The Actuarial Profession
making financial sense of the future

Unlocking the secrets of the swaptions market

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Agenda

- Types of swaptions
- Case studies
- Market participants
- Practical consideratons
- Volatility smiles
- Real world and market implied probabilities
- Future development of market
- Questions

Types of swaption

- Fisher equation tells us the theoretical relationship that connects the rate, inflation and real rate markets
- (1+nominal rate)=(1+ inflation rate) x (1+real rate)
- Nominal rate ~ inflation rate + real rate

	Underlying	Payoff
Interest rate option (Swaption)	Interest rate swap Zero coupon or Par	Payer: max[0, PV (floating LIBOR leg) – PV(fixed leg at strike K)] Receiver: max[0, PV (fixed leg at strike K) – PV(floating LIBOR leg)]
Inflation option	(RPI) Inflation swap Spot or forward starting inflation base	Payer: max[0, PV (RPI_{n+t} / RPI_t) – PV(fixed leg at strike K)] Receiver: max[0, PV (fixed leg at strike K) – PV (RPI_{n+t} / RPI_t)]
Real rate option	Real rate swap Spot or forward starting inflation base Underlying can be a zero coupon swap or a linker style profile i.e. with coupons	Payer: max[0, PV (floating LIBOR leg) – PV ((1+K)^n x RPI_{n+t} / $RPIt_t$)] Receiver: max[0, PV ((1+K)^n x RPI_{n+t} / RPI_t) – PV (floating LIBOR leg)] Spot inflation base (2-month lagged from trade date of swaption) is a bullish view on inflation during the expiry period if you are long the receiver and a bearish view if you are short the payer Forward inflation base (2-month lagged from the expiry date) is effectively a bearish view on inflation if you are long the receiver and a bullish view if you are short the payer

Typical strategies using swaptions

- An end-user with fixed and real (RPI-linked) risk exposures (liabilities, debt, market-making) will typically consider the following option strategies
- Terminology tip: payer and receiver refers to the position of the option buyer with respect to the fixed or real leg.
  - the buyer of a payer (interest rate) swaption has an option to pay a fixed rate (the strike) and receive a floating rate LIBOR
  - the buyer of an inflation receiver has an option to receive a fixed rate and pay RPI
  - the buyer of a real rate receiver has an option to receive the real rate (the strike) and pay a floating rate LIBOR

	Option strategy
Monetise triggers	Sell interest rate or real rate payer Sell inflation receiver
Tail-risk hedging	Buy interest rate or real rate receiver Buy inflation payer
Risk management	Buy and sell payers and receivers

Monetising inflation-hedging triggers

Who was the end-user?	UK pension scheme
	Client wished to monetise a trigger to hedge (RPI) inflation at 3.2% by selling away the opportunity to benefit from a fall in RPI inflation below 3.2%.
How did they do it?	Sold an (RPI) inflation receiver swaption. - Underlying was a zero coupon (RPI) inflation swap - Strike rate was ATMF-30bps (Forward starting RPI base) - 2y5y/10y/30y/50y - large (underlying swap PV01) - Swap settled, collateralised with third party valuations
What was the outcome?	Unexpired

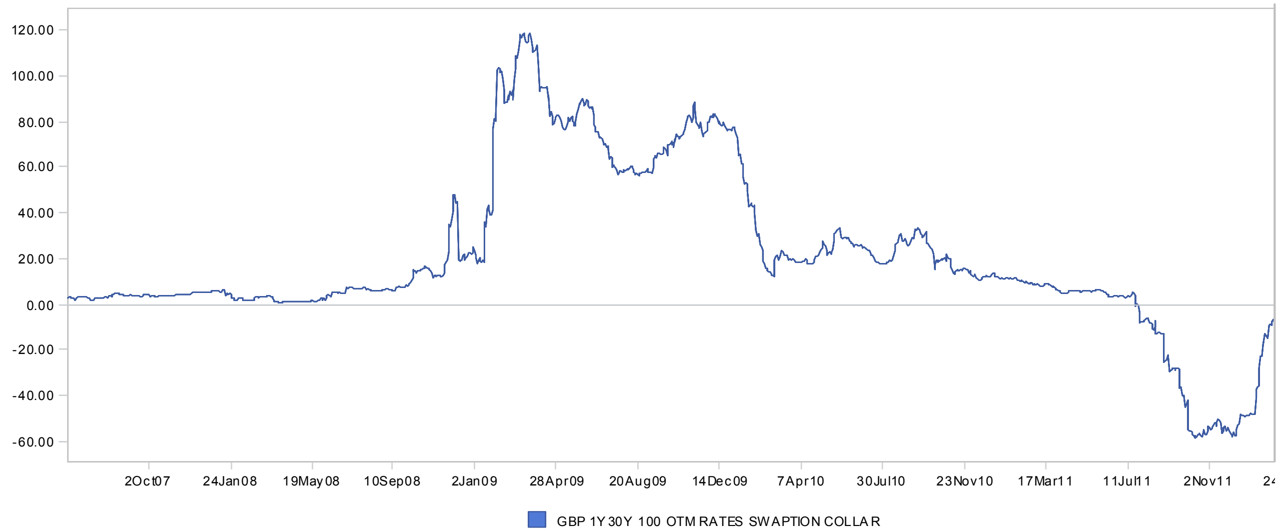
Protecting against a fall in real rates

Who was the end-user?	Buy-out insurance company	Pension scheme (British Nuclear Fuels - BNF)
Why did they transact?	Insurer wished to protect itself from a fall in real yields of more than 25bps relative to those assumed in the buy-out price.	Corporate was concerned about an increase in the accounting deficit as a result of falling real yields. An imminent change in sponsorship meant that BNF would not, however, benefit from a rise in real yields.
How did they do it?	Bought a real rate receiver swaption - Underlying was a zero coupon real rate swap - Strike rate was ATMF-25bps - 3m20y - 50k (underlying swap PV01) - American exercise - Swap settled and collateralised	Bought a real rate receiver swaption, financed by the sale of a real rate payer swaption such that structure was zero premium. - Underlying was a zero-coupon real rate swap - Strike rates on the swaptions were symmetrically 17bps wide of the ATMF - 1y20y - 400k (underlying swap PV01) - Cash settled and uncollateralised
What was the outcome?	Real rates fell slightly → structure finished in-the money Client satisfied that structure delivered what was “on the tin”	Real rates rose slightly → structure finished out-of-the-money Client satisfied that structure delivered what was “on the tin”

Protecting against a rise in real rates

Who was the end-user?	**Corporate with inflation-linked revenue stream**
Why did they transact?	Planned index-linked bond issuance and so concerned about a rise in real yields which would increase their cost of financing. Uniquely, the bond issuance was contingent on a non-market event (e.g. competition authority ruling) and so their hedge was contingent – i.e. no premium would be paid by the client or trade entered into with the bank if the contingent event failed to materialise.
How did they do it?	Contingent real rate swap. End user would not necessarily recognise the contingent swap as a swaption but this is how the contingent trade is risk managed. - underlying was a (linker-style) real rate swap - Strike rate was ATMF+20bps - 3m25y - large (underlying swap PV01) - Uncollateralised. Swap settled if contingent event took place
What was the outcome?	Contingent event took place and swap was entered into. Swaption expired and the bank's potential loss should trade not take effect was limited.

Other market participants – hedge funds



Why?

– Motivated by i) alpha and ii) tail risk hedging against extreme macro events

How?

– Shorter expiries for liquidity but will play in longer-tails so are a liquidity provider for the types of trades pension funds and insurers are considering

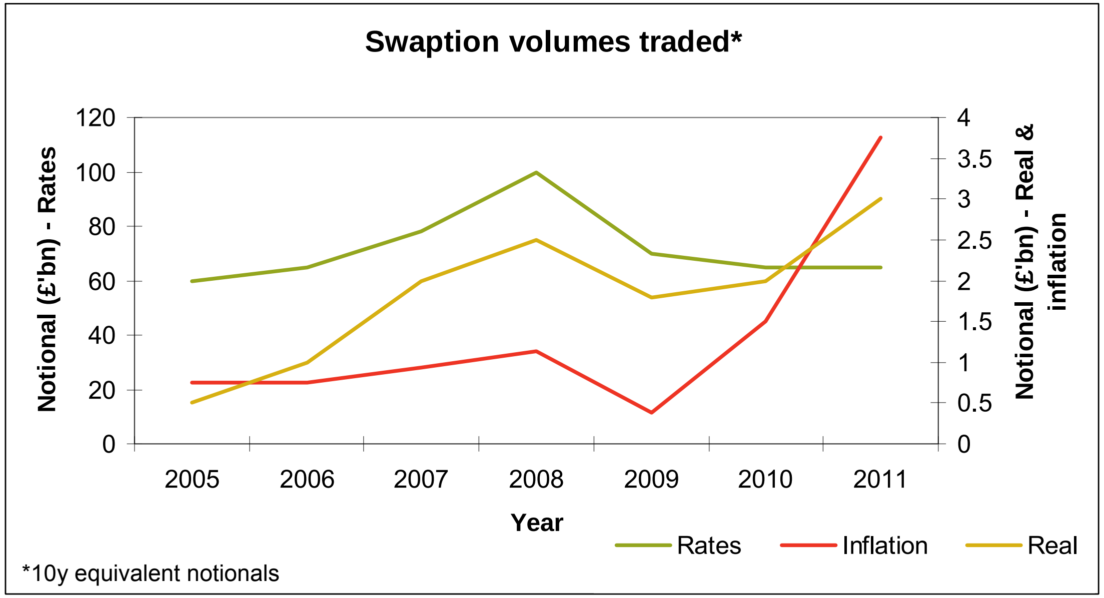
– RV trades on volatility surface

– Increasingly trading rates and inflation markets via options

Outcome?

– Short-term distortions in rates and inflation vol and skew creates opportunities for pension funds and insurers

Other market participants – banks and dealers



Why?

– i) non-interest rate trading desks (eg CVA, inflation, vol desks) are hedging (mainly) for risk management and ii) dealers are market-making for profit

How?

– CVA desks buy rates receivers and inflation payers

– Inflation trader hedges an inflation swap's cross gamma risk to real interest rates using conditional real rate instruments

– Strip options from sterling corporate linkers e.g. puttable and callable bonds

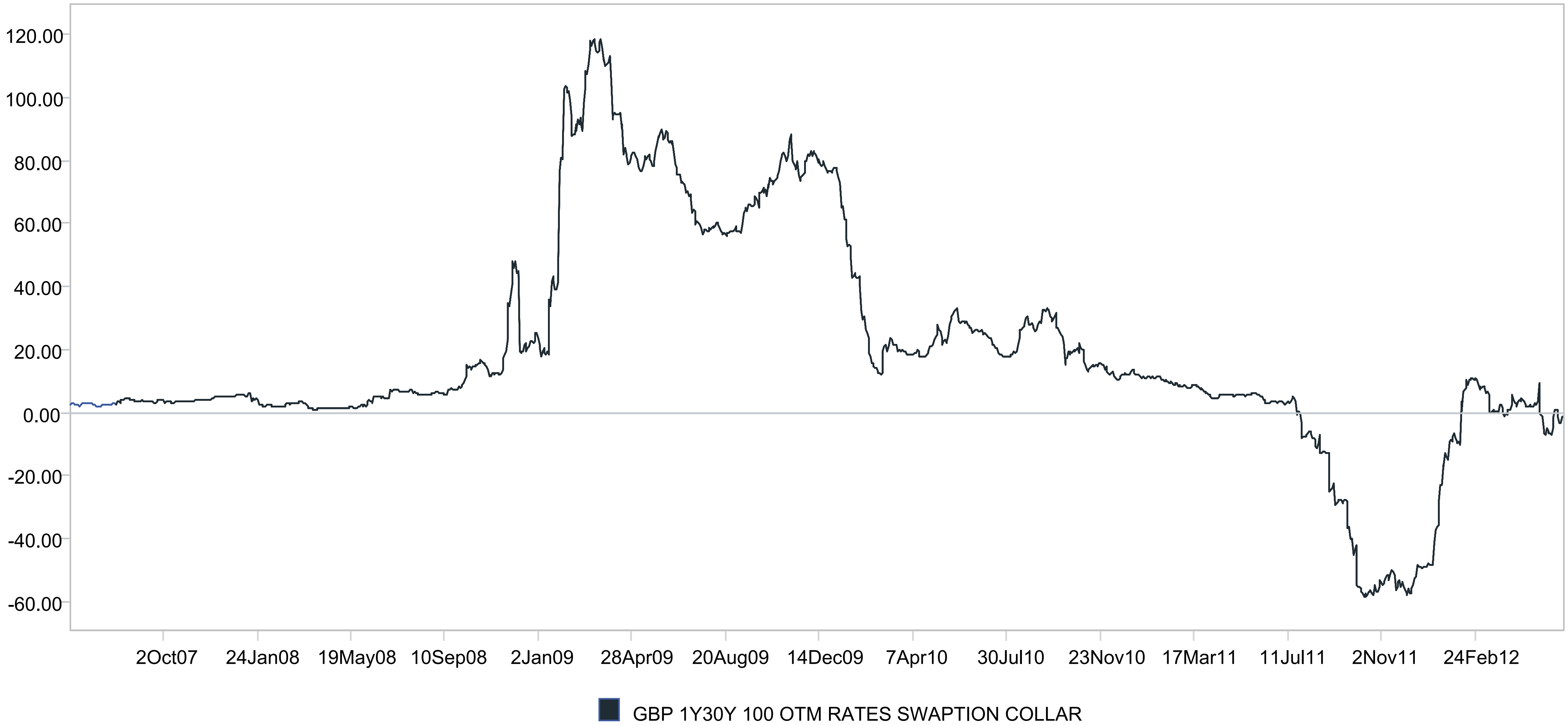
Outcome?

– Creates supply/axes for pension fund and insurer's transactions

Practical considerations

Designing a programme	- Extend the toolkit and measure fund manager against a liability benchmark. Fund manager should have a clear view on the discretion they would like but should be expected to commit to a benchmark. - Additional risk of conditional hedging can be controlled and allowed for when setting a tracking error for the manager's portfolio - Manager should then be expected to assess and make the following decisions: -Type of swaption to use (rates, inflation, real) -Choice of expiry / tail -Proportion of liabilities to be covered by swaptions vs. swaps/linear instruments
Execution	Sterling vol market can lurch between being "bid" and "offered". Price discovery Discretion Don't comp large trades Large programmes may mean splitting the delta and the vega trading and running the "gap" risk
Ongoing risk management	Bilateral collateralisation – no central clearing Independent valuations

Evolution of GBP Rates 1y30y 100 wide collars



Evolution of GBP Rates 1y30y 100 wide collars

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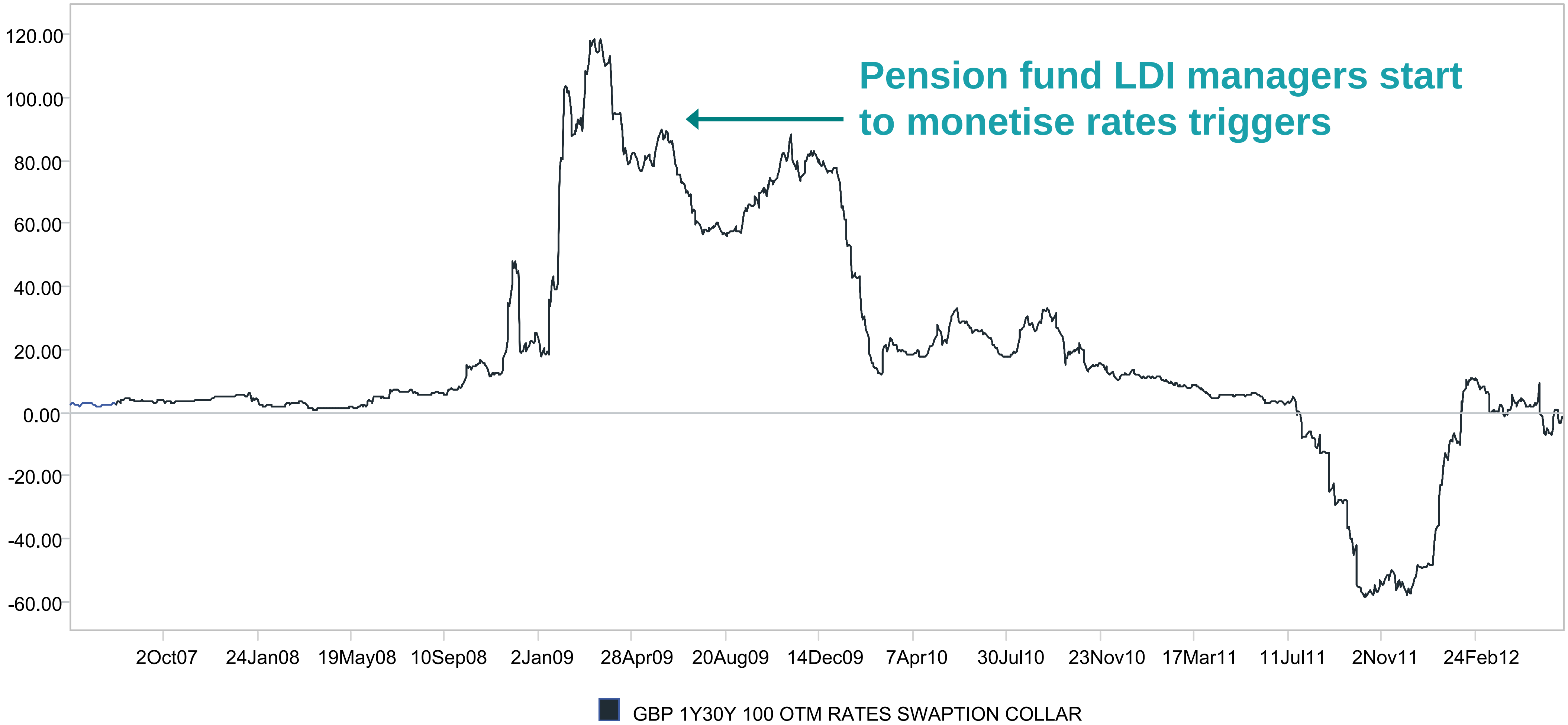

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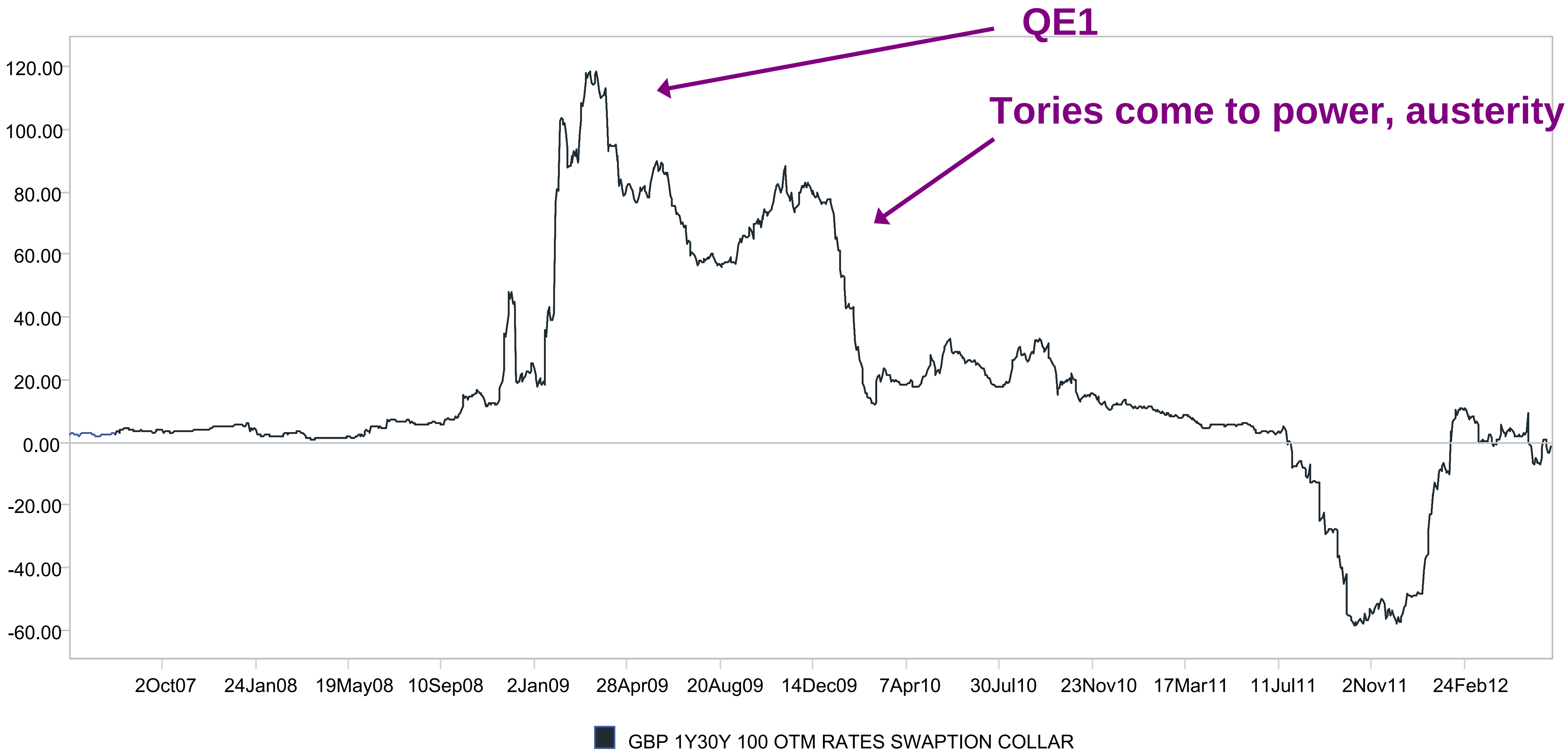
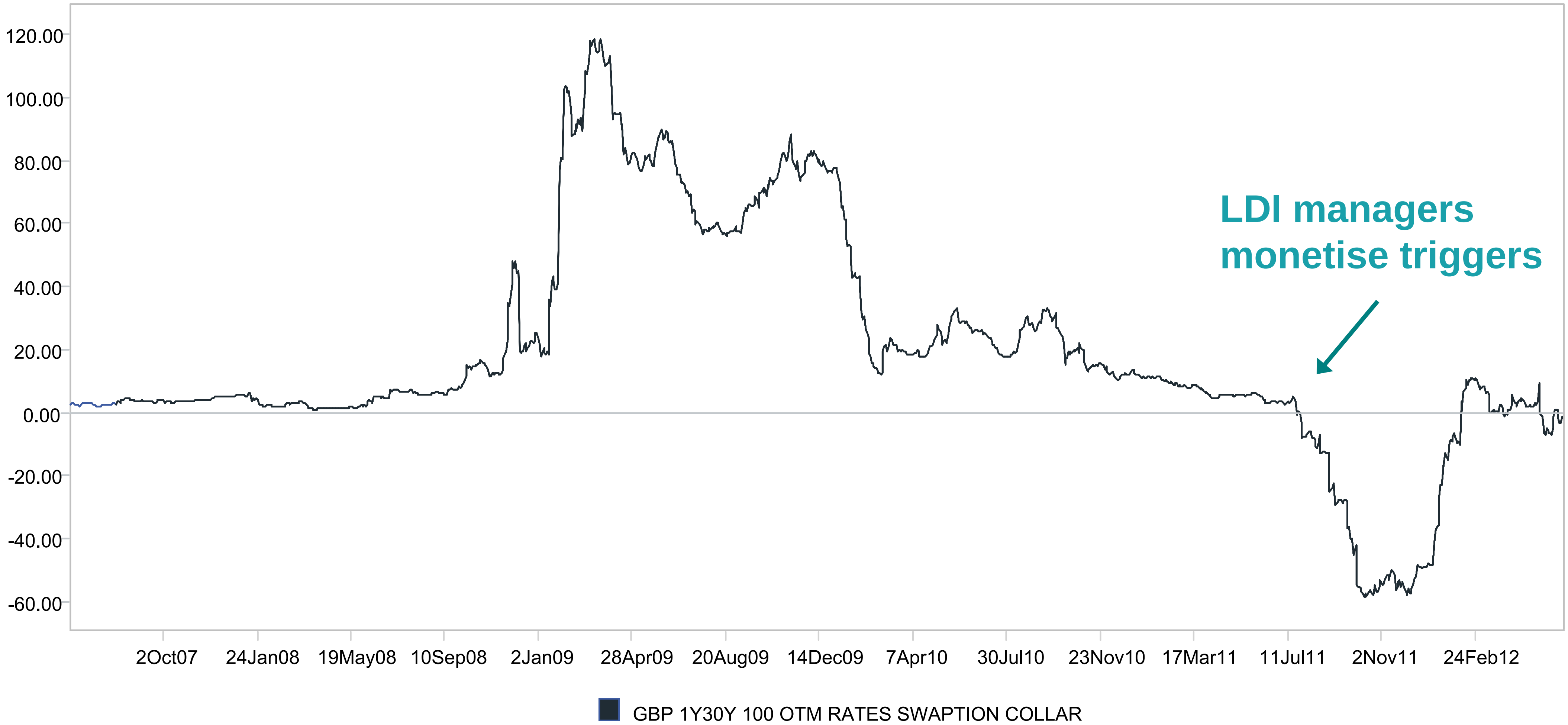


Evolution of GBP Rates 1y30y 100 wide collars



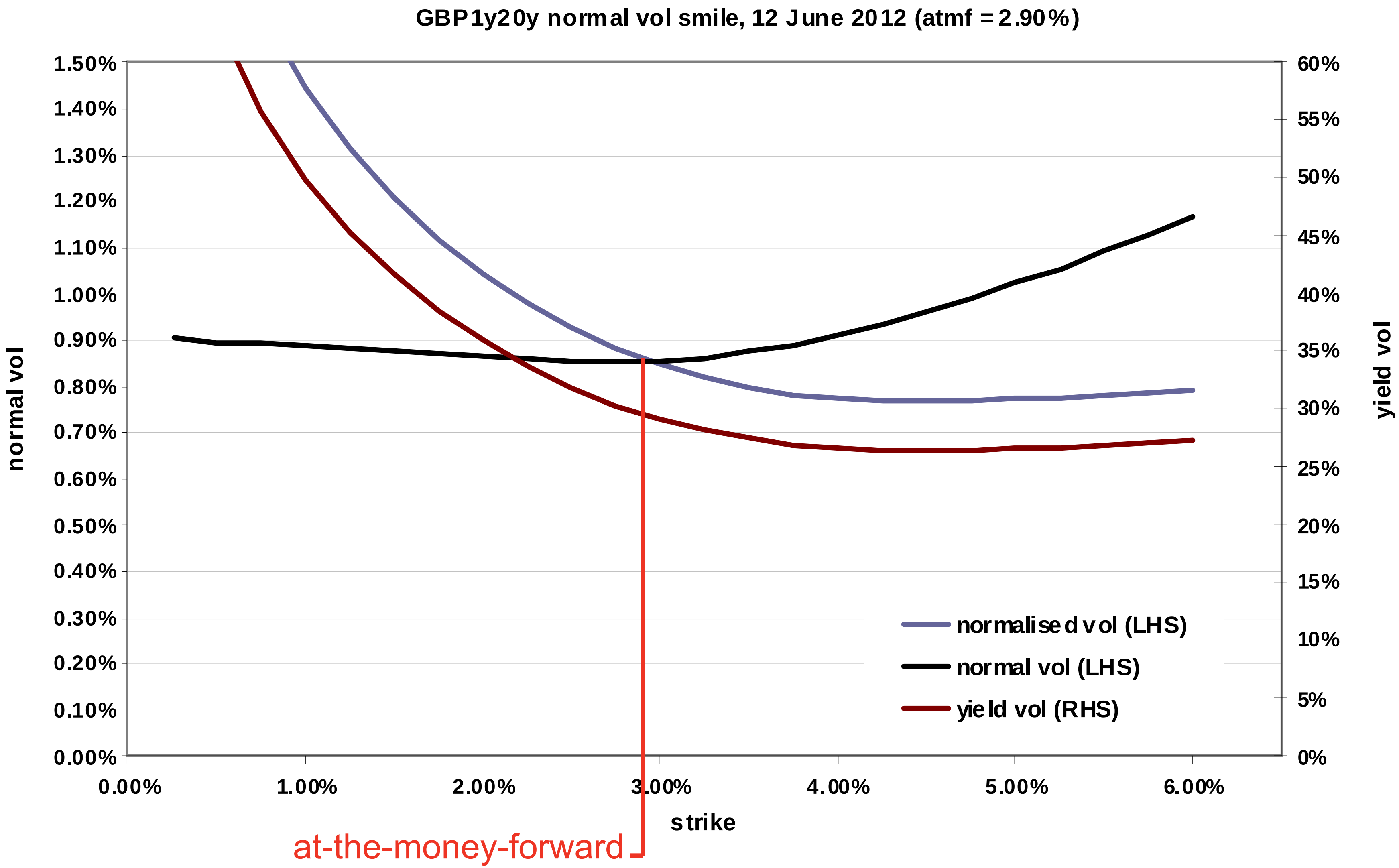
Volatility smiles: vanilla rates swaptions

Vanilla rates swaption is the option to pay fixed ("payer") or receive fixed ("receiver") in a standard (par) interest rate swap

What volatility?

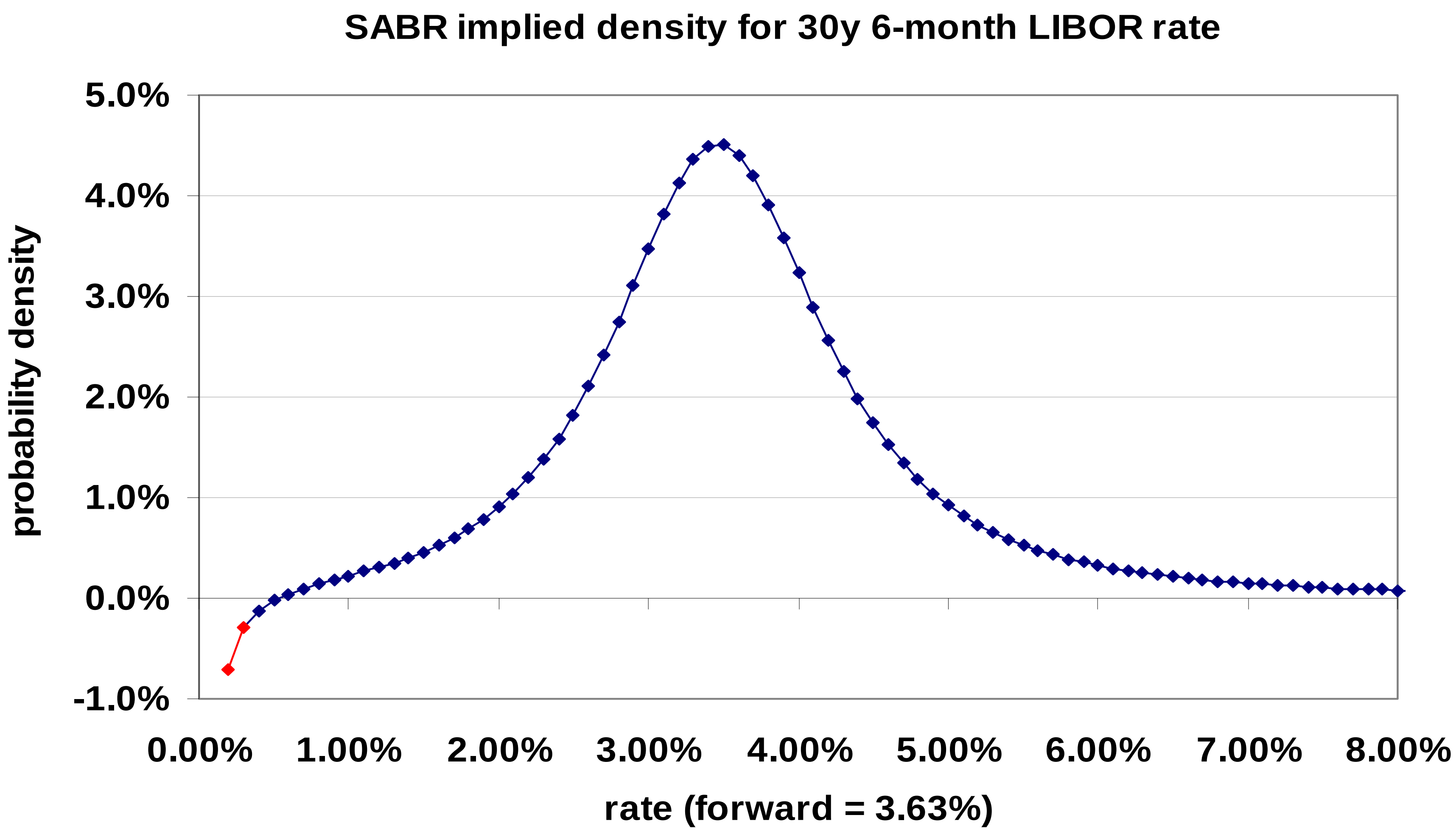
"% Yield vol" σ_Y if swap rate ~ lognormal	e.g. 30%
"normalised vol" = *swap rate* * σ_Y	e.g. 0.75%
"normal vol" σ_N if swap rate ~ normal	e.g. 0.75%
"bp/day vol" = 10000 * bp normal vol / $\sqrt{250}$	e.g. 4.7/day

Volatility smiles: vanilla rates swaptions



Volatility smiles must avoid arbitrage e.g. SABR model negative probabilities

SABR model implied density for F=3.63%, α=1.25%, β=50%, ρ=15%, ν=22%



Volatility smiles: vanilla rates swaptions

Payoff of an interest rates payer swaption at expiry

$$= \max\big[\, 0,\ \mathrm{PV}(\mathit{LIBOR\ leg}) - \mathrm{PV}(K\ \mathit{fixed\ leg})\,\big] \quad ;\ K = \text{strike rate}$$

So swaption price =

$$\underbrace{E_Q\big[\max[\, 0,\ (\mathit{fwd\ market\ rate} - K)\,]\big]} * \mathrm{dv01}$$

$\mathit{BlackScholes}(\sigma_Y(K))$ if *market rate* follows geometric BM; or

$\mathit{normal\ option\ formula}(\sigma_N(K))$ if *market rate* follows BM

Black Scholes pricing formulae

Black Scholes(1976) *option pricing formula*:

The value of a call option for a non-dividend paying underlying stock in terms of the Black–Scholes parameters is:

$$C(S,t) = N(d_1)\,S - N(d_2)\,Ke^{-r(T-t)}$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$

The price of a corresponding put option based on put-call parity is:

$$P(S,t) = Ke^{-r(T-t)} - S + C(S,t)$$
$$= N(-d_2)\,Ke^{-r(T-t)} - N(-d_1)\,S.$$

Normal option pricing formula based on Black Scholes assumptions but Brownian motion not geometric Brownian motion, *e.g. Bachelier* (1900), *Iwasawa* (2001)

$$C = e^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}]$$

$$P = e^{-r(T-t)}[(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}]$$

Volatility Smiles: zero coupon rates swaptions

Zero coupon rates swaption is the option to pay compounded fixed ("payer") or receive fixed ("receiver") against a compounded LIBOR floating leg, i.e. enter a zc interest rate swap

Payoff of a zc rates payer swaption at expiry

$$= \max\left[\ 0,\ PV\left(\prod_{i}^{n} (1+LIBOR_i)-1\right) - PV\left(\prod_{i}^{n}(1+K)\right)\ \right]$$

So swaption price =

$$E_Q\left[\max\left[\ 0,\ (1+\mathit{fwd\ zc\ market\ rate})^n - (1+K)^n\ \right] *\ DF\right]$$

Volatility Smiles: zero coupon rates swaptions

e.g. 1y20y zero coupon payment $(1 + \textit{zc market rate})^n$ can be replicated with a set of 1y20y par swaps:

	1y20y	bucketed positions	par atm vols
5y	1,090	1,090	0.78%
10y	976	976	0.90%
15y	1,533	1,533	0.88%
20y	-11,821	-11,821	0.87%
25y	-3,253	-3,253	0.85%
30y	0	0	0.84%
40y	0	0	0.80%
50y	0	0	0.76%
	-11,475		

zc rate volatility derived from basket of european par swaptions with same expiry dates and zc swaption priced as:

$$E_Q\left[\max\left[\,0,\ \textit{fwd zc market rate} - K\,\right]\right] * \textit{dv01}$$

Volatility smiles: inflation swaptions

Payoff of a zc inflation payer swaption at expiry t

$$= \max\left[0, \mathrm{PV}\left(RPI_{n+t} / RPI_t \right) - \mathrm{PV}\left((1+K)^n \right)\right]$$

So swaption price =

$$E_Q\left[\underbrace{\max\left[0, (1+fwd\ zc\ market\ rate)^n - (1+K)^n \right]}_{} * DF \right]$$

$BlackScholes(\sigma_Y(K))$ if RPI_{n+t} / RPI_t follows geometric BM; or

$normal\ option\ formula(\sigma_N(K))$ if *zc market rate* follows BM

Volatility smiles: inflation swaptions calibration

Note the zc inflation swaption vol model should recover index option implied vols since the underlying is similar:

swaption = $\max\left[0, \text{PV}(RPI_{n+t} / RPI_t) - \text{PV}((1+K)^n)\right]$ at time t

(fwd) index option = $\max\left[0, RPI_{n+t} / RPI_t) - (1+K)^n\right]$ at time $n+t$

Volatility smiles: real rate swaptions

Payoff of a zc real rate payer swaption at expiry *t*

$$= \max\left[0, \mathrm{PV}\left(\prod_i^n (1+LIBOR_i)\right) - \mathrm{PV}\left((1+K)^n * RPI_{n+t} / RPI_t\right)\right]$$

where *K* = *zc real rate strike.*

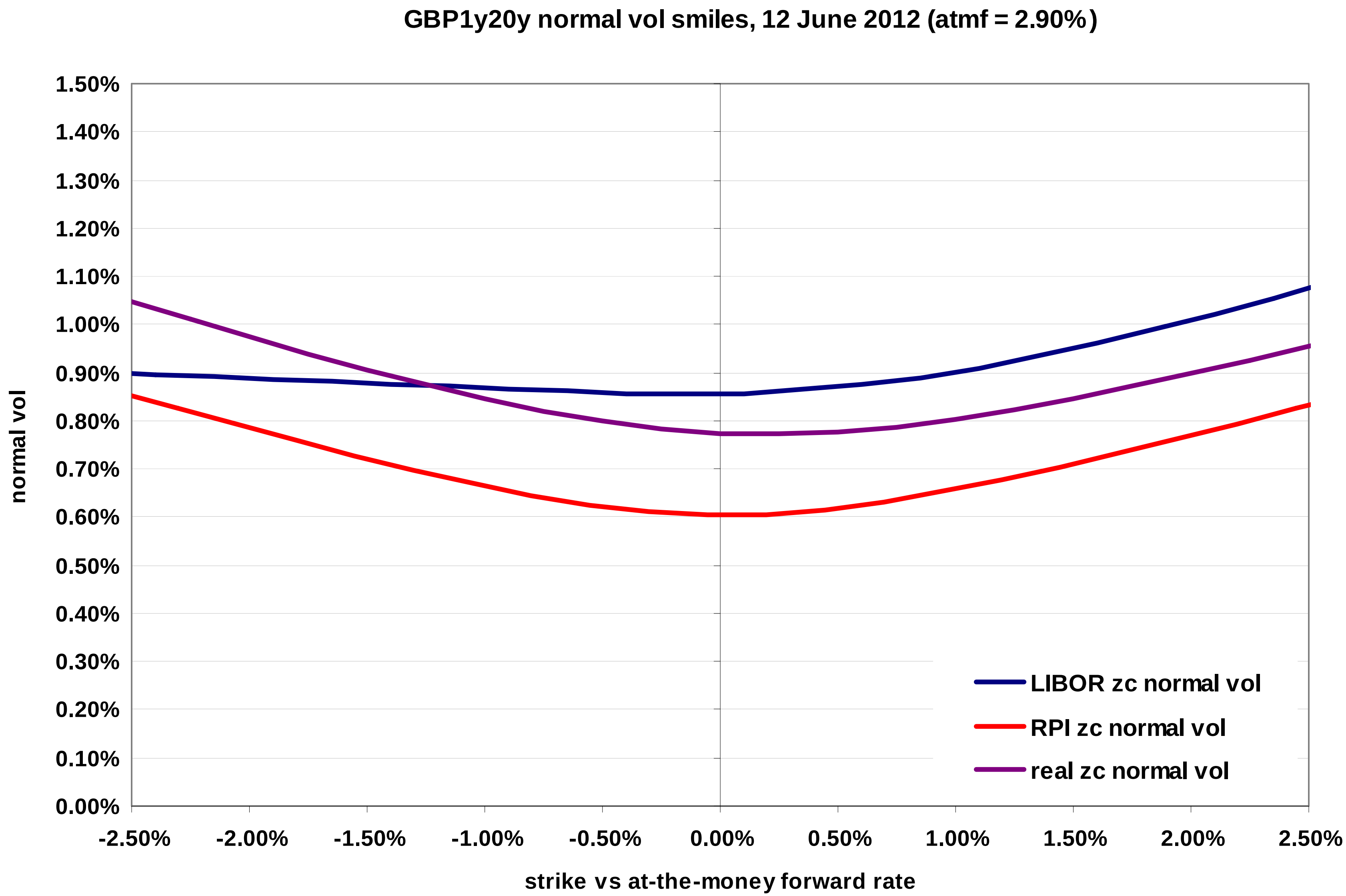
This is a spread option between interest rate and inflation legs, with implied vols from their respective zc swaption markets. So,

real = nominal – inflation

$$\Rightarrow \sigma_{\mathbf{real}}^2 = \sigma_{\mathbf{nominal}}^2 + \sigma_{\mathbf{inflation}}^2 - 2\rho\, \sigma_{\mathbf{nominal}}\, \sigma_{\mathbf{inflation}}$$

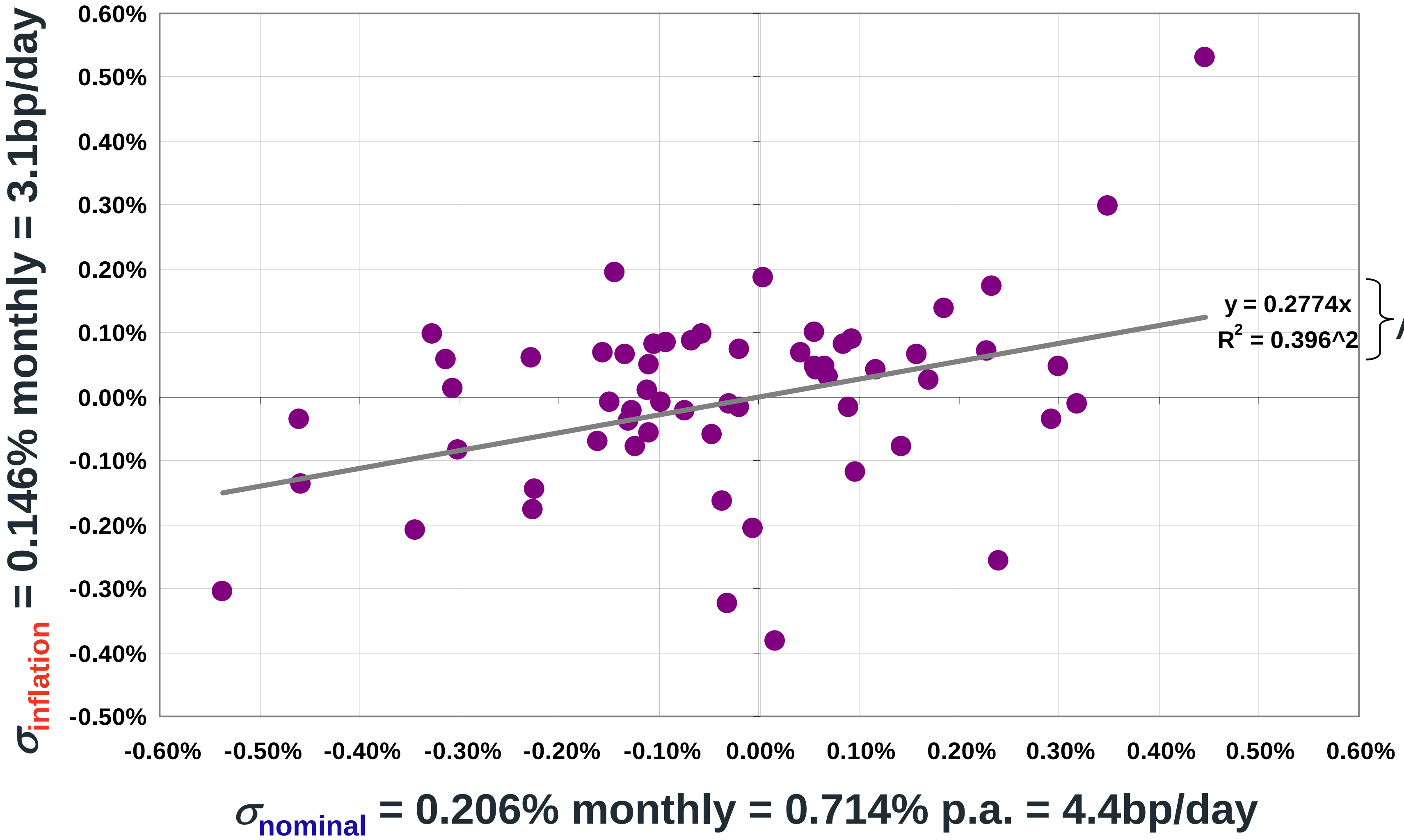
where $\sigma_{\mathbf{inflation}}$ is scaled by $(1+K)^n$

Volatility smiles: LIBOR, RPI and real



Volatility smiles: rates inflation correlation

There is a market in rates versus inflation correlation for expiries using correlation swaps.



$\sigma_{inflation} = 0.70\sigma_{nominal}$

Monthly data Bloomberg composite close, May2007 to May2012

Volatility smiles: real rate vol

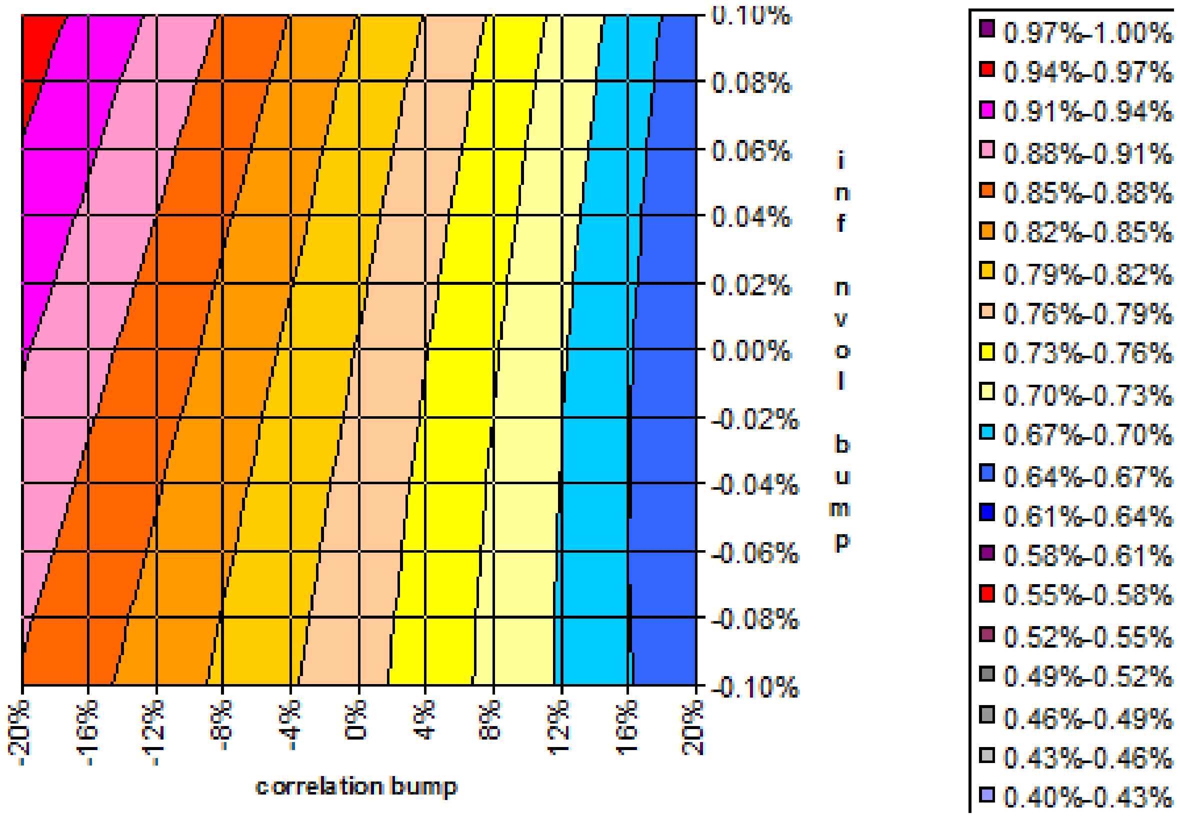
Real rate swaption spread volatility is dominated by the higher of interest rates and inflation volatility for correlation $\rho \approx$ 45% and inflation vol around 60% of 0.86% rates normal vol

inflation normvol bump		-20%	-16%	-12%	-8%	-4%	0%	4%	8%	12%	16%	20%
	-0.10%	0.88%	0.86%	0.84%	0.81%	0.79%	**0.77%**	0.75%	0.72%	0.70%	0.67%	0.64%
	-0.08%	0.88%	0.86%	0.84%	0.82%	0.80%	**0.77%**	0.75%	0.72%	0.70%	0.67%	0.64%
	-0.06%	0.89%	0.87%	0.85%	0.82%	0.80%	**0.78%**	0.75%	0.73%	0.70%	0.67%	0.64%
	-0.04%	0.90%	0.88%	0.85%	0.83%	0.80%	**0.78%**	0.75%	0.73%	0.70%	0.67%	0.64%
	-0.02%	0.91%	0.88%	0.86%	0.83%	0.81%	**0.78%**	0.76%	0.73%	0.70%	0.67%	0.64%
	0.00%	**0.91%**	**0.89%**	**0.87%**	**0.84%**	**0.81%**	**0.79%**	**0.76%**	**0.73%**	**0.70%**	**0.67%**	**0.64%**
	0.02%	0.92%	0.90%	0.87%	0.85%	0.82%	**0.79%**	0.77%	0.74%	0.71%	0.67%	0.64%
	0.04%	0.93%	0.91%	0.88%	0.85%	0.83%	**0.80%**	0.77%	0.74%	0.71%	0.68%	0.64%
	0.06%	0.94%	0.91%	0.89%	0.86%	0.83%	**0.81%**	0.78%	0.75%	0.71%	0.68%	0.64%
	0.08%	0.95%	0.92%	0.90%	0.87%	0.84%	**0.81%**	0.78%	0.75%	0.72%	0.68%	0.65%
	0.10%	0.96%	0.93%	0.90%	0.[illegible]8%	0.85%	**0.82%**	0.79%	0.76%	0.72%	0.69%	0.65%

real rate normal vol ranges between 85% and 99% of rates norm vol

Volatility smiles: real rate vol

Real rate swaption spread volatility is dominated by the higher of interest rates and inflation volatility for correlation $\rho \approx 45\%$ and inflation vol around 60% of rates implied vol



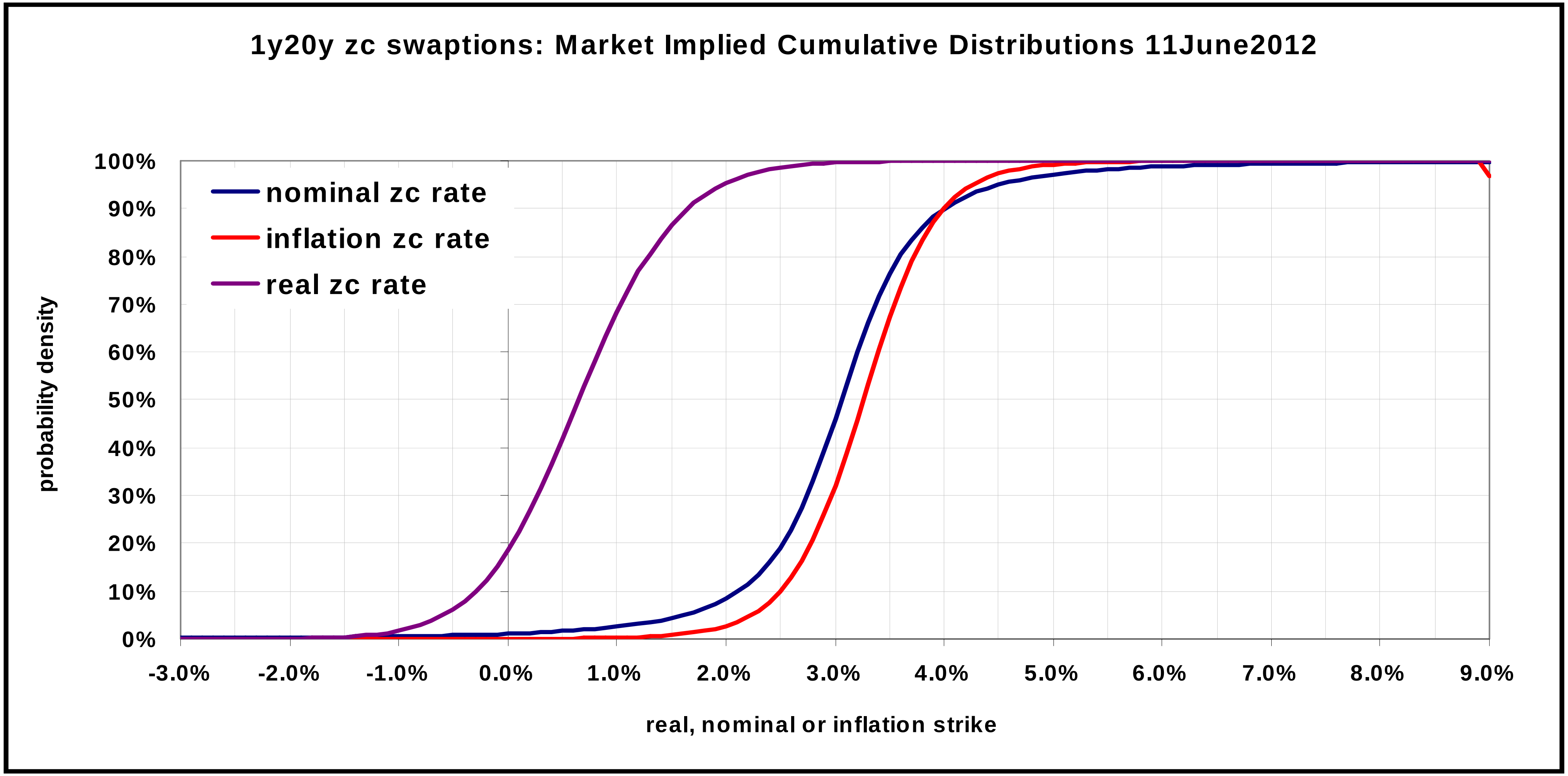
Real world and market implied probabilities

Payer spreads (i.e. call spreads) can be used to derive CDF and density for forward nominal, inflation and real rates:

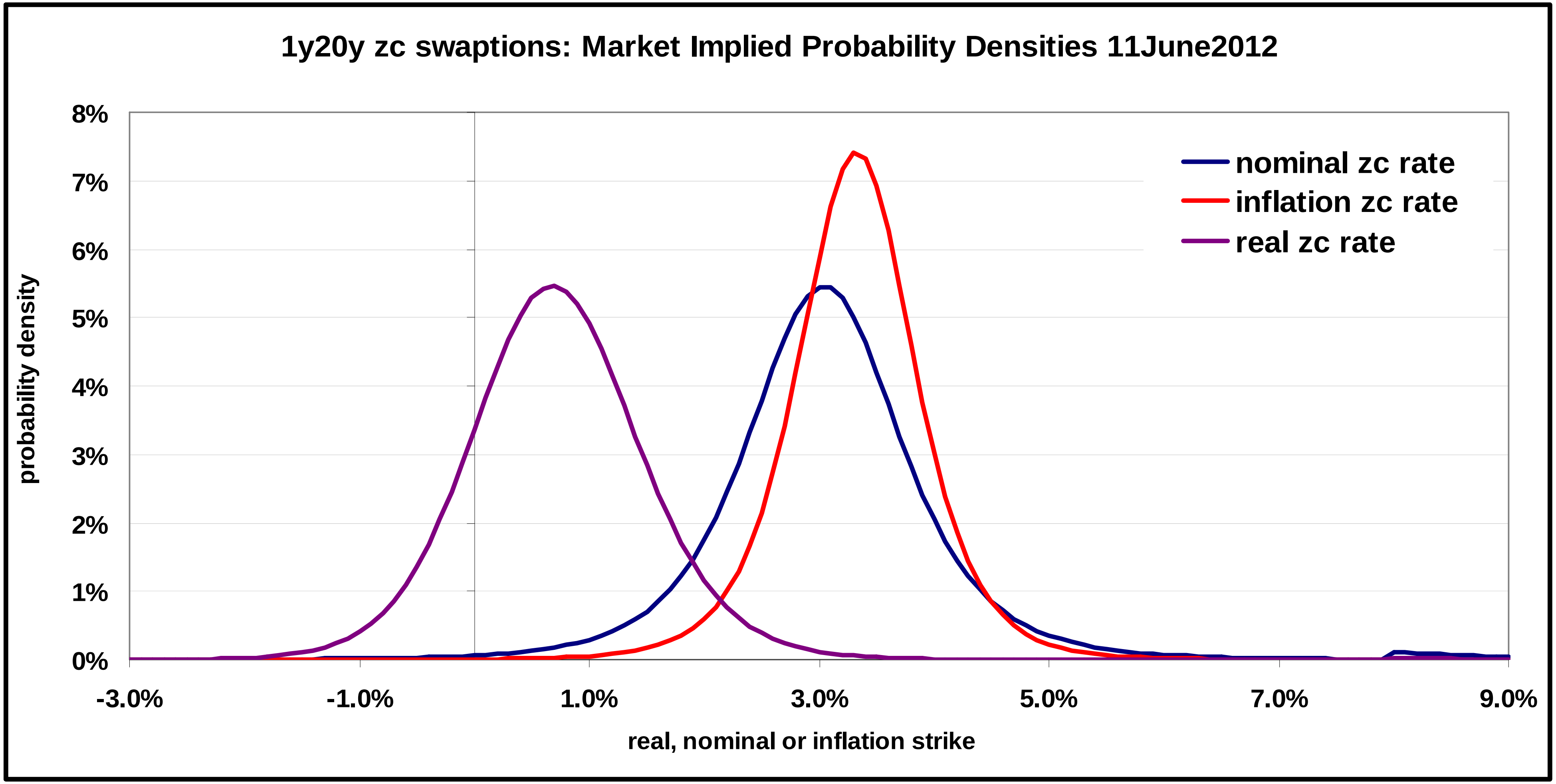
e.g. Pr[1y20y RPI fwd rate>K]

$$\approx \frac{(\text{1y20yPayer}(K\text{-0.01\%}) - \text{1y20yPayer}(K\text{+0.01\%}))}{\text{fwd swap dv01} * 2}$$

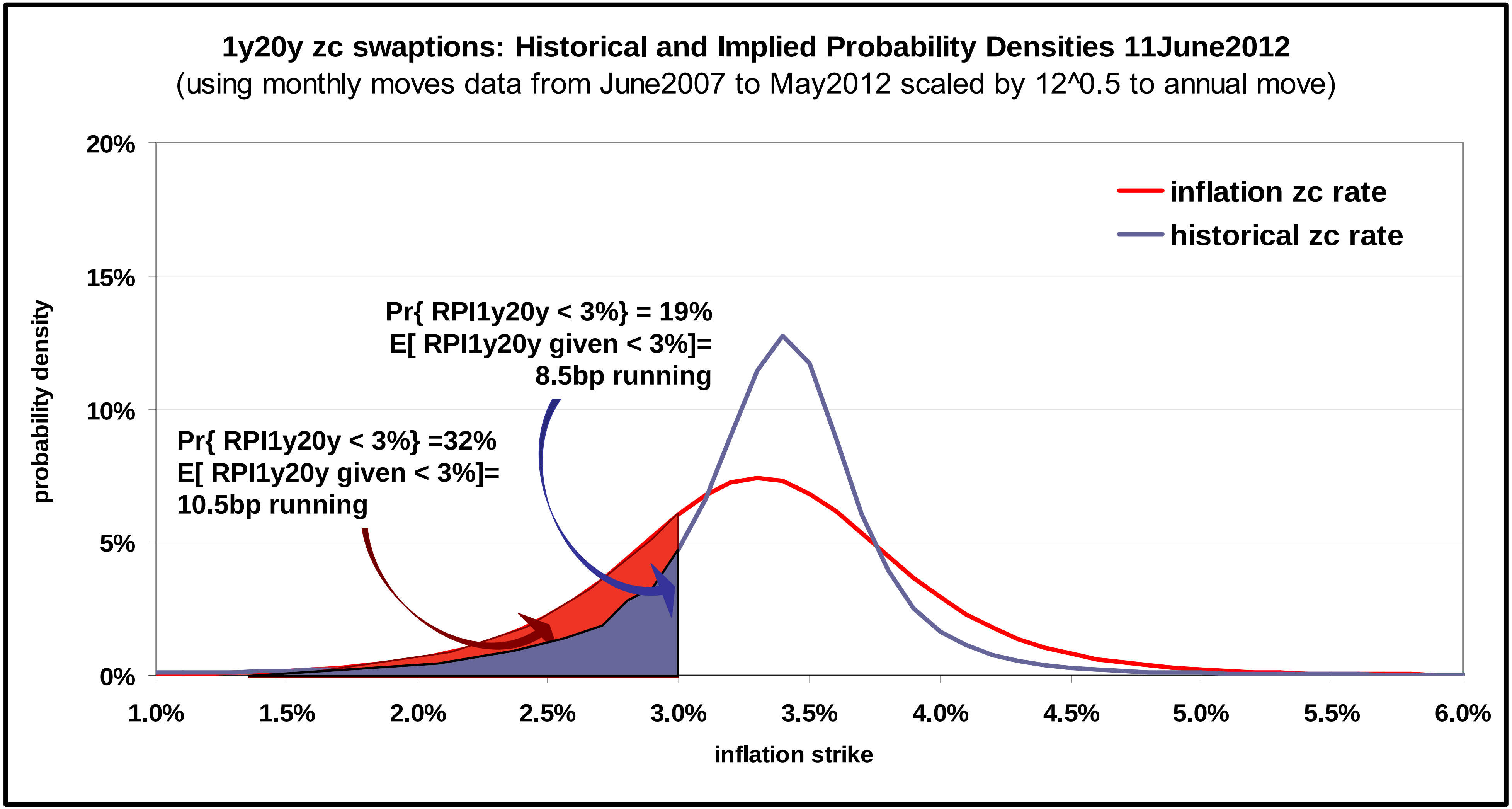
Real world and market implied probabilities



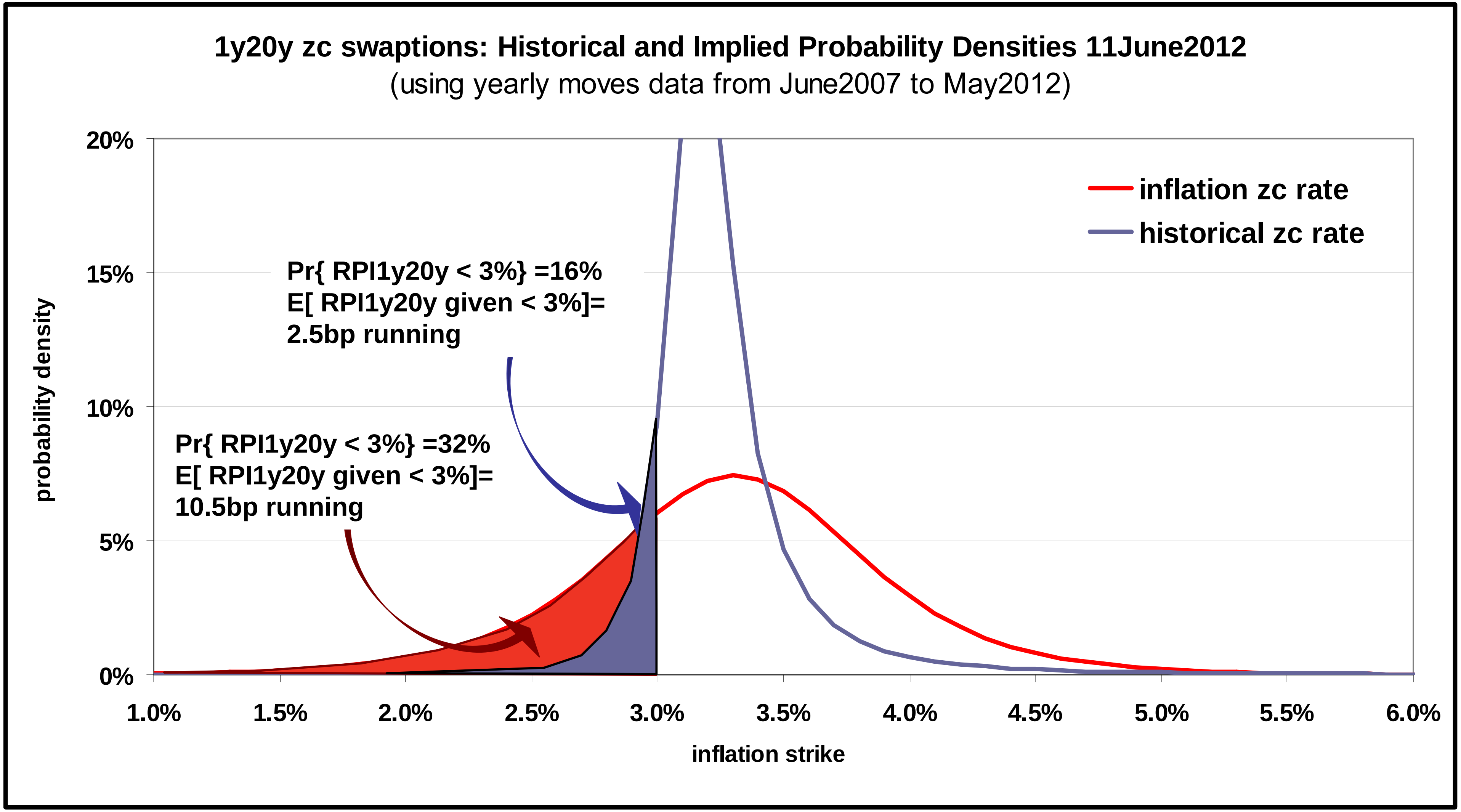
Real world and market implied probabilities



Real world and market implied probabilities



Real world and market implied probabilities



Strategies based on risk neutral probabilities

Volatility smiles imply unique risk neutral probability distribution functions for nominal, real and inflation forward rates.

These probabilities and expectations can be compared with investors' subjective views to appraise strategies.

Sophisticated end users (e.g. LDI asset managers) are very informed about structure of supply and demand in underlying swap markets.

Implied volatility >> historical volatility may motivated covered writes.

High inflation swap rate mean reversion means dealers must capture gamma from on intra-day moves, something difficult for end users to do.

Outlook and future development of market

Sufficient natural flow for viable rates, inflation and real swaptions markets

Virtuous liquidity cycle is building

Quick reactions to market conditions advantageous

Credit and capital concerns can be mitigated in practical terms

References

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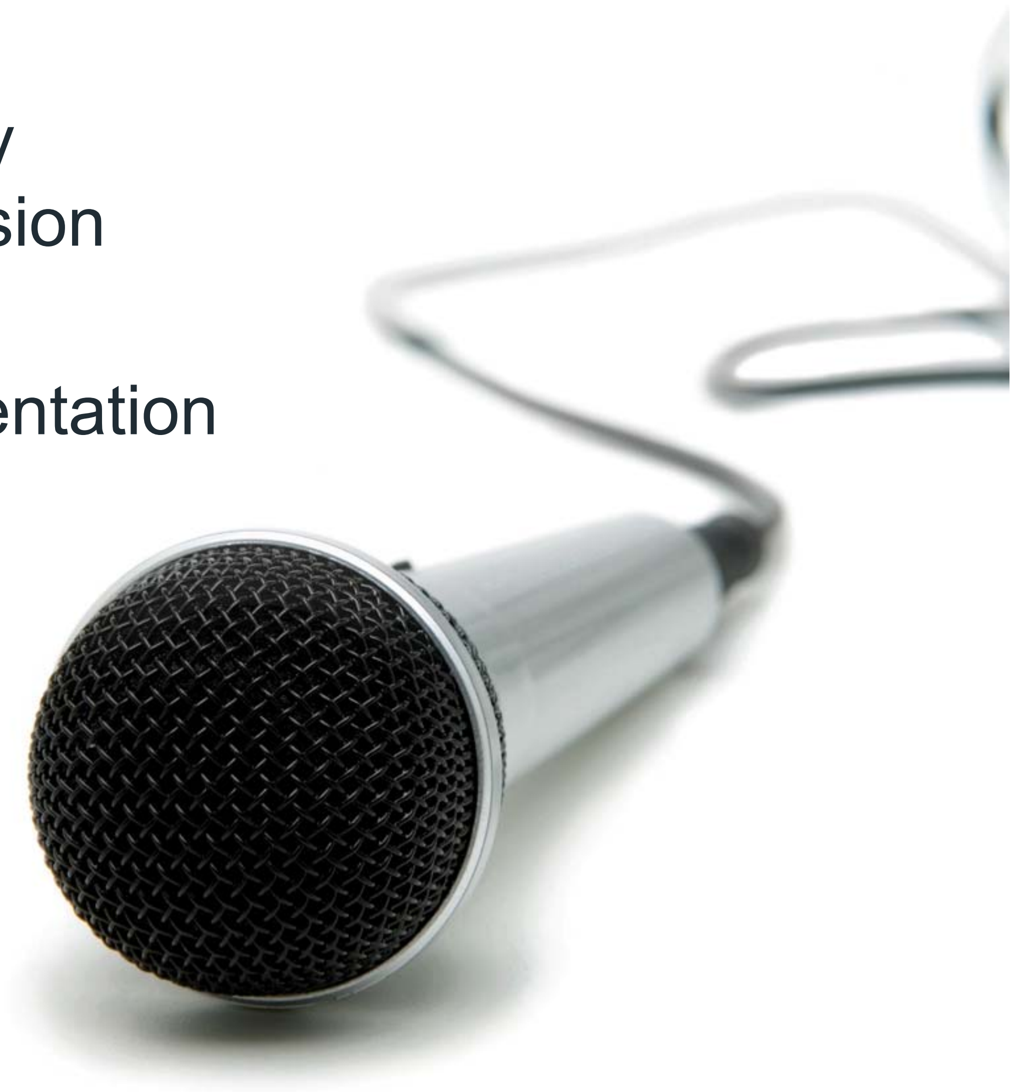
Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.



Slides reserved for potential discussion points

- ☞ *rates swaptions realised vs implied volatilities*
- ☞ *historical bp/day swaption vol grid*
- ☞ *swaption pricing in Bloomberg*
- ☞ *market implied correlation*
- ☞ *indicative pricing*

Swaption pricing in Bloomberg (SWPM <GO>)