A New Approach to Risk-Neutral Scenarios

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Brief History of ESGs

1. Regulatory!
2. Other Uses
   ✓ ALM (Pensions, Life, General)
   ✓ Liability hedging strategies
   ✓ Strategy Decisions
   ✓ Product Pricing & Communication

- 1995 Wilkie Model Update
- 2003 FSA Realistic Balance Sheet
- 2003 Barrie & Hibbert TSM
- Watson Wyatt Tillinghast In-House
- 2005 Commercial ESG Models in Europe MCEV
- 2007 DFA Capital GEMS
- 2007 South Africa PGN-110 requires market consistent valuation
- 2008 * SCOR Bootstrap Model
- 2009 Solvency II mandates market consistent valuation of insurance liabilities.
Two Types of ESG

Risk Neutral

- **Purpose**
  - Calculate Market Consistent Valuation of Liabilities

- **Traditional Models**
  - Banking Models, arbitrage-free models

- **Pros**
  - Easy to satisfy accounting regulations by perfectly replicating market prices

- **Cons**
  - Unintended consequences, e.g. negative / exploding rates
  - Limited availability of key market parameters, e.g. implied volatilities

Real World

- **Purpose**
  - Realistic dynamics of market prices and estimation of extreme events

- **Traditional Models**
  - Statistical Models, Mean Reverting models

- **Pros**
  - Includes features of markets that management believes in, e.g. mean reversion, fat tails

- **Cons**
  - May be difficult to get within required tolerance for market data
Outline

- Valuation in Insurance
- Real-World Features
- Risk-Neutral Puzzle
- A New Approach to RN and RW Scenarios
Two alternative valuation methods for assets and liabilities:

- **“Realistic”** valuation based on real-world scenarios
- **Market-consistent** valuation based on risk-neutral scenarios or other market-consistent techniques

<table>
<thead>
<tr>
<th></th>
<th>Real-world</th>
<th>Market-consistent</th>
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</thead>
<tbody>
<tr>
<td><strong>Pro</strong></td>
<td>Realistic probabilities and distributions of projected risk factor values including <strong>tail events</strong></td>
<td>Expected values in line with markets at valuation time, including derivative markets</td>
</tr>
<tr>
<td></td>
<td>suitable for <strong>risk management</strong>, economic <strong>capital</strong> assessment</td>
<td>Theoretical solidity: martingale property eliminates all forms of risk premium</td>
</tr>
<tr>
<td><strong>Con</strong></td>
<td>Discounted expected value may deviate from the market value of a replicating asset portfolio: <strong>not market-consistent</strong></td>
<td><strong>Unrealistic</strong> distributions such as strongly negative interest rates in risk-neutral scenarios</td>
</tr>
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<td></td>
<td></td>
<td>Leading to erroneous triggering of Life insurance <strong>guarantees</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Risk assessment in terms of <strong>VaR</strong> or economic <strong>capital</strong> is <strong>not supported</strong> as distributions are not realistic</td>
</tr>
</tbody>
</table>
Application Areas of Valuation

- Current value of assets/liabilities, e.g. Market-Consistent Economic Value (MCEV)
- Value of assets/liabilities at a solvency horizon, typically after 1 year, e.g. Solvency II and Economic Capital

Nested stochastic is computationally not efficient, therefore the need for proxy modelling.
# Valuation Techniques Used

<table>
<thead>
<tr>
<th>Approach</th>
<th>Cash-flow based</th>
<th>Portfolio based</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCEV</td>
<td>- Projected cash-flows (guarantees) depend on real-world scenarios</td>
<td>- MCEV = initial market value of portfolio</td>
</tr>
<tr>
<td></td>
<td>- MCEV of cash-flows is obtained using risk-neutral scenarios</td>
<td></td>
</tr>
<tr>
<td>Solvency</td>
<td>- <strong>Real-world</strong> scenarios for solvency period</td>
<td>- Market-Consistent <strong>Capital</strong> required for an asset portfolio (which may also be a replicating liability portfolio)</td>
</tr>
<tr>
<td></td>
<td>- Starting at the conditions of each real-world end point: risk-neutral scenarios for conditional MCEV calculation → full distributions at the solvency horizon, risk and capital measures</td>
<td>- <strong>Real-world</strong> scenarios for solvency period</td>
</tr>
<tr>
<td></td>
<td>- Cash flows depending on real-world economy (e.g. guarantees) may be inaccurate if risk-neutral scenarios used</td>
<td>- Portfolio valuation based on these scenarios → full distributions at the solvency test horizon, risk and capital measures</td>
</tr>
</tbody>
</table>
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- Risk-Neutral Puzzle
- A New Approach to RN and RW Scenarios
Realistic RW scenarios should exhibit those features that are observed in historical time series.

<table>
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<tr>
<th>Feature</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Heavy tails</td>
<td>Tails of observed return data deviate from normal or lognormal behavior</td>
</tr>
<tr>
<td>Asymmetric tails</td>
<td>Negative returns often exhibit fatter tails than positive ones</td>
</tr>
<tr>
<td>Tail dependence</td>
<td>Observed dependencies suggest weaker dependence under normal market conditions but higher dependence under stressed market conditions</td>
</tr>
<tr>
<td>Mean reversion</td>
<td>Some variables exhibit mean reversion property, such as interest rate, inflation, or credit cycle</td>
</tr>
<tr>
<td>Volatility clusters</td>
<td>High volatility events tend to cluster in time, e.g. equity indices, FX rates</td>
</tr>
<tr>
<td>Absence of Arbitrage</td>
<td>Simulated scenarios should not allow for arbitrage opportunities, e.g. interest rate parity, positive forward rates</td>
</tr>
<tr>
<td>Stationarity</td>
<td>Invariance of statistical properties of the returns in time</td>
</tr>
<tr>
<td>Absence of autocorrelation</td>
<td>Autocorrelation of investable risk factor returns is insignificant</td>
</tr>
</tbody>
</table>
The course of economy is subject to crises. Realistic economic scenarios should represent both normal and stressed market conditions.
Tail Patterns

**CDF of Monthly Returns of MSCI UK (1970-2010)**


Heavy Tails

Asymmetric Tails

Empirical data

Lognormal fit
Dependency Patterns

Tail Dependence

Mean Reversion and Clustering Patterns

Mean Reversion of Real Interest Rate Based on USD 10Y Treasury and US CPI

Volatility Clusters: MSCI US annual moving average volatility
Cyclicality of Credit Risk

Historical default rates & migrations exhibit co-movement with credit cycle

- Default and migration probabilities exhibit time dependence—credit cycle
- Defaults can be 10 times higher in bad years compared to good years
- Ratio of downgrades to upgrades can be 4 times higher in bad years relative to good years

Data source: S&P
Risk-neutral scenarios will not reproduce these features, so they **miss reality in many different aspects**

«Real-world» scenarios derived from RN scenarios by a simple addition of a **risk premium** are not sufficient to represent reality

Real-world scenarios have to be generated such that they **reproduce all the observed features**

This does **not** imply that the simulated RW scenarios are bound to reproducing historical behaviour only
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The following elements fully determine the dynamics of RN scenarios:

<table>
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<th>Risk-free <strong>yield</strong> curves at valuation time</th>
<th><strong>Implied volatility</strong> surface at valuation time</th>
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<td><strong>Correlation</strong> between different risk factors</td>
<td><strong>Martingale</strong> property</td>
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Challenges of Risk-Neutrality (I)

- Definition of “risk-free” reference required
- **Government bond yields may be slightly negative** *(compatibility with swaption pricing)*
- Forward rates in the long-term limit are illiquid
- Simple interest rate models (e.g. 1 short rate and 1 long rate) cannot render actual form of yield curve

**Risk-free yield curves at valuation time**

- Implied volatility values available only for liquid options
- For illiquid markets or non-traded assets (real-estate, hedge funds) models or judgment are used
- **Moneyness dimension of interest rate derivatives often neglected** *(flat smile)*

**Implied volatility surface at valuation time**
Challenges of Risk-Neutrality (II)

- Martingale property
  - Martingale condition for all investment strategies satisfied approximately
  - In practice, martingale property met only for simple investment strategies
  - Martingale condition does not easily reconcile with mean reversion of interest rates
  - Deriving RW scenarios from RN scenarios is not an obvious task

- Correlation between different risk factors
  - Correlations cannot be derived from derivative markets
  - Therefore a correlation model or judgment is required
  - Imposing correlations means adding more conditions to an already large set of conditions
For many risk-factors, the notion of “market consistent” should be revised to “model or judgment consistent”, in particular in case of:

- Less traded combinations of strike price and expiry periods
- Assets with no derivative markets such as property, hedge fund, private equity indices
- Correlation parameters
- Very volatile market data (often smoothed out for robustness of results)

Martingale property satisfied only approximately (e.g. for 10k scenarios)

- Well satisfied for static strategies or simple rollover strategies
- Often not satisfied for strategies with more complex rollovers
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Different aspects of valuation and risk assessment require real-world as well as risk-neutral scenarios.

Example—Life liabilities:

- RW scenario values are used to check the trigger conditions of guarantees and to calculate the ensuing cash flows.
- The corresponding RN scenario values are then used for the market-consistent valuation of the cash flows.
Possible ways to generate consistent real-world and risk-neutral scenarios:

- Generate RN scenarios \(\rightarrow\) derive RW scenarios (sophisticated risk premium model)
- Generate RW scenarios \(\rightarrow\) derive RN scenarios (imposing martingale conditions)
- Generate RW and RN scenarios through a joint algorithmic process

Precondition for the discussed method: RW scenario generator
Two prevailing approaches to transform RW to RN:

**Adjusting Scenario Values**

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<th>RW/RN Probabilities</th>
<th>RW Scenario</th>
<th>RN Scenario</th>
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<td>( p_1 )</td>
<td>( S_1 )</td>
<td>( \tilde{S}_1 = S_1 + \Delta S_1 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( p_n )</td>
<td>( S_n )</td>
<td>( \tilde{S}_n = S_n + \Delta S_n )</td>
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- Market replication: \( \sum_{i=1}^{n} p_i \cdot \text{Option}^k (S_i + \Delta S_i) = \text{Market price of Option}^k \)
- Ranking preservation: \( \text{CDF}(S_i) = \text{CDF}(\tilde{S}_i) \)

**Adjusting Scenario Probabilities**

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- Market replication: \( \sum_{i=1}^{n} \tilde{p}_i \cdot \text{DCF} \cdot \text{Option}^k (S_i) = \text{Market Price Option}^k \)
- Probability measure: \( \sum_{i=1}^{n} \Delta p_i = 0 \)
- Minimal distortion: \( \min \sum_{i=1}^{n} \Delta p_i^2 \) (2\textsuperscript{nd} order approx)
The implied volatility surface determines the distributions of risk-neutral scenarios.

These distributions are used to construct risk-neutral scenarios from real-world ones.

The construction process keeps RW scenarios consistent with the corresponding RN scenarios.

The generated risk-neutral scenarios are:

- Consistent with the volatility surface and market prices of derivatives
- Consistent with correlation assumptions
- Martingale conditions are fulfilled for simple and complex investment strategies with rollovers as far as a limited number of scenarios permits
Benefits of the Approach

- Consistency between RW and RN scenarios leads to consistency between asset and liability modeling.
- RN scenarios inherit those features of RW that are not conflicting with martingale property.
- Provides an intrinsic approach to construct RN scenarios for risk factors with no derivative markets.
- Macro-economic variables e.g. GDP can be included in RW scenario sets:
  - Regulators define stress scenarios in terms of macroeconomic variables.
  - Firms perform portfolio valuation contingent to those stress scenarios.
  - Stressed RN scenarios can be obtained through corresponding stressed RW scenarios.