Preliminary Report of the

CAPITAL ALLOCATION WORKING GROUP

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1. EXECUTIVE SUMMARY

- Capital enhances security, for which a higher premium may be charged.

- Risk can be fully characterised using a statistical distribution.

- Capital Allocation and Capital Setting are key to good management in the insurance industry.

- Current risk measurement techniques, which essentially collapse or summarise the risk distribution, are inadequate.

- Good capital allocation methods, best suited to different parties with different perspectives on risk control, already exist.

- Capital allocation and capital setting lie at the heart of risk transfer, providing a common basis for the insurance and banking industries.

- Convergence between insurance and banking is proceeding, but differences persist (and can be expected to do so for some time).

- Risk management and capital management are basically a *means* of achieving a profitable return.

- The “risk : return” relationship is captured by the investment and reinsurance markets.

- (Re)insurers should diversify within their sectors, as investors may not be able to do so, simply through asset allocation.

2. PURPOSE OF PAPER

This is a preliminary report. In presenting this paper we seek to:

- describe the foundations of capital allocation;
- identify and discuss the perspectives of interested parties;
- begin to discuss the key issue of balancing risk and return;
- describe and critique some commonly used methods; and
- suggest what future research is required.

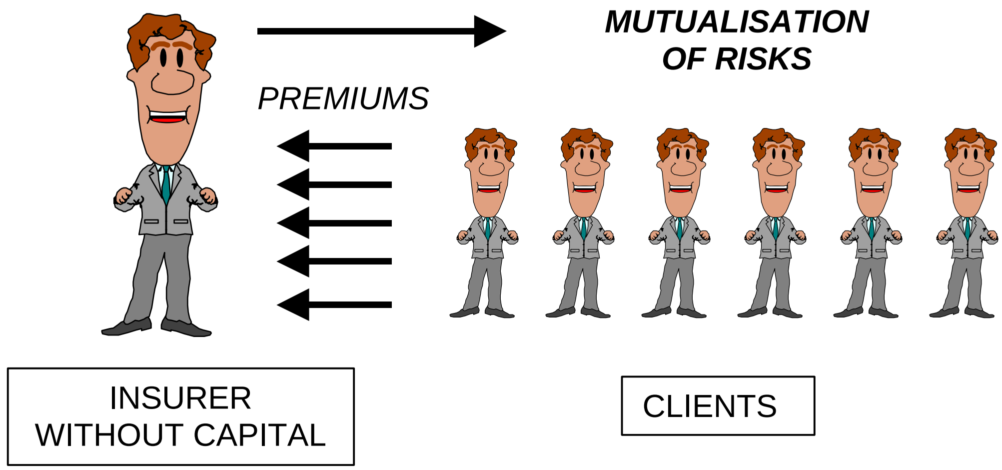
We also hope that this paper will act as a simple guide to the subject, for newcomers, and as a basic reference for practitioners.

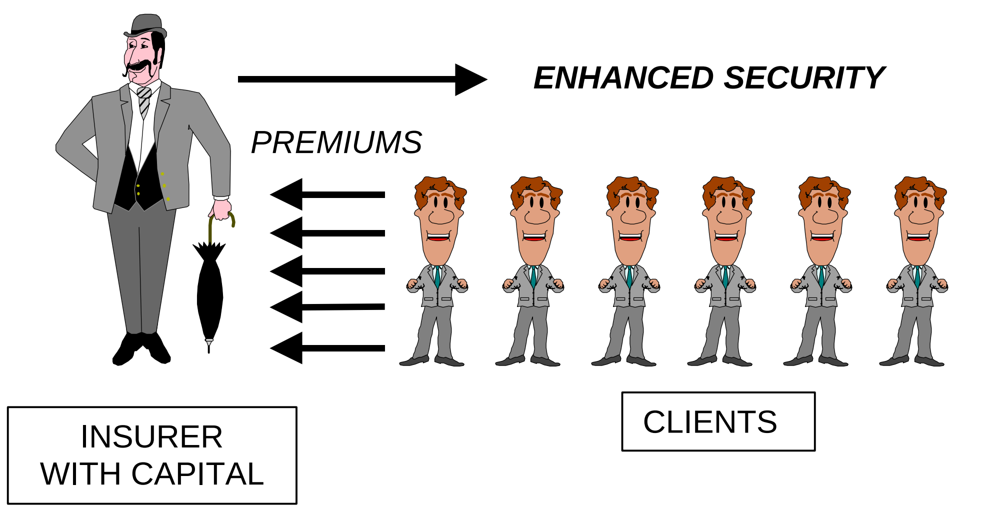
We have tried to produce useful definitions for some of the terms relating to capital allocation. We do not claim that these are perfect or deserve the label "definitive", but hope that they will provide a useful foundation from which discussion can proceed.

3. SETTING THE SCENE

3.1 Why Add Capital?

Capital enhances security. A rich individual can more readily recover from loss than others and might offer to act as a "guarantor" to a less wealthy person. Of course, a community group or society might pool their resources to help an unfortunate member recover from a loss. However, the security that such "mutualisation" of risk can offer will never be greater than the total resources committed by the group.





Enhanced security provides a more valuable insurance product, for which a higher premium can be charged, or more stringent terms imposed. The importance of security, in differentiating between market players, is borne out by the high profile role of rating agencies in the market.

3.2 Definitions

Capital Allocation

The hypothecation (notional *distribution*) of the capital or free assets of an insurer between specific insurance operations, whether by sector, underwriter, individual contract or source of risk. Conceptually, this may be expressed as:

OPERATION	RISK	CAPITAL
1	RISK _1	CAPITAL_1
2	RISK _2	CAPITAL_2
3	RISK _3	CAPITAL_3
4	RISK _4	CAPITAL_4

CAPITAL ALLOCATION

Capital Setting

The determination of the *total* amount of capital or free assets required by an insurer to support the writing of business at an acceptable level of risk (whatever that may mean). Conceptually, this may be shown as:

OPERATION	RISK	CAPITAL
1	RISK_1	CAPITAL_1
2	RISK _2	CAPITAL_2
3	RISK _3	CAPITAL_3
4	RISK _4	CAPITAL_4

$\Sigma \longleftarrow$ **CAPITAL REQUIREMENT**

(Monetary) Return on Underwriting Operations

The excess of funds remaining once an underwriting result has fully developed, determined as the sum of the premiums received plus investment income received on the premiums (the insurance fund), less expenses and claims. Where capital is used to enhance security, the investment return on the allocated capital may also be included. Conceptually, the return on underwriting operation “1” may be shown as:

RETURN_1

= PREMIUM_1

+ INVESTMENT_RETURN_ON_INSURANCE_FUND_1

+ INVESTMENT_RETURN_ON_CAPITAL_1

– EXPENSES_1 – CLAIMS_1

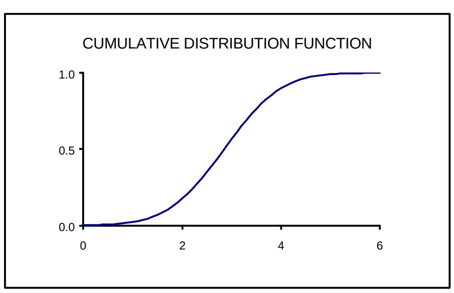
(Percentage) Return on Capital (ROC)

Dividing the monetary return on underwriting operations by the amount of capital put at risk under the capital allocation process will give the return on capital figure. For operation A, this may be shown as:

ROC_1 = RETURN_1 / CAPITAL_1

Risk

A loss, the occurrence of which is uncertain. Although this is a useful definition for understanding what risk is, a numerical representation is required as a basis for capital allocation. Here, we can characterise RISK_1, for underwriting operation “1”, as:



$F(X_1)$

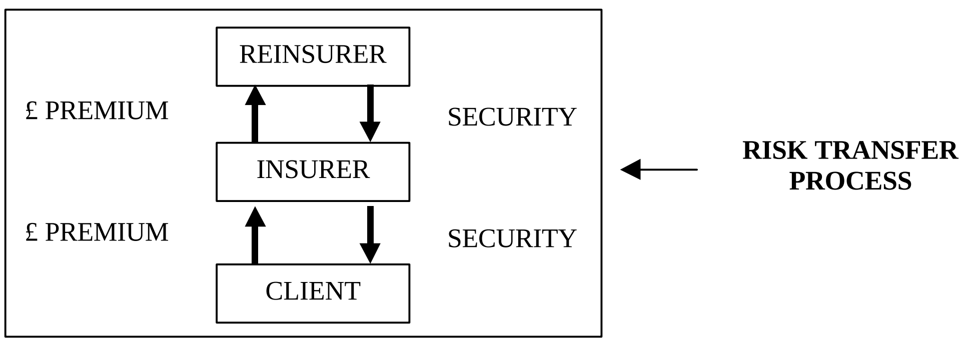
So, if the losses resulting from underwriting operation “1” are described by the random variable X_1 then “RISK_1” can be described by the above graph of the cumulative distribution function of X_1, $F(x_1)$ = Probability that X_1 is less than or equal to x_1.

Risk Management

The assessment, mitigation, transfer and retention of risk. From this definition, it can be seen that insurance and reinsurance are risk management "tools", and that capital allocation is an aspect of the retention of risk, which is part of risk management. It is useful, at all times and for all parties, to refer back to the broader consideration of risk management.

Risk Transfer

Passing risk (an uncertain loss) to another party, often in return for certainty of payment. This process lies at the heart of insurance and reinsurance. Parties who are either unwilling or unable to bear a risk can swap the uncertainty of a loss for the certainty of a cost in rewarding the new bearer (insurer or reinsurer) of the risk. In essence, the insured or reinsured is buying security.



3.3 Simplifying Assumptions

The Aggregate Loss Distribution

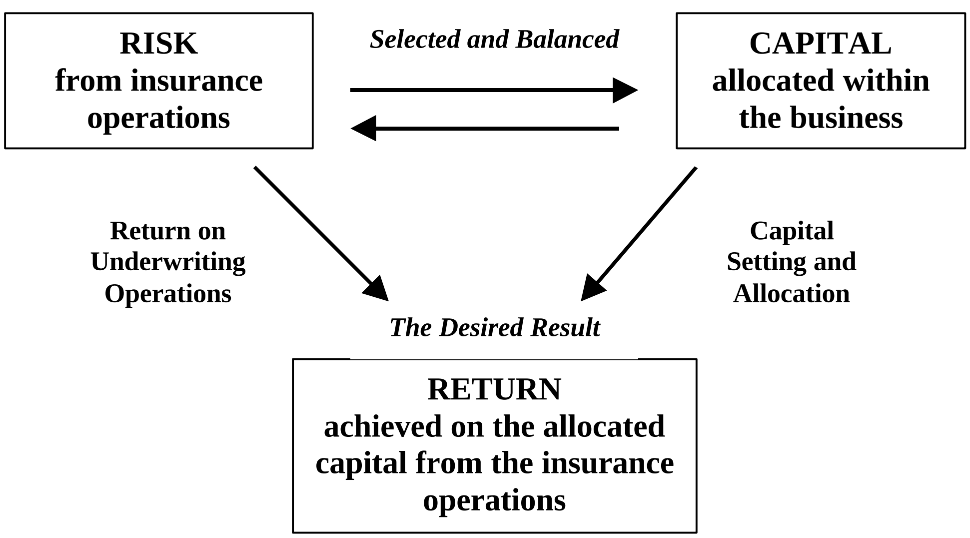
All capital allocation is built upon a foundation of the knowledge of the risks written by an insurer. It could be argued that complete knowledge of the individual risks, corresponding to each business segment that has capital allocated against it, and of the sum of those risks, the "Aggregate Loss Distribution" (ALD), is essential before any allocation can be made.

However, generally, this information is not available and it is common for working assumptions to be made in carrying out a capital allocation exercise. These assumptions are based on estimates of likely loss distributions and of correlations between the business segments, allowing an assessment of the likely ALD.

The proper measurement and expression of the risks underlying insurance business is a crucial challenge for the industry. However, for the purposes of this paper, we shall assume complete knowledge of both the individual risk segments and of the ALD. It is our hope that, by discussing how the risk information will be used, we can contribute towards an improved understanding of the form and quality of information that will be required to support the allocation of capital.

The Interdependence of Risk, Capital and Return

Much discussion within the working group concerned the elements of an insurer's business that can be controlled and those that can not. Although the following framework did not conclude the debate, it may be useful in explaining the context of much that follows.



As can be seen above, we have viewed RISK and CAPITAL as "means" and RETURN as an "end". However, this view does not ignore the problem that an *appropriate* target rate of return will very much depend on expectations regarding the business that will be underwritten and the amount of capital within the business.

It is fair to say that any one of these three aspects of insurance business can be fixed or varied, depending on the perspective and aims of the party concerned.

The Time Dimension

At any point in time, the underwriting results from many insurance contracts, written over a number of years, may still be developing. Therefore, business written over a number of different underwriting years may present an underwriting loss, which will lead to a call on the capital allocated to the business. Moreover, the same capital may be "at risk" in respect of a number of underwriting years at exactly the same time.

The working group has spent a great deal of time discussing how this aspect can be incorporated within the capital allocation process. For now, we have decided that:

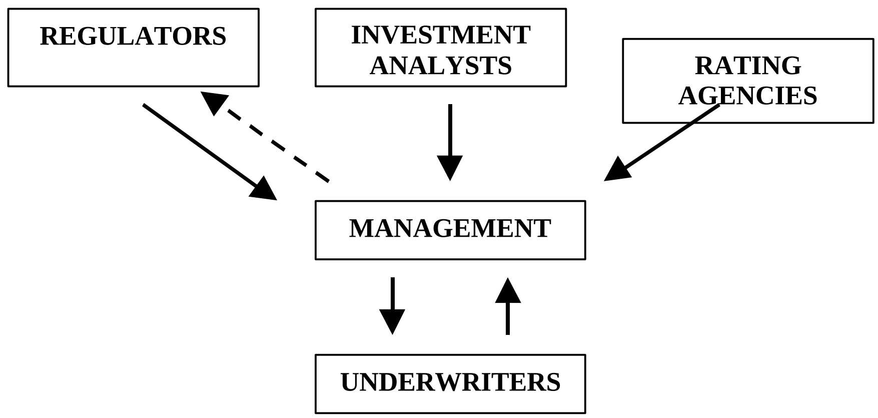
- individual underwriting years may be considered in isolation;

- capital can be allocated solely on the basis of the risks implicit in the business written during that underwriting year; and

- the capital (or free assets) to be allocated, is that available at the start of the underwriting year, having allowed for calls (i.e. significant underwriting losses) arising from all previous underwriting years.

4. INTERESTED PARTIES

This chapter discusses five main groups who have an interest in capital allocation. They each have a role to play within the (re)insurance market:



Each party is subject to a great number of external and internal influences, but we shall restrict our comments to the main ones shown above. Management can be seen to be the hub of many influences and we discuss its position and interests first. For simplicity, we have restricted our comments to insurance companies, but similar comments will apply for the reinsurance market.

4.1 Management

By "management" we refer to both the executive function (providing leadership and guiding strategy) and the operational management functions (implementing the business plan to achieve the strategic aims).

The balancing of risk and return is embedded within management. Management in all industries has been making decisions about risk and return since the beginning of trade and commerce. All business issues and opportunities revolve around these two aspects.

Moreover, there are many facets to risk: operational, management, investment, business development, reputation, political and employment, to name but a few. Having said all this, although risk is a critical issue, management is fundamentally concerned with achieving a return and we should not focus solely on risk, to the detriment of our overriding strategic aims.

Regulators control entry and participation in the market. Management needs to satisfy the regulators that they have an adequate capital basis, appropriate personnel and transact business appropriately. From a capital allocation perspective, the regulators will be concerned to see that capital is optimally used within the business to ensure security for policyholders.

Investment analysts will influence what levels of risk are acceptable and what rate of return is required. Later, we discuss the influence of the investment market more fully, in defining the "risk : return" relationship.

As well as serving shareholders (whose views we take as being represented by the investment analysts) management will also have personal incentives, which link performance to share price.

The investment analysts will try to place a value on the future proceeds of the business. Their focus will be on the optimal use of capital and a sophisticated approach that suggests a competitive advantage may be well received by the investment community.

Rating agencies play a crucial role in the market. Reputation and security are often the two most important issues for a prospective customer, in making a risk transfer purchase. Management will need to be able to demonstrate to the rating agencies that they are achieving maximum security for a given level of capital and target rate of return.

Underwriters need to be given appropriate guidelines and incentives to help them achieve aims that are consistent with the company's. As the capital allocation process will have a significant effect on the freedom that underwriters enjoy, it should be transparent, rational and practical (in terms of theoretical and administrative simplicity). Ideally, the capital allocation system should be objective. However, for relationship management and other reasons, some flexibility may be essential for the system to receive the support of underwriters.

More importantly, the system needs to be robust, so that it can capture and reflect different risk profiles, regardless of the "non risk" aspects of the business that is transacted. For example, in a "soft" market, premium rates may fall, despite increases in exposure. Some independent measure of risk would be invaluable in this environment. However, this is not a trivial matter!

So, management will be looking for a realistic, transparent system, which can be implemented in the existing culture (and, probably, with existing systems). The support and acceptance of staff (particularly as it affects underwriting guidelines, performance measurement and incentives) will be essential. Also, ideally, there should be consistency between business lines, so the capital is effectively employed in the business as a whole.

4.2 Underwriters

Underwriters are essentially transaction-based. Success is judged by business quality (whether measured by volume, profit or value). Most underwriters operate in the market and peer views and pressures have a significant impact on their behaviour.

Management will impose guidelines and define performance measures and incentives for the underwriters. Capital allocation may be used to help focus aims and it could be argued that this is the best means of focussing underwriters' attention on returning the best results for shareholders.

However, with its foundation in risk, which is (historically) a secondary issue (to price) for underwriters, the relevance of capital allocation may take some time to gain general acceptance. This cultural barrier should not be ignored when developing a capital allocation system.

Underwriters will look for consistency, transparency, simplicity, efficiency and, above all, a rational basis for the constraints and guidelines imposed on them.

4.3 Regulators

Regulators are concerned with preserving a market, to attract and retain good players, for the benefit of consumers, industry and commerce. They need an efficient system of control, to keep their costs down and impose the minimum burden on companies.

However, when push comes to shove, regulators are most concerned with avoiding insolvencies in the market. The optimal use of capital to avoid insolvency (whether through adequate backing for risky business, or through restricting the amount of risk that a company takes on) is critical to regulators.

So, regulators will look for a transparent, rational, generic(ish), consistent (both temporally and between insurance sectors), conservative and objective system, which is capable of audit.

However, as suggested above, the management of insurance companies can put pressure on regulators, to reduce information and capital requirements. For this reason, it seems unlikely that regulators will see a capital allocation system that meets their needs, without the general support of management within the insurance industry.

4.4 Investment Analysts

As discussed above, investment analysts will look for a sophisticated, competitive and rational approach to capital allocation. However, they will probably not play as important an influencing role as underwriters and regulators, in finding the balance between freedom and control. Their role, in defining the “risk : return” relationship, is discussed in the next chapter.

4.5 Rating Agencies

Rating agencies, as for regulators, are concerned with security. However, they are not just concerned with insolvency and will look to see a balance between optimal use of capital to grow the business and to protect its solvency.

For a rating agency, a capital allocation system should be capable of review (they have to rate all companies in the market), so consistency with other systems is preferable. Also, a subjective assessment needs to be made, so consistency and an objective basis for measuring risk will be important.

4.6 Conclusions

No system can be all thins to all people. However, the above discussion highlights the need for a universally applicable means of measuring risk and the agreement of standards by which capital allocation systems can be judged.

5. KEY ISSUES

5.1 Role of Capital in the Wider Context and Target Rates of Return

This section compares the role of capital in insurance with that in other industries and discusses how to ascertain a target Return on Capital (ROC).

The role of capital in an insurer is different from most other industries. Capital is needed to support business once it has been written. This contrasts with, say, the construction industry, which needs capital to purchase raw materials before sales are made. The costs of raw materials are reasonably well known (or a quantity surveyor can tell you). Capital allocation exercises attempt to ascertain the cost of supporting insurance business *once it has been sold*.

5.2 Risk and Reward

The arguments all centre on risk and reward.

We have already defined risk as a loss, the occurrence of which is uncertain. We can use a number of measures (all of them "summary" measures, given our assumption that a risk can be fully characterised using a statistical loss distribution) to describe risk, such as: expected policyholder deficit, probability of ruin or standard deviation.

The reward for accepting risk, is return, which we may loosely define as "a gain, the occurrence of which is uncertain". It is rational to assume that the expected return offered to an investor should be higher for taking a greater risk.

The "risk : return" relationship is borne out in the investment markets, where risk-free rates of return (say on government bonds) are lower than those on unsecured debt. However, it is not always true that such clear-cut comparisons can be made in insurance.

From a shareholder's perspective (we generally assume that we are dealing with companies that have permanent share capital) the ideal amount of capital would be zero. There would therefore be no potential downside. Such an extreme case would approximate to a license to print money.

However, regulators (reflecting demand from policyholders) require that companies hold capital. Probably more important than this, we need to recognise that the global insurance (and especially reinsurance) industry is one where reputation and security ratings are almost everything.

Analysts and rating agencies will take account of capitalisation in assessing a company. Without a positive assessment, a company will be unable to write any (profitable) business. Further, a higher rating does generally lead to higher premiums.

Having said this, there is a danger of going too far in the opposite directions, and holding too much capital. Over-capitalisation may lead to tax and expense inefficiencies, and management may be more concerned with ancillary issues (e.g. acquisitions) than running an insurance business.

5.3 How do we Quantify Risk and Reward?

We first need to be clear what our measures of risk and return are.

It would tend to be true that required capital is established from regulators', policyholders' and rating agencies' viewpoints. The owners of the company may also have a say in this, such as targetting a triple-A rating. The question is then, how much risk is represented by the business and what return is required?

Because, ultimately, return goes to shareholders, we could use a shareholders' risk measure, such as the standard deviation of returns. This may be adjusted to allow for "utility" (e.g. being equally keen to avoid a small loss as to make a large gain), either explicitly or implicitly, as with the proportional hazards transfer described later.

Other parties' interests may be captured in other risk measures:

- Policyholders, Regulators and Rating Agencies may look to the "Expected Policyholders Deficit" (see next chapter);

- Regulators, Management and Staff may look to the "Probability of Ruin".

The need to consider each party's risk tolerance (both its form and quantity) should be part of the basis of development of any capital allocation/setting system.

According to traditional financial market theory, we could derive an "efficient frontier", consisting of portfolios of assets that minimise risk for a given level of expected return. The potential investment in the insurer would then be compared with this efficient frontier.

The efficient frontier is defined in relation to risk-free assets and the market's loading for riskiness (the "risk premium"). So, we could look to the market to prescribe the required target return for a given level of risk.

In the remainder of this section we shall discuss the applicability of CAPM and the extent to which the management of an insurance company should diversify risks and smooth cashflows.

5.4 Should Insurers try to Minimise Volatility?

We assume in general that we are dealing with companies that are supported by permanent share capital, and also ones where shares are actively traded.

Without permanent capital (such as Lloyd' syndicates which are currently backed by venture capital on an annual basis) different issues come into play, concerning equity between subsequent generations of capital providers.

Where a company is owned by just one investor (such as Berkshire Hathaway owning General Re) different investor aims come into play – standard financial theory (including CAPM) does not apply, and you just ask your owner what he wants!

Some argue that insurers should concentrate on what they do best – underwriting insurance – and leave diversification and smoothing of returns to investors. This would mean presenting a risk/return profile to the market that included both diversifiable and undiversifiable risk. The idea, then, is that investors use this stock as one in their portfolio so they can remove diversifiable risk.

However, there are some strong arguments for insurers attempting to smooth their profit flow:

- The ability to manage the cashflows of a company competently will give investors greater confidence in the management. This is also to do with management of expectations and communication – insurers should try to ensure that investors and analysts understand the nature of the risks they are running.

- It is easier for investors to ascertain correlations between sectors than between companies. It is therefore helpful if a company in a sector (e.g. insurance) diversifies its own exposures as much as possible. This goes some way to removing the diversifiable risk, and is a more efficient way of doing it than if the investors attempt to. The result is an investment where it is clearer what the risks are – the market likes transparency. As an example of investors' current views on (re)insurance stocks, the Financial Time on 15 May 1999 described the traded Lloyd's vehicles as "one of the most arcane corners of the London Stock Exchange".

- Some diversification may not be possible by investors. For example, precise geographical diversification is likely to be very difficult, because most insurers will not write exclusively in one country. The access and ability of the companies to diversify their own exposures, as well as their expertise to do so, will be far greater than external investors outside the insurance market.

- In an industry where reputation and ratings are hugely important, it is a fact that the analysts who assess these factors do take into account the volatility of an insurer's profit flows relative to its peers.

5.5 The Circularity of it All

There is no clear-cut split between the investors' decisions on capital allocation and insurers' choices. In discussing macro-level risk/return arguments, we cannot get away from "micro" arguments focussing on specific insurance-related issues.

- Shareholders decide how much of their money to invest in an insurance company and what return they require. An insurer decides how much of its capital it wishes to invest in a particular project (e.g. a reinsurance contract) and what return it requires. Both decisions are related, but operate at different levels of scale.

- Diversification can be done by either the insurer internally or by the investors in that insurer, in their portfolio, to the extent discussed in 5.4.

- Reinsurance is a proxy for capital and, conversely, more capital means less reinsurance is needed. This parallel between the use of "outwards" reinsurance and capital management gives us the opportunity to compare the two. For example the cost of capital could be compared with the profit margin charged by reinsurers, and the frictional costs of moving and raising capital with the frictional costs of buying reinsurance (e.g. brokerage). In this case, the reinsurance "market" will influence the "risk : return" relationship and consistency with investment market perceptions is to be expected.

It is a fact that the (re)insurance industry is not seamless:

- Brokerage is substantial;

- Expenses are high;

- Maintaining relationships is essential and some apparently non-optimal business decisions are often made;

- There are often barriers to entry and restrictions to competition (e.g. the US surplus lines system).

These factors, together with uncertainties in pricing many lines (especially reinsurance) mean that the (re)insurance market is not a commodity market. Because of the arguments above, this observation flows through to *investment* in the (re)insurance market.

A lot of arguments from financial theory depend on the assumptions about the homogeneity of investors and that we are dealing with a commodity market. These assumptions do not hold here.

It is also not an arbitrage-free market (especially when we consider that barriers between the insurance and financial markets are being broken down). This further reduces the power of the CAPM arguments.

In general, this strengthens the case that insurance companies should attempt to diversify within their own sector and present investors with some smoothing of profits, if they are to satisfy the expectations of the investment community.

5.5 Comparisons with Other Industries

The most meaningful comparisons can probably be made with the banking sector, where capital requirements have traditionally been based on total potential downside, calculated from total business volumes.

However, banks try to offset their exposures to get a balanced book. Ultimately, their aim will not be to make money on taking huge amounts of risk, but in exploiting anomalies in the pricing of risk in the market. For this reason, capital requirements are becoming more focussed on the value that banks are actually placing at risk, when their risk transfer transactions (e.g. interest rates swaps and derivative purchases) are fully accounted for.

Insurers, on the other hand (certainly at first sight) deliberately take on and retain risk to make a profit. Capital requirements should (still) therefore be based on expected payouts, having taken account of risk transfer arrangements (e.g. reinsurance). Insurers can still expect to suffer catastrophic losses, that will filter through to reduce the profitability of the whole industry. Insurance is not a “zero sum game”.

Having said this, there are as many similarities as differences in the above descriptions of banking and insurance. We are also seeing products which sit comfortably in both sectors (e.g. weather risk options), further evidence of convergence in distribution channels and continuing developments in the Alternative Risk Transfer market.

With these trends, we may expect the development of standards, more keenly priced products in standard lines and more active trading which may validate no-arbitrage pricing models.

5.7 Conclusions

The arguments above can be summarised in the following bullet points:

- Insurance markets are different from other financial markets (but they are probably converging). The difference is essentially due to how different from pure commodities the products are, reduced by the extent to which active trading takes place. This also leads to different criteria for capital requirements.

- It is impossible to separate neatly the issues of investing in insurance stock and the operations of the insurers themselves.

- Investment in insurance companies does not satisfy the assumptions underlying most financial market models (no arbitrage, low frictional costs, rational and homogeneous investors, etc).

- It therefore follow that, to assess capital requirements for insurance stocks, CAPM as it stands should not be blindly adhered to.

- It also follows that insurance companies themselves should attempt to smooth their returns through diversification and relatively stable profit declarations.

So, what about target returns? An adaptation of CAPM may the answer, through splitting diversifiable risk:

- “Intra-sector” diversifiable risk, is that which can be diversified within the insurance sector, most effectively by insurers themselves; and

- “Inter-sector” diversifiable risk, best diversified by investors.

Assuming this responsibility is passed to insurers, it is left to investors to apply their CAPM models to the insurance sector. The arguments presented against applying CAPM to insurers’ stocks would be largely negated if we talk about it applying to a sector.

Insurers would then be responsible for managing the capital of the business to:

- Diversify as much as possible within the insurance sector, by exploiting their greater expertise and access to the markets;

- Targetting investors’ required returns for the insurance sector.

In the foregoing, we have avoided defining the insurance sector too closely, because the arguments have been based on principles rather than concrete mathematical calculations. It may be appropriate, for example, to consider the insurance and reinsurance sectors separately.

6. METHODS FOR CAPITAL ALLOCATION

6.1 Introduction

Section 6 gives a broad overview of various methods for setting and allocating capital. The methods described are based on an assessment of the aggregate loss distribution. We are not primarily concerned here with the estimation of aggregate loss distributions, but it should be noted that if capital allocations are to allow appropriately for diversification benefits, correlations between and within different lines of business must be taken into account when assessing aggregate loss distributions for capital allocation purposes.

Also, ideally, the “aggregate loss distribution” should incorporate:

- prospective business for which capital is to be allocated (and premiums set),
- business already written for which there are unexpired periods of cover (unexpired risk reserve),
- business already written for which the period of cover has expired, but the losses are not fully developed (outstanding claims reserve).

In other words, the “aggregate loss distribution” should ideally be the probability distribution for the total of all amounts which may become payable in future, and should allow for the uncertainty in each component and the correlations between the various components. It is therefore constantly changing as uncertainty is eliminated through the settlement of claims, and as uncertainty is introduced through the writing of new business. In principle, each such change should lead to the notional release or allocation of capital.

We do not attempt to describe a full system which allows for all these complexities of an ongoing insurance operation. Instead, we attempt to describe - in broad conceptual terms - different approaches and methods of capital allocation, taking an aggregate loss distribution as the starting point.

To apply these methods for an ongoing insurance operation in which the aggregate loss distribution is dynamic, requires dynamic financial analysis (DFA). The simplicity and purity of the methods as described here will not often be attainable in real-life. Nevertheless, we feel it is useful to have a clear description of alternative approaches which can be aimed for (if not achieved) in real-life applications. This should provide a guiding framework and ensure a degree of self-consistency in any real-life application.

We have distinguished between capital setting and capital allocation. Capital setting determines the overall level of capital to support the aggregate operations of an insurer. Capital allocation distributes the overall capital between different lines of business, or even between individual insurance contracts.

Capital setting is of fundamental importance to policy-holders, regulators and shareholders. Policy-holders and regulators because the aggregate amount of capital (in relation to the aggregate loss distribution) determines the security of the policies, and shareholders because it affects the level of risk and expected return on capital.

Capital allocation is of secondary importance. It is a device for efficient running of a multi-line operation. It is a means for providing underwriting guidelines, and for performance measurement.

6.2 Two General Approaches and Four Methods

Given an aggregate loss distribution, two general approaches to capital setting and allocation are considered:

1. Determine the total fund (aggregate premium plus shareholders capital) necessary to support the business. This is then split between premium and capital to give the desired level of expected return on capital, (subject to constraints imposed by price elasticity of demand).

2. Determine the total premium directly from the aggregate loss distribution, then set and allocate capital to give the desired expected ROC.

In either case, a method is needed for getting a single monetary amount from a loss distribution. Under approach 1, this amount represents the total fund (premium plus capital), under approach 2 it represents premium only. In either case, the number obtained should normally exceed the expected loss by an amount which takes account of the “degree of risk” represented in the loss distribution. There are various methods for doing this. Four considered here are:

a) second moment method,
b) quantile method,
c) expected exceedence method,
d) proportional hazards transform (PHT).

Any of these methods could be used under either approach 1 (to determine total fund from loss distribution) or approach 2 (to determine total premium from loss distribution), even though in the existing literature, some methods have been more closely associated with one of these approaches than the other.

For example, the “quantile method” means determining a single monetary amount from a loss distribution as that amount which has a certain (pre-set) probability of being exceeded. This method is more strongly associated with approach 1 (where it is called probability of ruin) than approach 2, but in principle, the aggregate premium could be set directly as a quantile of an aggregate loss distribution (i.e. approach 2).

Similarly, the “expected exceedance method” has been more closely associated with approach 1, (where it is called expected policy-holder deficit), but again the same method for getting a number from a loss distribution could quite reasonably be used directly as a premium rating principle (i.e approach 2).

Conversely, the proportional hazards transform is regarded primarily as a superior premium rating principle (approach 2), but many of its advantages carry over if used for the setting (and allocation) of the total fund (premium plus capital), i.e. approach 1.

In this paper, we regard all these methods as alternative ways of getting a single risk-loaded monetary amount from a loss distribution. They all require the level of the risk-load to be set by choice of the value for a single parameter which we denote ρ. We use the convention, for all methods, that a higher value of ρ corresponds to a higher risk-load, hence a larger monetary amount. In general, if we are using approach 1 (determining the total fund from the loss distribution) then we would use a higher value of ρ than if we were using approach 2 (determining the aggregate premium only).

Capital allocation is not an exact science. We currently have no definitive theory saying that one method is better than another. The best that can be done now is to choose a method that looks sensible and gives a reasonable answer. Without a compelling basis for objective judgement, subjective considerations will play a significant role. For example, if the actuarial department required a favour from a particular underwriter, they may be able to construct a method which minimises the capital allocated to that underwriter at the expense of his peers.

6.3 Description of the Four Methods

The four methods listed in 6.2 for getting a risk-loaded number from a loss distribution are briefly described, initially in the capital setting context, and initially using approach 1 (i.e. determining the total fund, premium plus capital, needed to support the aggregate operations of an insurer).

We suppose that we have an assessment of the probability distribution of the aggregate loss (claims plus expenses discounted at risk-free rate) of an insurer over the next accounting period. The cumulative distribution function for the aggregate loss x is denoted F(x). (Note that F(x) denotes the aggregate loss distribution for the entire operation. Later, when we consider capital allocation as opposed to capital setting, we consider how F(x) is made up of constituents representing different lines of business etc.)

The four methods described are alternative ways of determining the total fund size A (comprising premium P and risk capital C) from (i) the loss distribution F(x) and (ii) the risk-load parameter ρ. In each case, we have chosen a convention which makes *A* an increasing function of ρ; that is, a higher value of ρ means "more secure".

Whichever method is used, under approach 1 (setting the total fund) the requirements of regulators (and/or policy-holders) would determine a minimum value for ρ, but in practice, the value of ρ might be chosen so that the total capital required is equal to what the company actually has in its coffers.

(a) Standard Deviation Method

$A = \mu + \rho.\sigma$ where μ and σ are the mean and standard deviation of the aggregate loss distribution F(x). Example value: $\rho = 5$.

(b) Quantile Method

A is such that $F(A) = \rho$, or in other words, the ruin probability is $1-\rho$. For example, $\rho = 0.999$.

(c) Expected Exceedence Method

A is such that: $E(\max\{X-A,0\}) = (1 - \rho).\mu$, (where E() denotes expected value). The left side of this equation is the expected policy-holder deficit (EPD): if the aggregate loss X exceeds the fund A then the insurer is ruined and policy-holders will suffer a deficit (unpaid valid claims) of X-A. The method as specified here requires that this is a small proportion (1-ρ) of the expected aggregate loss μ. For example, ρ = 0.999.

It should be noted that the EPD method can also be applied by setting the EPD to a small proportion of the premium (rather than a small proportion of the expected loss). Here, we use expected loss μ as this gives a method for obtaining the total fund from the loss distribution.

(d) Proportional Hazards Transform Method

The total fund A is set to the expected value of the transformed distribution G(x) defined by:

$1 - G(x) = [1 - F(x)]^{1/\rho}$ where $\rho > 1$.

This is the so-called proportional hazards transform of the distribution F(x). The condition $\rho > 1$ ensures that 1-G(x) is greater than 1- F(x) (for all x where $0<F(x)<1$) so the expected value of G(x) (hence the fund A) necessarily exceeds the mean aggregate loss μ. Example value: $\rho = 3$.

For the sake of clarity, we repeat here that although the above definitions of the methods have all been given in terms of the total fund A, all these methods can equally be applied to determine the total required premium P from the aggregate loss distribution. These two alternatives are what we are calling approach 1 and approach 2. For approach 2, simply replace A with P throughout the above definitions. Typical values the risk-load parameter would then be smaller than those given above.

6.4 Comparison of Methods

Whether we are using approach 1 (setting the total fund) or approach 2 (setting the premium), the method used should satisfy certain criteria, such as:

- the amount (required total fund or premium) should always exceed the (discounted) expected loss,
- the amount (required total fund or premium) should always be less than the (discounted) maximum possible loss,
- there should be a diversification benefit of pooling blocks of business for which the losses are not perfectly correlated.

Both second moment and the quantile methods have serious flaws in these respects such that they cannot be regarded as generally applicable methods for setting capital (or premiums, or reserves for that matter).

The most obvious problem with the standard deviation method (or any similar method with a risk-load based on the second moment of the loss distribution, e.g. variance instead of standard deviation) is that the value given can exceed the maximum possible loss. Given any value greater than zero for the security parameter ρ, however small this might be, it is possible to find an aggregate loss distribution such that this absurdity occurs. It occurs for short heavy-tailed aggregate loss distributions, as would obtain under whole account stop loss reinsurance, for example.

The main problem with the probability of ruin method is that it does not necessarily give diversification benefits. When two blocks of business are combined into one operation (with the fund pooled, so that claims arising from both blocks of business can be paid from the pooled fund), then we would not expect to need more capital than we started with; in fact we might even hope the aggregate capital required after the merger would be strictly lower than before. This is because, with a common fund, poor claims experience on one block of business can be partly covered out of the portion of the fund notionally belonging to the other block of business: it is unlikely that both blocks will suffer high claims simultaneously. There should be some diversification benefit whenever the losses on the different blocks of business are not perfectly correlated. The greater the degree of mutual independence between different blocks of business, the greater the diversification benefit should be.

However, using the probability of ruin method, there may be a diversification penalty (rather than a benefit), and this can occur even in the most favourable situation of two mutually independent blocks of business (i.e. zero correlation). This is illustrated by a simple example in the appendix to this paper.

6.5 Capital Allocation - Approach 1

The discussion of diversification benefits leads naturally from capital setting to capital allocation. The aggregate loss distribution for an insurer F(x), would in practice be built up from assessments of the aggregate loss distribution for each line of business separately, together with an assessment of the degree of correlation between different lines of business. (There are good practical algorithms for the numerical calculation of the distribution of the sum of correlated random variables - but these are outside the scope of this paper.)

The aggregate loss distribution for each separate line of business would depend, among other things, on the volume of business expected in each line, together with appropriate assumptions regarding the degree of correlation/independence between different units of business within each line.

We use subscripts to denote the different lines of business, thus $F_1(x_1)$, $F_2(x_2)$ are the aggregate loss distributions for lines of business 1 and 2. The overall aggregate loss is $X = X_1 + X_2 + \ldots$and F(x) is the probability distribution function of this (taking into account the correlation between X_1 and X_2 etc).

Whichever capital adequacy principle is used (e.g. the EPD method, the PHT method, or any other method which gives a sensible diversification benefit and which can be specified in terms of a single security parameter ρ), we use ρ_0 to denote the value of the security parameter selected for the aggregate operations of an insurer. (This value would be a decision of the management and/or shareholders, subject to the minimum requirements of regulators and/or policyholders.)

If we are using approach 1, we can use A_0, A_1, A_2 to denote the fund as given by the selected principle for X, X_1, X_2 respectively, in each case using the same value ρ_0 for the security parameter, then the diversification benefit is manifested in: $A_0 <= A_1 + A_2$. That is, when two blocks of business are combined and a pooled fund is used to pay all claims, the size of the pooled fund A_0 is (usually) less than the total of the two funds appropriate for the two blocks separately. Thus pooling (diversification) leads to a reduction in either premiums required from policyholders, capital required from shareholders, or both.

As already pointed out, this inequality ($A_0 <= A_1 + A_2$) is not guaranteed under the ruin probability method, however we believe it does always apply under the other three methods outlined above. For these methods, equality occurs only if X_1 and X_2 are perfectly correlated, in which case F(X) is merely a scaled up version of F_1 (or F_2). In practice, there will never be perfect correlation between different blocks of business, so there is always a diversification benefit (i.e. strict inequality). Broadly speaking, the smaller the correlation, the greater the diversification benefit.

Given the inequality, how can the total fund A_0 be allocated between the constituent lines of business? There is no uniquely correct answer to this, but there is a neat approach which gives plausible allocations whichever capital adequacy method is used. This is to apply the selected method to each constituent loss distribution, using a reduced parameter value (ρ_0' say, where $\rho_0' < \rho_0$ and the same ρ_0' is applied to each line), such that the resulting funds for each constituent (A_1', A_2' etc) add to the total required fund A_0.

Although the standard deviation method is flawed (discussed above) it does give a diversification benefit, and has the advantage over the arguably sounder EPD and PHT methods that the calculations necessary for capital allocation are considerably simpler. For the standard deviation method, we have:

$$A_0 = \mu_0 + \rho_0.\sigma_0$$

where subscript zero indicates quantities for the overall aggregate loss distribution.

For each constituent block of business (subscript i = 1, 2 etc) we have:

$$A_i = \mu_i + \rho_0.\sigma_i \quad \text{(the same parameter } \rho_0 \text{ for all constituents)}$$

and:

$$A_i' = \mu_i + \rho_0'.\sigma_i \quad \text{(the same parameter } \rho_0' \text{ for all constituents)}$$

The required value ρ_0' is easily found from the requirement $A_0 = \sum A_i'$:

$$\rho_0' = \rho_0 \,.\, \sigma_0 / \sum\sigma_i$$

The other methods require numerical trial and error approaches to fund the common parameter value ρ_0' such that the equality $A_0 = \sum A_i'$ is satisfied.

6.6 Capital Allocation - Approach 2

Under approach 2, the selected risk-load method (e.g. expected exceedence method, or PHT) is applied, using the selected parameter value ρ_0, to the overall aggregate loss distribution F(x) in order to determine the total premium required. The same method can then be applied to each of the constituent loss distributions $F_1(x_1)$, $F_2(x_2)$.. using a common value of the risk-load parameter

($\rho_0' < \rho_0$), such that the premiums sum to the value obtained from F(x). Having obtained the aggregate premium P_i required for each constituent loss distribution in this way, capital can be allocated to give the required expected return on capital r. In the simplest case of claims settled at the end of one year, the equation to be solved for capital C_i is:

$$P_i = \mu_i + r.C_i.$$

In more realistic cases, DFA is needed to allow properly for the timing of premium receipts and claim payments.

6.7 Premium Rating and ROC - Approach 1

Using approach 1, having determined the allocation of the total required fund A_0 into components A_i' for the constituent lines of business, how does this relate to premiums and return on capital for each line?

The allocated fund, A_i' comprises total premiums P_i and allocated shareholders' capital C_i, that is:

$$A_i' = P_i + C_i$$

Clearly, if the total of premiums P_i is high (well in excess of expected loss), then C_i will be relatively low, and the expected return on capital will be high. Conversely, if total premiums are low (close to expected loss) then C_i will be relatively high and expected return on capital will be low. Therefore, if the expected return on capital is set at a required value r, the split of the total fund A_i' between premium and capital can be determined.

In the simplest case of claim settlement after one year, the solution of the two equations:

$$A_i' = P_i + C_i \qquad \text{and} \qquad P_i = \mu_i + r.C_i,$$

is: $$P_i = (\mu_i + r.A_i)/(1+r) \qquad \text{and} \qquad C_i = (A_i - \mu_i)/(1+r)$$

In practice detailed numerical modelling (DFA) is needed to properly allow for the timing of premium receipts and claim payments, hence the time when the capital C_i is tied up.

Having determined P_i and C_i in this way, the premium can be expressed as a multiple of the expected loss. This provides an underwriting guideline which can be applied to each contract written.

6.8 Effect of Volume of Business Written

Note that to carry out the calculations described, an initial assumption is needed on the volume of business to be written in each line. This is necessary in order to assess the aggregate loss distribution $F_i(x_i)$ for each line, hence the overall aggregate loss distribution $F(x)$ (taking into account correlations between different lines of business), hence the total required fund A_0 by whatever method is selected.

If the volume of business actually written in any line differs materially from the initial assumptions, then the aggregate loss distribution will be different from that assumed for the line concerned, and the diversification benefit will differ from that assumed for all lines. Therefore, a range of assumptions need to be tried, and the underwriting guidelines (ratio of target premium to expected loss) should be linked to volume of business written.

In a sophisticated application of these techniques, it may also be possible to take price elasticity of demand into account, that is, the volume of business that can be written in each line will depend on the premium rate. Thus the following iterative optimisation process is envisaged:

1. Initial estimates of volume of business to be written in each line.
2. Assessment of aggregate loss distribution $F_i(x_i)$ for each line, hence overall aggregate loss distribution $F(x)$ (taking account of correlations between the x_i).
3. Hence, using the selected method (eg expected exceedance or PHT) and selected value ρ_0 of the parameter, calculate the overall fund (approach 1) or premium (approach 2) required and the allocation of this to lines.
4. Hence calculate the premium rate needed in each line to give the required ROC for each line.
5. Consider the price elasticity for each line to estimate the volume of business which can be written at this premium rate: these replace the initial estimates of step 1.

Iterate steps 2 to 5 as necessary. Having carried out this optimisation at time zero, it should periodically be updated to reassess capital allocations and premium rates in the light of volumes of business actually achieved.

Taking this approach further might even suggest that capital allocation should be a dynamic system, adapting to changes in risk over time. However, the monitoring and systems implications of such an approach are easily outside the reach of most companies at the present time.

APPENDIX - Critique of Risk Loads based on Probability of Exceedence

In 6.3, we stated that the "exceedence probability" method of risk-loading can lead to a diversification penalty rather than the diversification benefit which is generally held to be appropriate in practice.

This means, for example, that if the total fund of insurers is set on the basis of probability of ruin (e.g. such that there is a 1% chance of ruin next year), then the total fund required on merging two insurers may exceed the sum of the two separate funds. This is a diversification penalty.

For example, suppose the probability distribution for a loss is as follows:

loss	probability
£2	0.0099
£1	0.6000
£0	0.3901

If the current level of security loading is the 99th percentile, then the fund required to support this business would be £1 (there is less than a 1% chance of a higher loss).

But if we combine two independent loss distributions, both as above, then the aggregate loss distribution is:

aggregate loss	probability
£4	0.000098 (= 0.0099 * 0.0099)
£3	0.01188 (= 0.6 * 0.0099 + 0.0099 * 0.6)
£2	etc

and again, basing the fund on the 99th percentile, gives a fund of £3, which is greater than £1 plus £1. This is a diversification penalty.

We conclude that probability of exceedence is not a sound general method for calculating capital or premium requirements.

There are other simpler observations which back this up:

- the quantile (e.g the ρ-percentile) does not necessarily increase with the risk load parameter (ρ),
- no account is taken of the severity of exceedences,
- in particular, the quantile (e.g. the 99th percentile) might be zero, even when very damaging exceedences are possible (e.g. nuclear risks).

7. NUMERICAL EXAMPLES (TO BE ADDED)

We have prepared numerical examples of the methods described based on shifted log-Normal distributions for the aggregate loss in each line $F_1(x_1)$, $F_2(x_2)$ etc. This is not intended to suggest that the shifted log-Normal should be used in practice, it is merely a convenient, mathematically tractable, model for illustrative purposes. In practice, numerical calculations can be carried out using any appropriate assessment of the aggregate loss distributions, which need not be representable analytically.

These examples have not yet been fully checked and it is our intention to demonstrate these at the Conference in October.