Forecasting of U.K. Population Mortality

with allowance for age, period and cohort effects

English Life Tables (ELT) : Period Expectation of Life at birth
Forecasting U.K. Population Mortality

Poisson Distribution Model for Deaths

\[ \text{Probability}\{D(x,t) = D\} = \frac{e^{-E(x,t)m(x,t)} \cdot \{E(x,t) \cdot m(x,t)\}^D}{D!} \]

Forecasting U.K. Population Mortality

Two-dimensional Lee-Carter Model
No cohort graduation effect

\[ m(x,t) = e^{a(x) + b(x) \cdot k(t)} \]

\[ \sum b(x) = 1 \text{ and } \sum k(t) = 0 \]
Forecasting U.K. Population Mortality

Data: 1961-2007
Test of whether there is a cohort effect

\[
1 - \frac{\text{Actual Deaths}}{\text{Expected Deaths LC}}
\]
Forecasting U.K. Population Mortality

Two Dimensional Lee Carter Extended
i.e. With Cohort Effect

\[ m(x, t) = e^{a(x) + b(x) \cdot k(t) + \lambda \{89 - (x - t)\}} \]

\[ \sum b(x) = 1, \quad \sum k(t) = 0 \]

\[ \sum \lambda(\text{all cohorts}) = 0 \]
Forecasting U.K. Population Mortality

Test of whether the cohort effect goes away

\[
1 - \frac{\text{Actual Deaths}}{\text{Expected Deaths LCExt}}
\]

Forecasting U.K. Population Mortality

LCExtended : allowing for the cohort effect
i.e. projecting lambda using ARIMA(0,1,0)
Males LCExt i.e. with allowance for Cohorts

Females LCExt i.e. allowing for Cohort effect
Forecasting U.K. Population Mortality

GAD: Period Expectations of Life at Birth
Males and Females

Forecasting U.K. Population Mortality

GAD: Period Expectations of Life at Birth: Males and Females versus LC model and LCExtended
Forecasting U.K. Population Mortality

Data 1961-2007 projected by 2\textsuperscript{nd} degree curve - meet in 2033?

Expectation of Life : Cohort exceeds Period by

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<th>LC</th>
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