ARTIFICIAL INTELLIGENCE AND STOCKMARKET SUCCESS

by

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“I have never hesitated to question commonly accepted theories when they appeared to me to be founded on hypotheses which implied consequences incompatible with observed data. Dominant ideas, however erroneous they may be, end up, simply through repetition, by acquiring the quality of established truths which cannot be questioned without confronting the active ostracism of the establishment.”

Maurice Allais

INTRODUCTION

As your eyes scan these first few lines of text to help you decide whether it is worth reading any further, your brain - without any conscious effort on your part - is correcting for the inbuilt “crossed wiring” of your visual system. The lenses at the front of our eyes cause an inverted image of what we look at to be projected onto the retinas at the back of our eyes, but as very young babies we soon learn from trying to touch objects near to us that the real world is an inverted image of what we appear to see. Our minds then reprogramme themselves as if by magic to allow us to “observe” things as they really are, and for the rest of our lives the inbuilt “crossed wiring” within our visual system causes us no practical problems whatsoever. Convincing evidence, some might think, of the truly remarkable power of the human mind to learn from observed real world experience and hence, in particular, of our ability to act “rationally” in our investment affairs in accordance with the “rational expectations” cornerstone of economic science. Or is it?

If the principles underlying the “rational expectations” approach were correct, then - as with chess grandmasters and world class tennis players - we would expect to see the general recognition of numerous investment “superstars” who had both the innate ability and mental fortitude to apply these “rational expectations” principles far more effectively than most other fallible human beings and thereby to demonstrate consistent stockmarket success over a number of years while at the peak of their investment careers. But, with the notable exception of Warren Buffett, no such “superstars” have been identified in recent decades, and furthermore the perceived impossibility of achieving consistently superior stockmarket performance has given considerable respectability to an academic conjecture that Warren Buffett has dismissed as “absolute rubbish” - the “Efficient Market Hypothesis” formulated by Fama (1970) and other financial economists around thirty years ago after their statistical investigations appeared to suggest that consistently superior performance could not be achieved using publicly available information.

The first objective of this paper is to demonstrate that in terms of understanding stockmarket investment the wiring of the human mind is so badly crossed that it is generally only after many years of painful and costly practical experience, if at all, that we begin to learn to overcome the wealth-destroying patterns of behaviour that are caused by this crossed wiring. The second, and principal, objective is to describe a two-part “artificial intelligence”
approach whereby individual and institutional investors alike can break the mould of the subconscious thought patterns that very often lead to an investment performance markedly poorer than that of a relevant stockmarket index. The third and final objective is to explain why the intellectually appealing, and allegedly “scientific”, approach of mathematical economics seriously hinders, rather than helps, our quest for a “more intelligent” approach to stockmarket investment. I shall define “artificial intelligence” very broadly as any means of achieving a standard of real world behaviour that is superior to what is possible with the unassisted human mind in conventional patterns of human activity.

Identifying, and then publicly advocating, a “more intelligent” investment framework than the much criticised “active investment” approach would in my opinion be an outstanding achievement for the UK actuarial profession in line with our stated public duty objective of “making financial sense of the future”, and indeed the importance of the investment field to actuaries was emphasised last autumn by Fraser Low and Paul Thornton in their respective Presidential Addresses. Fraser Low highlighted the need for a sound practical approach which embraces the most appropriate techniques, regardless of who invented them. Paul Thornton urged us not only to unravel the “true dynamics” of investment markets but also to find ways of anticipating - and possibly even of averting - market bubbles and market crashes. Both Presidents also stressed the desirability of learning wherever possible from the practical success stories of other numerate professional bodies. The main physical sciences parallel I shall use relates to the work of Harold Edwin Hurst. As Mandelbrot (1977) observes:

“Hurst’s name is likely to survive because of a statistical method he initiated and used to discover a major empirical law concerning long run dependence in geophysics.”

1. ARTIFICIAL INTELLIGENCE AND ACTUARIAL INTELLIGENCE

1.1 Six Pillars of Actuarial Intelligence

Having defined “artificial intelligence” very broadly as any means of achieving a standard of real world behaviour that is superior to what is possible with the unassisted human mind in conventional patterns of human activity, I shall begin with a discussion of six multiple choice questions, each one corresponding broadly to one of what I perceive as the six most important pillars of my training and experience as an actuary. These six pillars are:

(i) Real world applications of probability theory.
(ii) Conceptual vision as to the guiding principles of real world behaviour (often equivalent to “general reasoning”).
(iii) Rigorous evaluation of basic probabilities.
(iv) Rigorous evaluation of underlying rates of investment return.
(v) Wisdom derived from the practical experience of others (Bacon’s “I hold every man a debtor to his profession” motto, which appeared on the cover of the Journal of the Institute of Actuaries, is very relevant here).
(vi) Actuarial judgement based on first hand practical experience and on as many of the other five pillars as are appropriate in the specific circumstances.
To reflect Fraser Low's exhortation in his Presidential Address that some actuaries should offer a "high street" advisory service to individuals, my last three questions relate to investment advice to an individual. To reflect Paul Thornton's exhortation in his Presidential Address that actuaries must, as a matter of urgency, seek out the true dynamics of investment markets, my fourth question relates to differentials in investment returns between two different philosophies of equity investment. The reader is strongly encouraged to note down what he or she sees as the correct answer in each case before going on to read my analysis of it.

1.2 Donald's Paradox

There are 34 people in a room. Assuming that birthdays are uniformly distributed over the calendar year, and ignoring leap years to simplify the analysis, what is the probability that two or more people have the same birthday? Is it:

(a) around 0.1,
(b) around 0.5,
(c) around 0.8?

D.W.A. Donald, a former President of the Faculty of Actuaries, used this apparently simple problem to demonstrate that most people, even numerate professionals, cannot evaluate compound probabilities correctly. Nearly all non-actuaries (and, I suspect, a significant number of actuaries!) who have not encountered this problem before will tend to opt for (a), whereas the actual probability, namely 0.795, corresponds to (c). I see this as a classic example of how the untrained human mind jumps subconsciously to the erroneous conclusion that a linear model (in this case involving simple proportion) will be adequate, at least as a first approximation. If we denote the reciprocal of 365 by x, then this approach suggests that, for a small number, n, of people in the room, the required probability is of the order of nx. However, this fails to reflect the dependencies which make the situation grossly non-linear: when the number of people (who can be assumed to come into the room sequentially) increases from n-1 to n, the multiplier for the revised inverse probability that no-one has the same birthday is, to the first order of small quantities, l-(n-1)x, not 1-x, so that, again to the first order of small quantities, the required probability for n people in the room is:

\[ x + 2x + 3x + \ldots + (n-1)x \]

or

\[ \frac{n(n-1)x}{2} \]

rather than the seductive linear value of nx. For n=10, the above first order of small quantities expression gives the required probability as 0.123, which agrees to two significant figures with the actual probability of 0.117.

1.3 Two Bombs on an Aeroplane

The probability that there is a bomb on board a certain type of aeroplane, which can be assumed always to have its full complement of 100 passengers on board, is one in a million. Given that I have taken a bomb on board, what is the probability that there is at least one other bomb on board? Is it:
No-one with any grounding in probability theory should fall into the trap of thinking that the required probability is one in a million million, and hence many will opt for (b), on the grounds that the behaviour of the other passengers may be thought to be totally independent of my behaviour. There is also a seemingly plausible case for (a) in that there are fewer (99 rather than 100) other passengers who might take a bomb on board. However, in conformity with my "conceptual vision" pillar, common sense and observation of what happens in the real world both indicate that one of the most important factors affecting the likelihood of a bomb being on board is the effectiveness or otherwise of the security checks. Having been able to take a bomb on board myself, I conclude that the security checks were dangerously lax and accordingly that the probability of another bomb being on board is significantly greater than the guideline average of one in a million, and this corresponds to answer (c). An observation by Keynes (1921) corroborates this "conceptual vision" approach.

"James Bernoulli's second axiom, that in reckoning a probability we must take everything into account, is easily forgotten in these cases of statistical probabilities. The statistical result is so attractive in its definiteness that it leads us to forget the more vague though more important considerations which may be, in a particular case, within our knowledge. To a stranger the probability that I shall send a letter to the post unstamped may be derived from the statistics of the Post Office; for me those figures would have but the slightest bearing on the situation".

1.4 Christmas Day

What is the probability that Christmas Day fell on a Sunday in the year in which I qualified as a Fellow of the Faculty of Actuaries? (I was born in 1945 and studied for the actuarial examinations after reading mathematics at the University of Glasgow). Is it:

(a) significantly less than 1 in 7,
(b) around 1 in 7,
(c) significantly higher than 1 in 7?

I shall use this example to illustrate two crucial truths. First, the expression "let p(x) be the probability density function of ....." is virtually meaningless in many real world situations. Second, the unassisted human mind is incredibly weak at performing elementary algorithmic routines involving numbers.

Since there is inadequate information given above to determine with certainty the year in which I qualified, some might think that a simplistic "quantum theory" approach is appropriate and accordingly that the required probability is around 1 in 7, corresponding to answer (b). But again Bernoulli's second axiom should be kept in mind, and - on the assumption that my education followed a reasonably standard progression - I would have left high school at around 18, spent around 4 years on my mathematics honours course, and then taken around 4 years to complete the actuarial examinations, giving 1971 as a reasonable first guess as to the year in which I qualified. I now invoke Warren Buffett's comments as set out
in Hagstrom (1994) on the passive equity investment approach of index-matching. He fully accepts that for the vast majority of people who do not have a clue as to the true “drivers” of long term investment performance it is eminently sensible to follow an indexed approach; in particular, they will not be exposed to the psychological trauma of experiencing periods of serious underperformance against the broad market indices. However, with masterly understatement he then suggests that once you understand the true dynamics of real world investment behaviour “a little homework” should enable you to achieve a performance well in excess of that of most other investors. In the present example, “a little homework” shows not only that Christmas Day fell on a Saturday in 1971 but also that Christmas Day did not fall on a Sunday in any year from 1967 to 1976. On the given information, and subject also to the “guiding principle” of a standard educational career, the required probability is significantly less than 1 in 7, which corresponds to answer (a). However, with a trivial further “bit of homework”, namely looking up the Joint List of Members and Students of the Faculty and Institute of Actuaries, it can be seen that my year of qualification was in fact 1970, when Christmas Day fell on a Friday, so that with “perfect information” the correct answer is again (a). Keynes (1921) was well aware that our degree of uncertainty decreases, and correspondingly the confidence we have in our estimates of probability increases, as our understanding of real world behaviour improves with the arrival of new relevant information. Perhaps his most succinct description of the resulting dilemma is the following:

“Bernoulli’s second maxim, that we must take into account all the information we have, amounts to an injunction that we should be guided by the probability of that argument, amongst those of which we know the premises, of which the evidential weight is the greatest. But should not this be re-inforced by a further maxim, that we ought to make the weight of our arguments as great as possible by getting all the information we can? ...... But there clearly comes a point when it is no longer worth while to spend trouble, before acting, in the acquisition of further information, and there is no evident principle by which to determine how far we ought to carry our maxim of strengthening the weight of our argument. A little reflection will probably convince the reader that this is a very confusing problem”.

In short, the required probability is not absolute but changes as we acquire whatever further information we believe is appropriate in the specific circumstances of the case.

To a pure mathematician, it is “obvious” that we can work forwards or backwards from knowing today’s date and day of the week to determine the day of the week that another date falls on; the well known “Thirty days hath September .....” rhyme and the rule for leap years provide all the necessary information. However, actuarial work is practical, not theoretical, and we must be able to obtain a practical answer to a real world problem within a reasonable timescale. The following is the eminently useful algorithm that I developed many years ago for any date in the twentieth century; I leave it as an exercise for the reader to eradicate the millennium bug and adapt it for next century. The mathematical formulation involves linear congruencies (modulo 7 and modulo 28).

1. Let \( w \) be the last two digits of the year (modulo 28).
2. Let \( x \) be the number of leap year days earlier this century for year \( w \) and the required date.
3. Let \( y \) be the month digit in the table below:
4. Let $z$ be the day of the month.

5. Then for Monday = 1, Tuesday = 2, etc., the day of the week is:

$$w + x + y + z \pmod{7}.$$ 

For example, for 19th October 1987 we have $w = 3$, $x = 0$, $y = 0$ and $z = 19$, so that:

$$w + x + y + z = 22$$

$$= 1 \pmod{7},$$

which was a Monday; "Black Monday" of the 1987 Stockmarket Crash to be precise. This evaluation can be done mentally in a few seconds, and is a very rare example of how the human mind can operate effectively along the lines of a digital computer, but only after learning the algorithm and becoming proficient through regular practice, just as in learning to play the piano. At the time of writing this, I know of no-one else who uses this elegant algorithm, but having now committed it to writing for the first time I suspect the situation will soon change.

1.5 US Equities

Your client, a widow aged 40 in good health but of only modest net wealth, has been bequeathed £1m by an eccentric uncle. There is a condition in the will that the money must be invested for the long term in US equities using one or other of two investment strategies – major stocks in the top decile of price-earnings ratio, or major stocks in the bottom decile of price-earnings ratio. Do you advise your client to invest in:

(a) high price-earnings ratio stocks

or (b) low price-earnings ratio stocks?

O'Shaughnessy (1996) shows that the answer is (b) without a shadow of doubt; over the 42 years to 1994, low price-earnings ratio stocks, rebalanced annually, gave an annualised return of 13.5%, as against 11.4% for large stocks in the S & P 500 Index and 8.9% for high price-earnings ratio stocks. But these trend rates of investment return are only the tip of the iceberg as regards O'Shaughnessy’s painstaking research. To quote from the book’s cover:

"His analysis makes clear that you can do vastly better than the market by consistently using stock selection strategies that have withstood the test of time. Moreover, four decades of results warn investors to steer clear of some popular investment methods that can be toxic to their wealth".
Although not an actuary, O'Shaughnessy pinpoints the way in which the fallibility of human judgement can be overcome by the use of an actuarial approach:

“David Faust writes in his revolutionary book *The Limits of Scientific Reasoning*: ‘Human judgement is far more limited than we think. We have a surprisingly restricted capacity to manage or interpret complex information’. Studying a wide range of professionals, from medical doctors making diagnoses to experts making predictions of job success in academic or military training, Faust found that *human judges were consistently outperformed by simple actuarial models*. Like traditional money managers, most professionals cannot beat the passive implementation of time-tested formulas”.

O'Shaughnessy then goes on to explain the psychological shortcomings in human nature that make my “fourth pillar” of actuarially determined trend rates of investment return indispensable in the quest for consistently superior investment performance:

“The majority of investors as well as anyone else using traditional, intuitive, forecasting are overwhelmed by their human nature. They use information unreliably, one time including a stock in a portfolio and another time excluding it, even although in each instance the information is the same. Our decision making is systematically flawed because we prefer gut reactions and individual, colourful stories to boring base rates. Base rates are among the most illuminating statistics that exist ..... When used in the stock market, base rates tell you what to expect from a certain class of stocks (e.g., all stocks with high dividend yields) and what that variable generally predicts for the future. But base rates tell you nothing about how each individual member of that class will behave”.

1.6 Lottery Gamble

The UK national lottery has been restructured to limit winners to a maximum prize of £1m, except that they may then elect to try (in a televised show!) to increase their winnings to £5m, the sting in the tail being that there is a possibility of losing everything. A winner trying for £5m has to select, blindfolded, one of 100 balls in a traditional probabilistic urn numbered from 1 to 100. If the number drawn ends in a 7 (i.e. 7, 17 ......., 97) the payout is £5m; if 13 is drawn, the “winner” loses everything; otherwise the payout remains at the bankable value of £1m. Your client (the same one as in Section 1.5) has won £1m and asks you whether she should try for £5m. Do you advise her to:

(a) settle for £1m,

or (b) gamble for £5m?

As regards this “fifth pillar” corresponding to the practical wisdom of others with many years of practical experience, an apposite actuarial opinion was expressed by Gwilt (1953), the then President of the Faculty of Actuaries, when two investment papers were being discussed at a Sessional Meeting:
"If you will forgive a brief presidential platitude, I think myself it is wise in the
investment of the assets of a life office to follow a policy of moderation and restraint,
and to resist any temptation to strive for a spectacular profit if there is any possibility
of a serious loss, if one's assumptions should prove to be wrong".

As I suggest in Clarkson (1994), the acute psychological distress perceived as likely to result
from the (small probability) chance of losing everything will for most people outweigh the
(moderate probability) chance of enhancing the already life-changing win of £1m to £5m, and
accordingly my advice - consistent with Gwilt's comments in a life office context - would be
to opt for (a). Keen students of utility theory or of Markowitz (1959) will recognise this
example as one half of what has become known as the Allais Paradox, first described in
Allais (1953). In his 1988 Nobel Lecture, Allais (1989) points out that the overwhelming
tendency of "reasonable men" to opt for (a), far from being paradoxical, is a practical
example of the eminently intelligent human trait that he describes as "the preference for
security in the neighbourhood of certainty".

1.7 Another Lottery

Your client, the same one as in Sections 1.5 and 1.6, has won through to the final stages of a
lottery run by an overseas country. She must opt either for Jackpot A, which will pay out
£1m with probability 0.11 and nil with probability 0.89, or jackpot B, which will pay out £5m
with probability 0.1 and nil with probability 0.9. Do you advise your client to:

(a) opt for Jackpot A.
(b) opt for Jackpot B?

While the probability of winning £1m is marginally higher than the probability of winning
£5m, there is a strong likelihood of not winning anything in either case. Accordingly, the
slight psychological distress that might result from losing after opting for Jackpot B is likely
to be far outweighed by the mental visions of the utopian life that would result from winning
£5 rather than £1m. The mathematical expectation, to the extent that this concept has any
relevance, is also far higher with Jackpot B. Accordingly, my advice would be to opt for
Jackpot B. Again keen students of utility theory or of Markowitz (1959) will recognise this
example as the second half of the Allais Paradox.

1.8 Questions for Actuaries

After translating this "actuarial intelligence" section into my two-part "artificial intelligence"
philosophy for the enhancement of human achievement in the intimidating field of equity
investment, I shall endeavour to demonstrate the extent to which I perceive "crossed wiring"
to exist in the minds even of many actuaries by discussing - in the light of my introductory
"guiding quotation" from Nobel Laureate Maurice Allais - my answers to the eight following
important investment questions:

1. Do most stockmarket investors behave "rationally" most of the time?
2. Are equity markets broadly efficient at the "semi-strong" level corresponding
to publicly available information?
3. Is the “no arbitrage” principle a reasonable mathematical foundation for models of stockmarket behaviour?
4. Is stochastic calculus an appropriate mathematical tool for the investigation of stockmarket strategies?
5. Is variability of return (i.e. a statistical measure such as variance or semi-variance of return) a reasonable proxy for investment risk?
6. Is the expected utility maxim an appropriate mathematical tool for resilience testing?
7. Are “linear” stochastic models appropriate for solvency investigations?
8. Should novice investors be left to learn from their own mistakes?

Unlike the situation with the multiple choice questions I used to illustrate my “six pillars” of actuarial intelligence, it would be unreasonable to seek a simplistic “yes/no” answer to these perplexing questions. A Keynesian “weight of evidence” approach based on a combination of formal training and practical experience is a far better way forward, and accordingly I strongly encourage the reader to note down his or her considered response to each question on a scale of 1 (emphatically no) to 10 (emphatically yes).

2. ACTUARIAL INTELLIGENCE AND SHARE SELECTION

2.1 A Conventional Equity Selection Model

It is fairly obvious on common sense grounds that – other things being equal – the price-earning ratio of an equity share will tend to increase with the likely long term rate of earnings growth, but practical models using this “second pillar” guiding principle, such as the Weaver & Hall model, required an immense and horrendously expensive analytic effort to obtain meaningful numerical estimates of future earnings growth and of other fundamental factors. In 1975 it occurred to me that the analytic effort and consequential expense could be reduced very significantly by using a few senior analysts to rate certain essentially independent features (namely prospects for volume growth, level and sustainability of profit margins, ability to finance growth, and operational flexibility) on a scale of, say, 1 to 10, with the ranking measure of earnings growth being the sum of these component ratings. The crucial other questions that had to be addressed before a “conventional” practical model for share selection could be constructed were whether to use historic or prospective price-earnings ratios, and what fundamental attributes should be incorporated using variable parameters to optimise the goodness of fit between actual and expected prices. Using my “fifth pillar” of the practical experience of others (mainly the construction of, and the Institute Sessional Meeting discussion on, the Weaver & Hall model) and my “sixth pillar” (my own practical experience as enhanced by the results of certain pilot tests I carried out), I decided to use (as with the Weaver & Hall model) prospective price-earnings ratios to twelve months hence, and three (as opposed to five in the Weaver & Hall model) variable parameters - one each for long term earnings growth, dividend payout ratio, and the proportion of profits earned overseas. The resulting model, which is described in detail in Clarkson (1981), overcame virtually all of the wealth - destroying inconsistencies that O’Shaughnessy (1996) highlights, and gave very satisfactory performance results.
These are highly significant “fourth pillar” results, namely empirical measures of the investment returns of certain well-defined subgroups of UK shares that can be identified using a structured common sense approach to enhance the judgmental skills of investment practitioners.

2.2 A Disequilibrium Equity Selection Model

In September 1996, when presenting my Clarkson (1996) paper at the Nuremberg AFIR International Colloquium immediately after the presentation of the Exley, Mehta & Smith (1996) paper on stockmarket efficiency, I commented that the then newly available Hemmington Scott REFS (“Really Essential Financial Statistics”) data for quoted UK companies could probably be used to prove beyond any possible shadow of doubt that the UK equity market is grossly inefficient at the “semi-strong” level corresponding to publicly available information. If we use “real time” (linearly interpolated) earnings to twelve months hence and assume that earnings growth rates for all companies are constant beyond this forecasting horizon, we can obtain two “second pillar” insights. First, for shares with the same prospective price-earnings ratio, the medium term share price performance will tend to increase with the expected rate of earnings growth over the next twelve months. Second, for shares with the same expected earnings growth over the next twelve months, the medium term share price performance will tend to decrease as the prospective price-earnings ratio increases. In short, high earnings growth is “good”, whereas a high price-earnings ratio is “bad”, other things being equal. I then tossed in a “sixth pillar” conjecture that price-earnings ratios are often a reflection more of recent earnings growth than of a realistic appraisal of future earnings growth, and – as described in detail in my Clarkson (1997a, 1998b) papers to the 1997 and 1998 Investment Conferences of the Faculty and Institute of Actuaries and in the Clarkson (1997c, 1998a) articles in the “The Actuary”, I developed a “strategy investment” approach to equity share selection based on the utility ranking measure:

\[ U = G - cR, \]

where \( G \) is the consensus twelve months earnings growth rate, \( R \) is the prospective price-earnings ratio, and \( c \) is a constant estimated in dynamic terms as the value which leads to the best predictive power rather than in static terms as the value which gives the best fit on a “least squares” statistical approach. The optimal value of \( c \) is indeed far higher than the “equilibrium” value that is consistent with the “no arbitrage” teaching that there is no dependence between price-earnings ratio and future return. This exceptionally strong evidence of disequilibrium is consistent with other very well documented “fifth pillar” findings. For instance, Benjamin Graham, the father of security analysis, warned that buying shares on high price-earnings ratios was an almost certain way of underperforming the broad market; Soros (1988) shows, in terms of his “reflexivity” principle, that stockmarkets overreact to recent events; and O’Shaughnessy (1996) shows that in the US high price-earnings ratio shares systematically underperform low price-earnings ratio shares. Once the monthly volume of data is available, it takes me around three hours, using simple algorithms conceptually similar to my “day of the week” enumeration, to translate the relevant fundamental data into a complete ranking of expected medium term price performance for the 70 FTSE-100 Index constituents that have the highest quality of earnings in terms of no recent serious setbacks. The “value added” in terms of the typical outperformance of my “model portfolio” (4% in each of my top ten shares, 3.5% in each of the next ten, and 2.5% in
each of the subsequent ten) is quite remarkable for an elementary common sense strategy using information that is readily available at modest cost to institutional and private individual investors alike.

2.3 A Classification of Equity Share Selection Models

Let us now move up a gear in conceptual terms and classify equity selection models using four numbers (mainly digits) in a similar manner to the descriptions such as (0,1,0) and (3,3,0) that are used for ARIMA (autoregressive integrated moving average) time series models. I use, for the first digit, 0 for the assumption of equilibrium (and hence generally a “least squares” method of estimation) and 1 for the assumption of disequilibrium (and hence generally a dynamic method of estimation corresponding to the best predictive power on empirical testing). I use, for the second number, the duration in years of the horizon to which earnings are explicitly projected. I use, for the third digit, the number of variable parameters that are employed to obtain the (static) best fit as between actual and expected prices. I use, for the fourth digit, the number of variable parameters that are employed to optimise the (dynamic) predictive power of the model. The Weaver & Hall model is of type (0,1,5,0), while the Clarkson (1981) model, which I suspect is slightly more powerful in that the smaller number of variable parameters picks up more extremes of undervaluation or overvaluation, is of type (0,1,3,0). The “factor models” described in the academic literature are generally of type (0,0, n, 0); I see these as being far less powerful in that they not only essentially ignore reasonably reliable earnings forecast to a year or so ahead but have so many variable parameters that price extremes tend to be “explained” by the models rather than being thrown up as exploitable opportunities. The Clarkson (1998b) model could be enhanced in two ways. First, the long term performance would improve on average, although the consistency of performance would deteriorate, if specific earnings forecasts were made to a longer horizon of, say, two years. Second, and more importantly, my type (1,1,0,1,) model, while perhaps suitable for an individual investor who wants to use only readily available information, takes no account whatsoever of likely earnings growth beyond the forecasting horizon. I see the main reason for the success of my Clarkson (1981) model as its ability to incorporate easily determinable estimates of longer term earnings growth prospects. Using a dynamic rather than a static parameter, such a type (1,2,0,2) model will probably be close to optimal as an “analytic engine” suitable for a structured approach to share selection by institutional investors.

2.4 Mean Absolute Deviation Analysis

A classic example of the successful adoption by actuaries of a numerical approach developed by another profession is the Mean Absolute Deviation approach described by Plymen & Prevett (1972). The crucial “multiplier” that determines how far above and below the moving average that the control limits should be set was determined by what worked best in practice rather than by statistical theory. For investment trusts, the optimal value was found to be 1.6. I was astonished to find that this value of 1.6 also worked well for both gilts and UK equities. I later realised that 1.6 is for all practical purposes the optimal value for a series which varies over time in simple harmonic motion, i.e. a pure sine wave.
3. HURST AND HIS EXPONENT

3.1 Hydrology Parallels

Harold Edwin Hurst (1880 - 1978) left school at 15, and, after attending evening classes to further his education, won a scholarship to Oxford, where — largely as a result of his very great strengths in practical work — he was awarded a first-class honours degree. Almost his entire working career was spent as a hydrologist in Egypt struggling with the problem of reservoir control. As Peters (1991) observes:

"An ideal reservoir would never overflow; a policy would be put into place to discharge a certain amount of water each year. However, if the influx from the river were too low, then the reservoir level would become dangerously low. The problem was: What policy of discharges could be set, such that the reservoir never overflowed or emptied?"

There are obvious parallels with pension scheme funding, where the investment return corresponds to the influx of water from unpredictable levels of rainfall within the catchment area, while the difference between payments to beneficiaries and contributions from employer and employees corresponds to the controlled level of discharge of water from the dam. The pension funding problem is to find a reasonably stable strategy that does not lead either to excess surplus (the dam overflowing) or to financial or regulatory insolvency (the reservoir emptying). Hurst studied how the range of the reservoir level fluctuated around its average level; if successive influxes were random (i.e. statistically independent) this range - as with variance in the Black-Scholes option pricing model - would increase over time in line with the square root of time. Hurst obtained a dimensionless statistical exponent by dividing the range by the standard deviation of the observations, and hence his approach is generally referred to as rescaled range (R/S) analysis. By taking logarithms, we obtain the Hurst exponent H from the following equation:

$$H \log(N) = \log(R/S) + \text{constant},$$

where N is the number of observations and R/S is the rescaled range. In practice the best way to obtain an estimate of H is to find the gradient of the log/log plot of R/S against N. In strict contrast to the "statistical mechanics" independence value of 0.5 for $H$, Hurst found not only that for almost all rivers the exponent for the influx was well in excess of 0.5 (0.9 for the Nile!) but also that for a vast range of other quite distinct natural phenomena, from temperatures to sunspots, the estimates of H clustered very closely around the value of 0.71, indicating the existence of a “long term memory” causal dependence. Two observations come immediately to mind. First, there is an uncanny (and obviously numerically inverse) resemblance to the seemingly universal value of 1.6 for the Mean Absolute Deviation analysis multiplier, as against 2 for the type of “random walk” implied by cornerstones of modern finance theory such as the Capital Asset Pricing Model, Arbitrage Pricing Theory, and the Black-Scholes option pricing model. Second, Hurst's main professional paper (at age 75!) is Hurst (1955), is in the Proceedings of the Institution of Civil Engineers, the journal of the professional body that I and other actuaries had the privilege of working with over the past few years to develop the highly acclaimed RAMP approach to project risk management.
3.2 **Equities**

Mandelbrot (1977), Peters (1991) and FitzHerbert (1994) all describe how individual equity prices and equity market indices have Hurst exponents very significantly in excess of the "random walk" value of 0.5 that would apply if all price changes were in response to the random arrival of new information. However, H is a symmetric measure - not unlike variance as a measure of risk - in that it does not tell us whether values higher than 0.5 are due to an increase in the maximum value of the range, or a decrease in the minimum value of the range, or a combination of the two. When I worked through the data for some constituents of the FTSE-100 Index, using either daily or weekly prices, I found that sometimes there were medium term "surges" of a series of rising prices, increasing the maximum of the range very significantly, and sometime these "surges" were in the opposite direction, corresponding to a marked medium term trend of price decreases. For any particular stock, a medium term surge in either direction tended to be followed by a relatively quiescent period, after which another medium term surge - often in the opposite direction to the previous one - would take place. These typical patterns of cyclical behaviour, which are non-periodic in that there is no fixed cycle length, show up very clearly in Mean Absolute Deviation charts. The remarkable similarity in the general patterns of the three charts for investment trusts, gilts and UK equities shown in Clarkson & Plymen (1988) tends to confirm the conclusions of Hurst exponent analysis, namely that there are characteristic non-linear patterns in stockmarket data as the result of some powerful underlying causal mechanism that mathematical economics cannot explain.

3.3 **Inflation**

When the Wilkie stochastic investment model, as described in Wilkie (1984,1986), first appeared, I arranged for linear time series of the ARIMA (m,n,p) type to be tested as a possible enhancement to the Mean Absolute Deviation techniques that I had been using for a number of years. I found, however, that these linear time series had virtually no predictive power in identifying likely maximum and minimum values that correspond to important changes of trend. In the discussion on Wilkie (1986), I described the crucial linear inflation model as "far too tame", and put forward the outline of a non-linear inflation model in which there was an upwards only "shock" every 17 years on average and there was a momentum-reinforcing term once a strong upward trend in inflation had become established. I subsequently learned from Peters (1991) that this type of non-linear process was uncannily similar to the "biased random walk" that Hurst had simulated using two decks of playing cards; there was a trend-reinforcing mechanism which tended to cause the average value to either increase or decrease until a "joker" appeared and the previous trend was reversed. Hurst also observed that, for a long series of hydrology data, the means and standard deviations estimated in different periods within it often differed markedly. This is very similar to the situation with UK inflation, where, for instance, rates were very low for much of the 'fifties and early 'sixties but increased markedly during the 'seventies, suggesting (on my "second pillar" guidance) that a linear time series model fails to reflect important characteristics of real world inflation such as the financially destructive surges that occurred in the early 'seventies.
3.4 Identifying the Dependencies

Just as litmus paper turns red or blue respectively when put into a clear liquid that is either acidic or alkaline, a Hurst exponent of significantly greater than or less than 0.5 is incontrovertible evidence respectively of trend-persistence or mean-reversion in distinct contrast to the 0.5 "statistical mechanics" value that should result from the mathematical economics approach. The values in excess of 0.5 that have been derived from both individual equities and equity market indices suggest that, in order to understand the "true dynamics" of stockmarket behaviour, we must be able to find a convincing "second pillar" explanation of the underlying causal mechanism. As a preliminary step, consider, as an example of a natural phenomenon likely to have a Hurst exponent of around 0.7, the frequency of serious avalanches in the Alps as a result of unusually heavy falls of snow. The avalanche tragedies of early 1999, following the heaviest falls of snow in living memory in certain ski resorts that had previously been thought to represent a very low level of avalanche risk, caused a complete reversal in human behaviour identical to the trend-reversing appearance of the "joker" in Hurst's decks of cards. Whereas skiers previously sought out (ignoring avalanche risk completely) the resorts with the highest recent snowfall, as being likely to offer the best skiing, they switched almost overnight to seeking the resorts where there had not been a high snowfall recently, as likely to remove most of the avalanche risk. My extended "second pillar" insight, based on several decades' practical experience in both areas, is that it is the same human mind that guides behaviour under conditions of uncertainty and financial risk in stockmarkets and behaviour when exposed to the dangers of natural hazards in potentially risky sports. Accordingly, in the next section I describe – with particular emphasis on skiing, which will be familiar to many readers – those sports parallels that I see as being most relevant in building up a much more credible description of the functioning of the human mind in a stockmarket environment than that offered by the "rational behaviour" cornerstone of mathematical economics.

4. SPORTS PARALLELS

4.1 In this Section I set out, very tersely, some observations and personal experiences relating first to the recognition, then to the mastery, of physical risk in various potentially dangerous sports. The main objective is to build up, from specific examples, an understanding of what I shall call "risk-intelligent" behaviour.

4.2 Rapid River Canoeing

4.2.1 Swimming

The first rule is "if you cannot swim, do not canoe". Unlike other intelligent mammals such as dogs, horses and deer, human beings have to be taught how to swim, otherwise we will panic and drown when thrown into deep water.

4.2.2 Capsize Drill

Since the human mind perceives acute danger when the head is unexpectedly under water, even novice canoeists who are strong swimmers will tend to panic if their canoe capsizes. An
elementary but absolutely essential aspect of training is to practise capsize drill – staying calm as the canoe is deliberately capsized (preferably in a heated swimming pool!) and only coming out of the cockpit after the canoe has come to rest upside down.

4.2.3 Eskimo Roll

Rapid river canoeists must learn how to perform, with virtually 100% success, the “Eskimo Roll” whereby the paddle is used to right the canoe in other than shallow water. Again this is best practised in a heated swimming pool.

4.2.4 Damage Limitation

Should it be necessary to come out of the canoe in rapids, the drill is to hold on to the stern of the upturned canoe and to float down, feet first, on your back; the most vulnerable parts of the body (the head and face, then the knees, then the elbows) will thereby be unlikely to suffer damage.

4.2.5 Degree of Difficulty

There is an international “white water” grading system (from Grade 1 - minor rapids, to Grade 6 - grave danger to life) based on easily recognisable features such as the height of standing or irregular waves. My most memorable and enjoyable descents have been Grade 4 with the occasional Grade 5 stretch.

4.2.6 Canoeing Alone

When canoeing alone on rivers, a useful rule of thumb is not to attempt anything worse than two grades below what you would regularly tackle in the company of others. Accordingly, when canoeing alone I would not go above Grade 2.

4.2.7 Physiological Responses

Theoretical knowledge in no way prepares you for the uncontrollable physiological responses that occur when you are exposed to danger on the water. I can identify four quite distinct types of physiological response.

4.2.8 Freezing

For the first few years you will tend to “freeze”, just like a rabbit caught in the headlights of a car, when a rock unexpectedly appears directly ahead of you in rapids. Since the canoe is usually strong enough to bounce off the rock undamaged, this unhelpful human response to unexpected danger is not in this instance likely to lead to personal injury.

4.2.9 Improved Reactions

After a few years your eye/brain/muscle co-ordination has advanced to such a degree that your arms instinctively move the paddle to take you clear of the obstacle, and it is only when
the rock is safely abeam of you that the “shiver up the spine” adrenaline twinge hits you, about one second after your brain first recognised the danger.

4.2.10 Competence Limit

When tackling rapids and waterfalls at the threshold of your competence, our “fight or flight” defence mechanism pumps large amounts of adrenaline into the bloodstream. Your heart pounds far faster than normal, and you tend to over-react in physical movements. It was precisely this type of over-reaction that caused me to capsize on the Nith some years ago at the start of the most difficult Grade 5 section. My canoe suffered irreparable damage, but the mentally rehearsed damage limitation plan described above allowed me to escape without a scratch.

4.2.11 Extreme Danger

Many years ago, when the normally tame Clyde was in violent spate, I was carried uncomfortably close to the very high weir at the David Livingstone Memorial Centre in Blantyre before being able to inch my way back upstream against the current to the safety of the river bank. Shortly after setting off upriver to return home, three-quarters of one paddle blade broke off and disappeared downstream and over the weir. Although I was by then in no danger whatsoever, my system became quite literally flooded with adrenaline, and for the next thirty minutes or so I was in a state of deep shock and had only a very dulled awareness of what was happening round about me. My body had reacted to imaginary mental visions of “what might have been” had the paddle blade broken earlier (a strong likelihood of being drowned in the foaming turbulence below the weir!) rather than accepting the “rational” forward looking scenario of no imminent physical danger.

4.3 Skiing

4.3.1 Level of Difficulty

The general level of difficulty of a piste, which for most practical purposes corresponds to the gradient of its steepest section, is indicated by a reasonably standardised qualitative classification. In Europe, the classification involves – in increasing difficulty – green, blue, red and black runs. In North America, the nomenclature is normally “easiest”, “more difficult”, “most difficult”, and “double diamond”, but some resorts split the “more difficult” classification into two. These qualitative classifications have been found to be far more effective for skiers of all levels of ability than purely numerical descriptors, and it is interesting to note in this connection that most skiers would seriously over-estimate the gradient of a red or black run. For example, the average gradient of the White Lady run at Cairngorm in Scotland is only 20 degrees, while the West Wall in Coire na Ciste – the steepest slope that can be found at Cairngorm without going off-piste – has a gradient of 30 degrees. Even the Swiss Wall at Avoriaz, affectionately known to many British skiers as “The Wall of Death”, has a gradient of no more than about 35 degrees.
4.3.2 Descent Times

While navigation in an area with marked pistes is relatively straightforward, at least in good weather, it is important to be aware of the dimension of time; if you miss a crucial uplift once the lift system begins to close down from around 4 pm onwards, the only way back to your hotel or chalet is likely to be a very long and horrendously expensive taxi journey. A rule of thumb that I find very useful is 5 minutes for every 1,000 feet of vertical descent. For example, the main run back to town from the top station at Les Deux Alpes, which takes in everything from long shallow schusses to a 30 degrees black run for the last 1,000 feet, represents a total vertical descent of 5,500 feet and hence should normally take me just under 30 minutes.

4.3.3 Elementary Precautions

To minimise the often serious, and sometimes fatal, consequences of a navigation error, whether in good or bad overhead conditions, I normally carry with me - even if there is not a single cloud in the sky when I set off - a spare piste map, a detailed contour map, a compass, a small pair of binoculars, and a whistle.

4.3.4 Avalanche Risk

On occasions I will ski on a recognised off-piste run, such as the very challenging Tour du Charvet route back to the Bellevarde area of Val d'Isère, but only if I have read up on the run in advance and - more importantly - only if the avalanche warnings posted at the main lift stations do not indicate other than a low level of avalanche risk. The European scale of avalanche risk goes from 1 to 5, with 3, for example, corresponding to “moderate risk of an avalanche on some steep slopes”. Very high avalanche risk normally exists after heavy falls of snow, since it takes time for the layer of new snow to become firmly consolidated with the previous snow base. Until this consolidation occurs, any minor physical stimulus - such as a change in temperature or a skier passing over - can upset the fragile equilibrium between the new and old layers of snow and trigger an avalanche of awesome destructive power. On a recent ski holiday at Les Deux Alpes, the avalanche warning notices indicated 3 (off-piste skiing “déconseillé”) or 4 (“vivement déconseillé”), but many visiting skiers would be blissfully unaware of the import of these warnings, either because their grasp of French was inadequate or they did not even realise that the avalanche warning system existed.

4.3.5 Ski Brakes

In skiing, as with many other sports, safety improves over time with the development of new equipment. For example, until around 30 years ago retaining straps between the ski and the skier’s ankle were standard as the means of preventing a loose ski from running off downhill and injuring or killing someone. After ski brakes were developed, national ski organisations promptly tested them and pronounced them to be far more effective in overall safety terms. Retaining straps are now as defunct as cable bindings and leather lacing boots.
4.3.6 Swiss Wall

While at Morgins in the Portes du Soleil area some years ago, I did the “circuit” anticlockwise on my own one day, spending several hours at Avoriaz, including three most enjoyable descents of the Juan Vuartes World Cup downhill run. After Avoriaz, I had an exhilarating run down the Swiss Wall, in perfect snow and weather conditions, on my way to Les Crosets and thereafter to Morgins. The following day, three of us repeated this circuit, but unfortunately the weather had deteriorated markedly. With visibility down to around 10 yards when we reached the top of the Swiss Wall, we decided that discretion was the better part of valour and descended to Les Crosets on the chairlift.

4.3.7 Moguls

Suppose that, after a very heavy snowfall, a large number of skiers ski down a fairly steep slope. Rather than cutting up and compressing the initially even surface of the new snow in a random fashion, leading to a “white noise” two dimensional array of small irregularities, a fairly symmetric pattern of large humps and depressions builds up, making it very difficult for less experienced skiers to negotiate the piste. It comes back to Tolstoy’s classic analysis of the conflict between freewill and necessity: the skier finds it necessary to underweight on the crests of the initially small humps, and in sliding sideways to an extent with increasing weight on the skis on the subsequent down slope scrapes more and more snow off towards the bottom, thereby increasing the depth of the depression. Piste machines are normally used to level out these impediments to carefree skiing, but some skiers positively revel in tackling giant moguls, and accordingly in some places - the Triftji slope at Zermat, for instance, and above the middle station at Aspen Highlands - the moguls are left untouched. At leading US resorts such as Vail you can check in for an afternoon’s “bumps clinic” to be taught the correct mogul technique.

4.3.8 Expert Tuition

Skiing, like swimming, is not a pursuit that the human mind and body can master within an acceptable timescale without the assistance of expert tuition. However, after perhaps one week’s expert tuition at the beginner level to build up confidence and develop basic skills, a further two weeks’ instruction at the intermediate level is often sufficient to build up a skier’s confidence and technical skills to such an extent that he or she is able to turn on, and thereby descend, the steepest slopes that are found both on piste and off piste at ski resorts. But many intermediate skiers, even after many years of enjoyable ski holidays, get into something of a rut as regards their technique. However, “breakthrough” tuition is available at many ski resorts to enable them to break these ingrained inefficient habits and to “move up a gear” to ski much nearer to their full potential, which generally involves not only much greater enjoyment but also far less risk of physical injury.

4.3.9 Fall Line Fear

Those who have tried to learn skiing the hard way, namely without expert tuition to build up basic confidence in very easy situations before progressing further, may well have experienced the dreaded ski syndrome of “fall line fear”. When a turn on a steep slope is being attempted, the unprepared human mind not surprisingly perceives acute physical danger
at that point in the turn when the skis are pointing straight downhill. The brain freezes in trepidation, in precisely the same way as a rabbit caught in the headlights of an oncoming car, the momentum of the turn is lost, and the consequent frightening acceleration straight down the mountain can only be stopped by a deliberate, and often painful, fall to the ground.

4.3.10 Grand Couloir

While at Courchevel for the first time, I could not resist trying the infamous “Three Couloirs” at Saulire, which are probably the steepest runs in the Alps officially classed as designated pistes. Having done the Emile Allais Couloir without incident one day, I could feel the adrenaline pumping and my heart pounding the next day as I left the safety of the cluster of other skiers that had come out of the Saulire cable car and made my very lonely way over to the top of the much longer, and much steeper, Grand Couloir. It was very icy and alarmingly narrow at the top. Towards the end of the narrowest part, I realised that with one more turn I would be into easier territory. I carried out a vigorous jump turn in classic textbook fashion, but – almost certainly as the result of the high adrenaline level in my system – my vigour was such that I knocked my left ski out of its bindings. I could only watch in horror as it shot off and somersaulted down the rutted ice before coming to rest, almost out of sight, in soft snow several hundred yards below me. Ski brakes simply do not work on such steep and icy slopes. Fortunately I was well aware from first hand personal experience in less dangerous situations that the plastic sole of a normal ski boot has no grip whatsoever on steep icy slopes, and that the seemingly obvious course of action of taking off my other ski and walking down could prove fatal. I would slide out of control down the icy moguls and could easily break my neck. Accordingly, I waited a minute or two to compose myself and then skied down, very gingerly and with as few turns as possible, on one ski.

4.3.11 Avoidable Fatality

About six years ago two skiers lost their bearings coming down the very icy Epaule de Charvet black run at Val d’Isère, and one fell over a cliff to his death on the rocks below, while the other just managed to stop in time. Nearby skiers saw the incident, raised the alarm, and very soon a rescue helicopter was approaching the other, absolutely terrified, skier. Tragically, he was unaware that the plastic sole of a normal ski boot has no grip whatsoever on steep icy slopes, and he took his skis off in a rational – as he saw it – endeavour to facilitate his rescue by the helicopter that was now hovering overhead. He too fell to his death over the cliff.

4.3.12 Actions of Others

During a recent ski holiday at Les Deux Alpes, where I skied down anything from 15,000 feet to 30,000 feet of vertical descent each day, my only mishap was when a snowboarder lost control and crashed into me from behind on a steep black run back to town. There was a violent clash of heads. Fortunately I was no more than very slightly concussed, but I could easily have suffered a fractured skull. Tragically less fortunate was the English lady at Tignes a few years ago who was hit from behind by a skier so recklessly out of control that she was tossed high into the air and then landed head first. She died almost instantly from a broken neck.
4.4 Ski Mountaineering

4.4.1 Avalanche Precautions

On the final day of a superb ski mountaineering tour across the Dolomites in 1978, our guide warned us that there would be a significant avalanche risk when crossing the steep slope below the Drei Zinnen and that we would need to take all possible precautions to minimise the risk of injury or death. We took our rucksack off one shoulder and undid the waist belt, to make it easier to discard. Similarly, we undid our ski retaining straps and took our hands out of the loops at the top of our ski poles. We then set off briskly across the slope (to minimise the time exposure) at intervals of around fifty yards apart; this meant that it was likely that no more than one of us would be caught up in an avalanche and that the rest of us would be close at hand for a rescue. If an avalanche started above us, we had to ski off straight downhill to gather speed rapidly and then try to ski out to one side or the other. If unfortunate enough to be caught up in an avalanche, we had to throw out the ball of avalanche cord from our anorak front pocket, discard all our equipment to avoid being dragged down too deep into the snow, make swimming motions while being swept downhill to keep near the surface, and finally put a hand in front of our mouth and nose just before we came to rest to make an air pocket and avoid being suffocated. We crossed without incident, but our state of mental preparedness was so acute that it was almost something of an anti-climax to reach the safer territory beyond the steep slope.

4.4.2 Quantifying the Risk

It was only ten years or so later, in developing my Clarkson (1989) downside approach to risk, that I made a numerical estimate of the risk that I had been exposed to on this Dolomites tour - roughly one chance of death in a thousand for each day on the mountains; one chance in a hundred of death over the total of ten days on the tour; and one chance in fourteen of at least one of our party of seven being killed. But virtually no-one taking part in potentially dangerous sports knows, or appears to be remotely interested in, even the order of magnitude of the quantitative risk of injury or death.

4.4.3 Framing of Choices

Suppose that for my Dolomites tour we equate the exhilaration at the time and the lasting sense of great achievement thereafter to the monetary sum of £5m. If I was offered the ski mountaineering equivalent of the monetary gamble in the first half of the Allais Paradox, namely receiving £5m with probability 0.99 and a probability of 0.01 of being put against a wall and shot, I would have absolutely no hesitation in declining. But, should the opportunity arise, I would set out tomorrow on another equally risky ski mountaineering tour.

4.4.4 Risk-Intelligent Behaviour

I have given considerable thought as to whether it was reckless for me to have participated in such a dangerous activity as my Dolomites tour, and my conclusion is emphatically no. It was (as I had expected before setting out) one of the most exhilarating and rewarding episodes of my life; I believed that the often very strenuous and sometimes very risky activities were within my physical and mental capabilities; and as a group we had taken all
reasonable precautions, such as employing a guide (who thereby became legally responsible for our safety) and learning well in advance what to do in various life-threatening scenarios such as an avalanche. I suggest that we define such behaviour under conditions of uncertainty and risk resulting from natural hazards as "risk-intelligent". I observe in passing that the RAMP approach to managing project risk can similarly be described as "risk-intelligent" in the financial, environmental and political spheres of risk.

4.5 Yacht Racing

4.5.1 Knocked Flat

During an overnight race round Ailsa Craig, our yacht was knocked flat (the crosstrees on the mast touched the water!) by a sudden downdraught when we were only twenty yards or so away from the sheer cliff face of the rock. No-one fell overboard or was injured, and there was no damage to the yacht, but it seemed to take a very long time before the yacht crept back to its normal upright position as a result of the stabilising influence of the heavy keel. The "risk-intelligent" correct course of action was to do nothing other than to stay calm and hold on.

4.5.2 Crewman Panic

The inverse "risk-stupid" behaviour was demonstrated on another overnight race when a crew member, who had been seasick for some time in the very heavy seas caused by a severe gale, became so mentally disturbed that he attempted to jump overboard to bring an end to his suffering. Fortunately he was caught in time by other crew members, and then tied down in the cockpit for his own safety until the storm abated some hours later and he returned to a normal state of mind.

4.6 Parascending

I first observed the new sport of being towed high in the air on a parachute behind a power boat some years ago at a Mediterranean holiday resort. Our innate fail-safe approach to risk in new and unfamiliar areas came into play and I thought "This looks great fun; I'd love to do it, but it is too risky for me". After watching numerous people enjoying the experience without mishap over the next few days, I found that I had switched to believing - on the basis that no accident had occurred or had seemed likely to occur - that is was not, after all, too risky for me. The next day I went up for a most enjoyable parachute trip high above the bay.

4.7 Hang-Gliding

4.7.1 Change of Mind

I feel that hang-gliding is too risky for me, and - to follow up on my risky sports approach as set out in Clarkson (1989) - I have asked numerous actuaries of an adventurous frame of mind whether they had ever participated. The nearest to actually taking part was an actuary I met at the 1994 AFIR International Colloquium in the United States. He had booked, and paid for, an introductory lesson, but when he returned a few days later to take to the skies he found that, of the three instructors at the company, one had a leg in plaster and another had an arm
in a sling, both as a result of hang-gliding accidents in the previous two days. He immediately asked for, and was given, a full refund for the lesson that he now perceived as being foolhardily dangerous.

4.7.2  **Mixture of Probabilities**

Suppose that windsurfing involves zero risk and that hang-gliding is half as risky again (in terms of the probability of being killed) as ski mountaineering on a daily participation basis. I have to choose between ski mountaineering with certainty, and a 50:50 chance of either wind-surfing or hang-gliding based on the toss of a coin. Without hesitation I will opt for the former since I regard the prospect of even a small likelihood of participating in hang-gliding as being unacceptable to me on risk grounds. On a “look-through” probability basis, however, this choice involves the higher risk in terms of the probability of death on a “mixture” basis.

4.8  **Private Flying**

A trainee pilot has to acquire a vast amount of book knowledge about navigation, meteorology, and aviation law, but clearly “hands-on” flying experience, initially accompanied by an instructor, is of crucial importance. A vital part of this practical instruction is learning (by not being able to see ahead through the windscreen!) to have sufficient confidence in the instruments to be able to fly by instruments alone. The unassisted and untrained human mind often makes serious errors of judgement in estimating speed, height above the ground, and direction, and – convinced of its infallibility – tends to jump to the erroneous and possibly life-threatening conclusion that the instruments are not functioning correctly.

4.9  **Hot Air Ballooning**

In certain very unusual combinations of wind and other weather conditions, hot air balloons that take off around ten miles away can pass close to where I live. On both of the two occasions that this has happened, neighbours’ dogs and cats – which are not in the slightest perturbed by motor vehicles or aeroplanes – reacted with outright terror at the appearance (albeit slow-moving and silent) of such an unusual and, in their relatively primitive minds, potentially dangerous object. It is salutary to remind ourselves that, until only a few centuries ago, many human beings reacted similarly to the appearance of a comet in the night sky. But, in line with Adam Smith’s demonstration in his “History of Astronomy” that the main purpose of “science” in human affairs is to quell the mental tumult that is often caused by events which we cannot “explain”, comets ceased to be a source of consternation once Halley applied Newton’s inverse square law of gravitation to brilliant practical effect and accurately predicted the next return of what we now know as Halley’s Comet.

5.  **REAL WORLD INVESTOR BEHAVIOUR**

5.1  **First Equity Investment**

When I first started in paid employment as an actuarial student in 1967, I wanted to put money aside for long term savings but believed that investment in ordinary shares was too
risky. I entered into a regular savings scheme with a building society. Within six months or so, being aware then that life offices invested a very large proportion of their funds in ordinary shares, and having seen the UK stockmarket rise strongly, I changed to believing that equity investment was not too risky for me. Despite the early surrender penalty, I withdrew from my contractual savings scheme and invested in a unit trust. I can still picture the two enticing features of the half-page newspaper advertisement that tempted me in: the headline figure of a 101% rise in the unit price over the previous twelve months, and the unit price graph that seemed to be heading for the sky. The parallels with my changed attitude to parascending, as set out in Section 4.6, are obvious.

5.2 Inconsistencies of Active Investment

The human mind is so painfully slow at arithmetic operations of an algorithmic nature that the vast majority of “active” investors look at one share at a time and – at best – classify its expected future relative return into one of a small number of (often very unequal in size) subgroups such as “very undervalued”, “moderately undervalued”, “fairly valued”, “moderately overvalued”, and “very overvalued”. However, unlike level of difficulty assessments in skiing, rapid river canoeing, and many other sports, where there is a consistent numerical underpinning, the criteria used by “active” investors not only tend to be qualitative rather than quantitative but also tend to change markedly over time in line with the prevailing consensus market sentiment. Not surprisingly, for intensively researched large capitalisation shares there is no convincing evidence that this unstructured approach can add value, even before transaction costs.

5.3 Downward Surges

At the individual share level, the sector level, and the overall market level, an initial unexpected item of bad news often sets off a three-stage downward surge in prices. First there is an initial mark-down by market-makers to establish a new very short term equilibrium point between buyers and sellers. Second, just like the actuary who changed his mind about going hang-gliding, investors look around them at similar things that could go wrong, conclude that the situation is far more uncertain and risky, and sell shares. Third, other investors panic at the sight of falling share prices and, fearing that there may be more bad news and price falls to come, sell at any price without any considered assessment of the probably high medium term returns that they would achieve by holding on. The parallels with my two yacht racing incidents need no further elaboration. Theoreticians will say that this analysis is flawed; some investors would spot the long term value before prices had fallen too far and buy. However, my Swiss Wall skiing analogy comes into play; the same bad news as the unthinking investors reacted to alerts thinking investors not only to the likelihood that previous earnings forecasts were too high but also that the economic background is more uncertain, but not necessarily much worse on a probability - weighted expected basis; they then decide that discretion is the better part of valour and that it would be unwise to buy before prices have fallen to an apparently oversold level to compensate for the now higher future uncertainty. Rather than being independent in their actions, the unthinking investors respond by trying to sell, while the thinking investors wait for lower prices to buy. The resultant market behaviour is a downward surge in prices over three days or more which leads to a Hurst exponent well in excess of 0.5; the state of equilibrium which mathematical
economists say will be established immediately after the initial item of bad news simply does not materialise.

5.4 Upward Surges

A similar three-stage process also works in reverse from oversold, rather than overvalued, price levels. Thinking investors first spot the long term value and buy; the resulting price rises and the accompanying bullish rationalisations in the financial press alert others to the opportunities, and there is further buying of selected shares on a “value” basis. Finally, unthinking investors, fearing that they may “miss the boat” in a rising market, stampede in by buying highly liquid “blue chip” stocks with hardly a glance at the underlying fundamentals or at indicators of value such as price-earnings ratios.

5.5 Mental Impediments

Just as even professional tennis players are mentally unable to serve as well on their second serve as on their first because of the fear of a self-inflicted setback in the very near future, investors – professionals as well as private individuals – are, to a greater or lesser degree, psychologically unable to buy shares that have fallen so much as to appear grossly oversold because of the fear of further short term losses. In presenting my Clarkson (1998b, 1998c) papers at Cambridge last September, when – with hindsight – we about three - quarters of the way through the severe 1998 setback in UK equities, I observed that my strategy investment calculations as at 28th August had indicated a sector switch from Great Universal Stores and Marks & Spencer (which were in my “bottom 10”) into the six engineering shares which were in my “top 30”. On consensus estimates, these engineering shares had, by far, better forecast earnings growth, lower prospective price-earnings ratios, and better quality of earnings (in terms of no serious setback in recent years). But on 28th August the market sentiment had been as black as I had been seen for some time. The BBC2 CEEFAX city news pages, for example, were crammed full of panic-inducing financial horror stories. For the record, and for the enlightenment of theoreticians who seem to believe that “information” available to investors consists of a structured and unemotional presentation of historic fundamental data and of considered numerical estimates of future fundamental data and of expected future returns, the three BBC2 CEEFAX city news pages at 3.40 pm on Friday, 28th August, when the FTSE-100 Index was down more than 100 points, were as below.

ASIAN MARKETS POUNDED BY RUSSIAN HAMMER

Asian markets crumbled as investors raced for the hills amid fears that the Russian bears will unleash a global economic meltdown.

Tokyo ended at a 12-year low, Singapore and Malaysia at 10-year lows, Manila at a 6-year low and Taiwan stocks plumbed a 21-month low.

Amid the market tumult, two top officials of Malaysia’s central bank quit after a major policy rift with Prime Minister Mahathir Mohamed, government officials said.
MIYAZAWA CALLS FOR CALM ON MARKETS

Tokyo was slammed by a triple whammy of domestic financial woes, contagion from Russia’s growing financial crisis, and concerns over a global financial slump.

“People are not sure about when the chain reaction will stop or whether it will ever stop, and that deepened previous market worries”, said Masatoshi Sakana, a manager at Kankaku Securities.

Japanese Finance Minister Miyazawa urged calm. “The most important thing basically is not to panic. There will be buyers in the market as well as sellers and as time passes the market will settle down.”

POST-COMMUNIST EUROPE COULD SUFFER

Analysts believe the global shockwaves caused by Russia’s crisis could rock the foundations of post-Communist Europe’s economic transformation.

At first, analysts had thought eight years of reform would protect them from any long term damage, but not now.

“Many countries are either in a position where their external accounts are precarious or their economies are weak,” said a Lehman Brothers expert.

Against such a news background, just an hour or so before the Bank Holiday weekend, it is perfectly understandable that almost everyone except Warren Buffett would conclude that a switch from the eminently safe UK retail stocks into engineering companies exposed to the vicissitudes of a potentially disastrous international economic situation represented an almost suicidal increase in the level of portfolio risk. By contrast, any investor who had the mental fortitude to follow the publicly available numbers rather than their emotions (my trainee pilot comments are relevant here) would have seen, ignoring transaction costs, a profit of more than 25% within a matter of weeks.

5.6 Strategy Investment

More generally, my “investment of the third kind” utility ranking approach leads to various insights into investor behaviour. First, over the long term, the shares that come out as most attractive will outperform the broad market indices by a comfortable margin. Second, when investor sentiment in the aggregate switches from bullish to bearish, investors attach far less weight than previously to earnings projections into the now highly uncertain and risky future, with the result that those shares that might be the best performers on an “intelligent” appraisal of the future are by far the worst performers in the changed environment where investors engage in a “flight to quality”. Third, the “quality” shares that are bought in such circumstances are driven to price levels that tend to lead to significantly below average relative performance over the short and medium term. A fuller commentary on these behavioural patterns is set out in Clarkson (1998c).
5.7 Panic by Investors

As Keynes (1936) describes, human nature is such that most people cannot wait long for good investment performance, whereas, for the reasons set out above, the inevitable swings in aggregate investor sentiment from unrealistic optimism to unrealistic pessimism often involve relatively long timescales of two to three years or more. After, say, two successive years of poor performance, an institutional investor will often abandon the previous strategy, just before it comes right, as a result of eminently understandable self-preservation instincts at both the personal and corporate levels. The highly successful move to a passive index-matching approach by some institutions who had previously been highly unsuccessful in their pursuit of the traditional active approach is an illuminating and amusing corollary.

5.8 Panic by Consultants

It is not only investors who panic after a period of poor performance. Several recent studies have shown that when pension scheme investment managers are sacked by trustees on the advice of their consultants (who probably steered the trustees towards appointing these managers in the first place!), the performance under the new managers is very often significantly worse than what would have resulted had the previous managers been left in place. This is one of the most serious sources of "implementational leakages" described by Addison & Shaffer (1997) in a paper to the Centenary Convention of the Institute of Actuaries of Australia. Their description of a not untypical sad case study concludes as follows:

"The client terminated at a time when the market moved to very strongly validate the strategy previously in place. The effect of this was that the client locked in the under-performance. The client went from one strategy that had underperformed but was about to come good, to a fund manager who had performed well, but was likely to under-perform in the changed market circumstance. Further, the client would have to make up the cost of the changeover."

6. MODIFYING THE WIRING IN THE INVESTOR'S MIND

6.1 Structured Share Selection

The first part of the "artificial intelligence" approach to stockmarket success that is suggested by the analysis earlier in this paper is to build up absolute faith in the mind of the investor that an appropriately structured, and essentially time-invariant, approach to share selection significantly enhances the long term returns that can be achieved.

6.1.1 Documented Approaches

Read O'Shaughnessy (1996) twice, omitting on the first reading the many lengthy schedules of numerical results. This will build up confidence in the existence of strategies that are successful over the long term. To obtain an understanding of practical approaches developed in a UK actuarial context, read Weaver & Hall (1967), paying particular attention to the comments of practitioners in the discussion, and Clarkson (1981), skimming through the
(very lengthy) formal paper itself but again taking note of the comments from expert practitioners at both the Faculty and Institute discussions.

6.1.2 Utility Rankings

O'Shaughnessy (1996) demonstrates that the combination of two or more successful single factor strategies can produce a far more powerful compound strategy. As an example of a powerful two-factor strategy read the short term growth/utility ranking approach set out in Clarkson (1998b). This is a type (1, 1, 0, 1) approach in terms of the classification system described in Section 2.3, whereas I am confident that a type (1, 2, 0, 2) approach which adds a long term earnings growth component will be even more powerful, but this has yet to be built. However, we must learn to walk before we can run, and "in the kingdom of the blind the one-eyed man is king".

6.1.3 Long Term Superiority

As an eminent philosopher once remarked, education is what remains when we have forgotten what we learned. Similarly, the myriad of technical detail that you have to plough through in reading these documented approaches to successful share selection will tend to fade away but leave indelibly printed in your mind what I have called a "second pillar" understanding, namely that what Warren Buffett describes as "a little homework" will open the way to long term stockmarket success.

6.1.4 Short Term Danger

Unfortunately, at present and for the foreseeable future, the apparently "intelligent" share selection approach described above will not only be far from optimal, in that few investors have either the technical skills or mental fortitude to implement it, but will also be positively dangerous as regards short term performance. It matters little whether we adopt the convention of driving on the left hand side of the road or on the right hand side of the road, provided everyone else follows the same convention, but chaos and carnage will result if different drivers follow different conventions. The frightening stockmarket parallel is that, at any one time, different investors are mentally conditioned to "drive on different sides of the road".

6.2 Investor Psychology

The second part of my "artificial intelligence" approach to stockmarket success accordingly relates to the realisation that "crossed wiring" in the minds of many investors will not only prevent them from being able to implement "intelligent" strategies but will also present extreme danger to investors who endeavour to behave in an "intelligent" fashion without taking into account the actions of those who behave in a quite different manner. My "actions of others" skiing analogy is of crucial relevance here.

6.2.1 The Inevitability of Price Falls

In rapid river canoeing, it is essential to realise that occasional capsizes will occur and accordingly that you should be able to swim, perform an Eskimo Roll, and know how to act
in a capsize. Similarly, in stockmarket terms, it is essential to realise that investors’ actions tend to be driven alternately by greed and fear, giving rise to behaviour that in statistical terms corresponds to a Hurst exponent significantly in excess of 0.5, rather than by “rational behaviour” of the nature predicted by mathematical economics. Accordingly, periods of falling prices are inevitable, yet many investors react to such eventualities in more or less the same manner as dogs and cats panic at the appearance of a hot air balloon. Warren Buffett goes so far as to suggest that no-one should contemplate stockmarket investment if they are not mentally prepared to lose 50% in the short run.

6.2.2 Improving the Return

As observed in the concluding section of Clarkson & Plymen (1988), if we improve the long term return sufficiently, then the investment risk due to swings in aggregate investor sentiment will tend to look after itself. As discussed earlier, my current thinking is that a type (1, 2, 0, 2) model will be best. But just as different tennis players achieve different results from the same racquets, different users will achieve vastly different performance from the same basic design of model.

6.2.3 Understanding the Psychology

A wealth of literature on the subject is readily available but generally ignored by those who approach investment mainly from a mathematical or statistical perceptive. As a start, I recommend Chapter 12 of Keynes (1936), Hagstrom (1994), O’Shaughnessy (1996) and FitzHerbert (1998).

6.2.4 Utility History

An “instrument reading” that I believe is very useful in determining whether stockmarkets are getting into overvalued territory, with the possibility of an “unexpected” setback arising, is a Mean Absolute Deviation analysis on a fixed percentile (such as the upper quartile) of the utility measure used to rank shares in order of relative attractiveness.

6.2.5 Convexity

Suppose that we fit a parabola to the relative performances of my seven groups of ten shares using the Clarkson (1998b) model. While a further period of assessing this measure is clearly required, a very sudden move from almost no convexity to being very strongly concave downwards, as happened around July 1998 in the UK equity market, would appear to be an early warning sign of an imminent market fall.

6.2.6 Tracking the Psychology

Clearly no purely quantitative approach will ever be an infallible route to stockmarket success. As an adjunct to numerical indicators, it is essential to track investor psychology - preferably on a daily basis - through a studious reading of market reports in the quality financial press. As a classic example, the UK market report in the “Financial Times” for Friday, 9th October 1998, shortly after the equity market turned, referred to strong buying of precisely those sectors that had suffered worst in the market setback. This corresponded to a
particularly strong performance that day by many shares that had for weeks come out as exceptionally attractive on my utility ranking approach. The sports analogy is that the yacht that had been knocked flat by a sudden squall was returning through natural corrective forces to its normal upright position.

7. ARTIFICIAL DISINTELLIGENCE AND MATHEMATICAL ECONOMICS

7.1 Algorithmic Behaviour and Equilibrium

Under mathematical economics, investors should assess investment opportunities in terms of an algorithmic and unemotional “risk/return” evaluation of available information, thereby leading to a state of equilibrium through “no arbitrage” arguments. But as discussed earlier, the untrained human mind that is characteristic of most investors is woefully weak at even the most elementary algorithmic operations, and accordingly there is no justification whatsoever for assuming – as is invariably taken for granted in mathematical economics – that stockmarkets are in equilibrium, even as a crude first order approximation.

7.2 Risk, Variability and Reality

Any mean-variance, mean-semi-variance or similar approach to the estimation of risk implies statistical independence between successive time periods that would result in a Hurst exponent of 0.5. The incontrovertible fact that the Hurst exponent is significantly in excess of 0.5 for both individual share prices and equity market indices indicates that conventional risk evaluations will seriously underestimate the probability of extreme outturns.

7.3 Inappropriate Statistical Formulations

Perhaps the most potent criticism that I have come across relating to the standard statistical methodologies of mathematical economics is in Mandlebrot (1977):

“Faced with a statistical test that rejects the Brownian hypothesis that price changes are Gaussian, the economist can try one modification after another until the test is fooled. A popular fix is censorship, hypocritically labelled “rejection of statistical outliers”. One distinguishes the ordinary “small” price changes from the large price changes that defeat Alexander’s filters. The former are viewed as random and Gaussian, and treasures of ingenuity are devoted to them ..., as if anyone cared. The latter are handled separately, as ‘non stochastic’.”

In modern finance theory parlance, potentially devastating real world market movements corresponding to such “non stochastic” events are subsumed into the euphemistically named category of “jump risk”. In a recent and commendably lucid article, clearly inspired by the well publicised difficulties of some “hedge funds” last autumn, Mandelbrot (1999) sets out his expert opinion on the failings of modern portfolio theory:

“Modern portfolio theory poses a danger to those who believe in it too strongly ... Though sometimes acknowledging faults in the present body of thinking, its adherents suggest that no other premises can be handled through mathematical modelling. This
contention leads to the question of whether a rigorous quantitative description of at least some features of major financial upheavals can be developed .... Revisionists correct the questionable premises of modern portfolio theory through small fixes that lack any guiding principle and do not improve matters sufficiently.”

7.4 **Stockmarket Efficiency**

By far the most pervasive teaching of financial economics is the pseudo-scientific conjecture that stockmarkets are broadly efficient at the “semi-strong” level corresponding to “publicly available” information. The sports analogy is the utterly nonsensical proposition that all human beings, regardless of age or previous (if any!) practical experience, are equally proficient skiers as a result of identical innate physical and mental abilities and of the ready availability of suitable equipment and expert tuition. George Orwell’s “all animals are equal” metaphor floats immediately into mind. I use the “pseudo-scientific” epithet to reflect my belief that much of the efficiency literature is fatally flawed at one or more quite distinct intellectual levels - mathematical, statistical and scientific. In terms of mathematical logic, it is often assumed that stockmarkets are always in equilibrium, whereas the Hurst exponent indicates otherwise. This false assumption renders any “conclusions” meaningless. In statistical terms, many of the tests assume that relevant distributions are normal in shape, despite vast bodies of empirical evidence to the contrary. In terms of broad scientific principles, researchers have not tested practical models such as the Weaver & Hall (1967) model and my Clarkson (1981) model to see if the superior performance claimed by their designers can also be obtained by others with sufficient practical experience to be able to use the models. I therefore regard most of the efficiency literature as a tissue of statistical irrelevancies; Warren Buffett dismisses the notion of efficiency as “absolute rubbish”.

7.5 **Psychology**

I strongly suspect that the extent to which apparently intelligent investors often deliver performance that is markedly inferior to that of an appropriate benchmark is closely related to their not having an adequate grasp of investor psychology. But the axiomatic approach of mathematical economics does not even begin to recognise the importance of psychological considerations such as those discussed earlier in this paper.

7.6 **The Scientific Process**

Kuhn (1970) describes how “normal science” consists of exploring a paradigm to the limits, with refinement after refinement being made to the initial basic formulation in an attempt (often unsuccessful) to “explain” stubbornly anomalous real world behaviour. My “second pillar” question of whether the paradigm offers a sound guiding principle in the first place is – in line with my introductory quotation from Nobel Laureate Maurice Allais – anathema to “the establishment”. It took almost two thousand years for the fundamentally flawed Ptolemaic system of astronomy to be replaced by the vastly more successful heliocentric system pioneered by Copernicus and Galileo.
7.7 The Expected Utility Maxim

It is assumed in calculating expected utility that a single-valued utility function always exists and that the probabilities (or probability density functions) used are independent of the particular scenarios being investigated. However, not unlike the situation in my "first pillar" probability paradox, there is a dependence which is assumed away by the axiomatic approach to utility. The "Allais Paradox", to which my "fifth pillar" and "sixth pillar" examples in Section 1 are together equivalent, shows that the expected utility maxim cannot account for the behaviour of thinking human beings that Allais (1953) describes as "reasonable men". Allais deduces – and I agree – that the expected utility maxim cannot be used in the foundations of any formal theory of economic behaviour.

7.8 "A Much More Refined System of Psychology"

If, as in Clarkson (1997b), we delve deep into the mathematical foundations to investigate which of the Von Neumann & Morgenstern (1944) axioms of utility is flawed, we find that it is the seemingly innocuous axiom that corresponds to the A(BC) = (AB)C associativity property of ordinary arithmetic and of matrix multiplication, such as in quantum theory. Markowitz (1959) did not even discuss this seemingly trivial axiom before adopting the expected utility approach, but Von Neumann & Morgenstern (1944) very perceptively observe that this may be the crucial axiom that has to be abandoned if we are to obtain "a much more refined system of psychology than the one now available for the purposes of economics". As shown in Section 4, "risk-intelligent" behaviour as I define it involves very subtle thought processes – obviously evolved by a "survival of the most intelligent" process over thousands if not millions of years – that do not conform to the mathematically essential associativity axiom. We now have a supreme irony: to obtain a mathematically tractable framework to guide human behaviour under conditions of uncertainty and risk, mathematical economists have "turned the calendar back" countless thousands of years to a far more primitive system of mental processes than now forms part of our innate intelligence.

8. INVESTMENT QUESTIONS FOR ACTUARIES

I now return to the eight investment questions for actuaries that I listed at the end of Section 1, and give – with terse reasons in line with my introductory quotation and the guidance from my "six pillars" of actuarial wisdom – my own personal answers on the suggested scale of 1 (emphatically no) to 10 (emphatically yes).

1. Do most stockmarket investors behave "rationally" most of the time?

The principal new theme in this paper is the proposition that hitherto unrecognised psychological factors over-ride the valiant attempts of virtually all investors to act in an "intelligent" manner consistent with the "rational behaviour" axioms of mathematical economics, with – in particular – many investors reacting to falling prices in a panic-stricken manner. Accordingly I give a score at 1 here.

2. Are equity markets broadly efficient at the "semi-strong" level corresponding to publicly available information?
The first part of my “artificial intelligence” approach relates to the existence of practical share selection systems that generate significantly above average performance over the medium and long term. Again I give a score of 1 here, but I fully accept that most investors do not possess the combination of technical skills and mental fortitude that is necessary to exploit these opportunities. Indeed, Warren Buffett observes that most investors behave like gamblers who play poker all night without once looking at the cards in their hands.

3. **Is the “no arbitrage” principle a reasonable mathematical foundation for models of stock market behaviour?**

As an immediate corollary to my response to the previous questions, my score is again 1.

4. **Is stochastic calculus an appropriate mathematical tool for the investigation of stock market strategies?**

After the flawed “no arbitrage” principle, the next most crucial simplifying assumption is the continuity-generating concept of “completeness”, namely that all desired strategies can be implemented instantaneously without affecting market prices. But as I discussed earlier, that is not how the real world works. When an item of unexpected bad news appears, unthinking investors panic and try to sell at any price, while thinking investors defer buying until it is perceived that there is an adequate margin of expected profit to compensate for the higher uncertainty. The result is a period of “free fall” of the type that so devastated some hedge funds last year that there were genuine worries at the highest governmental and regulatory levels that a global financial “meltdown” might result. My score here is again 1.

5. **Is variability of return (i.e. a statistical measure such as variance or semi-variance of return) a reasonable proxy for investment risk?**

My “first pillar” example of Donald’s Paradox applies. The dependencies that are highlighted by Hurst exponent analysis indicate that conventional statistical methodologies will seriously understate the incidence of extreme variations. Again my score is 1.

6. **Is the expected utility maxim an appropriate mathematical tool for resilience testing?**

In the light of earlier discussions and comments, again my score is 1. What I describe as my “sixth pillar” of actuarial judgement will often be superior to utility “solutions” to real world financial problems.

7. **Are linear stochastic models appropriate for solvency investigations?**

While essentially linear models such as the Wilkie model are an undoubted improvement over actuarial formulations which ignore stochastic variability and deal only with expected values, I see three serious dangers here. First, the “normal science” approach results in the standard models such as described in Wilkie (1986) being regarded as carved on tablets of stone, with “Bernoulli’s second maxim” caveats that might result from a careful study of underlying research work such as Wilkie (1984) being ignored. Second, while linear models might pass conventional statistical tests, such as a “runs test” as described in Wilkie (1984)
based on the sign (but not the magnitudes) of deviations from calculated values, my "second pillar" intuition, corresponding to my "two bombs on an aeroplane" example, suggests that any such "runs test" is invalid in the light of Hurst exponent analysis. Third, the model parameter values, being based on recent experience, will lead to highly misleading projections whenever, as will often happen in line with the teachings of Hurst exponent analysis, a change in the prevailing trend is just about to occur. Yet again my score here is 1.

8. Should novice investors be left to learn from their own mistakes?

The sports analogy is that, if I see someone about to ski inadvertently over a cliff, I will do my utmost to prevent the injury or death that might result from inaction on my part. Similarly, I see it as an essential part of our "making financial sense of the future" public duty motto as actuaries to translate what we see as the "true dynamics" of financial markets into advice that will enable others to reduce as far as is humanly possible the incidence of avoidable financial loss. My score here, yet again, is 1.

I now invite you to reassess your answers to these questions in the light of the analysis set out in the body of the paper. I imagine that your "second thoughts" aggregate score will be lower than your original one, and – if so – I shall have succeeded in modifying to some extent the "crossed wiring" in our minds that makes stockmarket success so difficult to achieve, regardless of our innate mathematical and other intellectual abilities. My minimum possible score of 8 over these questions is clearly extreme, but reflects not only my "six pillars" of actuarial wisdom but also the extensive "due diligence" and "verification" exercises that I have carried out in recent years in an earnest endeavour to determine what, if anything, a mathematical economics approach can contribute to actuarial science.
EPILOGUE

"Will you join in our crusade?
Who will be strong and stand with me?
Somewhere beyond the barricade
Is there a world you long to see?

Victor Hugo : Les Misérables (musical version)

The most general conclusions that I draw about real world stockmarket behaviour can be neatly portrayed in terms of the “cocked hat” pattern that results when compass bearings of three quite separate landmarks in very different directions from the observer are plotted on a chart to determine position. Taking first of all a “Hurst-Buffett-Mandelbrot” vector corresponding to their highly practical understanding of real world behaviour, and then adding a “strategy investment” vector and a “risky sports” vector, we find that these three vectors delineate a very small “cocked hat” triangle. A “modern finance theory” vector, on the other hand, would not pass anywhere near the position identified by the other three vectors. In navigation, a very small “cocked hat” triangle suggests that an accurate position has been found. In the investment context of the present paper, the parallels are that the “true dynamics” of stockmarket behaviour have been identified with more accuracy than has hitherto been possible and, furthermore, that these real world dynamics differ irreconcilably from those implied by the teachings of modern finance theory.

Down through the centuries, many of the classic works of literature and opera have been based on masterly insights into the comic (happy ending) or tragic (unhappy ending) fallibility of human behaviour. For example, in “Tam O’Shanter” Robert Burns describes very vividly and very amusingly how his hero falls foul of an overly complacent, but not untypical, approach to some fairly obvious natural hazards:

“When we sit boosing at the nappy
Getting fou and unco happy
We think not of the long Scots miles
The mosses, waters, slaps and stiles
That lie between us and our hame ....”

Similarly in Shakespeare’s “Macbeth” the hero becomes so recklessly overconfident after the successful outturn of the first of the witches’ three predictions (“you shall be king hereafter”) that he wrongly infers invincibility from the remaining two predictions relating to “till Birnam Wood shall come to Dunsinane” and “no man of woman borne”. Also, Mozart’s “Magic Flute” depicts the human struggle involved in the “quest for enlightenment” and in particular in the stark choice between “the guidance of wise men” (as personified by Sarastro) and “magic and superstition” (as personified by the Queen of the Night). The investor behaviour parallels are obvious, and a further bearing based on these masterpieces of the literary arts would also pass through the real world behaviour locus already identified in my “cocked hat” analogy.

A crucial theme of this paper is that the axiomatic “rational behaviour” approach of mathematical economics takes us backwards, not forwards, in our quest for stockmarket
success, whether as investors, trustees, directors or professional advisers. My suggested way forward is to abandon the intellectually appealing, but totally unrealistic, “theory of games” approach of universal rational behaviour and to develop instead the “theory of sports” approach which accords with the actual workings of the human mind when exposed to significant levels of risk. My “second pillar” guiding principle of “risk-intelligent” behaviour, which explicitly recognises the likelihood of “risk-stupid” behaviour on the part of some others, is deduced from real world experience relating mainly to rapid river canoeing, skiing, and ski mountaineering, but I am the first to admit that I have only “scratched the surface” in terms of exploring this new way forward. In risky sports, “risk-intelligent” behaviour is clearly the rule rather than the exception, but in stockmarket behaviour it is the exception rather than the rule, as otherwise the crucial Hurst exponent would be very close to 0.5 rather than, as can easily be seen to be the case, significantly higher.

As a mathematician myself, I can understand how some of the arguments set out in this paper may appear unconvincing, or even totally fallacious, to other mathematicians who have not been exposed to all three of the areas of harsh real world practicability that I discuss – passing the UK actuarial examinations, first hand experience of stockmarket investment both for your own and for other people’s money, and participation in a wide range of potentially dangerous sports. I refer such mathematicians to the Epilogue in Tolstoy’s “War and Peace”, which not only explains how long (around two millennial) it took for the “true dynamics” of planetary motion to be recognised after the general acceptance of the mathematically alluring (but totally wrong) geocentric approach, but also how successfully the mathematical foundations were rebuilt once it was realised that a major paradigm shift was required. But of even more importance is the fact that the quest for a better scientific theory of astronomy was driven not by altruistic notions of “pure science” but by the eminently practical and humanitarian objective of finding a better framework for navigation on the high seas that would reduce the unacceptably high rate of maritime fatalities. Similarly, the main driver of a better understanding of the “true dynamics” of stockmarket behaviour should be the imperative of reducing – for professional and private individual investors alike – the incidence of avoidable financial loss.
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