Double chain ladder with a touch of Bornhuetter-Ferguson

Agenda

- Introducing the problem: stochastic reserving
  - Current solution: chain ladder methods

- Motivating a model for the problem of stochastic reserving
  - Addressing the limitations of chain ladder methods

- Defining a model for the problem of stochastic reserving
  - Consistency with the chain ladder method
Agenda

- The double chain ladder estimation method

- New insights:
  - Estimating the tail
  - Separation into RBNS and IBNR
  - Introducing prior knowledge

- Simulation methods to obtain statistical distributions

- Conclusions

The individual claims mechanism

- The life of an individual claim in the general claims process:

- Three categories of claim:
  - Reported and settled
  - Reported but not settled, RBNS
  - Incurred but not reported, IBNR
The problem: stochastic reserving

• Outstanding liabilities are impacted by two types of delay during the claims process:
  – Reporting delay
  – Settlement delay

• Objectives:
  – Produce point forecasts for the outstanding reserve and cash flows
  – Produce accompanying distributions

Motivating a model for the chain ladder mean

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What is a method?

- A sequence of steps, specifically designed to produce particular results

- A method can be inflexible
  - It is hard to adapt it to deal with unsatisfactory results

- An example is the chain ladder method

The chain ladder method

- Current method for calculating loss reserves: chain ladder method (CLM)

- CLM in its most basic form suffers from three main drawbacks:
  - Unstable estimates
  - No information about the tail
  - Unable to separate RBNS and IBNR claims
What is a model?

- A mathematical framework that completely describes a real-life problem

- Translates a real-life problem into a language which we, as mathematicians, can understand and work with

- To apply to a specific data set, we also require an estimation method based on the model

Introducing the model: addressing limitations of CLM

- We will introduce a mathematical model which underlies the CLM

- Using this model we are able to:
  - Reduce the instability of the CLM in a natural way by introducing prior knowledge at a micro level
  - Automatically provide the tail
  - Separate into RBNS and IBNR claims

- With this model, we are creating a vehicle which can incorporate current actuarial techniques in a more natural manner
Summary

- The problem of stochastic reserving includes many dependencies
- These are implicit within the chain ladder method
- They will be made explicit in our model

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CL predictions for payments

Defining a model for stochastic reserving
The modelled data: two run-off triangles

- We model annual data triangles
  - Incremental aggregated payment data
  - Incremental aggregated counts data, which is assumed to have fully run off

Introducing index notation

- We index the data as follows:
  - Accident year, $i$
  - Reporting delay, $j$
  - Settlement delay, $l$
  - Development delay, $j$

- Note that $j = j' + l$
The parameters involved in the model

- **Accident year: \( \alpha_i \)**
  - Represents ultimate claim numbers

- **Reporting delay: \( \beta_j \)**
  - Represents the proportion of ultimate claims reported with \( j \) period delay

- **Settlement delay: \( \pi_l \)**
  - Represents the proportion of claims settled \( l \) years after being reported

The inflation parameters involved in the model

- **Inflation parameters**
  - \( \tilde{\mu}_{j,l} \) dependency on reporting delay and settlement delay
  - \( \gamma_i \) dependency on accident year

- **Individual claim payment mean** = \( \tilde{\mu}_{j,l} \times \gamma_i \)
The generality of the inflation parameters

- The inflation parameters can account for many dependencies, according to the choice of the practitioner
  - Dependence on the reporting delay: \( \tilde{\mu}_{j',l} = \tilde{\mu}_{j'} \)
  - Dependence on the settlement delay: \( \tilde{\mu}_{j,l} = \tilde{\mu}_l \)
  - Dependence on the development delay: \( \tilde{\mu}_{j',l} = \tilde{\mu}_{j'+l} \)

Deriving an expression for the mean

- Under our model the mean of the total of the incremental payments, for accident year \( i \) and development delay \( j \), is given by:

\[
E[X_{ij}] = \alpha_i \gamma_i \sum_{l=0}^{j} \beta_{j-l} \tilde{\mu}_{j-l,l} \pi_l
\]

- Is this consistent with the chain ladder method?
The chain ladder mean

- The chain ladder mean of the total of the incremental payments, for accident year $i$ and development delay $j$, can be formulated as:

$$ E[X_{ij}] = \tilde{\alpha}_i \tilde{\beta}_j $$

- $\tilde{\alpha}_i$ represents ultimate payment numbers
- $\tilde{\beta}_j$ represents the development delay

- For derivation of this result, see Mack (1991)

Rediscovering the chain ladder mean

- We impose the following relationships:

$$ \alpha_i \gamma_i = \tilde{\alpha}_i $$
$$ \sum_{l=0}^{j} \beta_{j-l} \mu_{j-l} \pi_l = \tilde{\beta}_j $$

- This ensures that our model has the same component structure as the one implicitly assumed by CLM
The double chain ladder estimation method

Introducing the double chain ladder method

- DCL is a method like CLM to produce estimations for the total of the incremental payments

- The classical chain ladder algorithm is applied twice to obtain estimates for all of the parameters in the model

- They can give the same value for the point estimates but DCL gives us more information
Over-parameterisation of the chain ladder mean model

- We aim to solve the problem using only two run-off triangles

Therefore, we have to restrict ourselves to:

\[ \tilde{\mu}^l_+ = \tilde{\mu}^l \]

- Given more data, this restriction may not be necessary

We rescale to obtain a constant mean:

\[ \mu = \sum_{l=0}^{m-1} \tilde{\pi}_l \tilde{\mu}_l \]

- \( \mu \) represents the mean of individual claim payments in the first accident year

We can now completely solve the problem

The parameters to estimate by DCL

- Ultimate claim numbers: \( \alpha_i \)
- Reporting delay: \( \beta_i \)
- Settlement delay: \( \pi_l \)
- Development delay: \( \tilde{\beta}_j \)
- Ultimate payment numbers: \( \tilde{\alpha}_i \)
- Severity inflation: \( \gamma_i \)
- Individual payment mean in first year \( \mu \)
The DCL method: estimating the parameters

• Apply CLM to **count data** from a toy example to get the estimates \( \hat{\alpha}_i, \hat{\beta}_j \).

<table>
<thead>
<tr>
<th>Estimated Counts</th>
<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
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</thead>
<tbody>
<tr>
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<td>( b_i )</td>
<td>( \hat{\alpha}_i )</td>
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<td>240.00</td>
<td>110.38</td>
</tr>
</tbody>
</table>

• Reminder:
  - \( \hat{\alpha}_i \) represents ultimate claim numbers in the \( i \)th accident period
  - \( \hat{\beta}_j \) represents the proportion of ultimate claims reported with \( j \) period delay

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The DCL method: estimating the parameters

• Apply CLM to the **payment data** to obtain the estimates \( \hat{\alpha}_i, \hat{\beta}_j \).

<table>
<thead>
<tr>
<th>Estimated Payments</th>
<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
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<td>7</td>
<td>2,500.0</td>
<td>1,629.0</td>
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• Reminder:
  - \( \hat{\alpha}_i \) represents ultimate payment numbers in the \( i \)th accident period
  - \( \hat{\beta}_j \) represents the proportion of ultimate claims that develop with \( j \) period delay

The DCL method: estimating the parameters

- Use the following relationships between the CLM estimates and the parameters to estimate the remaining parameters:

\[
\alpha_i \mu \gamma_i = \tilde{\alpha}_i \\
\sum_{l=0}^{j} \beta_{j-l} \pi_l = \tilde{\beta}_j
\]

- Reminder:
  - \(\pi_l\) represents the proportion of claims settled \(l\) years after reporting
  - \(\gamma_i\) represents the claims inflation in the \(i^{th}\) accident period
  - \(\mu\) represents the mean of individual payments in the first accident year

Solving the linear system gives the following values:

\[
\begin{array}{cccccccc}
\tilde{\pi} & 0.66 & 0.127 & 0.105 & 0.068 & 0.028 & 0.01 & 0.002 \\
\tilde{\gamma} & 1 & 0.967 & 1.311 & 0.951 & 1.055 & 1.168 & 1.033 \\
\tilde{\mu} & 15.28 & & & & & & \\
\end{array}
\]

- We’ve now estimated all the parameters, and can apply the formula derived from the model
Estimating the RBNS claims

- RBNS claims contribute to cells to the right of the paid data

- We predict RBNS reserve using estimated parameters and estimated count data from the upper triangle

- RBNS point prediction for cell (i,j): \( \hat{X}_{ij}^{rbs} = \sum_{l=0}^{\min(j,d)} \hat{N}_{i-j} \hat{n}_{l} \hat{n}_{i+j} \)

Worked example

- For illustration, we focus on payments in cell (1,11)

- RBNS estimation for (1,11) comes from reported counts in the previous six years:
  - We have chosen a maximum delay of six years
Consider the counts from six years ago – cell (1, 5)

Multiply by \( \hat{\pi}_6 \) which represents the proportion of claims for which a payment is made after six years

Gives an estimate for the number of claims reported six years ago that contributes to our cell (1, 11)

\[
2.523 \times \hat{\pi}_6 = 2.523 \times 0.0011 = 0.0028
\]

Proceed in the same way to find estimates for the number of claims reported four and five years ago that contributes to our cell (1, 11)

Sum to get the total estimate of the number of claims that contribute to (1, 11)
### Worked example

- We’ve estimated the total number of claims that contribute to (1,11) as 0.046

- Now we multiply by $\mu_t \times \gamma_t$, which represents the mean of claim payments which occurred in the first accident period

- This gives us our RBNS estimation for cell (1,11):

$$0.046 \times \mu_t \times \gamma_t = 0.710$$

### Estimating the IBNR claims

- Since the accidents are not reported yet, the IBNR reserves are derived from the lower triangle

- This fills in the paid triangle in the purple highlighted section:

#### IBNR point prediction for cell (i,j):

$$\hat{X}_{ij}^{ibnr} = \min(i - m + j - 1, d) \sum_{l=0}^{\min(i - m + j - 1, d)} \hat{N}_{(i-j-l)\mu} \hat{\gamma}_i$$
Worked example

• For illustration, we focus on payments in cell (3,11)

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<th>Estimated Counts</th>
<th>IBNR Estimates</th>
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<td>48.9</td>
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<td>110.4</td>
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</table>

• IBNR estimation for (3,11) comes from incurred but not reported counts in the previous six years:
  • We have chosen a maximum delay of six years

\[ 3.1 \times \pi_6 \]

\[ = 3.1 \times 0.0011 \]

\[ = 0.0034 \]

Worked example

• Consider the counts from six years ago – cell (3,5)

• Multiply by \( \pi_6 \) which represents the proportion of claims for which a payment is made after six years

• Gives an estimate for the number of claims reported six years ago that contributes to our cell (3,11)
Worked example

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<td>2.7</td>
<td>1.7</td>
<td>1.1</td>
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- Proceed in the same way to find estimates for the number of claims reported four and five years ago that contributes to our cell (3,11)
- Sum to get the total estimate of the number of claims that contribute to (3,11)

\[ 3.1 \times \hat{\pi}_6 + 1.9 \times \hat{\pi}_5 + 1.2 \times \hat{\pi}_4 = 0.056 \]

Worked example

- We’ve estimated the total number of claims that contribute to (3,11) as 0.056
- Now we multiply by \( \hat{\mu} \times \hat{\gamma}_9 \), which represents the mean of claim payments which occurred in the third accident period
- This gives us our IBNR estimation for cell (1,11):

\[ 0.056 \times \hat{\mu} \times \hat{\gamma}_9 = 1.122 \]
The estimated reserve: the chain ladder mean

Using the available information

- Currently, when calculating the RBNS, we use the formula:

\[ \hat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} \hat{N}_{i,j-l} \hat{\mu} \hat{\gamma}_l \]

which involves the estimated counts
  - This produces a result consistent with the CLM

- We could instead use the count data directly in this formula:

\[ \hat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} N_{i,j-l} \hat{\mu} \hat{\gamma}_l \]

- This leads to greater accuracy, since we are using actual count data rather than estimated counts
Predicting the tail through DCL

- With CLM, when a triangle has not run-off one needs to fit a tail
- DCL provides the tail prediction as an intrinsic part of the model

DCL tail

Paid data

CL prediction

DCL and introducing prior knowledge

- CLM (and therefore DCL) provides a prediction for the reserve which is heavily dependent on the figures in the bottom left of the triangle

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- The estimators from CLM seem to be unstable
- Methods such as the Bornhuetter-Ferguson method propose to improve the estimates for recent accident periods by incorporating prior knowledge
Locating the source of the instability

- The model breaks down the chain ladder estimates into their individual components
  \[ \tilde{\alpha}_i = \alpha_i \gamma_i \]
- The instability comes from the estimation of the severity inflation

Looking for information in the incurred data

- The proposed solution:
  Take a more realistic estimation of the inflation from the incurred triangle using BDCL (Bayesian Double Chain Ladder)
An example with real data

- We consider a liability dataset consisting of three triangles: payment, counts and incurred data.
- Apply DCL estimation method to obtain point forecasts for future calendar years.
- Total reserve estimated at approximately £14 million.

Comparison of inflation estimates

- The instability within the paid data can be seen in the estimates for the inflation in the last 2 accident years.
- The estimates from the incurred data are more stable in the final accident periods.
Using BDCL to obtain a more realistic reserve

- DCL reserve using estimates for inflation from the paid data
- BDCL reserve using estimates for inflation from the incurred data
- The total reserve is 13% lower using the incurred data to estimate the inflation

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<th>Future</th>
<th>BDCL</th>
<th>DCL</th>
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</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>10,893,235</td>
<td>1,561,518</td>
</tr>
</tbody>
</table>

The full statistical model
Obtaining a distribution

• So far we have only discussed point estimates of the individual payments

• We have at no point mentioned anything about the variance or the distribution of the reserve estimations

• Now we will discuss how the introduction of a model allows us to obtain full distributions based on our model assumptions

Parameters and distributions

• We will only introduce a single new parameter: the variance of the individual payments

• The following statistical distributions are assumed for each of the components in the model:

<table>
<thead>
<tr>
<th>Component</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count data</td>
<td>Poisson</td>
</tr>
<tr>
<td>Settlement delay</td>
<td>Multinomial</td>
</tr>
<tr>
<td>Individual payments</td>
<td>Gamma</td>
</tr>
</tbody>
</table>
Estimates for simulation

- We already have estimates for many of the parameters
  - Only need to estimate $\hat{\sigma}^2$ via the method of least squares
- Now we have all the information we need to simulate the data
- We derive empirical distributions of:
  - The cash flows
  - The total reserve

Empirical illustration

- Consider the following results produced from a motor dataset

<table>
<thead>
<tr>
<th>Simulated predictive distribution from BDCL</th>
<th>RBNS ('000s)</th>
<th>IBNR ('000s)</th>
<th>Total ('000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>97,508</td>
<td>9,127</td>
<td>106,635</td>
</tr>
<tr>
<td>5D</td>
<td>18,776</td>
<td>5,429</td>
<td>21,804</td>
</tr>
<tr>
<td>0.50%</td>
<td>61,165</td>
<td>1,221</td>
<td>65,882</td>
</tr>
<tr>
<td>1%</td>
<td>62,110</td>
<td>1,943</td>
<td>69,645</td>
</tr>
<tr>
<td>5%</td>
<td>70,856</td>
<td>2,908</td>
<td>76,602</td>
</tr>
<tr>
<td>10%</td>
<td>76,141</td>
<td>3,700</td>
<td>81,728</td>
</tr>
<tr>
<td>25%</td>
<td>85,040</td>
<td>5,401</td>
<td>91,913</td>
</tr>
<tr>
<td>50%</td>
<td>95,383</td>
<td>7,886</td>
<td>103,781</td>
</tr>
<tr>
<td>75%</td>
<td>107,979</td>
<td>11,661</td>
<td>119,122</td>
</tr>
<tr>
<td>90%</td>
<td>120,950</td>
<td>15,603</td>
<td>134,064</td>
</tr>
<tr>
<td>95%</td>
<td>130,938</td>
<td>19,248</td>
<td>146,686</td>
</tr>
<tr>
<td>99%</td>
<td>152,070</td>
<td>26,404</td>
<td>171,998</td>
</tr>
<tr>
<td>99.50%</td>
<td>165,542</td>
<td>32,460</td>
<td>183,404</td>
</tr>
</tbody>
</table>
Conclusions

- The chain ladder model is a solid framework for loss reserving
- Provides a natural method for introducing prior knowledge
- Intrinsic tail estimation
- Separates RBNS and IBNR reserves
- Gives distribution forecasts as required by Solvency II
- Does not rely on proprietary software
References

• Martinez-Miranda, M.D., Nielsen, J.P. and Verrall, R. (2011) Double Chain Ladder. Revised and resubmitted to *ASTIN Bulletin*
