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A New Approach to Risk-Neutral Scenarios

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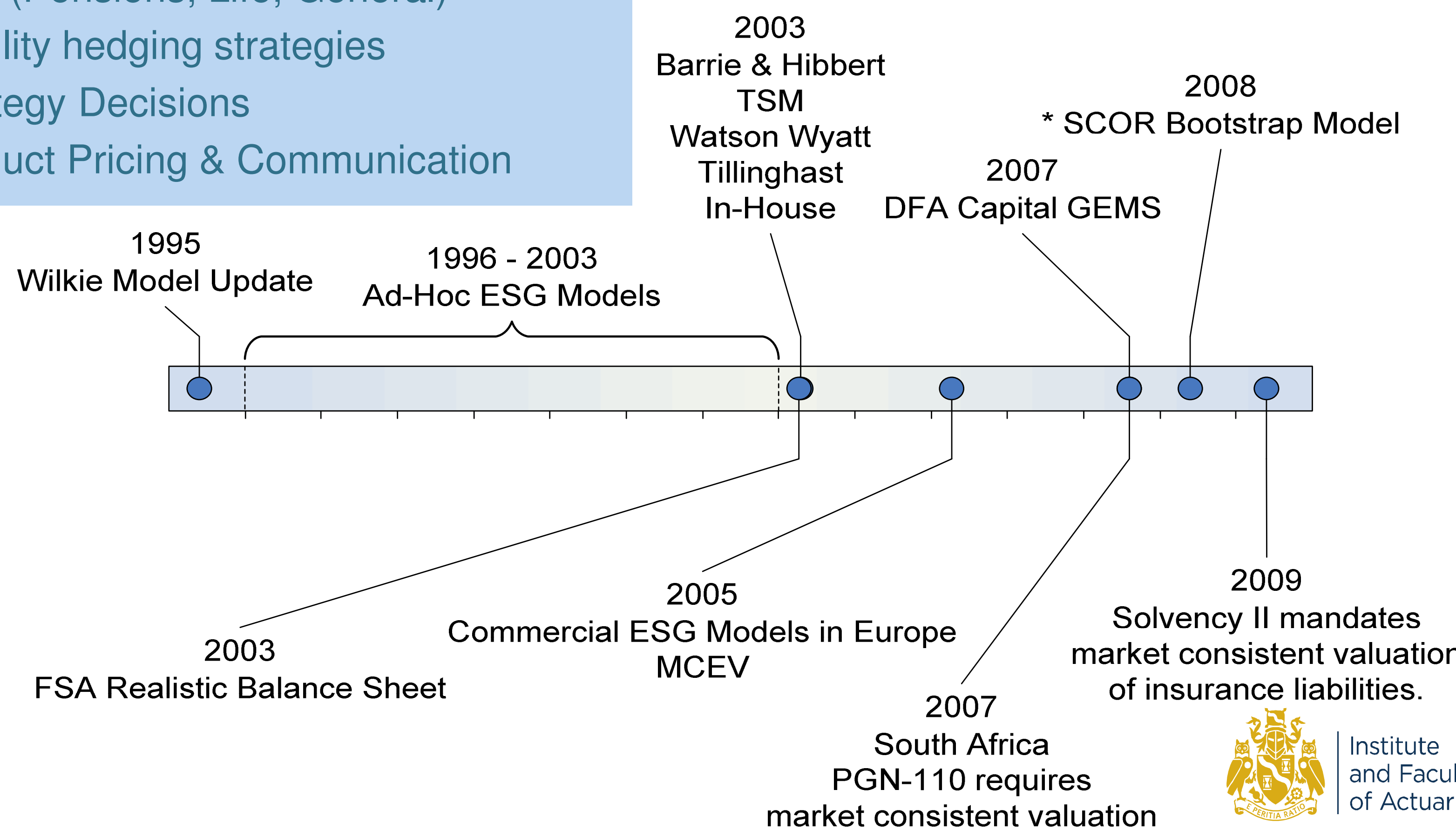
Brief History of ESGs

1. **Regulatory!**
2. **Other Uses**

- ✓ ALM (Pensions, Life, General)
- ✓ Liability hedging strategies
- ✓ Strategy Decisions
- ✓ Product Pricing & Communication



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Two Types of ESG

Risk Neutral

- **Purpose**
  - Calculate Market Consistent Valuation of Liabilities
- **Traditional Models**
  - Banking Models, arbitrage-free models
- **Pros**
  - Easy to satisfy accounting regulations by perfectly replicating market prices
- **Cons**
  - Unintended consequences, e.g. negative / exploding rates
  - Limited availability of key market parameters, e.g. implied volatilities

Real World

- **Purpose**
  - Realistic dynamics of market prices and estimation of extreme events
- **Traditional Models**
  - Statistical Models, Mean Reverting models
- **Pros**
  - Includes features of markets that management believes in, e.g. mean reversion, fat tails
- **Cons**
  - May be difficult to get within required tolerance for market data

Outline

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- **Valuation in Insurance**
- Real-World Features
- Risk-Neutral Puzzle
- A New Approach to RN and RW Scenarios

Types to Valuation

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Two alternative valuation methods for assets and liabilities:

– “**Realistic**” valuation based on **real-world** scenarios

– **Market-consistent** valuation based on **risk-neutral** scenarios or other market-consistent techniques

	Real-world	Market-consistent
Pro	▪ **Realistic probabilities** and **distributions** of projected risk factor values including **tail events** ▪ suitable for **risk mangement**, **economic capital** assessment	▪ **Expected** values **in line with markets at valuation time**, including derivative markets ▪ Theoretical solidity: martingale property eliminates all forms of risk premium
Con	▪ Discounted expected value may deviate from the market value of a replicating asset portfolio: **not market-consistent**	▪ **Unrealistic** distributions such as strongly negative interest rates in risk-neutral scenarios ▪ Leading to **erroneous triggering** of Life insurance **guarantees** ▪ Risk assessment in terms of **VaR** or **economic capital** is **not supported** as distributions are **not realistic**

Application Areas of Valuation

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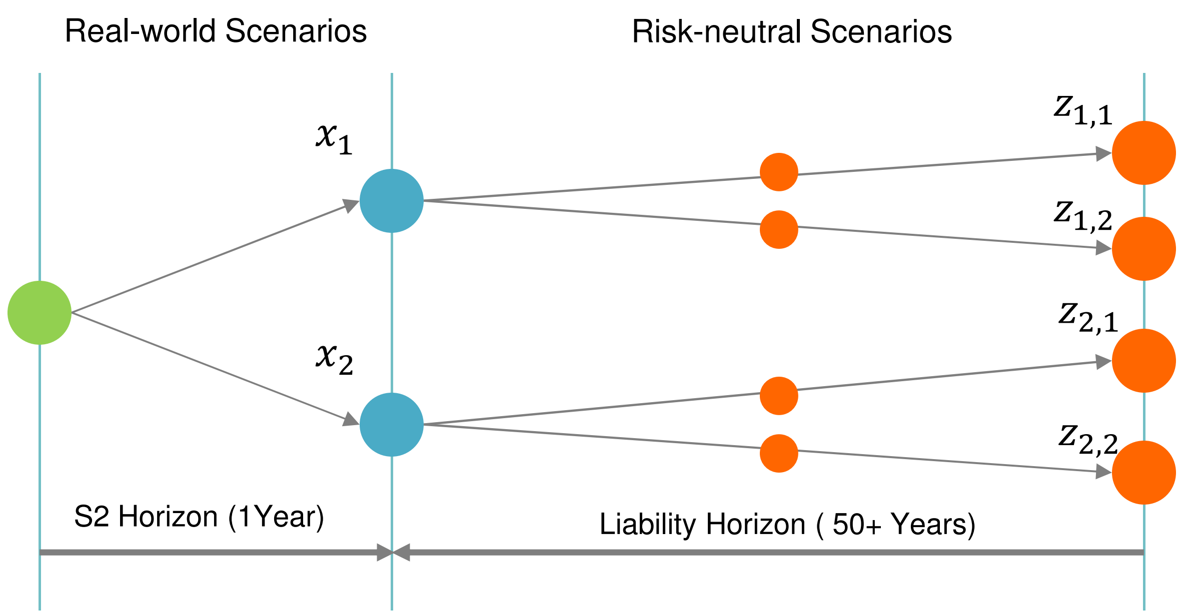
- Current value of assets/liabilities, e.g. Market-Consistent Economic Value (**MCEV**)
- Value of assets/liabilities at a solvency horizon, typically after **1 year**, e.g. **Solvency II** and **Economic Capital**



Nested stochastic is computationally not efficient, therefore the need for proxy modelling

Valuation Techniques Used

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Approach	Cash-flow based	Portfolio based
MCEV	▪ Projected cash-flows (guarantees) depend on **real-world** scenarios ▪ MCEV of cash-flows is obtained using **risk-neutral** scenarios	▪ MCEV = initial market value of portfolio
Solvency	▪ **Real-world** scenarios for solvency period ▪ Starting at the conditions of each real-world end point: **risk-neutral** scenarios for conditional MCEV calculation → full **distributions** at the solvency horizon, **risk** and **capital** measures ▪ Cash flows depending on **real-world** economy (e.g. guarantees) may be inaccurate if **risk-neutral** scenarios used	▪ Market-Consistent **Capital** required for an asset **portfolio** (which may also be a replicating liability portfolio) ▪ **Real-world** scenarios for solvency period ▪ Portfolio valuation based on these scenarios → full **distributions** at the solvency test horizon, **risk** and **capital** measures

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Observed Features of Market Data

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Realistic RW scenarios should exhibit those **features** that are observed in historical time series.

Feature	**Description**
Heavy tails	Tails of observed return data deviate from normal or lognormal behavior
Asymmetric tails	Negative returns often exhibit fatter tails than positive ones
Tail dependence	Observed dependencies suggest weaker dependence under normal market conditions but higher dependence under stressed market conditions
Mean reversion	Some variables exhibit mean reversion property, such as interest rate, inflation, or credit cycle
Volatility clusters	High volatility events tend to cluster in time, e.g. equity indices, FX rates
Absence of Arbitrage	Simulated scenarios should not allow for arbitrage opportunities, e.g. interest rate parity, positive forward rates
Stationarity	Invariance of statistical properties of the returns in time
Absence of autocorrelation	Autocorrelation of investable risk factor returns is insignificant

Crisis Events

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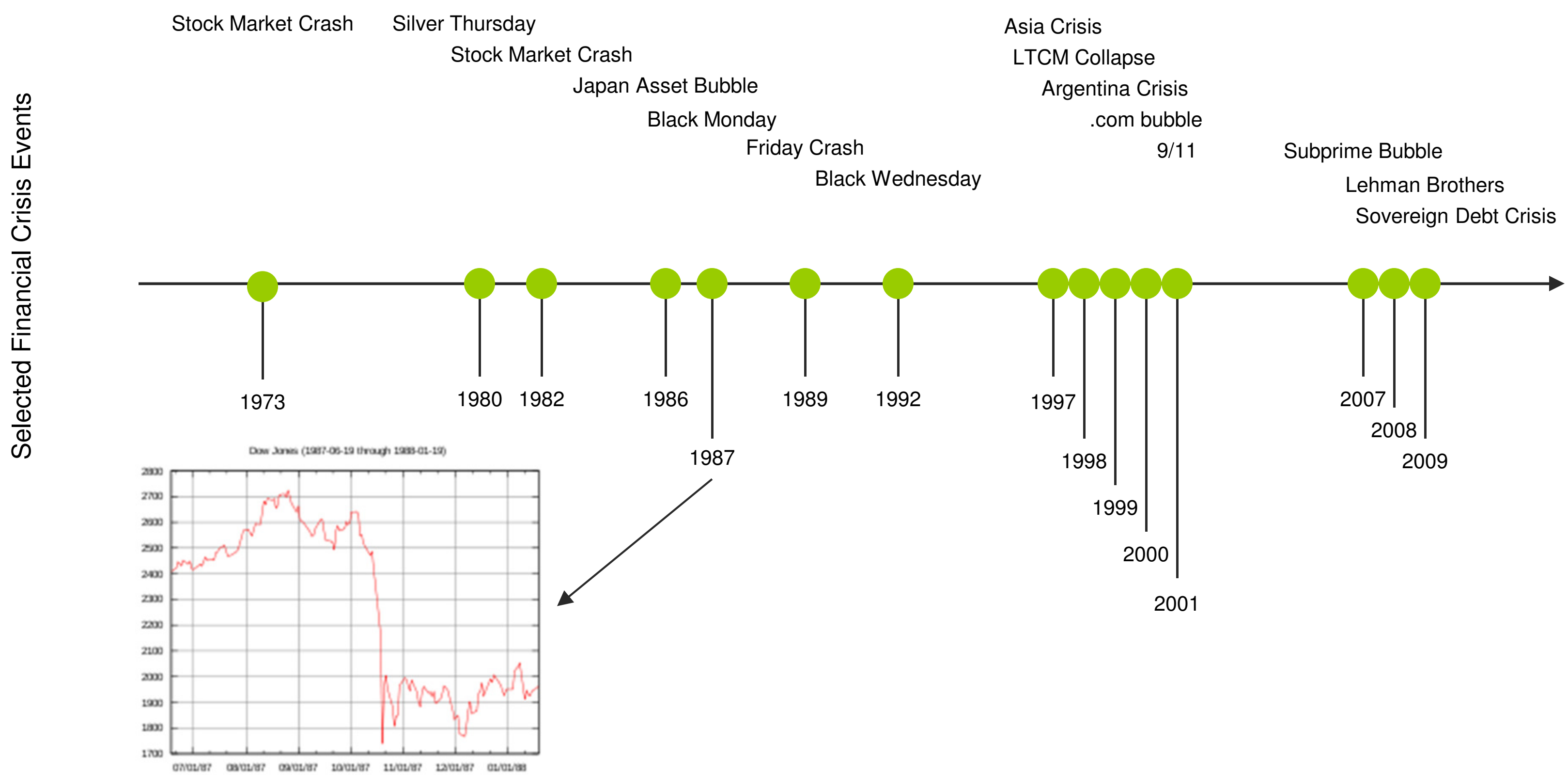
The course of economy is subject to **crises**. **Realistic** economic scenarios should represent both normal and stressed market conditions.



Tail Patterns

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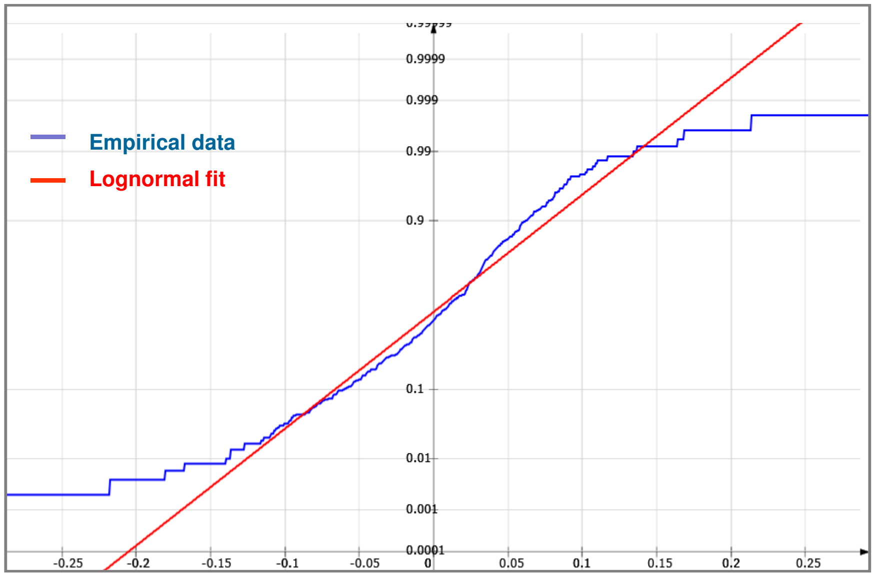
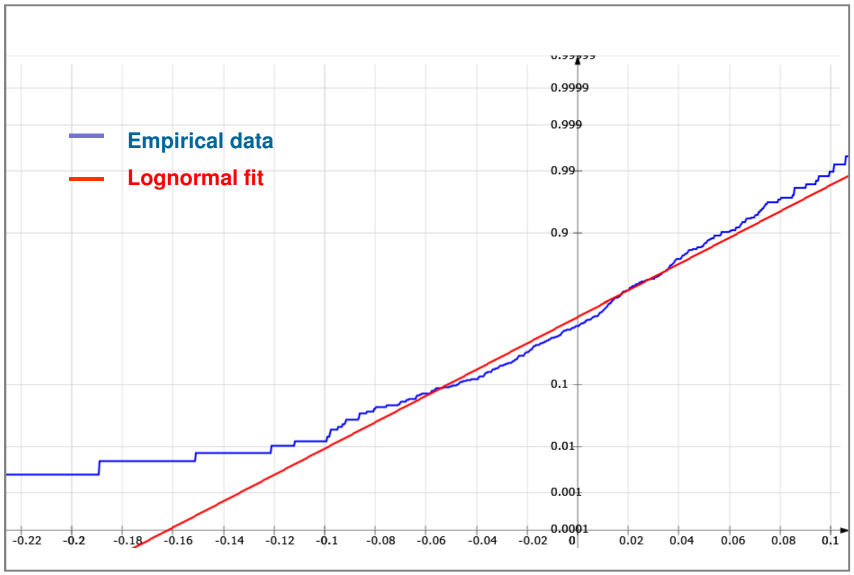
Heavy Tails



CDF of Monthly Returns of MSCI UK (1970-2010)

AsymmetricTails



CDF of Monthly Returns of MSCI US (1980-2010)

Dependency Patterns

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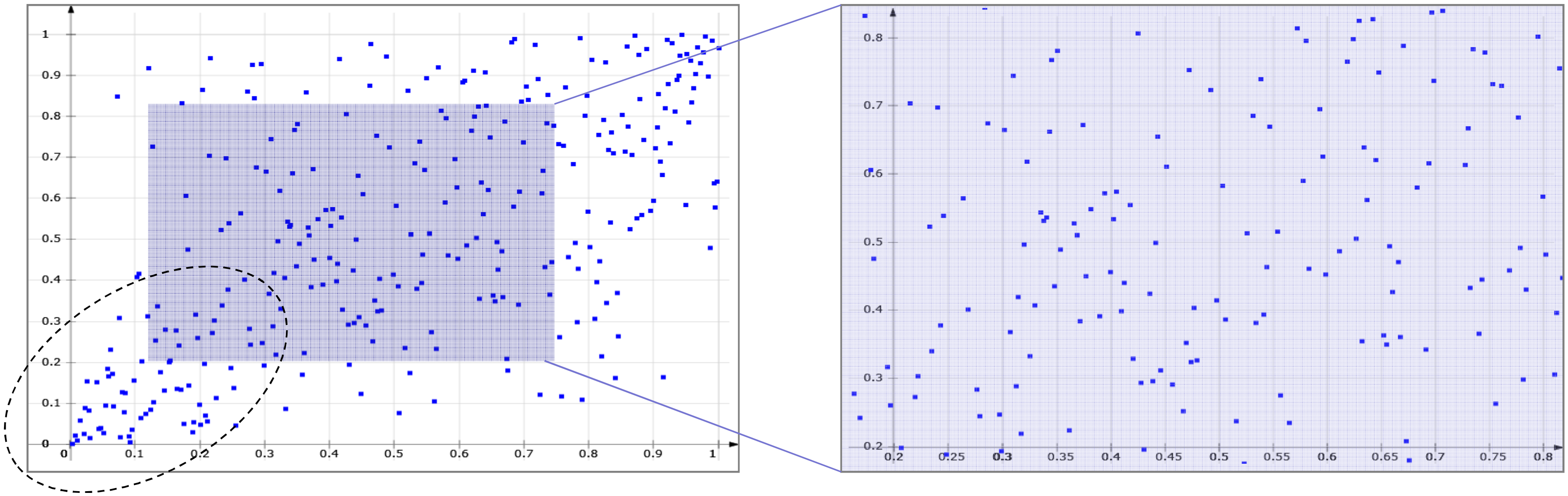
Tail Dependence



Rank Correlation of Monthly Returns MSCI US and UK (1980-2010)

Mean Reversion and Clustering Patterns

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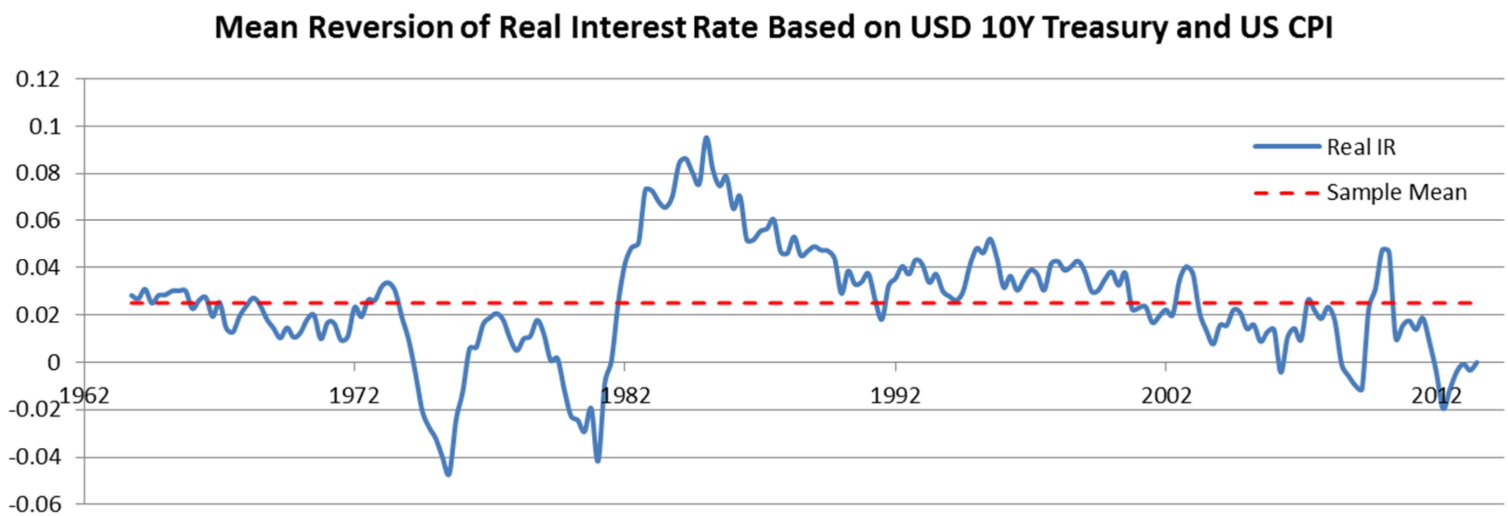

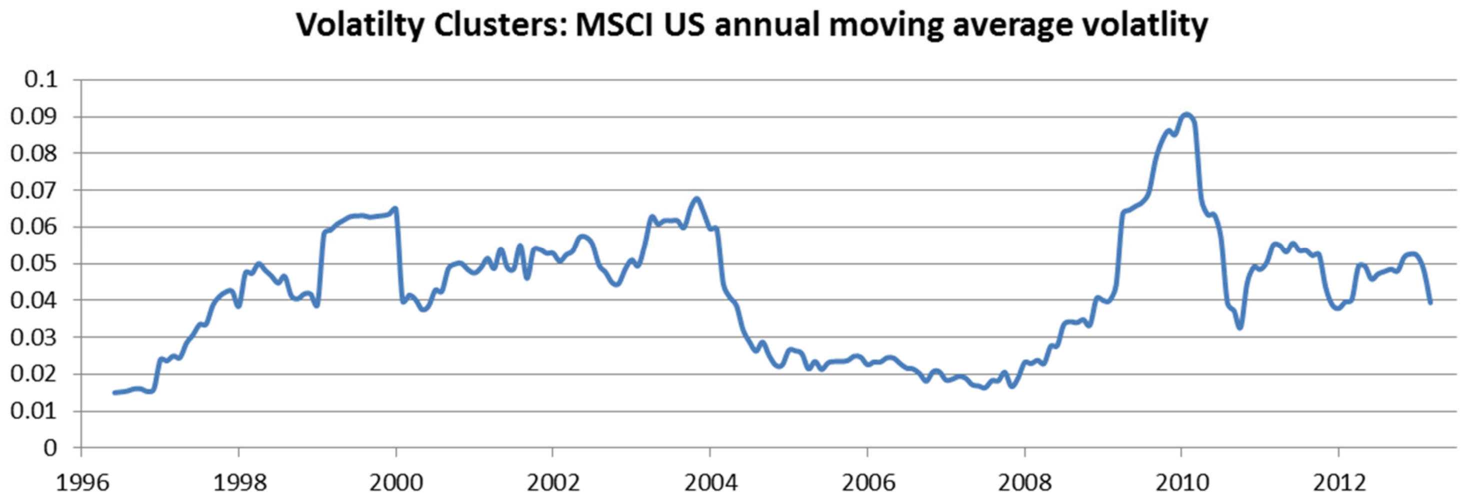

Cyclicality of Credit Risk

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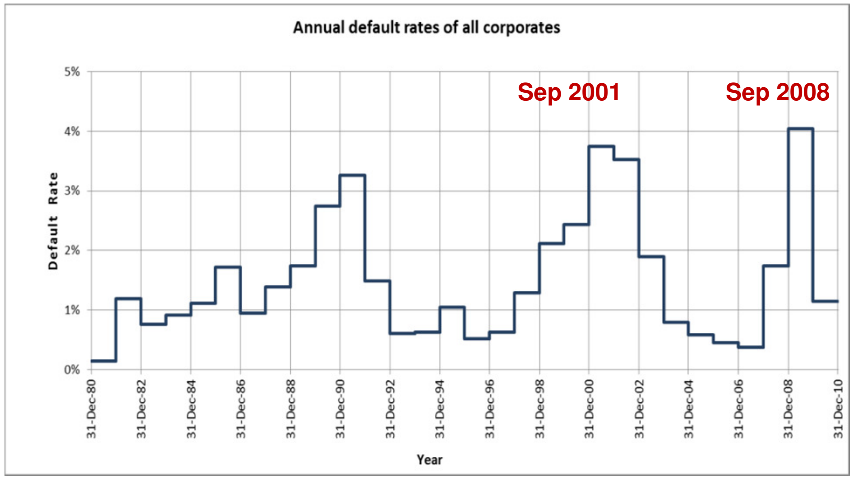
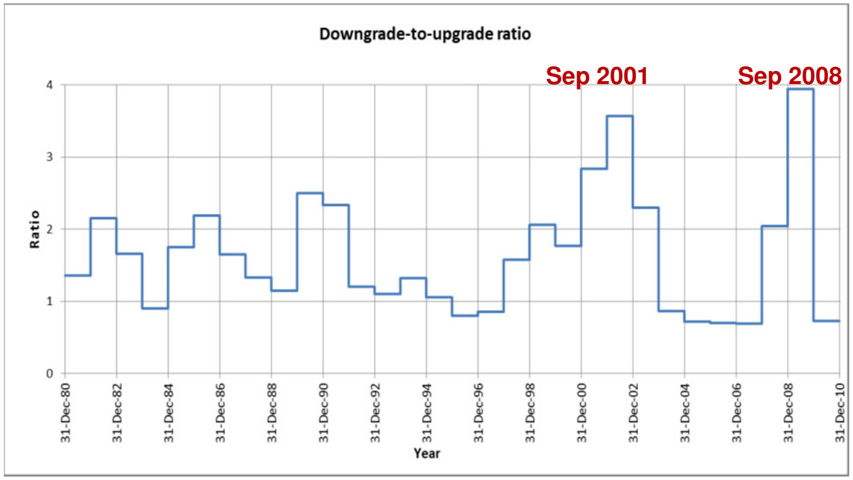
Historical default rates & migrations exhibit co-movement with credit cycle





Data source: S&P

- Default and migration probabilities exhibit time dependence—**credit cycle**
- Defaults can be **10 times higher** in bad years compared to good years
- Ratio of downgrades to upgrades can be **4 times higher** in bad years relative to good years

Reproducing Observed Features

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- Risk-neutral scenarios will not reproduce these features, so they **miss reality in many different aspects**
- «Real-world» scenarios derived from RN scenarios by a simple addition of a **risk premium** are not sufficient to represent reality
- Real-world scenarios have to be generated such that they **reproduce all the observed features**
- This does **not** imply that the simulated RW scenarios are bound to reproducing historical behaviour only

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Risk-Neutral Puzzle

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The following elements fully determine the dynamics of RN scenarios:

Risk-free **yield** curves at valuation time

Implied volatility surface at valuation time

Correlation between different risk factors

Martingale property

Challenges of Risk-Neutrality (I)

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Risk-free **yield** curves at valuation time

- Definition of “risk-free” reference required
- **Government bond yields may be slightly negative (compatibility with swaption pricing)**
- Forward rates in the long-term limit are illiquid
- Simple interest rate models (e.g. 1 short rate and 1 long rate) cannot render actual form of yield curve

Implied volatility surface at valuation time

- Implied volatility values available only for liquid options
- For illiquid markets or non-traded assets (real-estate, hedge funds) models or judgment are used
- **Moneyness dimension of interest rate derivatives often neglected (flat smile)**

Challenges of Risk-Neutrality (II)

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Martingale property

- Martingale condition for all investment strategies satisfied approximately
- **In practice, martingale property met only for simple investment strategies**
- Martingale condition does not easily reconcile with mean reversion of interest rates
- Deriving RW scenarios from RN scenarios is not an obvious task

Correlation between different risk factors

- **Correlations cannot be derived from derivative markets**
- Therefore a correlation model or judgment is required
- Imposing correlations means adding more conditions to an already large set of conditions

Risk-Neutral Paradigm in Practice

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- For many risk-factors, the notion of “market consistent” should be revised to “model or judgment consistent”, in particular in case of:
  - Less traded combinations of strike price and expiry periods
  - Assets with no derivative markets such as property, hedge fund, private equity indices
  - Correlation parameters
  - Very volatile market data (often smoothed out for robustness of results)

- Martingale property satisfied only approximately (e.g. for 10k scenarios)
  - Well satisfied for static strategies or simple rollover strategies
  - Often not satisfied for strategies with more complex rollovers

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Motivation

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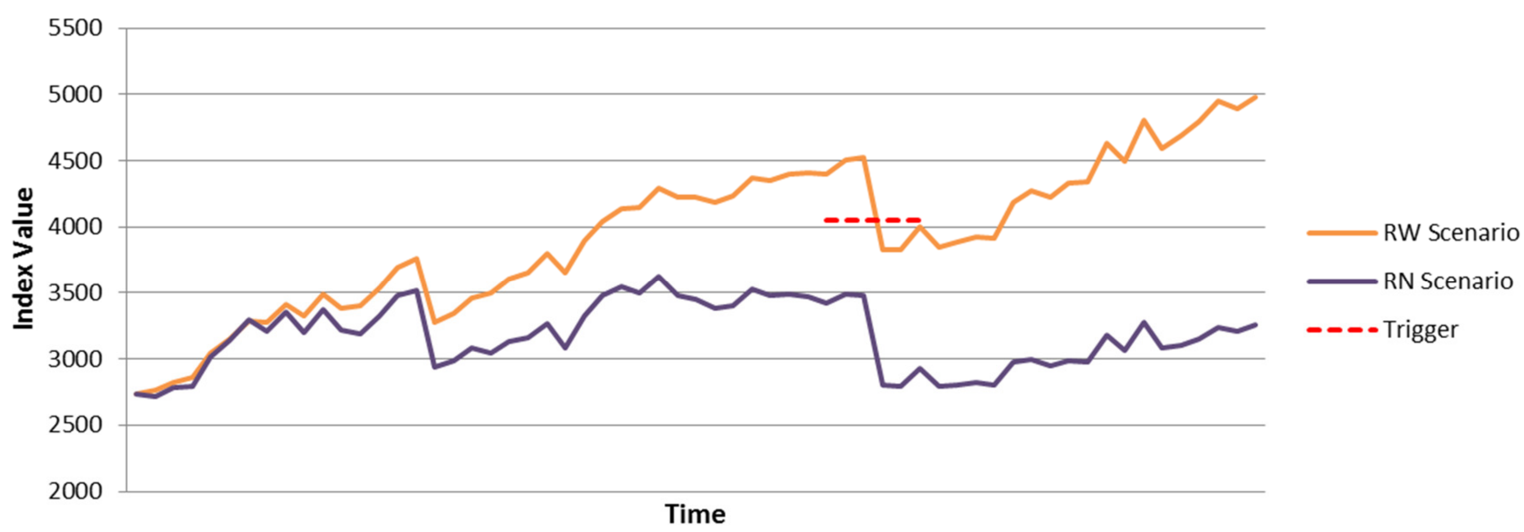
- Different aspects of valuation and risk assessment require real-world as well as risk-neutral scenarios
- Example—Life liabilities:
  - RW scenario values are used to check the **trigger conditions of guarantees** and to calculate the ensuing **cash flows**
  - The corresponding RN scenario values are then used for the market-consistent **valuation** of the cash flows



Generating Consistent RW and RN Scenarios

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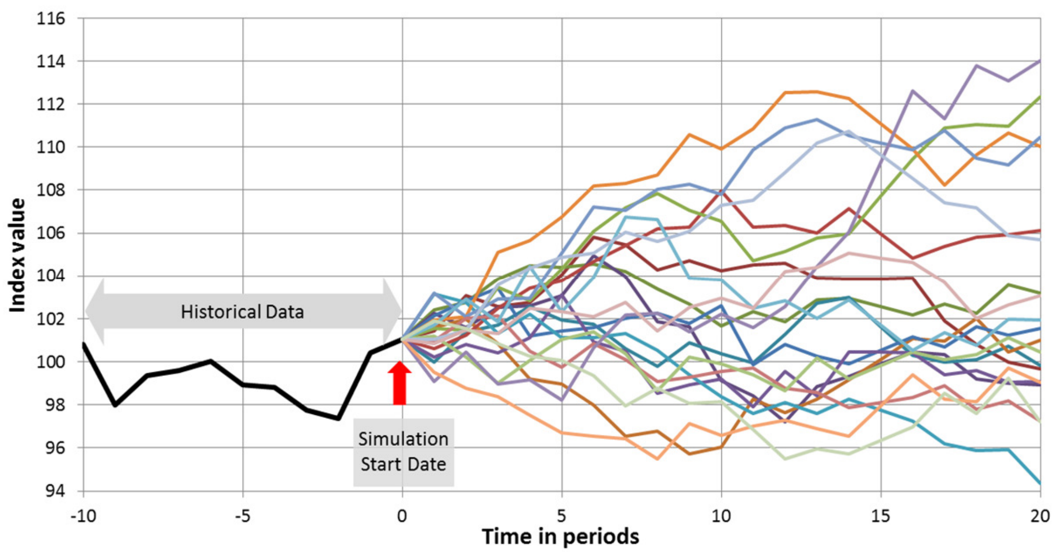
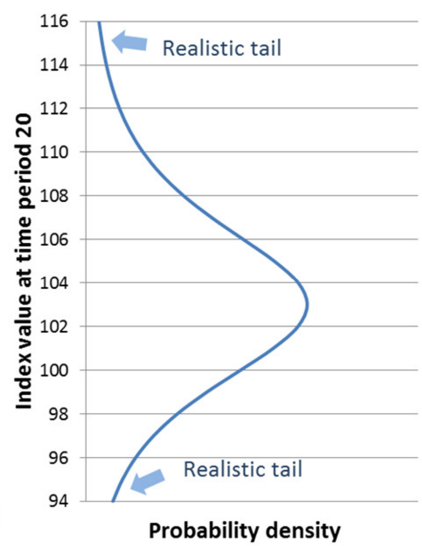
- Possible ways to generate **consistent** real-world and risk-neutral scenarios:
  - Generate RN scenarios → derive RW scenarios (sophisticated risk premium model)
  - **Generate RW scenarios → derive RN scenarios (imposing martingale conditions)**
  - Generate RW and RN scenarios through a joint algorithmic process
- Precondition for the discussed method: **RW scenario** generator





Different Approaches

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Two prevailing approaches to transform RW to RN:

Adjusting Scenario Values

RW/RN Probabilities	RW Scenario		RN Scenario
p_1	$\boldsymbol{S_1}$	$\longrightarrow$	$\tilde{\boldsymbol{S}}_1 = \boldsymbol{S_1} + \Delta \boldsymbol{S_1}$
$\vdots$	$\vdots$		$\vdots$
p_n	$\boldsymbol{S_n}$	$\longrightarrow$	$\tilde{\boldsymbol{S}}_n = \boldsymbol{S_n} + \Delta \boldsymbol{S_n}$

- Market replication: $\sum_{i=1}^{n} p_i \, DCF_{\text{Option}^k}(\boldsymbol{S_i} + \Delta \boldsymbol{S_i})$ = Market price of Option^k
- Ranking preservation: $CDF(\boldsymbol{S_i}) = CDF(\tilde{\boldsymbol{S}}_i)$

Adjusting Scenario Probabilities

RW/RN Scenario	RW Probability		RN Probability
$\boldsymbol{S_1}$	$p_1 = \frac{1}{n}$	$\longrightarrow$	$\tilde{p}_1 = \frac{1}{n} + \Delta p_1$
$\vdots$	$\vdots$		$\vdots$
$\boldsymbol{S_n}$	$p_n = \frac{1}{n}$	$\longrightarrow$	$\tilde{p}_n = \frac{1}{n} + \Delta p_n$

- Market replication: $\sum_{i=1}^{n} \tilde{p}_i DCF_{\text{Option}^k}(\boldsymbol{S_i})$ = Market Price Option^k
- Probability measure: $\sum_{i=1}^{n} \Delta p_i = 0$
- Minimal distortion: $\min \sum_{i=1}^{n} \Delta {p_i}^2$ (2^{nd} order approx)

From RW Scenarios to RN Scenarios

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- The **implied volatility** surface determines the distributions of risk-neutral scenarios
- These distributions are used to construct **risk-neutral** scenarios from real-world ones
- The construction process keeps RW scenarios **consistent** with the corresponding RN scenarios
- The generated risk-neutral scenarios are:
  - Consistent with the volatility surface and market prices of derivatives
  - Consistent with correlation assumptions
  - Martingale conditions are fulfilled for simple and complex investment strategies with rollovers as far as a limited number of scenarios permits

Benefits of the Approach

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- Consistency between RW and RN scenarios leads to consistency between asset and liability modeling
- RN scenarios inherit those features of RW that are not conflicting with martingale property
- Provides an intrinsic approach to construct RN scenarios for risk factors with no derivative markets
- Macro-economic variables e.g. GDP can be included in RW scenario sets
  - Regulators define **stress** scenarios in terms of macroeconomic variables
  - Firms perform portfolio valuation **contingent** to those stress scenarios
  - Stressed RN scenarios can be obtained through corresponding stressed RW scenarios