Presented to the Institute of Actuaries Students' Society

November 1973

CATEGORY B UNIT-LINKED POLICIES

by

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Introduction

1.1 The Category B unit-linked policy as defined by Grant and Kingsnorth in their paper to the Institute in 1967 includes any contract under which the Insurance Company retains the income from the units. With the proliferation of unit-linked policies, there is now a rich variety of contracts which fall under this heading, together with further groups in which either a substantial part of the income is retained or an annual deduction equal to a fixed percentage of the capital value of the units credited is made. Such policies exhibit most of the characteristics of Category B policies, and it is therefore appropriate to consider them as belonging to the same family.

1.2 The purpose of this paper is to highlight the peculiarities of Category B policies. Ideally, this should be done by considering each class of policy in turn. To do so, however, could prove tiresome to the reader who is interested only in the salient features, and attention has therefore been restricted to an endowment assurance, which appears to be the most common Category B contract issued.

1.3 The first aspect of Category B contracts examined is the relationship between the Units and the currency in which premiums are paid. In the light of this relationship, premium and valuation formulae are developed, and the need to modify them is discussed, with particular regard to Zillmerization. Finally, a number of associated subjects are considered.
2.1 Fundamental to any Unit-linked contract is, of course, the "Unit". Originally, this meant the units of an authorised unit trust, but it may now be taken to represent a share of an investment portfolio of any type, the value of a share at any time being the value of the pool of investments divided by the number of shares in the pool.

2.2 As the type of investment can vary, so can the income yield on the investments, and therefore on the shares. If the investments were fixed interest stocks a high yield should result, whilst if they were goods, e.g. paintings or wine, no income would accrue. Naturally, this latter type of unit is not directly susceptible of Category B treatment. The income which arises from most Units is variable, both when expressed in terms of cash, and also when expressed as a percentage of the value of the Units themselves. This variability of itself creates a number of problems, with which this paper is not directly concerned. In consequence, it will be assumed (except where the context shows otherwise) that the income arising from the Units which is retained by the office is a fixed percentage of the value of the Units at any time. The net rate assumed is 2½ per cent. - approximately that which may be obtained on the F.T. Actuaries All Share Index at the present time and therefore appropriate to Units of a fund invested in equities.
2.3 This concept may be used in practice as well as in theory, as may be shown using authorised unit trusts. Under Category B policies, investment would normally take place in what are, perhaps inappropriately from the point of view of the policyholder, commonly called distribution units, and the dividend income would be retained by the life office. Instead, it would be quite feasible for the office to deem that the investments would be made in accumulation units, and arrange that each year a deduction of $2\frac{1}{2}$ per cent, would be made from the number of units credited. Provided the dividend yield on the units averaged $2\frac{1}{2}$ per cent net in the long run, the proceeds under the two methods would be similar. If the income were more than $2\frac{1}{2}$ per cent, then the policyholder would automatically get the benefit of the excess, while if it were less the office would be protected against receiving less income than it had anticipated.

2.4 This approach is not widely adopted although some offices do have contracts which use the concept. Perhaps marketing considerations discourage its use, but it does have a number of important advantages. First, the office is assured of its income and therefore the problem of equity between the policyholder who is looking for growth and the office which requires an adequate level of income is greatly reduced; it is not totally removed, because the office may
still be seeking to relieve its own expenses from the tax paid on the income. Secondly, the company may take advance credit for the whole of the net income which it knows it will receive in the future, and there is thus no need to retain a margin in the rate of interest in the valuation basis as compared with the premium basis. It may be mentioned in passing that a number of offices recognise that there would be considerable inequity were the dividend yield on the Units to increase substantially, and a number of policies of this type do contain the provision for some or all of the extra yield to be given to the policyholder by way of additional Units. This may also be done by making the policy with profits, but it is debatable whether from the policyholders' point of view this is acceptable.

2.5 Under Unit-linked contracts, most of the proceeds are expressed in terms of Units. At the same time, some of the proceeds, and all of the premiums, will normally be expressed in fixed monetary terms i.e. in "Cash".

2.6 Taking the most common form of Category B endowment assurance as an example, premiums are payable in Cash, as are part of the benefits on death. All other benefits are payable in Units. The methods used is to establish a 'nominal sum assured' in terms of Cash, which sum is then translated into Units equally over the term of the contract. The terms of the transfer between Cash and Units is the market valuation made at the time when the transfer is made.
2.7 That Units and Cash are interchangeable at a given rate at any point of time is fundamental; it is however suggested that this interchangeability should be considered to be the only connection. Whilst it is convenient to retain the labels Cash and Units in order to keep firmly in mind the problems inherent in each, the two items could quite readily be, say, chalk and cheese. At any point of time it would be possible, through the market place, to equate a given amount of chalk with a unit of cheese. Such a relationship would not be a constant one, and would depend on the relative demand for each at any given time.
The Endowment Assurance Contract

3.1 At this stage it is helpful to set out the benefits of the policy under consideration. The sum assured will be taken as 100 Cash for each year of the endowment term. Premiums of a level Cash amount will be payable annually in advance throughout the currency of the contract. In return, 100 Cash will be deemed to be transferred into reversionary Units each year on the day when the premium falls due according to the rate of exchange then in force. At maturity the proceeds will be the Units so allocated. Should death occur during the term of the contract, the proceeds will be the Units already allocated up to the date of death together with an amount equal to the Cash which would otherwise have been converted into Units after the date of death. If the policy-holder wishes to stop paying premiums then the Units already allocated to him will be paid at maturity or earlier death, subject to a deduction from the units allocated in order to cover the initial expenses under the policy if the cessation of premium takes place during the early years of the contract. If, instead of this, he wishes to surrender the policy then a proportion of the paid up policy, depending upon the period to maturity and possibly on age attained, will be paid to him.

3.2 This policy has many similarities to a traditional non-profit endowment assurance, as can be clearly seen if it
is assumed that the value of a Unit stays constant in terms of Cash. However, the similarities tend to prove superficial, and can be misleading. To illustrate this, consider the effect of a rise in the value of investments. If the movement were permanent, then were a traditional policy underfunded at the time of the rise, the rise would, of itself, help the solvency of the office, as the capital gains could be used to cover the shortfall. Not so with the unit linked contract, as the gain on the Unit reserve belongs to the policyholder and the Cash amount of underfunding may be increased, if it is the Unit reserve which is underfunded.

3.3 Complications of the contract arise from cash guarantees, from a difference between the bid and offer value of the units, and from capital gains tax. All these items are best dealt with by suitable adjustments to the premium basis and the valuation reserves set up, and consideration of their influence has been deferred until later in the paper.

3.4 The contract now needs to be divided between its Cash and Unit elements. Taking first the Unit element, it will be seen that when a Unit is allocated to the policy the value of the corresponding liability may be taken as \( A_{x+t;\overline{\kappa}} \) at an appropriate rate of interest, which is 2½ per cent. given the assumption above that the income which will be received from the Units will be a constant 2½ per cent. of the value of the Units. Because of this constancy, it does not matter how the rate of exchange between Cash and
Units varies. Provided that the number of units purchased for the policy is equal to the allocated number multiplied by $A_{x+t:n-1}$ then this part of the policy will always be in a matched position.

3.5 The Cash element of the policy is rather more complicated. On the one hand, Cash is received as premiums each year. The amount of Cash which needs to be transferred into Units each year varies, and is evidently $100 \times A_{x+t:n-1}$.

In addition, there is to be provided from the Cash receipts not only the decreasing term assurance for an amount equal to the balance of the nominal sum assured not yet transferred to units, but also the expenses of the policy.

3.6 There is one item as to which it is not clear whether it should be considered a Unit element or a Cash element viz, renewal expenditure. This is, of course, payable in Cash but its level depends to a large extent on the inflation suffered. It is arguable that some Units should keep pace with inflation, in which case renewal expenses may be better expressed in Units. This latter suggestion may be considered to be strengthened if, in addition to receiving the income from the Units, the life office also receives a management charge from the Units. However, the pattern of receipt of such management charges will not follow the pattern of expenses incurred, and the equation of inflation with increases in the value of Units is dubious. For this reason, renewal expenses will be regarded as a Cash expenditure.
4.1 The affinity of this contract to a non-profit endowment assurance has already been noted, and were the difference between the Unit element and the Cash element to be ignored, premiums and valuation formulae would be derived in the traditional manner. The investment part of the policy would then be said to earn 2½ per cent. and the formulae would therefore probably be calculated by reference to this rate.

4.2 The premium calculated by taking into account the separate elements is likely to be close to that mentioned above, owing to the fact that the investment element forms the greater part of the premium. However, the form of the policy is such that not all of the premium which is received (and which is not required for expenses and life cover) needs to be transferred into Units in the early years of the contract, whilst in the later years more than the premium received would need to be transferred. Hence a positive Cash reserve is built up during the early years of the contract; it is then gradually dissipated throughout the rest of the term of the contract. The traditional approach, when using a rate of interest of 2½ per cent. throughout, assumes that this Cash reserve will earn 2½ per cent. interest. In current conditions this rate is too low, and it can be seen from the rates of premium at present being offered by life offices for ordinary non-profit endowment assurances, that
the rate of interest which it is anticipated may be earned is at a much higher level than 2½ per cent.

4.3 It would, however, obviously be imprudent to assume that the present high rates would continue over the full term of, say, a 30 year policy. On the other hand the nature of the policy is such that the bulk of the investments to be made in Cash will be made in the first five to ten years of the policy and it is therefore primarily this period over which we need to estimate the interest rate which will be available. For the purposes of illustration, it has been assumed in this paper that a rate of 5 per cent. net may be earned on any positive Cash reserve held in respect of these contracts, and this rate is therefore used in the premium basis.

4.4 Turning to the valuation basis, the rate of interest which may be assumed will depend upon the purpose of the valuation. Obviously, it would be convenient to be able to take 5 per cent. in the valuation as being the rate which may be earned on the Cash reserves as this would decrease the possibility of any initial strain arising on the policies. However, if the rate of interest of 5 per cent. has been chosen as being a realistic assumption of what may, on average, be expected in the future, then in for statutory returns it would be unwise to use a rate of interest which one felt had only a fifty-fifty chance of being achieved. A rate of interest of 4½ per cent. has been adopted in the valuation formulae for Cash reserves.
4.5 The adoption of a higher rate of interest on the Cash element of the contract than under the traditional approach means that the premium that may be charged will be less. It will be shown, however, that this does not always hold for the valuation reserves.

4.6 The use of a more stringent rate of interest in the valuation basis means that a strain will arise under the contract. This strain needs to be financed, and it should be immaterial whether this finance arises from free reserves within the office or whether it arises from extra capital raised from the shareholders. In either event it is capital being applied for a specific purpose, and the only reasonable criterion for whether its use is justified is whether or not the return which is being received is sufficient. For it to be sufficient, the premiums being paid must be adequate - and it is not possible to ensure this simply by taking margins in the premium basis and hoping that they will be sufficient to cover the return needed.

4.7 It is necessary, especially for a new life office, where the free reserves of the established life office are not yet in being and a conscious decision is being made to channel money into the life office business, to build a model office in order to investigate the cash flow arising under these policies, incorporating in this model the valuation basis which will be used in the statutory returns of the office.
4.8 This subject is one which has not received much attention within the Institute, although it is obvious that most if not all consultancies as well as a number of life offices are investigating new products in this way. One of the radical changes which this approach implies, is that before setting a premium rate one must look to the valuation basis which will be used, since the valuation basis will affect whether or not the rate of return on capital is adequate.

4.9 It is not the purpose of this paper to attempt to build up a model to investigate the premium rates of this type of contract and, indeed, the subject warrants a paper of its own. Moreover, the transition from a net premium to office premium is not attempted and in consequence the traditional sequence of premium calculation followed by valuation is used in this paper.
Formulae

5.1 The calculation of the premium is essentially the calculation of the Cash element of the premium, as the Unit element arises solely from the transfer from Cash, and as shown in paragraph 3.7, the assets and liabilities of the Unit element are automatically matched. It is intended to derive only the net premium, as the additions to arrive at the gross premium are, apart from those in respect of capital guarantees, no different from those under a traditional policy.

5.2 The benefits in Cash which we therefore need to consider are a reducing term assurance and the transfers required to buy the Units. The value of these benefits for a policy of term \( n \) years taken out at age \( x \) of 1 per annum converted to Units is:

\[
\left\{ (n.M_{[x]} - R_{[x]} + R_{[x]+n}) + \sum_{t=0}^{n-1} (P_{[x]+t}. A'_{[x]+t:n-t}) \right\} \div D_{[x]} \tag{1}
\]

where \( A'_{[x]+t:n-t} \) is at \( 2\frac{1}{2} \) per cent, and all other terms are at 5 per cent.

This format is inconvenient for calculation; however if the second term, but with \( A \) replacing \( A' \) is added to the first term and subtracted from the second, we obtain:

\[
n A_{[x]:n} + \sum_{t=0}^{n-1} \left\{ (P_{[x]+t} / D_{[x]}) (A'_{[x]+t:n-t} - A_{[x]+t:n-t}) \right\} \tag{2}
\]

\[= n.A_{[x]:n} + L_{[x]:n} \text{, say.} \tag{3}\]
Whence

\[ n \, \frac{A(x)}{\bar{a}(x)} = \frac{\nu}{\bar{a}(x)} \] (ii)

As is to be expected, if \( A' = A \) (i.e. the same rate of interest is earned on Cash as on Units), then the premium is that for an ordinary endowment assurance of sum assured \( n \).

5.3 The following table illustrates the effect that the higher rate of interest has upon the net premium, calculated according to A 49/52 (select) mortality, per 100 per annum sum assured.

Table I:
Comparison of Premiums

<table>
<thead>
<tr>
<th>Age</th>
<th>Term</th>
<th>Traditional Net Premium, 2½% throughout</th>
<th>Cash/Unit Net Premium, 5% on Cash</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
<td>70.9</td>
<td>68.1</td>
<td>.96</td>
</tr>
<tr>
<td>35</td>
<td>25</td>
<td>75.7</td>
<td>73.5</td>
<td>.97</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>80.8</td>
<td>79.3</td>
<td>.98</td>
</tr>
<tr>
<td>45</td>
<td>15</td>
<td>85.9</td>
<td>84.9</td>
<td>.99</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>90.6</td>
<td>90.1</td>
<td>.99</td>
</tr>
</tbody>
</table>

5.4 The valuation of the Unit liabilities is straightforward, being simply the number of units allocated to the policy multiplied by \( \frac{\bar{A}(x+t:n)}{\bar{a}(x)} \). It can readily be deduced that the future Cash liabilities are represented by:
\[(n-t) \cdot A_{x+t: \overline{n-t}} + L_{x+t: \overline{n-t}} \]  
when the Cash reserve is:

\[(n-t) \cdot A'_{x+t: \overline{n-t}} + L_{x+t: \overline{n-t}} - n \cdot \pi_{x+\overline{n}} \cdot a_{x+t: \overline{n-t}} \]  

(6)

If the value of the Units purchased had stayed constant in Cash terms, then the value at that time would be: \( t \cdot A'_{x+t: \overline{n-t}} \) combined with (6) it will be seen that if \( A' = A \), the two reduce to the standard formula:

\[n \cdot A_{x+t: \overline{n-t}} - n \cdot \pi_{x: \overline{n}} \cdot a_{x+t: \overline{n-t}} \]  

(7)

5.5 The following table shows the comparison between traditional reserves based on A49/52 (ultimate) and 2\% per cent. interest and reserves using a valuation rate of 4\% per cent. on the Cash element, and 2\% per cent. on the Unit element, with A49/52 (ultimate). All policies mature at age 60, have a Cash transfer of 100 per annum and it has been assumed that the rate of exchange of Units had remained constant.
Table II:
Comparison of Reserves.

<table>
<thead>
<tr>
<th>Age</th>
<th>Duration</th>
<th>Traditional Reserve</th>
<th>Cash Reserve</th>
<th>Unit Reserve</th>
<th>Cash + Unit less Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>5</td>
<td>365</td>
<td>81</td>
<td>277</td>
<td>-7</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>777</td>
<td>145</td>
<td>624</td>
<td>-8</td>
</tr>
<tr>
<td>45</td>
<td>15</td>
<td>1238</td>
<td>181</td>
<td>1053</td>
<td>-4</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>1751</td>
<td>176</td>
<td>1577</td>
<td>2</td>
</tr>
<tr>
<td>55</td>
<td>25</td>
<td>2329</td>
<td>118</td>
<td>2216</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>391</td>
<td>75</td>
<td>312</td>
<td>-4</td>
</tr>
<tr>
<td>45</td>
<td>10</td>
<td>828</td>
<td>123</td>
<td>702</td>
<td>-3</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>1315</td>
<td>133</td>
<td>1183</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>20</td>
<td>1863</td>
<td>94</td>
<td>1772</td>
<td>3</td>
</tr>
<tr>
<td>45</td>
<td>5</td>
<td>415</td>
<td>62</td>
<td>351</td>
<td>-2</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>876</td>
<td>88</td>
<td>789</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>15</td>
<td>1396</td>
<td>69</td>
<td>1330</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>437</td>
<td>42</td>
<td>394</td>
<td>-1</td>
</tr>
<tr>
<td>55</td>
<td>10</td>
<td>929</td>
<td>43</td>
<td>886</td>
<td>0</td>
</tr>
<tr>
<td>55</td>
<td>5</td>
<td>462</td>
<td>19</td>
<td>443</td>
<td>0</td>
</tr>
</tbody>
</table>

5.6 It will be seen that there is surprisingly little difference between the two reserves calculated by the different methods. The pattern is clear: the reserves in the early years of a contract are less when the two part formula is used, but slightly higher in the later years of the contract. The reason for this appears to be in the progression of $L_{x+n}$ with $n$ which complicates the decrease in reserves that usually follow from an increase in the interest rate.
5.7 When considering these results, it is as well to bear the following points in mind:

(i) the Cash/Unit reserves are lower at the most important time, that is in the early years of the policy when the initial costs of setting up the contract have to be borne.

(ii) in both cases the valuation premium is higher than the net premium used in the premium basis and the increase is more marked in the case of Cash/Unit reserves. Were the latter premium to be used, then the cash reserve in Table II would be increased by approximately \((3,5,7,9,10)\) if the policy had \((5,10,15,20,25)\) years to run respectively.

(iii) the net premium employed in the Cash/Unit basis is lower than that in the traditional basis. Consequently, for a given office premium, there is more room to manoeuvre on any Zillmer adjustment to be made. On the other hand, any such adjustment will be made to the Cash Reserve, so that the addition to the net premium for a given Zillmer will be more, because of the higher rate of interest assumed.
The Zillmer Adjustment

6.1 A Zillmer adjustment is made to enable the high initial expenses of the contract to be met out of the first premiums received, thus reducing the need for capital to meet the liabilities in the early years. It is essentially a capitalization of future margins in premiums charged over net premium required, and as such a number of constraints should be imposed upon the level of Zillmerization to be made. The most important of these constraints are:

(1) that the Unit element should not be Zillmerized.
(2) that the Zillmerized premium should not be more than the office premium, excluding the policy fee and any premium specifically required for other purposes, e.g. maturity guarantees.
(3) that the policy should not be treated as an asset.
(4) that the Zillmerized reserve should not be less than the paid up or surrender value of the policy.
(5) that the effect of expected inflation should not be such as to cause strains in the latter years of the policy in excess of foreseeable margins.

6.2 The first of these constraints, that the Unit element should not be Zillmerized, is possibly the most controversial. It is, however, consistent with the Unit approach. It has already been stated that Units and Cash should be separated, and the rate of exchange between Units and Cash can be volatile. Movement downwards causes concern about capital guarantees,
which, for better or for worse, are included as a benefit under such policies, owing to marketing pressure. The problems of such guarantees have recently been widely discussed and are also mentioned briefly later in this paper. The danger under Zillmerization is in unforeseen movement upwards in value, especially should these prove to be permanent re-adjustments, but many of the arguments pertinent to maturity guarantees still hold.

6.3 All margins in the valuation are Cash margins with the exception of (a) management charges receivable and (b) tax relief on Unit income in respect of past unrelieved expenses. Neither of these margins, is, however, susceptible of Zillmerization. Consequently, any attempt to Zillmerize the Unit reserve must be regarded as deliberate mis-matching of investments. Such action is not to be undertaken lightly in any circumstances; the volatility of the Units, and the need to make up any shortfall from Cash makes such an action more dangerous than usual. It is suggested that the risk involved in mis-matching Units should require an additional reserve to be set up for statutory valuations of such an amount as would negate the advantages gleaned from its adoption.

6.4 The statement that management charges receivable are not susceptible of Zillmerization depends on the definition of Zillmerization. It is sometimes legitimate to increase the rate of interest used for valuing Units by an amount not exceeding management charge, but it is held that this is not Zillmerization. The reserve still increases from zero at the outset of a policy to the full sum assured at maturity, whereas a Zillmer adjustment has the effect of creating a negative reserve at the outset.
6.5 To restrict the Zillmerized premium to the office premium seems self-evident; if it were not done, then unless there were other margins available, money would need to be provided from elsewhere to meet the shortfall in anticipated income, and the contracts could not be regarded as solvent. If there are other margins, it is suggested that either these will be concrete enough to be brought directly into account in the valuation, or else they will be too nebulous to be regarded as a sufficient reason to adopt a higher premium.

6.6 That a policy should not be treated as an asset is sound actuarial practice, and the arguments in its favour need not be repeated here. However, the reserve is now composed of two parts. So long as the policy is not entitled to a full surrender or paid up value, there is no need to interpret the statement to mean that the Cash reserve shall not be negative, but rather to allow also for the Unit reserve when applying the constraint. However, the question then arises as to what extent the Unit reserve may be used to offset a negative Cash reserve given that Units may depreciate in price. The problem is not easy to solve, and the extent must depend on the type of Unit involved. For practical purposes, it may be sufficient to restrict the size of the negative Cash reserve, to, say, 75 per cent. of the Cash applied in the purchase of the Units representing the Unit reserve. This approach has the merit that the adjustment allowed does not fluctuate with variations in Unit price. Very occasionally policies will be treated as assets, but only for a very short while, as more purchases of Units, at the depreciated price will rectify the position. This question should also be considered in the context of the next few paragraphs.
6.7 Again, that a policy should be valued at not less than might be paid out under circumstances beyond the control of the office is sound actuarial practice. Once a policy is entitled to full paid up and surrender rights, normally after a period of five years for policies of longer term, the liability arising upon cessation of premiums is the Unit reserve, and it follows that the Zillmer adjustment should not be so large as to cause the Cash reserve still to be negative. Before full rights are available, however, it is likely that there will be a period during which some smaller benefit will be paid upon cessation. During this period, it is reasonable to allow the Cash reserve to be negative up to an amount equal to, say, three-quarters of that proportion of the Unit reserve which has been deducted when determining the paid up or surrender value.

6.8 Where a new life office issues a Category B contract, it is likely that part of the business written will be reassured on an original terms basis. When this happens, it is often the case that the commission paid by the reassurers to the ceding office is greater than the commission which the ceding office is paying to the procurers of the business. It is quite possible for the reassurance commission to be such that it is sufficient to meet all the commission payable to the procurers. Under these circumstances, the expenses of the policy will be largely independent of size. It is traditional to express the Zillmer adjustment in terms of the sum assured. However, in this case, it would seem more appropriate to express the Zillmer adjustment in terms of the premium.
payable. This is because it is generally found that the average sum assured of a policy varies with the term of the policy, whereas the average premium paid per policy tends to be independent of the term.

6.9 Where the policies are being valued individually the adoption of this approach is not different. Nor is it difficult to ensure that the constraints mentioned in paragraphs 6.5 and 6.6 apply on an individual policy by policy basis. Where, however, the valuation is being made on grouped data this method is not so convenient, nor is it possible to ensure that the constraints are complied with on a policy by policy basis. It is suggested that when this occurs, it should be sufficient for the actuary to investigate the pattern of reserves set up for the various terms of policy being issued, and to ensure that the Zillmer adjustment does not lead to the constraints applying. If he is satisfied that the constraints are not reached then it should be sufficient for him to state this in the statutory valuation, and to value on the equivalent Zillmer, but expressed as a percentage of the sum assured and having no regard to the fact that for longer term policies (where a higher Zillmer adjustment is made than is justified) constraints (2) and (3) may be breached.

6.10 Renewal expenses are normally taken into account by charging a fixed policy fee per policy. It is important, however, not simply to dismiss renewal expenses by assuming that this policy fee will cover the future costs. It is now generally recognised that some level of inflation is likely to be
with us for the foreseeable future, and it would be imprudent not to have regard to inflation of expenses in both the premium basis and the valuation basis. As a first step, the policy fee may be set at a somewhat higher level than is initially required. This is an advantageous approach as there is then a smaller strain from any delay in receiving tax relief on the expenses during the early years of a contract. It would however mean that a reserve should be set up from the policy fees received in the early years of the policy to cover renewal expenses in the later years of the policy in excess of the policy fee. In practice it is difficult to quantify this reserve, and it is suggested that rather than set up a specific reserve for this contingency it is preferable to restrict the Zillmerization which is made on the policy to ensure that there will not be a negative cash flow under the policy in later years if the rate of inflation assumed in the calculations is incurred. This problem will not, of course, arise if the company is receiving a management charge from the Units and this has not been taken into account in the valuation basis.
Associated Problems

7.1 The advantage of setting up a model office to investigate the cash flow arising under the contract has already been mentioned. An office model is a powerful tool, not only for investigating such items as the Zillmer adjustment, but also for studying any of the subjects which have been mentioned hitherto and put on one side because they obscure the salient features of the policy.

7.2 The first such complication is that Unit income may be variable. This should make little difference to the premium basis; although the company is likely to be more cautious about the average Unit income yield which it may expect to receive. The Unit valuation basis, is, however, likely to be more stringent than the premium basis adopted in order to satisfy the Department of Trade & Industry that the policyholders have the degree of security which it feels is appropriate. As a result, the initial strain under such policies will be greater, and it is necessary to ensure that a reasonable yield is obtained on the capital employed.

7.3 The second important feature is that the rate of exchange of Units may vary in either direction. As far as maturity and death guarantees are concerned, there has been a considerable amount of research and discussion in recent years, and it is intended to make only a few observations on the subject.
7.4 The problem of deciding upon the correct premium to charge for capital guarantees almost defies solution. The main reason for this lies in the difference between the Units and Cash which has been one of the themes of this paper, and the attempt to guarantee a minimum rate of exchange of Units for Cash. The theory of capital guarantees is very similar to those of stop-loss reassurance and of reinsurance in the general insurance market. One is concerned not only with the tail of the probability distribution relating to the benefit payment but also with setting a premium which will not significantly disturb the solvency of the office.

7.5 So far, this appears to have been done for equity units by taking a very conservative view of the movement of equity indices, and assuming that the correct premium to be charged is the premium according to this view. The general insurance market has approached the subject from a different direction. There, the practice is to take a realistic view of the pattern of events and apply a probability distribution, determining the expectation and the variance of the amount to be paid out. The premium calculated by reference to the expectation is then regarded as being the minimum premium; and would be appropriate only if the company was in a position where it had just an evens chance of remaining solvent. Where, however the solvency level of the company was much higher, then the premium to be charged should be increased by such a multiple
of the variance as would ensure that the likelihood of the company being ruined is not increased due to the introduction of the benefit under consideration. When the probability of a payment being made is remote, the actual premium calculated using this criterion may be several times that according to the expected value.

7.6 It is admitted that this approach depends upon an adequate model of the Unit price being available. However, consideration of the problem in this way may help to show that the premiums calculated by the present method may not be unreasonable, and indeed may be too low, even though present methods do rely on very conservative estimates of the market behaviour of Unit prices.

7.7 Having determined the premium to be paid for the capital guarantee, the next problem is valuation. It appears only reasonable to use a retrospective approach in the early years of a policy (when the future is too uncertain as to provide a meaningful basis), and a prospective approach in the later years. It is therefore suggested that until the policy is within ten years of maturity, the premium charged be reserved, and accumulated with interest at the rate appropriate to Cash reserves; once the policy is within ten years of maturity, this reserve would be one tenth of the full
retrospective amount multiplied by the number of years to run
plus one tenth of the actual shortfall (if any) under the
policy multiplied by (ten less the number of years to run).
Any release in reserve involved would be placed in a contin-
gency reserve until that reserve was sufficient to cover a
fall of, say, 50 per cent. in Unit prices in respect of all
policies within ten years of maturity.

7.8 When the traditional approach to valuation is being
adopted, it is usual for the policy to be valued as the sum
of an ordinary endowment assurance and the reversionary value
of the appreciation of the Units allocated. Very often,
depreciation on any individual policy is not allowed as a
negative value, on the grounds that the capital value of the
Units is guaranteed. No doubt such action is approved by the
policyholders and by the D.T.I., but it is questionable whether
it is fair on the shareholders. This is especially so in the
case of a young life office which may find itself forced to
raise further capital in unpredictable quantities if Unit
prices fall. It is suggested that, provided the premium
charged for the capital guarantee is adequate, there should be
no need to exclude depreciation if to do so would require more
capital to be raised, although a moderate increase in the
capital guarantee fund may be required to preserve the solvency
level at its former position. Alternatively, if it is felt
that to include all depreciation weakens the valuation basis too far, then at least depreciation should be allowed to the extent of the capital guarantee fund.

7.9 This problem is intensified where the office purchases Units at a price in excess of the bid value credited to the policyholder. The difference produces an automatic depreciation in value which, when the first approach mentioned in the previous paragraph is adopted, may cause a strain. Strictly speaking unless it were assumed in the valuation basis that the Units would appreciated in value this difference should be met from the premiums as they were received, and the Zillmerized premium thus further restricted to the office premium less this margin and less any premium required for other purposes.

7.10 The total liabilities for unit-linked contracts within a life office will be composed of Unit reserves for each different type of Unit offered by the company. The different types of Units can no more be mixed than can Cash and Units. In consequence it is suggested that it should be obligatory to show each type of Unit and Cash separately in Statutory returns and likewise to differentiate between them in the assets. As far as Cash reserves are concerned, however, there is no
essential difference between Cash reserves held for different types of unit linked contract or between those held for capital guarantees and capital gains tax. The admission of negative Cash reserves for individual policies may mean that, for a recently introduced contract, the overall Cash reserve is negative. This is of no consequence (presentation apart), and the only constraints should be that the total Cash reserves be positive. Care must however be taken to ensure that the interest rates assumed on positive Cash reserves are compatible with those assumed on the offsetting negative Cash reserve.
Conclusion

8.1 The paper only scratches at the surface of the subject. Some areas where further papers would be helpful have been indicated, and there are doubtless others, together with areas where more research is required. Perhaps the area where the need is most pressing is that of capital guarantees. This subject is controversial, and has been the object of much heated discussion amongst. Actuaries for some little time. A reasonable solution is urgently needed, for adoption in statutory returns to avoid the arbitrary imposition of unsatisfactory methods.

Author's Note

The opinions expressed in this paper are my own, and do not necessarily reflect those held by my partners, to whom I am, however, indebted for the full discussion of ideas which has led to this paper. I would also like to thank those people who have read a draft of this paper and have been good enough to comment on it.

November 1973