Introduction:  
The Basis Risk Question

Pension/Annuity Fund  
Fixed Premiums (hedge cost)  
Indexed Payments based on E&W mortality

Annuity Payments

Investors/reinsurers

What is the risk that indexed payments fall short of annuity/pension payments?
Introduction
The Basis Risk Question

- We consider two populations with forces of mortality $\mu_1(x,t)$ and $\mu_2(x, t)$ at age $x$ in year $t$, that is from time $t-1$ to time $t$.
- Assume that the first force of mortality is associated with the population for England and Wales.
- The second force of mortality can be associated with the actual lives underlying a portfolio identified for hedging,
- The problem is: given a pre-defined hedging instrument whose value is governed by changes in $\mu_1$, how effective is that instrument for hedging the liabilities of the pension fund or insurance company?

The Group

- LBRWG set up in December 2011 to think about the problem
- Joint group consisting of LLMA members and Actuarial Profession Affiliates

<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
<th>LLMA or IaFoA representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevin Armstrong</td>
<td>AVIVA</td>
<td>LLMA</td>
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<td>Peter Banthorpe</td>
<td>RGA</td>
<td>IaFoA</td>
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<td>Robert Bugg</td>
<td>Milliman</td>
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<td>Andrew Gaches</td>
<td>Hymans Robertson</td>
<td>IaFoA</td>
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<tr>
<td>Torsten Kleinow</td>
<td>Heriot Watt University</td>
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<tr>
<td>Jessica Mosher</td>
<td>AXA</td>
<td>LLMA</td>
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<tr>
<td>Pretty Sagoo (Chair)</td>
<td>Deutsche Bank</td>
<td>LLMA</td>
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</table>
Overview:
UK Longevity Market

- Proportion of buy outs/buy ins/indemnity swaps show that popularity of swaps is growing but index based hedging remains limited
- Of deal flow shown, only £70m notional in 2011 was index-linked (Pall Pension deal – E&W Population linked)
- AEGON transaction in March 2012 on €12bn was also index-linked (Dutch Population Mortality)

Source: Hymans Robertson

Benefits of Index Based Swaps:
Why aren’t they more popular for hedging risk?

Benefits
- Lower Cost
- Faster Execution
- Potential for liquidity
- Transparent
- Flexible structure

S-forward Rows
Notional x Realised Survival Rate

S-forward payoff at maturity
Obstacles to Index Based Swaps: Progress made and remaining barriers

Addressed
• Standardization and transparency
  – Standardization of basic derivatives (q/s-forwards)
  – Launch of LLMA Indices
• Knowledge and education
  – Mainstream exposure
  – LLMA pricing framework
• Long-term nature of risk
  – Emergence of innovative structures appealing to both hedgers and investors

Outstanding
• Basis Risk
  – Cost-benefit analysis
  – Assessment of hedge effectiveness
  – Allowance for capital reduction (Solvency II)

Types of Basis Risk: Three Main Categories

Demographic
• Socio-economic status
• Lifestyle
• Geography

Sampling
• Small population
• Large annuity amounts

Structural
• Choice of reference age and genders
• Duration of hedge
Assessment of Basis Risk: Main Steps and Questions to Answer

Historical Analysis
• What relationship have the two populations demonstrated in the past?
  – Empirical assessment
  – Establish an underlying link

Modeling
• What do you expect the future relationship to be?
  – Stochastic models
  – Two-population models

Measurement
• What measure to use for hedge effectiveness?
  – Deterministic quantification
  – Select appropriate metrics

Longevity Basis Risk

Types of Basis Risk
- Difference between target and reference (Demographic)
- Structure of Hedge (Structural)
- Variability from size (Sample)

Assessment
- Historical Analysis
- Stochastic Modeling
- Deterministic Quantification

Metrics
- Mortality Rates
- Improvements
- Survival Rates
- Life Expectancy
- Liability Values
- VAR
Parallels with the Past: The early days of Inflation Risk hedging

Types of Basis Risk
- Difference between scheme inflation and market inflation
- Structure of Hedge (Structural)
- Variability from size (Sample)

Assessment
- Historical Analysis
- Stochastic Modeling
- Deterministic Quantification

Metrics
- Inflation Rates
- Liability Values
- Tailored Metrics, e.g., liability values

Assessment of Basis Risk: Historical Analysis

Objective
- Empirical assessment to link reference and target populations
- Understand key differences

Assessment
- Historical analysis of key metrics
  - Mortality rates
  - Life expectancy
  - Liability cash flows

Limitations
- Availability of historical data
Assessment of Basis Risk: Modeling

Objective
• Predict future evolution of mortality for the two populations
• Model the underlying stable, long-term relationship

Assessment
• Stochastic two-population models
  – Derive long term trends from general population
  – Estimate difference in short term trends for the target population

Limitations
• Complexity of existing models
• Limited data for the target population to calibrate models

Assessment of Basis Risk: Measurement

Objective
• Determine hedge effectiveness
• Cost-benefit analysis

Assessment
• Quantify risk reduction with metrics
  – Variance in value: $1 - \frac{\sigma^2_{(Hedged \ Liability)}}{\sigma^2_{Liability}}$
  – Value at Risk: $1 - \frac{\text{VaR}_{(Hedged \ Liability)}}{\text{VaR}_{Liability}}$

Limitations
• Necessary to select a metric consistent with hedging objectives
• Several dimensions should be considered in the decision to hedge
Assessment of Basis Risk: What’s missing?

- Easily transferable assessment techniques
- Long time series of historical portfolio data, especially for small schemes
- ‘Rules of thumb’ for estimating basis risk given basic inputs (size, demographic, index)
- Widespread knowledge of indices and hedging instruments available
- Knowledge of complex two-population mortality models
- View on hedge metrics aligned with hedging objective

The Longevity Basis Risk Working Group:
The LLMA and the Institute and Faculty of Actuaries

Objective
- Define a practical methodology to assess basis risks for longevity transactions which is easily accessible to market practitioners

Work Performed
- Review of existing literature
- Identification of available inputs and desired outputs
- Simplified spreadsheet analysis

Conclusion
- Significant amount of research required
- Necessary to involve a consultancy or academic institute
What are we trying to achieve?

(Very!) Simplified Case Study

<table>
<thead>
<tr>
<th>Hedge Portfolio Assumptions</th>
<th>Projection Details</th>
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<tr>
<td>Hedge Instrument</td>
<td>Model</td>
</tr>
<tr>
<td>S-Forward</td>
<td>Lee Carter</td>
</tr>
<tr>
<td>Maturity</td>
<td>Calibration Period</td>
</tr>
<tr>
<td>10y</td>
<td>1946-2005</td>
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<tr>
<td>Reference Age</td>
<td>Hedge Population Calibration Data</td>
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<tr>
<td>60y</td>
<td>E&amp;W</td>
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<tr>
<td>Strike (survival) rate</td>
<td>Annuity Population Calibration Data</td>
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<td>83.8%</td>
<td>E&amp;W</td>
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<td>Number of Simulation Paths</td>
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<td>£205,000</td>
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<th>Annuity Portfolio Assumptions</th>
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<tr>
<td>Annuitant Ages</td>
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</tr>
<tr>
<td>60, 65, 70, 75, 80, 85</td>
<td></td>
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<tr>
<td>Annuity Amounts</td>
<td>£1000</td>
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<tr>
<td>Total Annuity Value after 1 year</td>
<td>£60,400</td>
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Hedge Portfolio Assumptions

- Hedge Instrument: S-Forward
- Maturity: 10y
- Reference Age: 60y
- Strike (survival) rate: 83.8%
- Hedge Contract Notional: £205,000

Annuity Portfolio Assumptions

- Annuitant Ages: 60, 65, 70, 75, 80, 85
- Annuity Amounts: £1000
- Total Annuity Value after 1 year: £60,400

Projection Details

- Model: Lee Carter
- Calibration Period: 1946-2005
- Hedge Population Calibration Data: E&W
- Annuity Population Calibration Data: E&W
- Number of Simulation Paths: 1000

Distribution of Annuity Values at t=1 year

**Pre- Hedge Overlay**

- PERCENTILE: 99.5%
- Average Annuity Level: £60,397
- VaR level: £77,734
- Capital Required: £17,337

**After Hedge Overlay**

- Average Annuity Level: £60,886
- VaR level: £69,429
- Capital Required: £8,543
- Hedge Effectiveness: 50.73%

No demographic basis risk!
No sampling risk!
Lots to do........
What are we trying to achieve?

The Biggest Challenge

- The biggest challenge is how to model the demographic risk.
- Given the typical inputs for a pension scheme or annuity book:

<table>
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<tr>
<td>Target Population Size – Number of individuals</td>
</tr>
<tr>
<td>Target Population Annuity/Pension Amounts</td>
</tr>
<tr>
<td>Geographic location</td>
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<tr>
<td>Historical mortality experience information if available</td>
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- How do we simulate the two populations? (Hedge and portfolio)
- How are their mortality diffusions related?
- If $\mu_1(x, t)$ is the force of mortality for E&W, we need to generate $\mu_2(x, t)$ (mortality for the pool).
- If $\mu_2(x, t) = f(\mu_1(x, t))$ where $f$ is an appropriate function, what form should $f$ take?

The Longevity Basis Risk Working Group: Proposal for Research Funds

Stage 1
- Invitation to tender, inviting consultancies and academic institutions to submit detailed proposals for research

Stage 2
- Given approval of funding sponsors:
  - Phase 1
    - Delivery of a detailed methodology for measuring longevity basis risk and a thorough description of process, strengths and limitations
  - Phase 2
    - Specification of metrics and a practical framework for quantifying longevity basis risk
The Longevity Basis Risk Working Group: Deliverables

**Phase 1**
- **Background research**
  - Review evidence of the differences in mortality improvement for various sub-populations
    - Socioeconomic groups
    - Geographic locations
  - Review existing models to structure relationship between the mortality of the two groups
- **Projection methodology**
  - Produce detailed specifications of proposed modeling methodology
  - Detail the strengths and limitations of proposed methodology given objectives of the working group

**Phase 2**
- **Quantification**
  - Define metrics and framework to quantify results
- **Case Study**
  - Apply model and framework on a practical, realistic, illustrative example

The Longevity Basis Risk Working Group: Next Steps

- **Produce a tender document to distribute to respondents**
  - Background and expectations for the assignment
  - Research and deliverables required
  - Expected timelines
  - Required information in response to tender
- **We will approach multiple parties (Mainly Consultants and Academic Institutions)……**
The Longevity Basis Risk Working Group: Next Steps

- Would you like to tender?
- Please get in touch!

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.