Capital (Cost) Allocation Leading Practices
A brief tour

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Capital Cost Allocation Best Practices

• Design-Driven Approach
• Core Elements:
  – Realistic framework of insurer capital usage
  – Explicit risk preferences and reward appetite
  – Key sensitivities: the Three R’s
• Operational buffer
### Capital (Cost) Allocation
### Leading Practice Process

<table>
<thead>
<tr>
<th>Leading Practice Step</th>
<th>Rationale</th>
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</table>
| 1) Design driven approach | Decide what to reflect and ignore
|                       | Employ sensitivity testing |
| 2) Realistic capital usage costs | Insurer capital is a shared asset with two distinct types of usage, Rental and Consumption
|                       | Allocate the costs of its true usage to contributing lines |
| 3) Consumption Costs via Risk Preference function | Every risk metric has an implicit risk preference function underlying it
|                       | Assess capital consumption costs using risk preference function |
| 4) Key sensitivity tests: the Three R’s | Reserves, reinsurance and return periods |
| 5) Create an operational buffer between the capital model and the field | Use a sophisticated method to produce percentage allocations which are then applicable to any total
|                       | Only allocate cost of capital as far down in the organization as necessary
|                       | Translate cost of capital into familiar terms – e.g., % load in target combined ratios |
Capital Cost Allocation System Design
Begin with the End in Mind

• The CFO is operating an internal capital market
  – An unconstrained market of one capital supplier and numerous consumers

• Price access to this capital by any means necessary
  – What to reward and punish, emphasize and ignore

• Decide in that pricing policy whether (and how much) to reflect:
  – Time and history
  – Fact and intuition
  – Return periods
  – Risk factors

• There is nothing inherently right or wrong about any approach
  – Only the algorithmic expression of the risk preferences

Desirable Features Of Capital Cost Allocation Approach
Actual Example

1. Drill-Down and Roll-Up (linear)
2. Produce Strictly Positive Allocation (DM pet criteria)
3. Explainable (to key opinion leaders) Methodology (Use Test)
4. Focus on Downside not simply Volatility
5. Measure Risk at the Portfolio Level
6. Stable and Robust (particularly w/r/t updating one business unit’s results)
   
5 and 6 are mutually exclusive
## Desirable Features Of A Good Allocation Metric = Covariance

1. Drill-Down and Roll-Up  
   - Yes – additive

2. Produce Strictly Positive Allocation  
   - Yes – Risk Charge In Proportion Of Contribution To Total Variance

3. Explainable (to key opinion leaders) Methodology  
   - ≈ - Implicit risk preferences are buried

4. Focus on Downside not simply Volatility  
   - No – Volatility only

5. Measure Risk at the Portfolio Level  
   - Yes – Total variance

6. Stable and Robust  
   - No – Changes to one segment affect others

## Desirable Features Of A Good Allocation Metric = Shared Asset

1. Drill-Down and Roll-Up  
   - No – Interaction effects

2. Produce Strictly Positive Allocation  
   - Yes – Rental + Consumption charges

3. Explainable (to key opinion leaders) Methodology  
   - Yes – Intuitively Related To Opportunity Cost Of Capacity

4. Focus on Downside not simply Volatility  
   - Yes – Downside based

5. Measure Risk at the Portfolio Level  
   - Yes – Risk preference function defined at portfolio level

6. Stable and Robust  
   - No – Changes to one segment affect others
2) Realistic Capital Cost Framework

*Shared Asset – a reminder*

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**Insurer Capital is a Shared Asset**

**Asset Owners**
- Control Overall Access Rights
- Preserve Against Depletion From Over-Use

**Shared Asset**
- Reservoir, Golf Course, Pasture, Hotel, ...
- Insurer Capital

**GLOBAL**

**LOCAL**

- User 1: Consume On Standalone Basis
- User 2
- User 3: Tunnel Vision - No Awareness Of The Whole
- User 4
Shared Assets Can Be Used Two Different Ways

**• Consumptive Use**
- Example: RESERVOIR

**• Permanent** Transfer To The User

**• Non-Consumptive Use**
- Example: GOLF COURSE

**• Temporary** Grant Of Partial Control To User For A Period Of Time

**• Both Consumptive and Non-Consumptive Use**
- Example: HOTEL
- **Temporary** Grant Of Room For A Period Of Time
- Guest could destroy room or entire wing of hotel, which is *Permanent Capacity Consumption*

An Insurer Uses Its Capital Both Ways

**1. “Rental” Or Non-Consumptive**
- Returns Meet Or Exceed Expectation
- Capacity Is Occupied, Then Returned Undamaged
- A.k.a. *Room Occupancy*

**2. Consumptive**
- Results Deteriorate
- Reserve Strengthening Is Required
- A.k.a. *Destroy Your Room, Your Floor, Or Even The Entire Hotel*

*Charge Each Segment for Its Capital Usage*
Two Kinds Of Charges:

1. **Rental** = upfront fee for right to (possibly) use the Guarantee
   
   → *Occupying underwriting capacity*
   
   *BCAR, SPCAR, RBC, SCR, …*

2. **Consumption** = contingent fee for using the Guarantee

   → Function of **Potential for Deficit** (*Consumption*)

   *Risk appetite / preference / riskiness leverage function*
**Evolution of Decision Making**

### #1: Deterministic Project Analysis

**Information**
- Single-Value Forecasts
- Intangibles

**Calculation**
- Cash Flows
- ROE or IRR
- Sensitivities

**Decision**
- Review
- Decision


### Next Step: Risk Analysis

#### #2: Risk Analysis

**Information**
- Forecasts of Ranges of Outcomes
- Intangibles

**Calculation**
- Simulation of Range of Cash Flows
- Return Distribution

**Decision**
- Review
- Decision

- Similar to DFA or Monte Carlo processes
- Uncertainty in variables is quantified
  - Only info which is impossible/too costly to quantify remains intangible
- Judging the acceptability of alternatives (“Risk Judgment”) is intuitive and specific to the decision maker
Next Step: Risk Preference Function

#3: Risk Preferences

- An extension of Risk Analysis
- Intuitive risk judgment, which is applied in Risk Analysis, is quantified by means of a corporate Risk Preference function
- Risk preference function does not replace judgment, but simply formalizes it so it can be applied consistently

Every Approach Has an IMPLICIT Risk Preference VaR

VaR

Risk Aversion

CARE!!

Don’t Care

Size of Loss

VaR Threshold
Every Approach Has an IMPLICIT Risk Preference

TVaR

---

Additional Care per $ of additional loss is constant

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Risk Aversion

Don’t Care

Size of Loss

TVaR Threshold

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Every Approach Has an IMPLICIT Risk Preference

“Zones of Impact” of Capital (Company X)

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Zones of Impact

Risk Aversion

Don’t Care

Size of Loss

---

Heights of the different boxes represent the firm’s RISK PREFERENCE FUNCTION
Riskiness Leverage Functions
Translating Risk Preferences into Capital Cost Allocation

• Rodney Kreps (2005)
• Simple idea: reflect risk opinion in a quantitative manner at the simulated scenario level
• More formally \( R = \int L(x)(x - \mu)f(x) \, dx \) where \( R \) is the risk load and \( L \) is the leverage function
• Use the whole curve
• The Use Test in Action
• We will walk through a simple example

### Business Segment Losses

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</table>

| Average | 3,196 | 908 | 12,064 | 16,168 |
| Percentage | 20% | 6% | 75% | 100% |

Riskiness Leverage Functions
Simple Example

• We have ten realizations from a stochastic model for the overall business

• Sort the realizations in ascending order on total loss.

• The average total loss is the sum of the average loss for each segment.

• If we have zero aversion to risk, we could allocate capital to these lines of business based on the broken-out average.

• Equivalently, we are allocating capital based on the weighted average scenario, where the weights are each one.
VaR (Value-at-Risk) and Contribution Measures

• We may decide to assign the ‘most-important’ pain point a weight of one, and zero weight to all other realizations.

• That point would be called VaR (Value-At-Risk), in this case at the 90th percentile.

• The contributions to VaR from individual segments add up to the total VaR, because the realization is one complete scenario.

• The contributing average amounts are called co-VaR.

• The Risk Charge is the excess of the weighted average over the straight average.

• Co-VaR is generally an unstable measure for capital allocation.

Probability Transforms
An easy way to define smooth weights

• One way to define the weights is with a probability transform. The weights are defined by a curve that effectively makes adverse realizations more likely.

• The weights are a smooth way to recognize that the worst results are even more painful than the proportional size of their losses.

• Curve shape can be altered by changing parameter values, but only so much.

• In this example, we show a Wang transform. There are other curves.
Utility Transforms
Another way to define weights using total loss

- Another family of weighting schemes defines the curve with formulas that depend on total loss, in other words the pain-per-dollar is explicitly changing.

- It's still just a way to calculate this realization weights.

- These weights are an Esscher transform with h=.45.

- The curve has a different shape than that of the Wang transform, but we chose h=.45 to provide the same risk loading overall.

<table>
<thead>
<tr>
<th>Realization</th>
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<th>Business Segment Losses</th>
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<td></td>
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<thead>
<tr>
<th></th>
<th>Straight Ave</th>
<th>Wght Ave</th>
<th>% Allocation</th>
<th>Risk Charge</th>
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<td>3,196</td>
<td>2,537</td>
<td>6%</td>
<td>24,228</td>
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<td></td>
<td>908</td>
<td>1,120</td>
<td>3%</td>
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<td>12,064</td>
<td>36,739</td>
<td>91%</td>
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<td></td>
<td>16,168</td>
<td>40,397</td>
<td>100%</td>
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Weighted TVaR
Explanation of the Statistic

- TVaR<sub>50</sub> (Tail Value at Risk at the 50<sup>th</sup> Percentile) is the average total loss for all realizations larger than the 50<sup>th</sup> percentile.

- The arbitrary threshold of the 50<sup>th</sup> percentile is chosen to quantify risk preferences.

- Co-TVaR<sub>A</sub> is the average losses from business segment A over the same realizations. Note that these realizations are not in strict ascending order for segment A losses.

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TVaR<sub>50</sub>
Co-TVaR<sub>50</sub>
Percentage
**TVaR Thresholds (Return Periods)**

- If we chose the 80th percentile (i.e. 1 in 5 Return Period), the TVaR is larger.

- In this example, the tail risk is driven by Business Segment C. The allocation to C is more at the higher threshold.

- To allocate capital to support different levels of adverse loss events, we can weight the two TVaRs together. We will have to choose the weights.

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| Co-TVaR_{80} | 992 | 940   | 52,940 | 54,872 |
| Percentage   | 1.8%| 1.7%  | 96.5%  | 100.0% |

| Co-TVaR_{50} | 5,629 | 449   | 23,552 | 29,630 |
| Percentage   | 19.0%| 1.5%  | 79.5%  | 100.0% |

**TVaR Weighting**

- Let’s assign a weight of 43% to Co-TVaR_{80} and 57% to Co-TVaR_{50}. The resulting weighted total TVaR is 40,397, producing the same risk charge as in the previous examples.

<table>
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<tbody>
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<td>Co-TVaR_{80}</td>
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<td>940</td>
<td>52,940</td>
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<td>Percentage</td>
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<td>0.57</td>
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<tr>
<td>Weighted</td>
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<tr>
<td>Percentage</td>
<td>9.0%</td>
<td>1.6%</td>
<td>89.3%</td>
<td>100.0%</td>
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</table>
Weighted TVaR Under the Scenario View

- By using two TVaR measures we describe our preferences between different 'zones' of the loss distribution.

- The preferences 1 and 2.9 over the two zones can be directly calculated from the 43%/57% weights and the thresholds of 50\textsuperscript{th} and 80\textsuperscript{th} percentile.

- The realization weights are a step function. Each step (there can be more than two) occurs at an important capital management point, (e.g. earnings miss, single downgrade, solvency impairment).

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Straight Ave: 3,196 | 908 | 12,064 | 16,168
Wght Ave: 3,651 | 658 | 36,087 | 40,397
% Allocation: 9% | 2% | 89% | 100%

Risk Charge: 24,229

5) Operational Buffer
Operational Buffer  
AKA “Resist the urge to allocate capital to the policy level”

- Loaded terminology: allocation, capital, ROE
- Mixed stakeholder audiences: profit center heads, finance, actuarial
- Issues with “Allocating Capital”:
  - Balancing to published figures
  - Responding to changes during the year
  - Producing granular ROEs requires allocation of other things (e.g., investment income)
- What is the operational goal?
  - Risk-adjusted performance evaluation

- Best practice
- Allocate to the lowest necessary level but no further
- Treat the capital costs as risk-based overhead expense
  - Carry costs of the Shared Asset
- Below there, treat it like any other expense load
- Use your existing target PLR or CR frameworks
- Simplifies the transition and updating