CASH MECHANICS OF PROPORTIONAL TREATIES

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1.0 INTRODUCTION

1.1 When the Underwriter is presented with a slip covering reinsurance of a primary carrier on the basis of a proportional treaty he has no great scope for negotiation.

1.2 In the case of quota share treaties the number of underlying insurances is likely to be large, sometimes very large indeed, and there is unlikely to be much fluctuation in the underwriting results due to the effects purely of stochastic variation. In the case of small cedant companies the number of underlying insurances will be smaller but the fluctuations in results not all that large.

1.3 In the case of surplus or fac-oblig treaties the variation in results may be larger and will depend in large measures on the retention of the cedant office and on whether there has been selection against or in favour of the reinsurer.

1.4 The emergence of claims of a special type, however, may well influence the results somewhat more greatly. A winter of bad weather could be one such cause. Such factors may possibly be covered by catastrophe excess loss protections, which may be inbuilt to the treaty.
1.5 The main factors the Underwriter will bear in mind in deciding whether to accept the Risk are:-

(a) General underwriting results for primary carriers overall on that class of business world-wide and, in particular, within the country concerned.

(b) The primary carrier's own record and its managerial capability.

(c) Any legislation in the country concerned that might bear on underwriting results; pressures of consumerism, social attitudes and attitudes of the courts.

(d) Whether legislation requires the retention of premium reserves and how outstanding losses are to be covered.

(e) Inflation rates, strength of the currency, delays in settlement.

(f) The standing of the Broker bringing in the business, how valuable is his portfolio to the Underwriter, what other business he brings in, both for the same Reassured and overall.
1.6 All these are factors over which the Underwriter has no control. He must assess them and take a decision accordingly as to whether to reject the proposal outright or to go on to the next stage. This stage includes the negotiation of terms over which he can have some control:

(g) The actual detailed terms and conditions of the treaty; in particular, any exclusions.

(h) Whether the treaty is protected by an excess loss protection for joint account.

(i) The amount of commission allowed to the cedant office (there is usually less scope for negotiations over the rate of brokerage allowed to the Broker).

(j) The rate of interest allowed on premium reserves.

(k) (Perhaps) Whether O/S losses are to be covered by loss reserves retained or by a Letter of Credit (which in turn may make a considerable difference to the rate of interest earned on funds allocated for that purpose).
The analysis that follows tracks the effect of varying decisions in regard to points (i) to (k) above. In many portfolios of reinsuring offices the proportional treaties are smaller in number than facultative reinsurances or non-proportional treaties but are large in terms of premium income. The scope for profit is at best marginal as the reinsuring office is seeking to show a margin allowing for brokerage (from about 1 1/2% to 2 1/2%) on top of original rates which may themselves be "thin" in a highly competitive market. In such cases, the underwriter will often console himself with the thought that there will be considerable premium flow in the early stages to generate interest income. It would come as a considerable shock to him, then, if he saw demonstrated that there is likely to be a negative cash flow throughout. Hence the analysis is set out in such a way as to lead to the introduction of a relatively straight-forward computer program which could be used by an underwriter, by means of simple input, to obtain a picture of the expected cash flow under the treaty in terms of the conditions being suggested to him.
2.0 Computer Model

2.1 The model is designed to accept the following input data:

Symbol

(i) Ultimate Premiums \( UP \)
(ii) Ultimate Claims \( UC \)
(iii) Commission and brokerage percentage \( CB\% \)
(iv) Quarterly interest rate on reserves retained \( r \)
(v) Quarterly market rate of interest \( i \)
(vi) Loss reserves retained factor \( LRR \)
(vii) Premium reserves retained factor \( PRR \)
(viii) Cumulative premium development factor at quarter \( j \) \( WP_j \)
(ix) Cumulative paid claims development factor at quarter \( j \) \( PC_j \)
Cumulative notified claim development factor at quarter \( j \) \( NC_j \)

(x) Time lag of cash settlement from the quarter end \( t \)

2.2 Given this data it is possible to generate the quarterly development of premiums, paid loss and notified claims by applying the quarterly cumulative patterns respectively to ultimate premiums and claims as follows :-

Written Premium during quarter \( j \) \( = \) \( UP \times (WP_j - WP_{j-1}) \)
Paid Claims during quarter \( j \) \( = \) \( UC \times (PC_j - PC_{j-1}) \)
Notified Claims during quarter \( j \) \( = \) \( UC \times (NC_j - NC_{j-1}) \)
2.3 The loss reserves retained are a function of the known case reserves (the outstanding losses) prevailing at the quarter end. It is normal for this relationship to be 100% of the known case reserves although this can vary from 0% to 150%. This variation is accommodated within the loss reserves retained factor (LRR). A similar rationale applies to the premium reserves where it is common to have a reserve of 25% of the previous calendar year's premium. Again this can be anything from 0% upwards.

2.4 Given these restrictions on cash it is possible to simulate the quarterly cash flow of a proportional treaty as:

- Premium
- plus Interest on Loss & Premium Reserves Retained
- less Commission & Brokerage
- less Paid Losses
- less Change in Premium Reserves Retained
- less Change in Loss Reserves Retained
2.5 The generated monetary receipt are then lagged in accordance with normal market practice, let's say two quarters, and net present valued back to inception using the quarterly market discount rate $i$. The underwriter can then assess the true profit or loss in current monetary terms.

3.0 Main Results

3.1 Appendix 1 at the end of this paper gives an example of a typical set of results. The model has proved invaluable in measuring the impact on cash flow and its net present value of varying certain input variables whilst keeping others constant. Numerous linear relationships have been uncovered. From these a predictive theory of proportional treaty cash mechanics has been developed. The results of this research to date can be summarised as follows:

3.2 Result 1

Given a fixed development pattern of premiums and claims, fixed interest on reserves retained and a constant combined ratio;

(i) the change in the undiscounted total cash flow is directly proportional to the change in commission and brokerage. The gradient of change is constant and equal to:

$$\Gamma LRR \sum_{j=1}^{\infty} \left( NC_{j-1} - PC_{j-1} \right)$$
The change in the discounted total cash flow is directly proportional to the change in commission and brokerage. The gradient of change is constant and equal to:

\( \frac{\Delta \text{DCCF}}{\Delta \text{Comm + Brokerage}} = \text{Constant} \)

The gradient of the discounted cashflow is geometrically affected by the time lag of cash settlement.

### 3.3 Result 2

Providing the following are constant,

(a) loss ratio  
(b) commission & brokerage  
(c) interest on reserves retained  
(d) premium and claim patterns  
(e) loss reserves retained percentage

The change in the undiscounted cash flow is directly proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to:

\[ r \cdot \sum \left( WP_{i-1} - WP_{i-5} \right) \cdot UP \]
(ii) the change in the discounted total cash flow is inversely proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to:

$$\nabla \left[ \sum_{A} \sum_{i} (WP_{i-1} - WP_{i-5}) - \sum_{A} \sum_{i} (WP_{i} - WP_{i-1} + WP_{i-5} - WP_{i-4}) \right] \cdot UP$$

A full expose of the theory behind these results is given in appendices 2 and 3 respectively.

4.0 General Observations

4.1 General observations of the work conducted to date are best illustrated in graphical form. Results 1 and 2 described earlier can be seen on graphs 1 and 2 respectively.

4.2 Another observation is the effect of changing the interest payable on reserves retained. This is demonstrated in graph 3. Here again, we can clearly see how the underwriter when reviewing the undiscounted cash receipt can easily overstate the true profitability of his account.
4.3 Lastly, we investigated the undiscounted and discounted effect on cash of changing the combined ratio (graph 4). Here again, the monetary restrictions of proportion treaties cause the true profit or loss always to be less than that observed from historical undiscounted receipts.

4.4 The model is a simple but powerful tool allowing any underwriter to assess, given a set of assumptions, the undiscounted and discounted profit or loss. Furthermore, the model provides an easy way of interpreting the break even loss ratio associated with a proportional treaty, an essential indicator for management. This knowledge is critical if underwriters are to insist on the inclusion or exclusion of clauses which maximise cash flow and hence profit.

5.0 Conclusion

5.1 We have only just started to uncover the mysteries surrounding the cash mechanics of proportional treaties. Much work still remains to be done. We need to consider the implications of letters of credit, the allocation of administrative costs both direct and indirect, the influence of premium and claim portfolio transfers, together with further theoretical analysis behind our general observations.
5.2 The paper has been written with a view to stimulate an interest and understanding of the monetary implications of proportional treaties. We look forward to an active and lively discussion at this year's GIRO conference.
Given a fixed development pattern of premiums and claims, fixed interest on reserves retained, and a constant combined ratio, a change in commission and brokerage is proportional to

(i) The change in undiscounted cash flow

(ii) The change in discounted cash flow

PROOF:

Let:

- $WP_j =$ Cumulative Premium Development Factor at time $j$
- $PC_j =$ Cumulative Paid Claim Development Factor at time $j$
- $NC_j =$ Notified Claim Development Factor at time $j$
- $UP =$ Total Ultimate Premiums
- $UC =$ Total Ultimate Claims
- $TCB =$ Total Ultimate Commission of Brokerage
- $r =$ Interest on Reserves Retained (Quarterly Rate)
- $i =$ Commercial Rate of Interest (Quarterly Rate)
- $t =$ Time Lag of Cash Settlement from Quarter End.

Now:

- $\sum Prem \ T'put = UP;$
- $\sum Change Prem. Reserves Retained = 0$
- $\sum Paid Loss = \sum PC_j.UC - UC =$ Ultimate Claims
- $\sum Changes Loss Reserves Retained = 0.$
- $\sum TCB.(WP_j - WP_j ) = \sum C&B \ T'put = TCB$

Undiscounted Total Cash Flow = $\sum [Prem \ T'put - (change in P.R.R)]$

$UTCF =$ + Int. on reserves retained.
- C&B T'put - paid loss T'put
- change in L.R.R.]

$= UP + Int. on reserves retained$
- $TCB-UC$
Given a constant combined ratio we can anticipate the effect on the undiscounted cash flow of varying the Commission and Brokerage whilst keeping premiums, interest and development patterns fixed.

Therefore:

\[ \text{UP1} = \text{UP2} \]
\[ \text{TCB1} + \text{UC1} = \text{TCB2} + \text{UC2} \]
\[ \text{TCB1} - \text{TCB2} = \text{UC2} - \text{UC1} \]
\[ \text{LRR1} = \text{LRR2} \]
\[ \text{PRR1} = \text{PRR1} \]

Undiscounted Total Cash Flow Changes

\[ = \text{UTC2} - \text{UTC1} \]
\[ = (\text{UP} + \text{INT2} - \text{TCB2} - \text{UC2}) - (\text{UP} + \text{INT1} - \text{TCB1} - \text{UC1}) \]
\[ = \text{INT2} - \text{INT1} + (\text{UC2} - \text{UC1}) \]
\[ = \text{INT2} - \text{INT1} \]
\[ = r \sum \left[ (\text{WP}_{j-1} - \text{WP}_{j-5}) \cdot \text{UP} \cdot \text{PRR} + r \sum (\text{NC}_{j-1} - \text{PC}_{j-1}) \cdot \text{UC2} \cdot \text{LRR} \right] \]
\[ - r \sum \left[ (\text{WP}_{j-1} - \text{WP}_{j-5}) \cdot \text{UP} \cdot \text{PRR} - r \sum (\text{NC}_{j-1} - \text{PC}_{j-1}) \cdot \text{UC1} \cdot \text{LRR} \right] \]
\[ = r \cdot (\text{UC2} - \text{UC1}) \cdot \text{LRR} \sum (\text{NC}_{j-1} - \text{PC}_{j-1}) \]
\[ = r \cdot (\text{TCB1} - \text{TCB2}) \cdot \text{LRR} \sum (\text{NC}_{j-1} - \text{PC}_{j-1}) \]

Now \( r = \text{Constant}; \sum (\text{NC}_{j-1} - \text{PC}_{j-1}) \) is constant; \( \text{LRR} \) is constant.
Hence: The change in the undiscounted cash flow is proportional to the change in commission & brokerage.

The gradient of change is constant and equal to

\[ r \cdot LRR \sum_{j} (NC_{j-1} - PC_{j-1}) \]

(ii) The change in the discounted (net present value) total cash flow is:

\[ \text{DTCF}_2 - \text{DTCF}_1 = \sum_{j} \left( \frac{\mu_{j} \cdot \text{UP}_{2} \cdot (WP_{j-1} - WP_{j})}{\text{UP}_{2}} - \frac{\mu_{j} \cdot \text{PRR}_{2} \cdot (WP_{j-1} - WP_{j} \cdot WP_{j-1})}{\text{UP}_{2}} \right) \]

\[ + \text{DINT}_2 - \sum_{j} \text{TCB}_{2} \cdot \mu_{j} \cdot (WP_{j-1} - WP_{j}) \]

\[ - \sum_{j} \mu_{j} \cdot UC_{2} \cdot (PC_{j-1} - PC_{j}) - \sum_{j} \mu_{j} \cdot LRR \cdot (NC_{j-1} - NC_{j-1}) \cdot UC_{2} \]

\[ - \sum_{j} \mu_{j} \cdot UP_{1} \cdot (WP_{j-1} - WP_{j}) + \sum_{j} \mu_{j} \cdot \text{PRR}_{1} \cdot (WP_{j-1} - WP_{j} \cdot WP_{j-1}) \cdot UP_{1} \]

\[ - \text{DINT}_1 + \sum_{j} \text{TCB}_{1} \cdot \mu_{j} \cdot (WP_{j} - WP_{j}) \]

\[ + \sum_{j} \mu_{j} \cdot UC_{1} \cdot (PC_{j} - PC_{j}) + \sum_{j} \mu_{j} \cdot LRR \cdot (NC_{j-1} - NC_{j-1}) \cdot UC_{1} \]

\[ = \text{DINT}_2 - \text{DINT}_1 + (\text{TCB}_2 - \text{TCB}_1) \cdot \sum_{j} \mu_{j} \cdot (WP_{j-1} - WP_{j}) \]

\[ + (UC_2 - UC_1) \cdot \left[ \sum_{j} \mu_{j} \cdot (PC_{j} - PC_{j}) + \sum_{j} \mu_{j} \cdot (NC_{j-1} - NC_{j-1}) \right] \]

\[ = r \cdot \left( \text{PRR}_2 - \text{PRR}_1 \right) \cdot \sum_{j} \mu_{j} \cdot (WP_{j-1} - WP_{j}) \cdot UP \]

\[ + r \cdot LRR \cdot (UC_2 - UC_1) \cdot \sum_{j} \mu_{j} \cdot (NC_{j-1} - PC_{j-1}) \]

\[ + (\text{TCB}_2 - \text{TCB}_1) \cdot \sum_{j} \mu_{j} \cdot (WP_{j} - WP_{j}) \]

\[ + (UC_1 - UC_2) \cdot \left[ \sum_{j} \mu_{j} \cdot (PC_{j} - PC_{j}) + \sum_{j} \mu_{j} \cdot (NC_{j-1} - NC_{j-1}) \right] \]

Now: \( \text{TCB}_2 - \text{TCB}_1 = UC_2 - UC_1 \)

Therefore:

\[ \text{DTCF}_2 - \text{DTCF}_1 = (\text{TCB}_1 - \text{TCB}_2) \cdot \mu \left[ r \cdot LRR \cdot \sum_{j} \mu_{j} \cdot (NC_{j-1} - PC_{j-1}) \right. \]

\[ + \sum_{j} \mu_{j} \cdot (WP_{j} - WP_{j-1}) - \sum_{j} \mu_{j} \cdot (PC_{j} - PC_{j}) \cdot \sum_{j} \mu_{j} (NC_{j-1} - NC_{j-1}) \]
Now $v^t$ is constant if $t$ is constant
\[ r \cdot LRR \sum_{j=1}^{\infty} (CN_j - CP_{j-1}) \text{ is constant} \]
\[ (WP_j - WP_{j-1}) \text{ is constant} \]
\[ (PC_j - PC_{j-1}) \text{ is constant} \]
\[ (NC_j - NC_{j-1}) \text{ is constant} \]

Therefore: The change in the discounted total cash flow is proportional to the change in commission & brokerage.

The gradient of change is constant and equal to:

\[ v^t \left[ r \cdot LRR \cdot \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \frac{v^j}{\omega^i} (NC_{j-i} - PC_{j-i}) - \sum_{i=1}^{\infty} \frac{v^j}{\omega^i} (WP_{j-i} - WP_{j-i}) - \sum_{i=1}^{\infty} \frac{v^j}{\omega^i} (PC_{j-i} - PC_{j-i}) - \sum_{i=1}^{\infty} \frac{v^j}{\omega^i} (NC_{j-i} - NC_{j-i}) \right] \]

Note: The gradient of change is geometrically affected by the time lag $t$ of cash settlement.
Providing the following are constant:

(i) Loss Ratio
(ii) Commission & Brokerage
(iii) Interest on Reserves Retained
(iv) Premium & Claim Patterns
(v) Loss Reserves Retained %

The change in the undiscounted & discounted cash flow is proportional to the change in premium reserves retained factor.

Proof:

(i) We are given the following:
\[ UP_1 = UP_2 \]
\[ UC_1 = UC_2 \]
\[ TCB_1 = TCB_2 \]
\[ LRR_1 = LRR_2 \]

Now:

Undiscounted Total Cash Flow Change
\[
= UTCF_2 - UTCF_1 \\
= (UP_2 + INT_2 - TCB_2 - UC_2) - (UP_1 + INT_1 - TCB_1 - UC_1) \\
= INT_2 - INT_1 \\
= r \cdot PRR_2 \cdot \sum_j (WP_{j-1} - WP_{j-5}), UP_2 \\
+ r \cdot LRR_2 \cdot \sum_j (NC_{j-1} - PC_{j-1}), UC_2 \\
- r \cdot PRR_1 \cdot \sum_j (WP_{j-1} - WP_{j-5}), UP_1 \\
- r \cdot LRR_1 \cdot \sum_j (NC_{j-1} - PC_{j-1}), UC_1 \\
= r \cdot (PRR_2 - PRR_1) \cdot \sum_{j \in 1} (WP_{j-1} - WP_{j-5}), UP_2 \
\]
The change in undiscounted cash flow is proportional to the change in premium reserves retained factor. The gradient of change is constant and equal to

\[ r \sum_{\text{All} j} (WP_{j-1} - WP_{j-5}) \cdot UP \]
(ii) The change in the discounted (net present value) total cash flow is:

\[
\begin{align*}
\Delta TC F_2 - \Delta TC F_1 &= \sum_{j} \nu^{\tau, j} W_{P_{j}} - \sum_{j} \nu^{\tau, j} W_{P_{j}} - \sum_{j} \nu^{\tau, j} PRR_{2} (W_{P_{j}} - W_{P_{j}+1}) U_{P_{j}} \\
&\quad + \text{DINT}_2 - \text{DINT}_1 \\
&\quad + \sum_{j} TCB_{2} - \sum_{j} TCB_{1} \\
&\quad - \sum_{j} \nu^{\tau, j} UC_{2} (P_{C_{j}} - P_{C_{j-1}}) - \sum_{j} \nu^{\tau, j} LRR_{2} (N_{C_{j}} - N_{C_{j-1}}) UC_{2} \\
&\quad - \sum_{j} \nu^{\tau, j} UP_{1} (W_{P_{j}} - W_{P_{j}+1}) + \sum_{j} \nu^{\tau, j} PRR_{1} (W_{P_{j}} - W_{P_{j}+1} - W_{P_{j}+5} + W_{P_{j}+4}) UP_{j} \\
&\quad - \text{DINT}_1 + \sum_{j} TCB_{1} - \sum_{j} TCB_{2} \\
&\quad + \sum_{j} \nu^{\tau, j} UC_{1} (P_{C_{j}} - P_{C_{j-1}}) + \sum_{j} \nu^{\tau, j} LRR_{1} (N_{C_{j}} - N_{C_{j-1}}) UC_{1} \\
&= - (PRR_{2} - PRR_{1}) \sum_{j} \nu^{\tau, j} (W_{P_{j}} - W_{P_{j+1}} + W_{P_{j+5}} - W_{P_{j+4}}) UP \\
&\quad + \text{DINT}_2 - \text{DINT}_1 \\
&\quad + r \cdot (PRR_{2} - PRR_{1}) \sum_{j} \nu^{\tau, j} (W_{P_{j+1}} - W_{P_{j+5}}) UP \\
&\quad + r \cdot (LRR_{2} - LRR_{1}) \sum_{j} \nu^{\tau, j} (N_{C_{j}} - N_{C_{j+1}}) UC_{1} \\
&= (PRR_{2} - PRR_{1}) \left[ r \sum_{j} \nu^{\tau, j} (W_{P_{j+1}} - W_{P_{j+5}}) - \sum_{j} \nu^{\tau, j} (W_{P_{j}} - W_{P_{j+1}} + W_{P_{j+5}} - W_{P_{j+4}}) \right] \cdot UP \\
\end{align*}
\]

Everything inside the square brackets is constant.
Therefore the changes in the discounted total cash flow is proportional to the change in premium reserves retained factor.

The gradient of change is constant and equal to:

\[
\left[ r \sum_{j} \nu^{\tau, j} (W_{P_{j+1}} - W_{P_{j+5}}) - \sum_{j} \nu^{\tau, j} (W_{P_{j}} - W_{P_{j+1}} + W_{P_{j+5}} - W_{P_{j+4}}) \right] \cdot UP
\]