Plan for Talk

- Introductory remarks
- The Cairns-Blake-Dowd (CBD) model
  - Pros and cons
  - Assessment criteria
  - Extension to include a cohort effect
  - Backtesting

Introduction – CBD Model

- Model designed for:
  - Annuities and pensions – longevity risk
  - Not for short-term mortality risk

- Model for mortality at higher ages

- CBD model:
  - exploits relative simplicity of mortality curve at higher ages
  - Not designed for lower ages
Introduction

- Pensions e.g. 30 year old

  - Uncertainty in value of deferred annuity is mostly affected by post-60 mortality

  - Model for mortality below age 60 is relatively unimportant
    - E.g. \( \text{Prob}(\text{Survival to age 60}) = 0.96 \) with St.Dev. 0.005

Background

- Part of wider LifeMetrics research programme
  - Comparison of 8 models
  - Within sample fit
  - Out of sample performance/backtesting

  - Development of new models

- Focus here on specific models we have developed
Introduction

- Why do we need stochastic mortality models?
  - Data => future mortality is uncertain
- Good risk management
- Setting risk reserves
- Annuity contracts with embedded options
  - E.g. guaranteed annuity options
- Pricing and hedging longevity-linked securities
  - E.g. q-forwards
- Many models to choose from:
  - Limited data => model and parameter risk

Measures of mortality

- $q(t,x) = \text{underlying mortality rate:}$
  - in year $t$ at age $x$
- $m(t,x) = \text{underlying death rate}$

- Poisson model:
  Actual deaths:
  $D(t,x) \sim \text{independent Poisson}(m(t,x)E(t,x))$
  $E(t,x) = \text{central exposed to risk}$

Need good mortality forecasting model

- ‘Process-based’ models
  - Model process of dying
    - Not used much yet
- ‘Explanatory’ or ‘causal’ models
  - Model causes of death
    - e.g. heart disease or socio-economic factors
    - Not used much yet, but post-code modelling more common
- ‘Extrapolative’ projection models
  - Will only be reliable if the past trends continue:
    - Medical advances can invalidate extrapolative projections by changing the trend
### Models

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Lee-Carter (1992) LC</td>
<td>( \log m(t, x) = \beta_2^{(1)} + \beta_2^{(2)} \kappa_t )</td>
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<tr>
<td>Cairns et al. (2007) CBD-2</td>
<td>( \log m(t, x) = \kappa_t^{(1)} + (x - \bar{x}) \kappa_t^{(2)} + \left( (x - \bar{x})^2 - \sigma^2 \right) \kappa_t^{(3)} + \gamma_{t-2}^{(4)} )</td>
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### CBD-1 fit at higher ages

![Graph showing CBD-1 fit at higher ages](image)

### Model Notation

- Beta\((x)\) terms => Age effects
- Kappa\((t)\) terms => Period effects
- Gamma\((t-x)\) terms => Cohort effects
Main extrapolative models:
Philosophical differences

- Lee-Carter model:
  - No smoothness across ages or years

- CBD model:
  - Smoothness across ages in same year

- P-splines model:
  - Smoothness across years and ages

How to compare stochastic models

- Quantitative criteria
  - Log-likelihood; BIC
  - Pattern of standardised residuals (i.i.d. ???)

- Qualitative criteria
  - Robust relative to age and period range
  - Biologically reasonable
  - Forecasts are reasonable

- Suitability for specific applications

Models - LC

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</table>
Lee-Carter Model

**Pros:**
- Robust
- Simple one-factor model
- Good fit over wide age ranges

**Cons:**
- Lack of smoothness of age effect (esp. small populations)
- Cannot cope with improvements at different ages at different times
- Tendency to use only very recent data
- Possible underestimation of uncertainty
  - $\beta_x$ affects both trend and uncertainty at age $x$
  - Cannot decouple
- One-factor model
  - Perfect correlation across ages
- No cohort effect

Models – CBD-1

Lee-Carter (1992) LC

$$\log m(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$$

Cairns-Blake-Dowd (2006) CBD-1

$$\log q(t, x) = \kappa_x^{(1)} + (x - \bar{x}) \kappa_t^{(2)}$$

Cairns et al. (2007) CBD-2

$$\log q(t, x) = \kappa_x^{(1)} + (x - \bar{x}) \kappa_t^{(2)} + \left( (x - \bar{x})^2 - \sigma_x^2 \right) \kappa_t^{(3)} + \gamma_{t-1}^{(4)}$$

CBD-1 Model

**Our first model independent of LC**

Why?
- Pensions
- High ages
- Simple models

**Pros:**
- Robust
- Two correlated factors: level and slope
- Allows different improvements at different ages at different times
- Simple age effects
- Easy to incorporate parameter uncertainty

**Cons:**
- No cohort effect
- Good at big picture but overall fit not as good as LC
- LC better able to pick up small non-linearities in mortality curve
Residuals LC & CBD-1
• Violation of indep. Poisson assumption

Communicating risk:
(Cohort) Longevity fan chart for 65-year old males

(Cohort) Survivor fan chart for 65-year old males in 2003
Inclusion of parameter uncertainty

Cohort effect:
Black line: 1930 cohort

Models – CBD-2

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<td>Cairns-Blake-Dowd (2006) CBD-1</td>
<td>( \log \gamma(t, x) = \kappa_4^{(1)} + (x - \bar{x}) \kappa_4^{(2)} )</td>
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<td>Cairns et al. (2007) CBD-2</td>
<td>( \log \gamma(t, x) = \kappa_4^{(1)} + (x - \bar{x}) \kappa_4^{(2)} + \left( (x - \bar{x})^2 - \sigma_x^2 \right) \kappa_4^{(3)} + \gamma_{t,x}^{(4)} )</td>
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CBD-2 Model

- Developed to address deficiencies of earlier models (LC and CBD-1)
- Builds on pros of earlier models
- Key advance builds on Renshaw-Haberman
- Several cohort extensions investigated: CBD-2 model was best in terms of balance between goodness of fit, parsimony and robustness

Are standardised residuals iid?

CBD-1 versus CBD-2
CBD-2 extra terms

Backtesting
- Forecasts of 2004 mortality rates
- Fixed forecast date 2004
- Data: 1960-1980
  - Forecast for 2004
- Data: 1961-1981
  - Forecast for 2004
- ...
- Data: 1973-2003
  - Forecast for 2004
Expanding horizons

- Data from 1960-1980
  - Forecast for 1985
- Data from 1961-1981
  - Forecast for 1986
- Data from 1962-1982
  - Forecast for 1987
- etc.

CBD-1: Rolling 5-yr ahead prediction interval
Age 85, 75, 65

<table>
<thead>
<tr>
<th>Year</th>
<th>Age 85 (xL, xM, xU, n)</th>
<th>Age 75 (xL, xM, xU, n)</th>
<th>Age 65 (xL, xM, xU, n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>[1, 10, 0, 19]</td>
<td>[1, 16, 0, 19]</td>
<td>[10, 20, 0, 19]</td>
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</table>

Conclusions

All models had difficulty in capturing the change in trend

Crude projections based on data up to 1980
Conclusions

- Results between models are reasonably consistent
- Backtesting:
  - No model emerges as obviously better
  - Eg general year-on-year noise swamps subtlety of cohort effect
- Revert to other criteria:
  - Quantitative and qualitative
- Recapitulation:
  - CBD-2 is a good, robust model for higher ages
  - CBD-1 good at modelling the bigger picture
  - Alternatives or adaptations needed for lower ages

References

- Cairns, Blake and Dowd (2006)
  A two-factor model for stochastic mortality: Theory and calibration.
  J. Risk and Insurance, 73: 687-718
- Cairns, Blake, Dowd, Coughlan, Epstein, Ong and Balevich (2007)
- Cairns, Blake, Dowd, Coughlan, Epstein and Khalaf-Allah (2008)

See:
- http://www.ma.hw.ac.uk/~andrewc/papers/
- http://www.pensions-institute.org/papers.html