Evolutionary Reserving Models
Are particle filters the way to go?

Agenda
- What is an Evolutionary Reserving Model?
- Why is it interesting?
- What is a Particle Filter?
- Are Particle Filters the way to go for Evolutionary Reserving?
- What next?
What is an Evolutionary Reserving Model?

Conventional model
Development patterns change over time – reserving cycles, management policy, demographic changes, economic cycles.

What is an Evolutionary Reserving Model?

Conventional model
Looking at changes through the valuations – the development pattern is spreading out.
What is an Evolutionary Reserving Model?

Evolutionary Reserving model
• Fits the changes in the past as well as predicting the future
• Each accident period has its own development pattern
• Changes smoothly from one accident period to the next
• Produces a probability distribution for the reserves.

Challenge: getting the “right” number of parameters
• Uses a small number of parameters to describe the development pattern
• The smoothing parameter balances responding to noise and missing changes in the accident direction
• The amount of smoothing determines the effective number of parameters in the accident direction.
What is an Evolutionary Reserving Model?

Process
1. Choose a parameter structure for the development pattern
2. Choose variance weights to give homogeneous variance of residuals
3. Choose amount of smoothing for each parameter
4. Check for patterns in residuals
5. Check distributional assumption is satisfied
6. Check smoothing appears reasonable.

Parameter structure – fitting to the first accident period

What is an Evolutionary Reserving Model?

Parameter structure – fitting to the first accident period

Log of Payment per Claim Incurred

Development period

Actual

What is an Evolutionary Reserving Model?

Parameter structure – fitting to the first accident period

Log of Payment per Claim Incurred

Development period

Actual

Fitted
What is an Evolutionary Reserving Model?

Parameter structure – fitting to the first accident period

Fitted values for five of the accident periods
What is an Evolutionary Reserving Model?

Fitted parameter 1 - two different levels of smoothing

Why is an Evolutionary Reserving Model interesting?

Benefits of Evolutionary Reserving

- Automate the response to changes in the data
- Allow more frequent reassessment of reserves
- Provide a description of the past to inform decisions on the likely future
- More robust forecasts – less sensitivity to last accident period
- Wider range of models that reflect the reality of change
- Qualitative!
Why is an Evolutionary Reserving Model interesting?

Costs of any reserving model
- Set-up time
- Actuary time
- Explaining to management time
- Software cost
- Quantitative!

Costs versus benefits

Benefit

- Evolutionary reserving
- GLMs
- Stochastic monitoring
- Bootstrapping
- Chain ladder

Cost
What is a Particle Filter?

Some history of Evolutionary Reserving

- De Jong and Zehnwirth (1983) – state space models, dynamic linear modelling, Kalman filter – normal and lognormal error
- Zehnwirth (1994) – varying parameter, dynamic or credibility models. Avoiding the problem of multicollinearity. Parsimony – better forecasts can be obtained by using the optimal number of parameters
What is a Particle Filter?

Some history of Evolutionary Reserving

- The Analytical Filter takes significant development time to understand and implement
- In some cases, the Analytical Filter might not be sufficiently accurate
- Greg Taylor suggested Particle Filters might be a more robust alternative way to fit parameters. This is where I came in.

What is a Particle Filter?

Some history of Particle Filters

- Particle Filters are a solution method, not a new model
- Terminology comes from Control Theory
- A fast method of solving equations with no analytical solution, where there is noisy information and the answer changes as more data comes in
- Many different versions of a basic algorithm
- Examples: tracking a missile, controlling a robot.
What is a Particle Filter?

How do Particle Filters operate?
1. Generate a sample from a probability distribution based on current information (called particles)
2. Calculate weights on each particle based on new information
3. Resample to get more particles in the area of highest likelihood
4. Generate a sample from these particles
5. Repeat from step 2.

Example
1. Initial particles are generated from Normal(0, var=0.2)
What is a Particle Filter?

Example

2. Each particle is weighted by the likelihood of the first sample value(s), given the particle value.

![Particle weights diagram](image)

Example

3. Particles are resampled according to the weights.

![Particle weights diagram](image)
What is a Particle Filter?

Example
4. A sample is generated from these particles – and so on

How do Particle Filters operate on triangles?
1. Start with an initial set of particles (values) that represent a best guess for the probability distribution of the parameters
2. The weight of each particle is its likelihood given the payments in the cells in the first accident period
3. Resample to get equally weighted particles
4. Add random noise to the particles
5. Repeat from step 2 with the next accident period.
What is a Particle Filter?

How do Particle Filters operate on triangles?

• Most applications only interested in the current state – history is not important. However, we want a full picture of the history to forecast for each accident year, and the future tells us something about the past
• Need an extra step – back-smoothing (terminology from the Kalman filter)
• Gives the full joint probability distribution of all parameters
• Can be used for simulating the aggregate losses.

6. Sample one particle from the particles corresponding to the last accident period
7. Use the likelihood of that particle to weight the previous accident period
8. Sample one particle from that weighted accident period
9. Repeat from step 7 with the previous accident period.
Are Particle Filters the way to go?

What did I do?

- Tried a mixture of simulated examples and real data – four of each, enough to get a reasonable assessment of the practicality of the method
- Compared the normal case with the Kalman filter solution
- Simple examples worked OK but as the number of parameters increased, accuracy decreased rapidly
- The particles couldn’t keep up with changes between accident periods.

Are Particle Filters the way to go?

What did I do?

With a 5 parameter normal model, the effective number of particles dropped severely from the initial 1000:
Are Particle Filters the way to go?

What did I do?
This affected the accuracy, even with 10,000 particles:

![Graph showing Parameter 1 vs Accident period for different particle counts and a Kalman filter comparison.]

- Tried a slightly different version – generate the initial particles from a similar problem but with lognormal error, then weight by the relative likelihoods of the original to the approximate problem
- This worked a lot better
- BUT the Analytic Filter and the Particle Filter sometimes gave different results, especially for the variance of the loss distribution.
Are Particle Filters the way to go?

What did I do?
How could I tell which one was right?

![Graph showing coefficient of variation of aggregate losses against parameter variance. The graph compares AF and PF methods.](image)

- Tried another method – direct maximisation of the likelihood (ML) using SAS non-linear optimisation routines
- Also tried Markov Chain Monte Carlo (MCMC) using SAS routines
- MCMC is much slower but is fairly reliable in producing a solution
- ML is not guaranteed to produce a solution, and can take a lot of iterations if the starting point is poor.
Are Particle Filters the way to go?

What happened?
- ML worked well – usually converged quite quickly
- ML agreed with the average of MCMC simulations
- ML produced a similar or better solution, based on the likelihood, than the Analytical Filter or Particle Filter
- Also tried R as a free alternative to SAS, and it worked as well as SAS
- Triangle with 6 development parameters and 26 accident periods takes less than 20 seconds to fit.

Conclusion
- Particle Filtering is probably not the way to go
- BUT there are lots of variations on the basic algorithm that might perform better – lots of scope for research
- Direct maximisation of the likelihood performs better on the examples tried so far and appears to be a practical method of modelling.
Are Particle Filters the way to go?

Conclusion
• Have only tried gamma process error so far, but it is likely that a wide range of process errors could be fitted this way.
• Simple to implement
• Has the potential to bring down the cost of stochastic modelling while increasing the benefits due to the wider range of models available.

What’s next?

Application
• Try it yourself – R is free and can read and write to Excel spreadsheets.
What’s next?

Further research

- Simple method of choosing a parameterisation of the development pattern and variance weights
- Diagnostics for goodness of fit and outlier detection
- Other error distributions – Poisson for counts, gamma mixture with binomial to model zeroes, normal mixtures.

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.