Introduction

What is longevity risk?

→ The risk of underestimating mortality improvements
  - Trend risk
  - Systematic and non-hedgeable
Mortality Trend Model Requirements

Goal: Mortality model for solvency purposes with the following properties

- Simultaneous modeling of mortality and longevity risk
- Full age range (20 to 105)
- Consideration of several populations at the same time, e.g. males and females
- Quantification of risk over limited time horizons
  - 1 year for Solvency II or several years for strategic planning
  - Risk in realized mortality evolution and changes in long-term assumptions
  - Stochastic mortality trend
- Plausible tail scenarios
- Conservative calibration
- Epidemiological and demographic input

Model Specification

We model the logit of mortality rates

\[
\text{logit}(q_{x,t}) = \alpha_t + \kappa_t^{(1)} + \kappa_t^{(2)}(x - x_{\text{center}}) + \kappa_t^{(3)}(x_{\text{young}} - x)^+ + \kappa_t^{(4)}(x - x_{\text{old}})^+ + \gamma_{t-x}
\]

- \(x_{\text{center}} = 60\), \(x_{\text{young}} = 55\), \(x_{\text{old}} = 85\)
- \(\kappa_t^{(1)}\) describes the general level of mortality
- \(\kappa_t^{(2)}\) is the slope of the mortality curve
- \(\kappa_t^{(3)}\) and \(\kappa_t^{(4)}\) describe additional effects in young and old age mortality, respectively
  - \(\kappa_t^{(3)}\) can be omitted if older ages are considered only
Model Estimation

Model estimation via Generalized Linear Model theory

- Logit is canonical link function for Binomial distribution
- Number of deaths is binomially distributed given initial exposures

Multi-population setting

Important note: Even if one is only interested in a single population considering several populations is worthwhile

- Trend uncertainty can be significantly reduced
  - There is clearly a common trend
  - A model for several populations must account for that
  - Increment correlations cannot generate such parallel evolutions
  - We apply cointegration and an error correction model for deviations from the common trend
Model Simulation

Projection of $\kappa_{t, \text{total}}^{(1)}$ for the total population

- Linear trends with breaks in the historical data
  - Commonly used random walk with drift does not allow for trend breaks
  - Trend breaks and thus changes of the best estimate trend are crucial when working in finite time horizons
- New idea: Each year, fit regression line to historical data and forecast future best estimate mortality as $\kappa_{t, \text{total}}^{(1)} = l_t(t + 1) + \epsilon_{t+1}^{(1)}(\sigma^{(1)} + \bar{\sigma}^{(1)})$
  - $\bar{\sigma}^{(1)}$ is a volatility add-on, $\sigma^{(1)}$ is current (best estimate) volatility
  - This trend modeling approach reflects actuarial practice of updating a model (here: the long-term trend) when new data becomes available
  - To stress most recent mortality experience, the regression line is fitted with weights $w_t = \left(1 + \frac{h}{N} \right)^{-t}$

Model Simulation (ctd.)

- Weighting parameter $h$ has massive impact
- Plausible one-year and run-off scenarios
- Each run-off scenario is a combination of one-year scenarios
- Disentangling of one-year noise and long-term trend uncertainty
- Possibly more plausible confidence bounds than for a random walk with drift
Model Simulation (ctd.)

Calibration of weighting parameter $h$
- Adequate parameter calibration is difficult to find and also a question of desired conservatism
- Possible approaches for parameter fitting:
  - Fitting to (most severe) events/evolutions in the past
    - Example: Rapid increase in life expectancies of Dutch males in the 1970’s
  - Expert opinion (see mortality/longevity threat scenarios later)
  - Comparison with confidence bounds in other models (questionable!)

Model Simulation (ctd.)

Projection of $\kappa_{t;p}^{(1)}$ for individual populations
- For each individual population we project as
  - $\kappa_{t;p}^{(1)} = \kappa_{t;\text{total}}^{(1)} + a_p + b_p (\kappa_{t;\text{total}}^{(1)} - \kappa_{t-1;\text{total}}^{(1)}) + \epsilon_{t;p}$
  - $b_p$ denotes the „mean reversion speed“ (absolute value should be smaller than 1)
  - $a_p/(1-b_p)$ is the long-term difference between the total population and population $p$
- Different approaches of calibrating the long-term difference
  - Fitting of an AR(1) process to historical differences
  - Weighted/unweighted average of historical differences
Model Simulation for a Single Population (ctd.)

**Projection of $\kappa_i^{(2)}$, $\kappa_i^{(3)}$, and $\kappa_i^{(4)}$**

- No substantial trend obvious in the historical data
- Forecast as correlated 3-dimensional random walk
- No substantial correlation with $\kappa_i^{(1)}$
- Volatility add-on $\tilde{\sigma}^{(2)}$ for $\kappa_i^{(2)}$ may be appropriate to limit diversification between mortality and longevity risk

- Between populations, increments of $\kappa_i^{(1)}$ and $\kappa_i^{(2)}$ are correlated
  - Historical correlations should be checked carefully and possibly adjusted

Model Simulation for a Single Population (ctd.)

**Projection of $\gamma_{i-x}$**

- Cohort parameters should stay around zero
- Forecast as imposed stationary AR(1) process
- Cohort parameters are rather irrelevant for simulations over short time horizons
Epidemiological and Demographic Expert Opinion

Mortality/Longevity Threat Scenarios

- Mortality data is often very sparse, in particular with respect to tail scenarios
- Thus, stochastic models should be enriched by expert opinion
- Possible derivations of mortality/longevity threat scenarios:
  - Different shocks to mortality projections
  - Likely effects of finding of a cure for certain illnesses
  - Scenarios from cause of death models
  - Scenarios the stochastic model cannot generate due to structural limitations, e.g. diverging mortality trends
- Application of threat scenarios:
  - Calibration/adjustment of model parameters
  - Inclusion in set of model outcomes

Summary

A mortality trend model with several appealing properties

- Large variability in simulation outcomes due to 5 stochastic drivers
- Clear interpretation of the model parameters
- Multi-population setting
  - Coherent mortality scenarios
  - Realistic assessment of diversification and accumulation effects
- Stochastic mortality trend
  - Risk can be quantified over finite time horizons
  - Disentangling of short-term noise and long-term trend uncertainty
  - Plausible outcomes in one-year view and run-off view
  - Trend process could be applied in other models as well
- Inclusion of expert opinion via threat scenarios
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