Variable Annuities Risk Management

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Risk and Investment Conference
Leeds - June 28, 2012

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VA Key Features

What is a Variable Annuity?

- VAs are
  - Unit-linked products;
  - With customizable guarantees;
  - In 2 phases: Accumulation and Income (Annuitisation).

- 4 types of guarantees (that can be enhanced by additional options):
  - Death Benefit (GMDB)*: Minimum payout on death guaranteed;
  - Withdrawal Benefit (GMWB): Minimum annual withdrawals guaranteed;
  - Income Benefit (GMIB): Minimum annual payments guaranteed in the inc. ph.;
  - Accumulation Benefit (GMAB): Minimum payout guaranteed at the end of a
    guarantee period.

*GMxB: Guaranteed Minimum x Benefit
VA Key Features

Why Variable Annuities?

- Policyholder perspective:
  - Upside potential;
  - Guarantees on downside risk;
  - Flexibility;
  - Death Insurance and annuity payments.

- Insurance Company perspective:
  - High fees - both M&E/Administration fees (1.3% - 2.4%)* and guarantee fees;
  - Less capital intensive than traditional fixed annuities;
  - However, VAs are risky products: risk management and pricing are key.

- Distributor perspective:
  - Very high fees (even 50bps).

GMDB - Guaranteed Minimum Death Benefit: guaranteed lump sum received when the owner of the contract dies.

GMIB - Guaranteed Minimum Income Benefit: guaranteed minimum income stream upon annuitisation at a particular point in the future.

GMWB - Guaranteed Minimum Withdrawal Benefit: guarantee similar to the income benefit, but that doesn't require annuitising. The policyholder has the right to withdraw up to a maximum guaranteed amount every year.

GMAB - Guaranteed Minimum Accumulation Benefit: guaranteed lump sum at a certain point in the future.

*Source BoA
VA Key Features

Main Guarantees

- **Life Time Guarantee** - GMWB, GMIB;
- **Fund Switch**;
- **Step-up (Ratchet) Guarantee** - GMDB, GMWB, GMIB;
- **Roll-up Guarantee** – GMDB.

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**Life Time Benefit**
- **GMDB**
  - Guaranteed lump sum in case of death („Roll-up“)
- **GMAB**
  - Guaranteed lump sum at maturity
- **GMIB**
  - Guaranteed income agreed in year 0
- **GMWB**
  - Guaranteed amounts for withdrawals
### Post 2008 VA

**Re-design of the products**

- **Lower guarantees:**
  - Accumulation and Income benefits limited/no longer offered;
  - Guaranteed withdrawal and roll-up rates reduced, Step-up periodicity increased.

On the other hand:

- In Europe, premiums are still explicitly guaranteed (GMIB widespread).

- **Higher fees (especially guarantee fees).**

- **Less risky investments:**
  - Reduced equity proportion (US VA Avg EBR: 45.6% in 2010 vs 60% in 2009)*;  
  - Target volatility funds (reduced hedge cost);
  - Indexed funds (reduced basis risk).

*Source Munich Re*
VA Risk Profile

Risk Overview

- **Model Risk**
- **Market Risks**
  - Equity
  - Equity Volatility
  - Interest Rates (Rho and Duration)
  - Credit
- **Behavioural Risks**
  - Lapses
  - Fund Switching
- **Biometric Risks**
  - Mortality (GMDB)
  - Longevity (Life time Opt.)
- **Hedge Risks**
  - Counterparty
  - Basis
  - Hedge Securities Liquidity

Scylla and Charybdis

Model Risk
Risk Management

General approaches, which may occur together

- Product design:
  - Pricing (e.g. dynamic fees), guarantees fine tuning, asset allocation;
  - If over engineered, it may imply an higher model risk.

- Hedge:
  - Static/dynamic, global/local (Tail), over all/some risk factors;
  - It needs resources and expertise.

- Investment banks hedge packages:
  Losing margins to Investment banks and potential hedge inefficiency.

- Reinsurance:
  Losing margins to reinsurer.

Market Risk Hedge

Types of Hedging Strategies

Hedging is based on the following Taylor approximation:

\[
\Delta f(S, t, r, \sigma) = \frac{\partial f}{\partial S} \Delta S + \frac{\partial f}{\partial t} \Delta t + \frac{\partial f}{\partial \sigma} \Delta \sigma + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} (\Delta S)^2 + \ldots
\]

Depending on which terms are hedged, the different approaches can be classified into:

- Delta hedge,
- Delta, gamma hedge,
- Delta, gamma, rho hedge,
- Delta, gamma, rho and vega hedge, . . .
Market Risk Hedge

**Delta Hedge**

- The idea is to use a hedging portfolio for which the partial derivative with respect to equity price $S$ is the opposite of the liability options portfolio.

**Delta Hedge vs Delta-Gamma Hedge**

- If the stock market does not move significantly before the updating of the replicating portfolio, the gamma impact is much smaller than the delta effect.
- Hence, in a low volatility market the delta hedge is not considerably less effective than a delta-gamma hedge.
- Conversely, the delta hedge is considerably less effective than a delta-gamma hedge when the equity volatility is high.
- Furthermore, the gamma impact is higher when the underlying is around the option strike and when the option is closer to its maturity.

- VA liabilities usually are out of the money long term options. Hence, generally the delta hedge is a reasonable approach.
- However, the hedge has to be monitored carefully because the gamma impact can surge in fast moving markets.
Market Risk Hedge

Hedging equity volatility (vega)...

- On the one hand, higher volatility causes an increase of the present value of the VA liabilities (economic loss).
- On the other hand, the fund performance does not depend explicitly on the equity volatility. Hence, if no correlation is assumed between the equity volatility and the fund return, a high equity volatility can not cause the guarantee to go in the money.

- The hedging options can be short or long dated:
  - Long-term options usually match VA liabilities better. However, they are expensive and illiquid.
  - Short-term options are cheaper and liquid. However, they have to be rolled over. In this case, higher volatility implies a higher hedge cost, i.e. an actual loss due to the higher premiums paid to roll over the hedge.

- Hence, hedging vega is not a trivial choice.

Market Risk Hedge

... and hedging interest rates (rho)

- Decreasing IRs cause an increase of the present value of the VA liabilities (economic loss).

- The fund performance depends on both equity and fixed income investments.
- If no correlation is assumed between equities and IRs (which is a tough assumption) the equity fund return is not affected by low IRs.
- Conversely, the fixed income investments generally have to be rolled over. Hence, lower IRs imply an actual loss due to the lower cash flows received in the future.

- Hence, hedging rho is not a trivial choice.

- It is worth noting that, since the VA investments have a lower fixed income proportion than the traditional annuities investments, the IRs impact on VAs cash flows is lower.
Market Risk – Current Scenario

Current scenario Impact on VA

- Low IRs:
  - On an economic basis, low interest rates are strongly increasing the value of guarantees, especially in the long term (rho risk);
  - Low IRs impact the fixed income investments roll-up;
  - However, it will likely take more than 10 years for economic claims from new VAs.

- Low equity level:
  - Guarantees in the money;
  - May be seen as positive for new VA business (Upside potential).

- High equity volatility:
  - Increases the value of the guarantees on an economic basis;
  - Increases the hedge costs.

Market Risk Management in the current scenario

- A 3 greeks (Delta, Vega, Rho) hedge approach is widespread among insurers.

- Some insurers are under hedging rho risk (assuming that IRs will rise) to avoid locking-in IRs at historical lows.

- A study by Oliver Wyman suggests that hedging only the tail risk offers a higher risk based return than an engineered hedge*.

- The Hedge transactions in place mainly consists of short term (<5Y) puts and futures**.

*Source UBS
**Source BoFA
Variable Annuities Risk Management

Conclusions

- VAs are unit-linked products with customizable guarantees;
- Potentially VAs are very profitable products;
- Despite the post 2008 re-design, VAs are still very risky products;
- Risk management and pricing are key;
- VAs are exposed to many types of risk, which can be managed by different RM strategies;
- Exposure needs to be monitored carefully: Greeks and hedge effectiveness;
- Hence, have the right people and tools!
Thank You

Any Questions?

Appendix A

VA Option Valuation – An Example
VA Option Valuation – An Example

Valuation

In general terms the value of the option \(\pi(Y)\) is the expected value \(E^Q[\cdot]\) of the present value of the future cash flows due to the option. In formulas:

\[
\pi(Y) = E^Q \left[ \sum_{\tau} (1 + r(\tau))^{-\tau} \cdot X(\tau) \right]
\]

where

- \(X(\tau)\) is the cash flow at time \(\tau\), which is typical of the particular option;
- \(r(\tau)\) represents the risk free interest rate between [0; \(\tau\)].

This calculation is normally performed by means of simulation.

VA Option Valuation – An Example

Withdrawals and GMWB

- In the Accumulation Period the policyholder can withdraw an amount up to the Account Value. This amount has not a guaranteed minimum, unless the policyholder buys a guaranteed minimum withdrawal benefit (GMWB).
- Any withdrawal before the end of the withdrawal charge period (\(T_{WC}\)) implies a fee.
- If the policyholder buys a GMWB, he is entitled to:
  - A total guaranteed amount available for future periodic withdrawals during the Accumulation phase, the Guaranteed Withdrawal Balance (GWB);
  - A Guaranteed Annual Withdrawal Amount (GAWA), that is a percentage of GWB (depending on contract and age at the date of first withdrawal).
VA Option Valuation – An Example

GMWB - Valuation

- For the GMWB, each cash flow (withdrawal) is

\[ X(t) = \min\{ \min\{R(t), GAWA(t)\}, GWB(t) \} \]

Usually
- \( GAWA(t) = GWB(0) \times f(t_W) \);
  - \( f(t_W) \) is a percentage which depends on the time (Ph’s age) of the first withdrawal \( t_W \).
- \( GWB(0) = \) Initial Premium;
- \( R(t) \) is the actual amount withdrawn, that is
  - \( R(t) = 0 \) for \( t < t_W \),
  - \( 0 \leq R(t) \leq GAWA(t) \), for all \( t \in [t_W, T] \),
  - and \( R(t) = GAWA(t) \), for all \( t \geq T \), assuming the "for life option".

Appendix B
Lapse Risk and Step-up Options Strategy

- Guarantees Out of the Money are likely to imply lapses, hence loss of future profits;
- The insurer can keep the guarantees about At the Money selling step-up options;
- If markets go down, the new guarantees go deep In the Money. To avoid losses, the insurer has to hedge buying step-up options;
- The cost of this hedge may be passed to the client through higher fees.

However

- In a volatile market this hedge cost will be very high, and so the fees. Would clients still buy the product?
- This strategy is considerably subject to model risk, increase in volatility and availability of the required options in the market.

Market Risk Hedge

Variable Annuities – Greeks in the Black-Scholes Model

\[
\begin{align*}
\Delta_P &= \frac{\partial P}{\partial S} \\
&= \Phi(d_1), \\
\Gamma &= \frac{\partial^2 P}{\partial S^2} \\
&= \frac{\Phi'(d_1)}{S \times \sigma \times \sqrt{T}}, \\
\Lambda &= \frac{\partial P}{\partial \tau} \\
&= S \times \Phi'(d_1) \times \sqrt{T - t}, \\
\PP &= \frac{\partial P}{\partial \tau} \\
&= -(T - t) \times K \times e^{-r(T - t)} \times \Phi(-d_2).
\end{align*}
\]
Model Risk

A well known example: Fat tails

- On Thursday 6.5.2010 the NYSE (Dow Jones Industrial Average) fell temporarily over 9%, as a consequence of such fears and automated trading.
- On Monday 10.5.2010 the Euro Stoxx index performed 10.35% within one day, after the announcement of a EUR 750bn bail-out plan.
- Assuming a volatility of 20% and log-normally distributed equity-market returns, this represents a 9.8 \( \sigma \)–event.
- Such an event has a return period of \( 5.9 \times 10^{17} \) years. This number is considerably bigger than the age of the universe which is \( 1.4 \times 10^{10} \) years.
- Hence, the log-normally distributed model is not correct in the tails.
- Many VA hedge strategies are based on the log-normal distribution assumption. This is a clear example of model risk.

VA US Market (up to 2010)

Some numbers

- The US market is definitely the largest one for VA:
  - 2010 Assets under Management: $1.5 trillion (US GDP – $14 trillion);
  - 2010 Sales: $138bn (about 90 times UK VA Market)*.
- VA market has been progressively exceeding FA sales over the past 15 years.
- VA sales track equity market.

\*YE2010 GBP/USD = 0.6432

2010 UK VA Sales ~ £1bn
2010 Europe VA Sales ~ €2.6bn