Unexpected developments (extreme events)
    • E.g. a medical cure against an important cause of death.
Uncertainty Trend

• Therefore uncertainly trend should be partly based on information and ideas out of the medical world to get an insight of the extreme events related to longevity

• Also historical changes in trend can help to get information about model choices and data used.
The following calculations are based on the newest Dutch Prognosis.
For the uncertainty in the long term life expectancy the goal table we use a recent study of the Dutch CBS. In that study also several international conclusions are used.
It is stated that a 95% confidence interval means an increase of plus and minus 5 in the life expectancy.
  - This is based on both statistical and medical information and ideas.
Using this, by adjusting the mortality rates of the BE long term projection in such a way that the life expectancy is adjusted with 5 years a shocked projection table (both upper and lower) can be created.
Playing with a goal table

• The uncertainty can be solved using the following formula:

\[
q(x; j + t) = q(x; t) \times f(x; j)^t \times e^{\frac{\alpha(x)t(t+1)}{2}}
\]

Original trend

For \( j+t \) is final year of projection the adjusted goal table this formula can be solved
Playing with a goal table

- Solving $\alpha(x) < j+t$ is the last year of the projection:

$$\alpha(x) = \frac{\log q(x; j+t) - \log q(x; t) - t \times \log f(x; j)}{\frac{1}{2} \times t \times (t + 1)}$$
Uncertainty Trend (goal table)

Impact shock in goal table
95% conf e(0).
Uncertainty Trend

• Each model
• Based on the history it can be analysed how different 10 year trends look like
  – I used here Dutch figures
• This gives an idea how also in the future short term trend can change or what the impact can be for other assumptions
• By setting all those different trends at the beginning of the prognosis some conservatism will be introduced
  – With each trend observed in the past a separate generation table can be calculated
  – I used 12 trends
• See next graph
Uncertainty Trend

impact possible trends on development e(0)
How to use this?

• An insurance company and pension funds need to know what the impact is on the Best Estimate liabilities of the uncertainty around the trend.

• Because a complex dependency between the development for the several ages it is not advised to calculate liabilities directly with the at a certain confidence level shocked mortality rates.

• Better is to use each of the generation tables (11 for start trend and 1 for the shocked goal table) plus the BE generation table to calculate liabilities.
Uncertainty capital starting trend

With each of the 12 historical 10 year trends a generation table is derived and with each of them liabilities can be calculated:

<table>
<thead>
<tr>
<th>Year</th>
<th>Trend Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.09420843</td>
</tr>
<tr>
<td>2</td>
<td>13.33246787</td>
</tr>
<tr>
<td>3</td>
<td>13.066988</td>
</tr>
<tr>
<td>4</td>
<td>13.17006177</td>
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<tr>
<td>5</td>
<td>13.33750594</td>
</tr>
<tr>
<td>6</td>
<td>13.02018636</td>
</tr>
<tr>
<td>7</td>
<td>13.07364983</td>
</tr>
<tr>
<td>8</td>
<td>13.02874271</td>
</tr>
<tr>
<td>9</td>
<td>13.48099007</td>
</tr>
<tr>
<td>10</td>
<td>13.09073366</td>
</tr>
<tr>
<td>11</td>
<td>13.25972385</td>
</tr>
<tr>
<td>12</td>
<td>14.0581669</td>
</tr>
</tbody>
</table>

Standard deviation: 0.293
Distribution: Student with 11 DoF
97.5% upper conf. level: 2.2*0.293
= 4.9% of the mean

Because the local trend is extreme at the moment in the Netherlands no capital is needed for this part.
Uncertainty trend

- Often developments are conditional.
  - In calculating uncertainty this should be counted for.
Uncertainty trend

• Assuming 100% correlation between the uncertainty goal table and uncertainty start trend the total capital can be set at the sum.

• The results will be highly dependent on the discount rate!

• Most likely the ratio’s mentioned can be used for other countries with not enough data available to make own calculations.
Standard model Solvency II
Standard Model

• For longevity just a simple shock of 20% on the mortality rates
  – Not duration dependent
  – One year time horizon
Comparison S II with internal model

Internal model versus SM

% of BE liabilities

age

SM
LOW (0.9)
HIGH (0.7)
Standard model

• IMO:
  – Standard model less suitable for risk management
  – Perhaps OK for SCR calculations for average portfolio’s