The use of Econometric Time Series Modelling Techniques in ERM

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Topics

- Introduction
- Spurious Relationship
- Stochastic Trends
- ARIMA Modelling
- Case Studies
- Conclusions
- Q&A
Introduction
Motivation

- Empirical analyses of many financial and business time series data sets reveals autoregressive nature of dependency structures over time e.g.
  - Annual RPI, Annual NAE, Annual FTSE All Share Return etc (see later)
  - Underwriting cycle in non-life insurance

- Fitting distributions to many years, months or days worth of data effectively loses any potentially valuable information that might be in such patterns over time

- ICA – Conditional Stress Tests
  - Equities – After a large stock market fall ~ 30 - 50%. Is an ICA Equity Stress Test of a further 40% price fall realistic?
  - Credit Spreads – 2008 saw a large widening in credit spreads. Should an existing ICA Credit Spread Stress Test credit spread movement be reduced?
Introduction

Objectives

- Two different methodologies:
  - Multivariate Methods – These methods seek relationships between the target and explanatory variable using linear or multiple regression techniques
  - Univariate Methods – These methods use only the time series of the target variable and exploit the non-independence of successive observations

- This presentation investigates the use of Univariate Methods only

- The following topics are outside the scope of this presentation:
  - Multivariate modelling or partial Univariate / Multivariate models
  - ARCH / GARCH modelling
  - Back-testing
Spurious Relationship
Two independent random variables $X$ and $Y$

- Consider two independent random variables $X_t$ and $Y_t$
  
  $$X_t = X_{t-1} + \varepsilon_t$$
  $$Y_t = Y_{t-1} + \delta_t$$

  $\varepsilon_t$ and $\delta_t$ are $N(0,1)$ distributed
  
  $X_0 = Y_0 = 5$

- Generate a random sample of 100 values for $X_t$ and $Y_t$ for $t = 1$ to $100$

- Using this output the linear correlation and $R^2$ have been calculated

- $X$ and $Y$ are not related and yet it is common, in repeated runs, to observe very high correlations far in excess of those expected from sampling error in the $N(0,1)$ values
Spurious Relationship
Scatter Diagram – Linear Regression

Linear Regression Y vs X

\[ y = 0.7705x - 1.0664 \]

\[ R^2 = 0.7593 \]

Linear Correlation = 87.1% ; DW = 0.283
Spurious Relationship
Time Series Diagram
Spurious Relationship
Residuals Diagram

Significant autocorrelation in residuals
Spurious Relationship
Residual Assumptions

- Actual$_t$ – Fitted$_t$ = Residual ε$_t$

- Quality of parameter estimates and validity of significance tests rely upon the residuals ε$_t$ ~ N(0,σ)

- Residuals must be
  - Normally distributed
  - Independent (no autocorrelation)
  - Same variance (no heteroscedasticity)

- Intuitively residuals should be simple randomness that remain after the deterministic part of the variation in a target variable has been modelled
  - Any systematic component in the error terms should really be in the model
  - If each residual is related to its predecessor they are described as autocorrelated
Spurious Relationship
Trending Variables

- The stochastic trends in $X_t$ and $Y_t$ are unrelated so linear regression cannot explain the variation of one with the other.
  - The residuals contain both stochastic trends – hence autocorrelation

- Establishing existence of trend is important for univariate modelling. Trend must first of all be removed. There are two types of trend:
  - Deterministic: e.g. $y_t = a + bt$
  - Stochastic: e.g. random walk $y_t = y_{t-1} + \epsilon_t$

- Most trending series in economics and business are not deterministic but are stochastic i.e. they exhibit random walk type behaviour
  - The identification of stochastic trend is a test for stationarity
  - A stochastic trend is removed by differencing e.g. converting an RPI value at month $t$ to an annual RPI return at month $t$ is in effect differencing the variable.
Stochastic Trends

Autocorrelation

- Let the variable $y$ at time $t = y_t$ and lagged variable $y$ at time $t-k = y_{t-k}$
  - 1st order autocorrelation $r_1 = \text{corr}(y_t, y_{t-1})$
  - 2nd order autocorrelation $r_2 = \text{corr}(y_t, y_{t-2})$
  - $k$th order autocorrelation $r_k = \text{corr}(y_t, y_{t-k})$

- The Autocorrelation Function ("ACF") measures the correlation between 2 variables $y_t$ and $y_{t-k}$.

- The Partial Autocorrelation Function ("PACF") measures the additional effect of $y_{t-k}$ on $y_t$, once effects of $y_{t-1}, y_{t-2}, y_{t-(k-1)}$ have been accounted for.

- Autocorrelation Plot (Correlogram)
  - This is very useful for analysing time series data and determining the most appropriate time series model.
  - The correlogram displays 95% bounds at each lag that enable quick tests of whether each value is significantly different from zero.
Stochastic Trends
Uses for Autocorrelation

- $y_t$ Random
  - All autocorrelations are small

- $y_t$ Stationary
  - Autocorrelations rapidly decrease as lag increases

- $y_t$ Trending
  - Many large autocorrelations

- Checking residuals are simple randomness.
  - It can be impossible to eliminate all autocorrelations from residuals

- ARIMA Modelling (see later)
Stochastic Trends
Some Useful Time Series

- $y_t = \varepsilon_t$ Purely random process (‘white noise’)
  - $\varepsilon_t$ has the same mean and variance and no autocorrelation

- $y_t$ follows an autoregressive process if it depends linearly on past observations of $y_t$
  - $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \ldots + a_p y_{t-p} + \varepsilon_t$
  - $\varepsilon_t$ is white noise as above
  - Simplest case is autoregression of order one $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$

- Let $y_t = \phi y_{t-1} + \varepsilon_t$
  - If $\text{Mod}(\phi) > 1$ then $y_t$ is said to be non-stationary – these are easy to spot
  - If $\text{Mod}(\phi) < 1$ then $y_t$ is said to be stationary (mean reverting) – the forecast function converges to the mean
  - If $\text{Mod}(\phi) = 1$ then $y_t$ is non-stationary – it meanders stochastically and is known as a random walk
Stochastic Trends
Some Useful Time Series – $y_t = 0.9y_{t-1} + \varepsilon_t$

Example of a mean reverting trend
Stochastic Trends
Some Useful Time Series – $y_t = 1.03y_{t-1} + \varepsilon_t$

Example of a non-stationary trend
ARIMA Modelling
Stationarity

- Time Series Modelling requires knowledge of the mean, variance and autocorrelations

- A series $y_t$ is said to be stationary if it has constant mean, constant variance and constant autocorrelations at each lag

- If a series is stationary, modelling can proceed by estimating the mean, variance and autocorrelations from significantly long time averages of the series

- A stationary series is not necessarily completely random as it can have autocorrelation

- The most fundamental property is stationarity in the mean
ARIMA Modelling
ARIMA \((p,d,q)\)

- Box-Jenkins is a univariate forecasting approach
  - It involves the careful examination of time series in order to identify the underlying data-generating process
  - The choice of best model can be systematically made using this approach

- It is useful to restrict the search for models to the class of **AutoRegressive Integrated Moving Average Models** – ARIMA\((p,d,q)\)

- An ARMA\((p,q)\) model for variable \(y_t\) is a combination of an autoregressive process of order \(p\), AR\((p)\) and a moving average process of order \(q\), MA\((q)\) where:

  \[
  AR(p), \text{ARMA}(p,0) \text{ process } y_t = a_1y_{t-1} + a_2y_{t-2} + a_3y_{t-3} \ldots + a_p y_{t-p} + \varepsilon_t
  \]

  \[
  MA(q), \text{ARMA}(0,q) \text{ process } y_t = b_1e_{t-1} + b_2e_{t-2} + b_3e_{t-3} \ldots + b_q e_{t-q} + \varepsilon_t
  \]
ARIMA Modelling
ARIMA \((p,d,q)\)

- An ARIMA\((p,d,q)\) process:
  \[ y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \ldots + a_p y_{t-p} + b_1 e_{t-1} + b_2 e_{t-2} + b_3 e_{t-3} + \ldots + b_q e_{t-q} + \varepsilon_t \]
  - If a variable must be differenced \(d\) times in order to achieve stationarity it is said to be integrated or order \(d\).
  - \(d = 1\) would mean that the variable now being modelled = \(\Delta y_t = y_t - y_{t-1}\)

- An AR model of sufficiently high order can usually be found to model any business series
  - If a large number of parameters are required for a good fit, forecasts can be poor. This motivates working with a broader class of models
  - Since the amount of data is limited it is preferable to fit a model involving as few a parameters as possible
  - This is known as the “Principle of Parsimony”.

- Experience suggests that an ARMA\((p,q)\) model may achieve as good a fit as an AR\((p')\) model but with fewer parameters i.e. \(p+q < p'\)
ARIMA Modelling
Box-Jenkins Methodology

- Differencing a time series to achieve Stationarity

- Identification of a model to be tentatively used
  - Inspection of the Autocorrelation function ("ACF") and
  - Partial autocorrelation function ("PACF") at different lags

- Estimating the parameters of the model
  - Maximum Likelihood, Least Squares etc.
  - This amounts to the minimisation of a complicated non-linear function of parameters that involves iterative numerical procedures

- Diagnostic Evaluation – Is the model adequate
  - t-statistics (and p-values); Durbin-Watson ("DW")
  - Residuals; Ljung-Box Q-statistic; AIC, SIC, Adj. $R^2$ etc.
ARIMA Modelling
Comparing the fit of different models

- Adjusted R² (“Adj. R²”)
  \[ \text{Adj. } R^2 = \frac{1}{(n-k-1)} \sum_{i=1}^{n} e_i^2 / \frac{1}{(n-1)} \sum_{i=1}^{n} (y_i - E(y))^2 \]

- Akaike Information Criterion (“AIC”)
  \[ \text{AIC} = 1 + \ln(2\pi) + \ln(\text{SSR}/n) + 2k / n \]

- Schwartz Bayesian Criterion (“SBC”)
  \[ \text{SBC} = 1 + \ln(2\pi) + \ln(\text{SSR}/n) + k \ln(n) / n \]

Sum of Squared Residuals (“SSR”)
\[ \text{SSR} = \sum_{i=1}^{n} e_i^2 \]

n = number of observations; k = number of explanatory variables
ARIMA Modelling
Durbin-Watson (“DW”) Statistic

- The DW Statistic evaluates autocorrelation for residuals placed in the same order as the data observations

- \( DW = \sum_{i=2}^{n}(e_i - e_{i-1})^2 / \sum_{i=1}^{n}e_i^2 \)

- \( DW \sim 2(1-r) \) where \( r \) = autocorrelation
  - \( DW = 2 \) – no autocorrelation
  - \( DW > 2 \) – negative autocorrelation
  - \( DW < 2 \) – positive autocorrelation

- The DW statistic is used instead of \( r \) because strict tests exist to examine whether DW is significantly different from 2
ARIMA Modelling
Autocorrelation diagnostic evaluation

- Residuals should be white noise
- The ACF of residuals should be investigated
- Can test for autocorrelation in residuals for several lags together
- Under null hypothesis of no autocorrelation in the first $m$ lags, the Ljung-Box Q-statistic has a chi-squared distribution with d.f. = $(m-p-q)$

$$Q(m) = n(n+2) \sum_{i=1} \frac{r_i^2}{(n-i)} \sim \chi^2_{m-p-q}$$

where $r_i = \text{corr}(e_t, e_{t-i})$
Case Studies
RPI Case Study – Data

- **Data**
  - Monthly data has been used
  - $\text{RPI}_{\text{Index}}(t)$ – RPI at the end of each month for the period Jan 1970 to Dec 2008 as provided by the Office of National Statistics (“ONS”).
  - Constructed an historical time series of a month rolling value of RPI(t) at month t, where:
    
    $$\text{RPI}(t) = \text{Annual RPI Change} = \frac{\text{RPI}_{\text{Index}}(t)}{\text{RPI}_{\text{Index}}(t-12)} - 1$$

- **ARIMA(2,[12]) Model Fit**
  - Monthly data Jan 1987 to Dec 2008
  - Box-Jenkins Diagnostic Evaluation tests OK
  - Large residuals in 2008
  - Simulation of 5,000 path-dependent scenarios of length 120 months

$$\begin{align*}
\text{RPI}(t) &= 0.02187 + Y(t) \\
Y(t) &= 1.37756 \ Y(t-1) - 0.38514 \ Y(t-2) - 0.7521 \ e(t-12) + e(t) \\
e(t) &\sim N(0.00000,0.00296)
\end{align*}$$
Case Studies
RPI Case Study – Annual RPI Data (1/70 to 12/08)

RPI with 5 year Moving Average

Value
0.0000 0.0500 0.1000 0.1500 0.2000 0.2500 0.3000

Year
Case Studies
RPI Case Study – Actual vs Fitted (Last 10 years shown)
Case Studies
RPI Case Study – Residuals (Last 10 years shown)
Case Studies
RPI Case Study – Residuals Distribution (All years)

Residuals Distribution (All years)

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<tr>
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<td>Sample No.</td>
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<td>Mean</td>
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<td>Std Dev</td>
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<td>Skewness</td>
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<td>Kurtosis</td>
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Case Studies

RPI Case Study – Model Fit and Future Projections

### RPI

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<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>Probability</th>
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<td>Y(t-1)</td>
<td>1.37756</td>
<td>22.084</td>
<td>0.00%</td>
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<td>Y(t-2)</td>
<td>-0.38514</td>
<td>-6.204</td>
<td>0.00%</td>
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<tr>
<td>e(t-12)</td>
<td>-0.75210</td>
<td>0.041</td>
<td>0.00%</td>
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- **Adj R²**: 97.9%
- **Durbin Watson**: 2.0022
- **SSR**: 0.0023
- **AIC**: -8.7805
- **SC**: -8.7261

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<th>12 / 09</th>
<th>12 / 10</th>
<th>12 / 11</th>
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<tr>
<td>Forecast</td>
<td>1.39%</td>
<td>2.29%</td>
<td>2.27%</td>
<td>2.26%</td>
<td>2.25%</td>
<td>2.24%</td>
<td>2.23%</td>
<td>2.23%</td>
<td>2.22%</td>
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<td>2.23%</td>
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<td>Standard Deviation</td>
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<td>1.57%</td>
<td>1.55%</td>
<td>1.53%</td>
<td>1.54%</td>
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<td>1.54%</td>
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<tr>
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<td>-3.94%</td>
<td>-2.82%</td>
<td>-2.81%</td>
<td>-3.72%</td>
<td>-3.20%</td>
<td>-3.62%</td>
<td>-3.48%</td>
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<td>Maximum</td>
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<td>7.62%</td>
<td>7.65%</td>
<td>7.70%</td>
<td>7.62%</td>
<td>7.38%</td>
<td>7.84%</td>
<td>8.08%</td>
<td>8.49%</td>
<td>7.99%</td>
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- **Percentile**
  - 0.5%: [-2.50%, -1.78%, -1.87%, -1.58%, -1.68%, -1.70%, -1.64%, -1.86%, -1.70%, -1.78%]
  - 1.0%: [-1.99%, -1.41%, -1.42%, -1.26%, -1.28%, -1.30%, -1.36%, -1.32%, -1.35%, -1.42%]
  - 5.0%: [-1.04%, -0.29%, -0.36%, -0.24%, -0.30%, -0.28%, -0.36%, -0.31%, -0.29%, -0.34%]
  - 25.0%: [0.39%, 1.23%, 1.23%, 1.26%, 1.19%, 1.17%, 1.21%, 1.22%, 1.22%, 1.20%]
  - 50.0%: [1.36%, 2.31%, 2.31%, 2.26%, 2.23%, 2.31%, 2.24%, 2.26%, 2.24%, 2.22%]
  - 75.0%: [2.39%, 3.32%, 3.30%, 3.29%, 3.25%, 3.34%, 3.28%, 3.30%, 3.29%, 3.23%]
  - 95.0%: [3.87%, 4.89%, 4.76%, 4.80%, 4.79%, 4.82%, 4.82%, 4.82%, 4.84%, 4.76%]
  - 99.0%: [4.88%, 5.91%, 5.86%, 5.85%, 5.77%, 5.94%, 5.87%, 5.80%, 5.92%, 5.72%]
  - 99.5%: [5.25%, 6.34%, 6.22%, 6.30%, 6.06%, 6.28%, 6.22%, 6.18%, 6.18%, 6.37%, 5.95%]
Case Studies
RPI Case Study – Future Projections

Actual and Forecast RPI - 2004 to 2012

Year
Case Studies
RPI Case Study – Four Random Scenarios (Press F9)
Case Studies
FTSE All Share Case Study – Data

- Data
  - Monthly data has been used
  - $\text{FTSEASTR}(t)$ – FTSE All Share Total Return Index at the end of each month for the period Jan 1987 to Dec 2008 as provided by Bloomberg
  - Constructed an historical time series of a month rolling value of $\text{FTSEAS}(t)$ at month $t$, where:
    $$\text{FTSEAS}(t) = \frac{\text{FTSEAS Annual Return}}{\text{FTSEASTR}(t)} / \text{FTSEASTR}(t-12) - 1$$

- ARIMA(1,[12]) Model Fit
  - Monthly data Jan 1987 to Dec 2008
  - Box-Jenkins Diagnostic Evaluation tests OK
  - Relatively largish residuals but still random
  - Simulation of 5,000 path-dependent scenarios of length 120 months

$$\begin{align*}
\text{FTSEAS}(t) &= 0.07479 + Y(t) \\
Y(t) &= 0.97975 Y(t-1) - 0.93485 e(t-12) + e(t) \\
e(t) &\sim N(0.00000,0.05191)
\end{align*}$$
Case Studies
FTSE All Share Case Study – Annual FTSEAS Data (1/87 to 12/08)
Case Studies
FTSE All Share Case Study – Actual vs Fitted (Last 10 years shown)
Case Studies
FTSE All Share Case Study – Residuals (Last 10 years shown)
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FTSE All Share Case Study – Residuals Distribution (All years)

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<td>Sample No.</td>
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Case Studies

FTSE All Share Case Study – Model Fit and Future Projections

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<tr>
<td>Y(t-1)</td>
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<td>e (t-12)</td>
<td>-0.93485</td>
<td>-78.133</td>
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| Adj R²   | 90.1%       |
| Durbin Watson | 1.8982    |
| SSR      | 0.7080      |
| AIC      | -3.0567     |
| SC       | -3.0160     |

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<td>9.08%</td>
<td>8.73%</td>
<td>8.46%</td>
<td>8.25%</td>
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<td>8.47%</td>
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<td>63.85%</td>
<td>74.63%</td>
<td>70.24%</td>
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<td>0.5%</td>
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<td>-32.75%</td>
<td>-34.93%</td>
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Case Studies
FTSE All Share Case Study – Future Projections

Actual and Forecast FTSEAS - 2004 to 2012

- Forecast
- Expected
- 1%
- 25%
- 75%
- 99%

Year
Case Studies
FTSE All Share Case Study – Four Random Scenarios (Press F9)
Case Studies
Underwriting (“UW”) Cycle Case Study – Risk Drivers *

- Target variable $y_t$
  - The concern here is price. If a company cannot compete at the prevailing price then it will lose money or business, yet price is multidimensional
  - Most analyses focus on some form of profitability measure such as the loss ratio or combined ratio with possible adjustments for the time value of money

- There are many potential explanatory variables:
  - Prior period values of profitability and its components
  - Other internal financial variables such as reserves, investment income, catastrophe losses, total capital and reinsurance
  - Regulatory / ratings variables – especially upgrades and downgrades
  - Reinsurance section financials
  - Economic variables such as inflation, unemployment and GNP
  - Financial market variables such as interest rates and stock market returns

* Enterprise Risk Analysis for Property & Liability Insurance Companies”; (2007); Guy Carpenter
Case Studies
UW Cycle Case Study – Data

- Data
  - Annual data has been used
  - Annual Underwriting Profit as % of Net Written Premium for the FSA Motor insurance class grouping at an overall UK industry level.
  - [Data by FSA insurance class grouping was provided to me. I have not been able to verify independently the data. The analysis therefore is more for illustration purposes only]

- ARIMA(2,[3]) Model Fit
  - Annual data 1987 to Dec 2005
  - Box-Jenkins Diagnostic Evaluation tests OK
  - Not a large volume of data
  - Residuals OK but do not appear as random, more a data volume issue

\[
\text{Motor}(t) = -0.09598 + Y(t)
\]
\[
Y(t) = 1.37739 Y(t-1) - 0.81563 Y(t-2) - 0.98131 e(t-3) + e(t)
\]
\[
e(t) \sim N(-0.00370, 0.02046)
\]
Case Studies
UW Cycle Case Study – Actual vs Fitted (All years)
Case Studies
UW Cycle Case Study – Residuals (All years)
**Case Studies**

**UW Cycle Case Study – Model Fit**

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Adj R²: 92.2%
Durbin Watson: 1.8374
SSR: 0.0228
AIC: -4.5397
SC: -4.3409
Conclusions

- Time Series modelling techniques can provide an informative insight
  - It is helpful if target variables are functions of explanatory variables or prior values of itself that have economic or business rationale
  - Avoid over-parameterised models – in-sample vs out-of-sample testing

- A visual inspection of the data is key to any analysis

- Models fits need to be supported by rigorous statistical diagnostics:
  - It is far too easy to determine optimal models and parameters that fail basic statistical tests such as those for t-statistics and autocorrelation in residuals
  - If the Model fails these tests one needs to try a different model

- Test sensitivity of the model parameters and forecasts to different start and end periods
Q&A
Questions ?