

Ambiguity Aversion and Insurance*

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Abstract

This paper considers financial markets for uncertain cashflow streams when participants are averse to model or parameter uncertainty, or more generally averse to ambiguity. Motivated by the desire to better understand why it is difficult to sell rainfall insurance in the developing world, this paper provides theoretical foundations for a type of constraint on private insurance markets intuitively understood by practitioners but not yet satisfactorily incorporated into theory. It is argued that prudential requirements and information asymmetries cause financial institutions to be better modelled as ambiguity averse decision-makers than expected profit maximisers. The model presented also offers explanations for the almost exclusive use of traditional insurance policies which closely match the risk to be insured, and the absence of indexed products as suggested by Shiller (2003).

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1 Introduction

Economists have convincingly argued that the primary function of a financial system is to allow the consumption streams of economic agents to be made less similar to their income streams; agents borrow and save to smooth consumption over time, and diversify, insure and hedge to smooth consumption over states of nature.¹ However, even highly developed financial systems are far from perfect. This paper considers a significant constraint on the ability of financial institutions to manage uncertainty, intuitively understood by many practitioners but not yet satisfactorily incorporated into theory. By drawing on recent advances in the study of decision-making under uncertainty, I also hope to offer insights of interest to financial professionals.

This paper considers decision-making of financial institutions and individuals when some events have obvious probability distributions and some do not. The former type of uncertainty is usually termed risk, and the latter is termed Knightian uncertainty or ambiguity. I argue that financial institutions may be modelled as ambiguity averse decision-makers, preferring to trade uncertainty when they have a good understanding of the odds, and demanding a premium to take on ambiguous uncertainty. A decision-maker who prices prudently under model or parameter uncertainty is an example of an ambiguity averse decision maker. I explain observed characteristics of financial markets in terms of ambiguity aversion of financial institutions and individuals, provide a technical result that develops the intuition provided by Mukerji and Tallon (2001) to non-linear financial contracts, and offer two explanations for ambiguity averse behaviour of financial institutions. It is shown that ambiguity aversion of financial institutions acts to reduce the ability of the financial system to pool and manage uncertainty.

Much has been written about the constraints that cause imperfections in financial systems. Indeed, such constraints are of more than just academic interest; they may cause people to suffer more than they have to. In the extreme, the inability of the poorest to protect themselves from adverse shocks can lead to unnecessary hunger and death. An insurance actuary might be aware of two theoretical constraints on private insurance markets that are of significant importance in practice. Firstly there may be informational constraints in the form of adverse selection (Akerlof 1970, Borch 1981) or moral hazard (Pauly 1974, Stiglitz 1983). Such constraints may lead to no trade in some risks, for example human capital, or may be partially overcome in a number of ways, including screening (Rothschild and Stiglitz 1976, Hellwig 1987) and signalling (Spence 1973). The constraints on products and equilibria in insurance markets are well understood and I shall not attempt to summarise the considerable literature here; for an overview of the effects of information asymmetries on insurance markets see the relevant chapters in Dionne, ed (2000) and for a more general survey of information asymmetries see Riley (2001).

¹For a textbook exposition of such arguments see Danthine and Donaldson (2002).

Secondly, there may be administrative costs of providing insurance. A key theoretical contribution is Raviv (1979) which develops a model of optimal insurance contracting to justify commonly observed features of insurance products. Specifically it is shown that when costs are not constant over the level of insurance coverage an optimal insurance contract exhibits coinsurance and a deductible clause. Such contracts only pay for a proportion of losses above a certain level, and are extremely common. Separately, Gollier (2003) has argued that high administrative costs of insurance may restrict demand for insurance products to such a degree that the added value of the insurance sector is surprisingly low.

This paper argues that a third significant constraint on private insurance markets arises from insurance companies or potential policyholders acting as though they were averse to ambiguity, preferring to engage in unambiguous risks with known probability distributions rather than ambiguous risks where beliefs are consistent with more than one probability distribution. Academics have written very little about why financial institutions might act as though they are averse to bad probability distributions, as opposed to just being averse to bad outcomes. Although full discussion is deferred until later, it is noted that statutory and professional reserving requirements of financial institutions are invariably more stringent in conditions of greater ambiguity, leading financial institutions to price prudently, as an ambiguity averse decision maker would.

As an example of how aversion to ambiguity may significantly constrain markets for uncertainty, neither information asymmetries nor costs adequately explain the absence of insurance against bets on nature where the seller and buyer have the same limited information. However, as is discussed in Section 2.1 in the context of rainfall insurance, financial institutions will often not be willing to contract on a particular source of uncertainty unless information is sufficiently informative.

It is often argued that financial institutions may be well modelled as risk-neutral expected utility maximisers, or equivalently expected profit maximisers. However, an expected profit maximiser would choose to sell a financial contract if the price were strictly larger than the perceived economic value and would buy the same contract if the price were strictly smaller than the perceived economic value. This perceived economic value is a single number, based on a single assessment of the probability of any event. Indeed, this economic value would be a single number even if the financial institution was a risk averse or risk loving expected utility maximiser. However, in practice we observe financial institutions displaying *portfolio inertia*, where they will buy a contract for any amount below a certain price, and sell it for any amount above a higher price, but will neither buy nor sell within a nondegenerate band of prices. Such a decision maker is not acting to maximise expected profits, based on a single best estimate belief.

When actuaries decide how much they are willing to buy or sell mortality risk for they will often include margins that cannot be fully explained by costs or selection of risks. For example, when selling an assurance product with higher payments on early death,

actuarial prudence would dictate that all calculations used mortality assumptions heavier (higher probability of death in a given year) than best estimate. When selling an annuity product with higher payments on later death, actuarial prudence would dictate that all calculations used mortality assumptions lighter (lower probability of death in a given year) than best estimate (Bühlmann 1970). Both methods overestimate the expected present value of liability outgo and are therefore considered prudent. Disregarding other margins and the technicality that best estimate might not be the same for individuals purchasing these two products because of different selection criterion, life insurance companies exhibit portfolio inertia over mortality risk, and indeed most risks that are ultimately borne by the life insurance company.²

The idea that ambiguity aversion may constrain markets is not new (Ellsberg 1961), but recent developments in the theory of decision making under ambiguity allow tractable and robust formalisations to develop intuition in the context of financial markets. As suggested in the pioneering work of Dow and Werlang (1992) and later generalised by Mukerji and Tallon (2003), there is a strong relationship between ambiguity aversion and portfolio inertia. Results have recently been demonstrated for specific economically significant problems, such as the absence of indexation of debt (Mukerji and Tallon 2004a) or wage contracts (Mukerji and Tallon 2004b), but these results assume a linearity between the decision maker's endowment and the financial product. A technical result in this paper suggests that portfolio inertia results carry through to financial products with payoffs a non-linear function of a noisy index.

Motivated by the difficulties of designing fire insurance, actuaries talked openly about the challenge of moral hazard long before economists began to think seriously about it.³ In a similar way, insurance companies and regulators developed methods of robust decision making under ambiguity before the recent positive formalisations by economists. Formal arguments are helpful in clarifying, developing, and contradicting intuition and are of particular use in academic discourse as assumptions or reasoning may be refuted. In this paper such a formalisation is offered for the effects of ambiguity aversion on trading uncertain cashflow streams. I begin by motivating this analysis with discussion of two contracts: Rainfall insurance and livelihood index insurance.

²Risks not ultimately borne by the life insurance company would usually be priced on a market basis, which would implicitly include any prudence margins of those ultimately bearing the risk.

³For example Crosby's (1905) paper on insurance and fire prevention was half a century before Arrow's (1963) interpretation and formalisation of moral hazard.

2 Motivating Examples

2.1 Rainfall Insurance

It is readily apparent that, in a global context, risk sharing is far from perfect; the fortunes of individuals do not rise and fall as one. However, risk sharing in smaller groups within the developing world seems to be surprisingly good, despite highly volatile incomes. Most empirical studies reject a hypothesis of full consumption insurance within villages, households or other groups, but find consumption within groups to be highly correlated (Deaton 1992, Townsend 1994, Udry 1994, Ligon 1998, Fafchamps and Lund 2003).

That insurance within communities is statistically distinguishable from perfect is not surprising; it is costly to overcome informational and commitment problems, and complete risk sharing in such a world is unlikely to be optimal. That insurance within communities is good is also not surprising; poor individuals might be expected to have strong incentives to overcome problems and develop mechanisms that protect them from the consumption levels associated with very high mortality rates.

Strategies for managing the risks faced by a group may be separated into two categories: Amending the group's aggregate profile of real risks; and entering into a portfolio of financial contracts to offset the effects of some of these real risks. Real risks such as income, mortality and health risks may be reduced in severity by diversification of income sources within the group, migration or marriage of a trusted member, or engagement in low risk, low expected return activities. An offsetting strategy might include holding unproductive physical assets to be used in time of need (self-insurance) as well as participating in informal, formal and government insurance arrangements.

However, no amount of risk-pooling within a group will protect its members against shocks that harm all members of the group, known as *covariate* shocks. Also, large unlikely shocks, termed *catastrophic* shocks are often difficult to share through informal mechanisms as they require full enforceability of contracts. Informal enforcement is shallow and if a large shock hits, and a large transfer is required, the informal insurer is likely to successfully renege on the agreement. As any informal insurance cannot credibly offer full protection against catastrophic risks, such catastrophic risks will remain, at least partially, uninsured. Self-insurance, although widely used in the developing and developed world (Carroll 1997, Morduch 2004), is inefficient if unproductive assets are invested in, as is likely to be the case in the presence of an underdeveloped financial sector.

There is now a significant body of microeconomic evidence showing that the inability of the poor to protect themselves from catastrophic or group-level covariate risk is not only a result of but also a major cause of persistent poverty.⁴ Briefly, the lack of insurance leads

⁴For a recent survey see Dercon (2002).

households to make inefficient decisions whilst still leaving them vulnerable to catastrophic or covariate shocks. Much of the recent theoretical literature on microeconomic risk management in the developing world has focussed on trying to disentangle the process by which individuals and groups manage risk through informal arrangements. However, with neither informal insurance nor self-insurance well-suited to managing catastrophic or covariate risks, formal or public contracts must be considered.

For the rural poor, yield and revenue risk from agricultural produce is substantial and drives much decision-making (Besley 1993). Agricultural risks are also often covariate within a community and include catastrophic elements, and following the above reasoning optimal management of these risks would require some element of formal contracting with strangers. However, traditional insurance contracts such as full or partial indemnity crop insurance have not yet been shown to be sustainable due to moral hazard problems, even in rich countries with easily enforceable contracts. Glauber (2004) estimates that for every \$1 paid in premiums by farmers for the US Federal Crop Insurance Program, the federal government provided subsidy of \$1.23 in 1981 and \$2.32 in 2003, with an average over this period of over \$2. Skees et al. (2002) report corresponding figures for crop insurance schemes in Brazil (1975-1981: \$3.57), Costa Rica (1970-1989: \$1.80), India (1985-1989: \$4.11), Mexico (1980-1989: \$2.65), and the Philippines (1981-1989: \$4.74).

Motivated by the difficulty of providing traditional insurance products tailored to the needs of the poor, the World Bank has recently been trialling micro and macro index insurance schemes in a number of developing countries.⁵ In a typical index insurance contract the policyholder pays a premium to the company at the effective date of the contract and the company makes payouts at one or more specified future dates calculated as defined functions of one or more realisations of the index. Popular products offer payouts based on regional rainfall or price indices, protecting against rainfall falling below a specified level or prices rising above a specified level. As an example of a macro index insurance product, in 2005 the World Bank and United Nations World Food Program piloted a rainfall index-based product for the Ethiopian government, constructed with the aim of providing a payout related to ‘aggregate Ethiopian rainfall-induced livelihood losses’ in the event of a major drought.

In many settings rainfall is the most substantial agricultural risk faced and large rainfall shocks can have substantial effects on welfare. Dercon et al. (2005) used Ethiopian data to estimate that experiencing a drought at least once in the previous five years lowers per capita consumption by approximately 20%. Menon (2006) used Nepalese data to show that higher levels of rainfall risk are associated with an increase in costly diversification away from rainfall risk. As an alternative to crop insurance, rainfall levels are surprisingly easy to directly insure using formal index insurance; there is no perceived moral hazard and insurers may protect against adverse selection by restricting the rainfall stations included in the index to those with a reliable history. The sparsity of suitable

⁵For an engaging overview of analysis and trials by the World Bank see Hess et al. (2005).

weather stations means that payouts may depend on rainfall in geographical areas a policyholder has no interest in and may therefore not well match the loss to be insured for some individual. However, this effect may be expected to be small as rainfall is often highly covariate within a region. Unlike aid or government support for which payouts are typically delayed and unreliable, rainfall insurance claims are cheap and quick to verify and administer. Successful design and implementation of a rainfall insurance product would not only bring direct gains due to the offsetting nature of insurance, but also allow individuals to choose more efficient real risk portfolios, which could then be further insured.

However, index insurance products have proved challenging to sell. It is instructive to consider the case of the BASIX Weather Insurance Program, India's first weather insurance initiative, launched on a trial basis in 2003 in association with the World Bank and ICICI Lombard. BASIX is an Indian microfinance institution with a focus on the agriculture sector and the trial involved a small weather insurance pilot program for groundnut and castor farmers in Andhra Pradesh (Gine et al. 2005). The trial is widely considered to be a success and has been followed by increased sales in subsequent years, but coverage is still sparse and largely limited to landowners close to existing rainfall stations (Manuamorn 2005).⁶

This paper will argue that rainfall insurance is difficult to sell because of ambiguity aversion of individuals and financial institutions. Insurance companies refuse to sell products based on rainfall levels from stations without a suitably long history, reducing the attractiveness of products to potential policyholders. Regardless of the cause of this behaviour, such an insurance company may be well modelled as an ambiguity averse decision-maker. Individuals hold ambiguous beliefs about how well the insurance product payoffs match their exposure. In the extreme, the ambiguity aversion of individuals or insurance companies can lead to no trade in such products.

2.2 Livelihood index insurance

To understand the future of index insurance in the developing world, it is helpful to consider the current state of such products in the developed world. There are examples of index insurance markets used to move risk between financial institutions such as insurance and reinsurance companies, but index insurance products sold to individuals in the developed world are conspicuous by their absence. Shiller (1993, 2003) suggests that this is due to a lack of imagination on the part of the finance industry. Consider Shiller's

⁶This is somewhat concerning. As noted by Morduch (2006), if only the landed purchase rainfall insurance we might expect price effects in the event of poor rainfall similar to those as analysed in Sen's (1981) work on the Great Bengal Famine: In the event of poor rainfall, landowners with rainfall insurance may drive consumer prices beyond affordability for the landless accentuating the effect of the poor rainfall on the lives of the landless.

hypothetical livelihood insurance product for an aspiring scientist who has committed to acquire a specialised education in a specific field. If the field is successful the scientist will prosper but, if not, the scientist will suffer, perhaps finding it impossible to switch occupations. Direct income insurance is impossible for standard moral hazard reasons: The scientist may work less hard if her lifetime salary is guaranteed. Shiller instead suggests an index insurance contract which would take in premiums if incomes in the field are higher than some agreed level and provide a payout if incomes in the field are lower than this level. The insurance contract is essentially a financial swap, swapping the index of average income in the field for a risk-free income stream. The scientist could pass a substantial amount of lifetime career income risk to the capital markets without any moral hazard problems (barring grand moral hazard where the individual is able to alter the index). Were such a product available, an aspiring scientist would gladly enter into the contract to transfer some lifetime career risk through the capital markets to investors the world over.

Ignoring the desire for career flexibility and possibility of unemployment, classical insurance theory would suggest that, even with independent noise in the swap product, a risk averse scientist would pay a premium to buy a positive amount of such an index product, as long as the product offered an element of insurance against lifetime income risk (Mossin 1968).⁷ This result is robust to the introduction of uninsurable background risks under plausible restrictions on preferences; indeed the intuition from theoretical and empirical work on background risks is that higher levels of uninsurable background risk corresponds to an increased willingness to pay for insurance against other insurable risks (Pratt and Zeckhauser 1987, Gollier and Pratt 1996, Guiso and Jappelli 1998, Heaton and Lucas 2000). Individuals could manage their risks like financial institutions do, intentionally hedging as best they can with imperfect instruments. However, this does not appear to happen. As with the absence of rainfall insurance, we will explain this behaviour in terms of ambiguity aversion of the individual or financial institution.

⁷Such a result is also proved in part (b) of Proposition 1.

3 Theoretical Foundations

3.1 Ambiguity

When modelling decision making under uncertainty, economists have developed valuable insight using Savage's (1954) elegant and tractable theory of subjective expected utility (SEU). A decision maker whose preferences satisfy Savage's axioms makes decisions as if she were maximising the expected value of her utility, for some utility function and some set of beliefs. A major implication of SEU, and in particular the linearity imposed by the independence axiom, is that the behaviour of a decision maker may be represented as if her subjective beliefs about uncertain events correspond to a unique probability distribution. Gollier (2001) defends this approach as follows:

If we relax the independence assumption, most problems presented in this book cannot be solved anymore... [T]he combination of facts showing that the independence axiom, and its related axioms of time consistency and consequentialism, make common sense and make most problems solvable is enough to justify the exploration of its implications.

However, although allowing significant tractability and being reasonable in a wide range of circumstances, SEU does not appear to be a good model of behaviour under ambiguity. According to Knight's (1921) distinction, there are two types of uncertainty: *risk* corresponds to situations in which events related to decision making may be assigned obvious probabilities; *Knightian uncertainty* or *ambiguity* as it was termed by Ellsberg (1961) corresponds to situations in which some events related to decision making may not be assigned obvious probabilities. Keynes (1921) introduced an additional distinction between judged probability and the weight of evidence.

The Ellsberg (1961) two-colour experiment elegantly clarified the intuition that decision makers behave as though they are averse to ambiguity whilst demonstrating that such behaviour may not be represented in Savage's SEU framework. The experiment runs as follows:

Urn *I* contains 100 balls, either red (*R*) or black (*B*), with unknown numbers of each. Urn *II* contains 50 red balls and 50 black balls. There are four possible lotteries: *BI*, *BII*, *RI* and *RII*. In lottery *CI* one ball is drawn at random out of urn *i*, $i = I$ or II , and the player receives \$100 if the color is *C*, $C = R$ or B , or \$0 otherwise.

Ellsberg correctly predicted that most people are indifferent to the choice between *RI* and *BI* and also to the choice between *RII* and *BII*, but that they strictly prefer *RII*

to RI and BII to BI .⁸ Such preferences violate the independence axiom and may not be represented in an SEU framework where the decision maker's subjective beliefs about uncertain events correspond to a unique probability distribution. Without entering a formal argument we may demonstrate this second point by noting that indifference between RII and BII implies that the subjective probability assigned to event RII must be $1/2$. However, a strict preference of RI over RII implies that the subjective probability of RII must be less than $1/2$.

Frisch and Baron (1988) suggests five possible explanations for such ambiguity averse behaviour: issues of blame, responsibility and regret are more salient in situations of ambiguity; an opponent may know more than you and have an advantage; a hostile opponent may bias the situation to your disadvantage; a series of identical ambiguous gambles (in which the missing information turns out the same) is more risky than a series of non-ambiguous gambles; and ambiguity may increase the attractiveness of 'delay'. The first explanation is most commonly appealed to, rephrased for an audience of economists as ex-post regret over ex-ante missing information. However, in section 3.4 I argue that the second explanation is equally credible in a situation of financial contracting as an alternative or supplement to detailed contracting, stratification of risks and signalling. In the same section I introduce a complementary explanation for a financial institution to act as though it was ambiguity averse: If the first priority of a regulator is for a financial institution to meet its liabilities as they fall due, prudent contracting and reserving requirements will be enacted which are more burdensome in conditions of greater ambiguity.

Ellsberg's work has stimulated a wealth of theory and a number of competing models of decision making under subjective uncertainty, most notably the Choquet expected utility (CEU) model of Schmeidler (1989) and the maxmin expected utility (MEU) model of Gilboa and Schmeidler (1989). In both models a decision maker's subjective beliefs are ambiguous if they cannot be represented by a unique probability distribution, but can be represented by a set of probabilities. For the purposes of this work we will apply the MEU model, as I believe it offers the clearest intuition in the context of a state space with infinite support. However, the results presented in this paper hold more generally within the class of ambiguity averse models. We note that the MEU model is a limiting case of the promising smooth model of decision making under ambiguity proposed by Klibanoff et al. (2005) and has strong similarities to the robust decision maker in Hansen et al. (1999). There is also a partial correspondence between MEU and CEU frameworks.⁹

⁸Indeed, Ellsberg (1961) reports Savage as holding these preferences on being confronted with the two-colour experiment.

⁹Specifically, for the class of non-additive probabilities there will correspond a unique set of priors for which the MEU criterion coincides with the CEU evaluation (Mukerji 1997).

3.2 Maxmin Expected Utility

Let Ω be a state space with typical element $\omega \in \Omega$ and Σ be the Borel- σ algebra on Ω . X is the set of all outcomes and $u : X \rightarrow \mathbb{R}$ is a von Neumann-Morgenstern utility over outcomes. Z is the set of all actions or acts, where an act is a Σ -measurable function $z : \Omega \rightarrow X$. In the SEU model with μ a subjective probability measure on (Ω, Σ) , the subjective expected utility of an act $z \in Z$ is:

$$\mathbb{E}_\mu[u(z)] = \int_\Omega u(a(\omega))d\mu(\omega)$$

In the MEU model the decision maker's prior Δ is a non-empty, compact and convex set of finitely additive probability measures on (Ω, Σ) . The Minmax expected utility of an act $z \in Z$ is:

$$\text{MEU}[u(z)] = \min_{\mu \in \Delta} \int_\Omega u(a(\omega))d\mu(\omega) = \min_{\mu \in \Delta} \mathbb{E}_\mu[u(z)]$$

Wald (1950) introduced the above MEU criterion as an intuitive solution to the problem of decision making under subjective uncertainty. An MEU decision maker evaluates actions based on the minimum expected utility over a set of probability measures and then chooses the best action from among them. The set of measures may correspond to all possible measures or perhaps just those that satisfy some threshold level of epistemic reliability (Gärdenfors and Sahlin 1982). Unlike more recent formulations of behaviour under ambiguity the model does not impose a separation between ambiguity attitude and beliefs. The narrative will ignore changes in ambiguity attitude and instead refer to a more inclusive set of priors as corresponding to more ambiguous beliefs. This corresponds to the intuition in Klibanoff et al.'s (2005) smooth model of ambiguity aversion in which MEU is a limiting case with infinite ambiguity aversion. However, it is possible to intuitively interpret differences in sets of priors as different attitudes.

Gilboa and Schmeidler (1989) provided an axiomatic justification for the MEU model based on the Anscombe and Aumann (1963) representation of SEU with independence replaced with two weaker assumptions: *c-independence*, under which (weak) preference rankings are preserved under mixing with constant acts; and *uncertainty aversion*, under which mixing may be allowed some hedging value. An alternative axiomatic justification was recently provided by Casadesus-Masanell et al. (2000) in a Savage (1954) framework with finite or infinite state space.

3.3 Basis Uncertainty

In the context of insurance, the *basis* is the difference between insurance payout and the policyholder's loss and *basis risk* refers to the risk that payouts from the insurance product do not exactly meet the potential policyholder's underlying exposure to be insured. For consistency in usage of the terms *risk*, *ambiguity*, and *uncertainty* this paper will refer to uncertainty in the basis that may be ambiguous as *basis uncertainty* as opposed to the potentially misleading *basis risk*. A product that matches a policyholder's exposure more closely will be said to have lower basis uncertainty and might be expected to have higher loading due to the difficulty of designing such a product whilst protecting the financial institution from adverse selection and moral hazard.¹⁰ Insurance products aimed at individuals are usually designed to have low or very low basis uncertainty as individuals do not appear to purchase insurance products with substantial basis uncertainty, even if they have much lower loading. Only sophisticated firms or institutions appear to be willing to offset risks using imperfect risky instruments, often employing sophisticated asset-liability modelling techniques to meet their objectives.

It should be mentioned that index insurance is a slight abuse of terminology: all financial contracts offer an indexed return in exchange for a premium, where the contract specifies the payout as a function of the indices. Rather, I consider the distinction between index and traditional insurance contracts to be due to differing levels of basis uncertainty. To give an extreme example of the difference between index and traditional insurance, consider annuity products intended to protect against living longer than expected. A traditional annuity product would provide a regular sum until death of the policyholder. A hypothetical index annuity product might provide a regular sum until an index falls below a certain level, where the index is calculated as the total number of individuals still alive from a specified group. Both products protect against living longer than expected, but the traditional product offers payouts that match the individual's exposure more closely. In practice, with little moral hazard and actuaries able to accurately select there is a large market in traditional annuities and very little need for a market in index-linked annuities, despite the slightly lower costs. However there are some losses for which profitable trade in traditional insurance is impossible but profitable trade in index insurance might be possible. It is these losses for which index insurance products are increasing being designed and trialled by the World Bank and microfinance institutions.

An actuary might argue that it is the presence of substantial perceived basis uncertainty that causes individuals not to buy Shiller's (2003) livelihood insurance product described in Section 2.2. This dislike of basis uncertainty may be explained to some extent within a SEU framework as the negative effect of additional basis uncertainty acts to offset the positive effect of increased insurance against the risk to be insured.¹¹ However, ambigu-

¹⁰We consider the premium loading factor or loading on an insurance product to be the λ that solves: Insurance Premium = $(1 + \lambda) \times \mathbb{E}[\text{Insurance Payout}]$.

¹¹To achieve portfolio inertia over an insurance product in an SEU framework we must introduce fixed

ity aversion acts to sharpen a decision-maker's dislike of the uncertainty in the basis if that uncertainty is ambiguous, and does so to a greater extent in conditions of greater ambiguity.

I consider ambiguity aversion to provide a strong intuition for aversion to uncertainty in the basis, in addition to any dislike arising from risk aversion. Unwillingness to purchase an insurance product is explained if the potential policyholder is ambiguity averse, has sufficiently ambiguous beliefs about the basis uncertainty and unambiguous beliefs about the risk to be insured, and the risk to be insured does not have too large a variance. Such a policyholder 'knows' the risk to be insured but doesn't 'know' how the payoffs from the insurance product relate to this risk. The proposed framework seems to explain observable characteristics of risk sharing. Sophisticated financial institutions do manage risk using products with large basis uncertainty, but these institutions are likely to invest in information to the point where their beliefs about the products are unambiguous, or have low ambiguity. In terms of an MEU framework, their set of prior beliefs may be considered to be small, and close to best estimate.

The framework also explains the partial success of rainfall insurance products in the developing world. Rainfall is a significant source of volatility in agricultural income in regions where these rainfall index insurance products are successful. However, this is not enough to ensure purchase; farmers must also hold sufficiently unambiguous beliefs about the basis uncertainty in the product. That products are first successfully sold to the better educated members of the community, who may have a better understanding of the insurance payouts from the insurance product, is not therefore surprising. That others later purchase insurance appears to be consistent with agents refining ambiguous prior beliefs on provision of additional information.

This paper's emphasis on ambiguity over basis uncertainty also explains the experimental results in Di Mauro and Maffioletti (2001) which suggest that ambiguity over the risk to be insured only has a weak impact on consumers' valuation of insurance. This paper argues that it is ambiguity over the basis uncertainty, not ambiguity over the risk to be insured, driving portfolio inertia in financial markets.

3.4 Prudence

Prudent behaviour, as understood by regulators and financial professionals, is behaviour that demonstrates some degree of caution. The training of financial risk professionals is laden with references to prudence under parameter or model uncertainty and this ap-

transaction costs or require that the insurance product may only be purchased in discrete lumps. Indeed, part (b) of proposition 1 shows that in the absence of fixed transaction costs or discrete insurance, a risk averse subjective expected utility maximiser would buy a positive amount of such a product so long as the product offered insurance against the individual's asset portfolio.

pears to be consistent with observed behaviour. Hogarth and Kunreuther (1992) and Kunreuther et al. (1993) report on a mail survey of 469 North American actuaries, hypothetically asked to act as consultants to a computer manufacturer concerning the price of a warranty. The suggested prices were considerably higher when probabilities were ambiguous than when they were well-specified. That actuaries behave in this way is uncontroversial within the profession. However, why don't firms just maximise expected profits, based on best estimate beliefs? We offer two normative explanations for this ambiguity averse behaviour in financial markets and argue that decision-making of financial institutions may be modelled intuitively by an MEU decision rule.

Actuaries are comfortable holding ambiguous beliefs about risks, using Bayesian techniques to incorporate new information, as introduced in an actuarial context by Whitney (1918) and later developed as the Empirical Bayes Credibility Model (Robbins 1956, Mayerson 1964, Bühlmann 1970). In addition to margins for expenses, cost of capital and profits, actuaries often perform calculations using a single probability measure calculated as the best estimate Bayesian measure but with the systematic addition of *prudence margins*, where the margins are higher in conditions of perceived greater ambiguity.¹²

When combined with selection or stratification of risks, this prudence may be considered as a practical way of combatting adverse selection. Following Akerlof (1970), consider a used car market, but where a series of tests enable buyers to correctly stratify by quality, θ . If the buyer is only able to tell that the quality of the car on sale is within some strata $\theta_L \leq \theta < \theta_H$ and is particularly averse to making an aggregate loss on a series of deals, perhaps it might be reasonable for the buyer to price the car based on the assumption $\theta = \theta_L$. This two step methodology of selection followed by prudence underlies actuarial pricing of risks, where the second step may intuitively be interpreted as decision making in an MEU framework where the set of beliefs about θ include all possibilities between θ_L and θ_H . Our set of beliefs is the set of all possible priors after selection and the MEU decision rule acts assuming the most pessimistic possibility for θ .

As well as providing for an additional margin against adverse selection, I consider prudence of financial institutions (FIs) to be a rational regulatory response to ambiguity. A statutory priority of most financial regulators is to maintain trust in the financial system and this priority is almost exclusively interpreted as requiring that FIs are able to meet their liabilities as they fall due with a high probability. For example in UK law the Financial Services and Markets Act 2000 lists 'maintaining confidence in the financial system' as one of the four statutory objectives of the Financial Services Authority (FSA), the primary regulator of financial institutions and professionals under UK law. In turn the FSA's General Prudential Sourcebook requires that:

¹²An exception to this was the pricing of UK defined benefit pension scheme liabilities in the 1990s, where actuaries routinely used optimistic pricing assumptions to calculate the employer contribution to the scheme. Such behaviour was never adequately justified theoretically, but seems to have come about due to weak regulation and a resulting 'race to the bottom' in actuarial assumptions used by actuarial consultancies (Gordon 1999).

A firm must at all times maintain overall financial resources, including capital resources and liquidity resources, which are adequate, both as to amount and quality, to ensure that there is no significant risk that its liabilities cannot be met as they fall due.

One method commonly employed by regulators to achieve this objective is to require financial institutions to hold statutory reserves as self insurance against risks borne by the institution. Reserving requirements for insurance companies in the UK started with the Life Assurance Companies Act of 1887 which required all insurance companies to deposit a sum of £20,000 with the Accountant-General (Raynes 1948). Reserving requirements have since become more complex, and are often more stringent in conditions of greater ambiguity. I now offer an intuition as to why this might be the case.

Consider a FI offering insurance on 100 independent identically distributed coin tosses. For each head the FI must pay £1. The regulator is concerned with the aggregate risk profile, requiring $\mathbb{P}(\text{insolvency}) < 5\%$ where insolvency occurs if the aggregate payout from these 100 insurance contracts exceed the reserves for these contracts. If the regulator is certain that the coins are fair an aggregate statutory reserve of £58 would be sufficient to keep $\mathbb{P}(\text{insolvency}) < 5\%$. However, now assume that expert opinion on the bias of the coins differs with some experts believing that the probability of a head is 0.6 and some believing that it is 0.4. If the regulator believes that each set of experts is correct with probability $\frac{1}{2}$, an aggregate statutory reserve of £66 is now required to keep $\mathbb{P}(\text{insolvency}) < 5\%$.¹³ Moreover, when the coin bias is known and as the number of contracts increase, the law of large numbers allows statutory reserves per policy to become close to £0.5. However, when the coin bias is unknown reserves per policy must not fall below £0.6 or the probability of insolvency would be greater than 5%.

Although the increased ambiguity does not change the best estimate probability distribution to be used, it acts to increase the subjective spread of outcomes. Moreover, when entering into independent identically distributed risks ambiguity does not ‘diversify out’; it is only by taking a large number of risks subject to independently ambiguous beliefs that allows ambiguity to be diversified out. Many financial institutions specialise in trading particular classes of risk and are unable to diversify the ambiguity over that class of risk. It is this undiversified ambiguity that I consider the cause of more stringent reserving requirements under conditions of greater ambiguity.

Even if the FI’s sole objective were to maximise expected profits, without any additional weight on minimising the probability of insolvency, the larger statutory reserving requirement under greater ambiguity requires an excess holding of liquid assets, generating an economic cost. This economic cost must be funded by a larger premium, corresponding to a pricing calculation on a more cautious set of assumptions. In practice the FI may also be averse to insolvency, perhaps because of the deadweight cost of insolvency or ca-

¹³Let $B_p \sim \text{Binomial}(100, p)$. Then $\mathbb{P}(B_{0.5} \leq 58) = 95.6\%$. Similarly $\mathbb{P}(\frac{B_{0.4} + B_{0.6}}{2} \leq 66) = 95.4\%$.

reer concerns of agent-employees. Indeed, it is common for insurers to hold assets that provide a margin well in excess of any required by the supervisory authority, although it is unclear as to whether this is driven by an aversion to insolvency or a reduction in the economic cost of reserving brought about by less stringent liquidity requirements in the presence of larger reserves (Faculty and Institute of Actuaries 2006). Even in a more general setup in which the FI may hold assets to offset some of its liability portfolio, an increase in ambiguity makes it more difficult to match ambiguous assets to independently ambiguous liabilities, acting to increase the subjective ex-ante probability of insolvency. If an FI were able to match its asset and liability portfolio exactly, passing all risk elsewhere, there would be no need for margins against ambiguity. However, in a world of costly contracting and limited liability ownership in which FIs bear residual risk, prudential requirements which are stricter in conditions of greater ambiguity may be a rational regulatory response to an extreme dislike of FI insolvency.

Detailed regulations relating to reserving requirements are often written in terms of a particular set of prudent assumptions. Similarly, decisions within a FI are often based on a single calculation using a single set of appropriately prudent assumptions. Such prudent decision making may be intuitively modelled in a MEU framework of ambiguity aversion, in which decisions are based on the most pessimistic probability distribution from a set of prudent prior beliefs about the aggregate risk portfolio.

4 The Model

We consider two agents, a financial institution (FI) and potential policyholder (PH), and any simple insurance contract:

Definition In a *simple insurance contract* between a FI and a PH the contract is agreed at time 0 and the premium payment, index realisation, and claim payout occur at time 1. The net payout at time 1 from FI to PH is given by $B \times f(I) - P$, where I is the random realisation of the underlying index at time 1. B, f and P are agreed at time 0 where f is a finitely valued weakly monotone function determining the shape of insurance, not constant over the support of I , B is a weakly positive constant and P is the premium for the contract.

Such a specification includes most financial products that could be thought of as offering insurance, such as financial options and stop loss or quota insurance with or without deductables. Common shapes for simple index insurance contracts include $f(I) = B - \max(I - K, 0)$ and $f(I) = \max(K - I, 0)$, for some strike index level K .

We will consider an index I given by the sum of two (statistically) independent random variables Y and Z . Y is a local shock which determines the PH's residual portfolio before purchase of insurance and Z is the source of noise in the insurance product. Y may be thought of as the risk to be insured and Z as the basis uncertainty to the PH in the insurance product. In the context of rainfall insurance, Y might represent the rainfall uncertainty as it affects the farmer's livelihood and Z the source of independent noise from other regional weather stations included in the index. In this model an *increase in basis uncertainty* corresponds to an increase in the perceived variability of Z .

The PH's uninsured portfolio of Y is clearly increasing in the realisation of Y . As we are interested in insurance products which act to partially offset the uninsured risk exposure, attention is restricted to simple insurance contracts with f weakly decreasing in I . The payoffs from the insurance contract are therefore weakly decreasing in the realisation of Y for a given realisation of Z .

We may write the FI's random profit variable Π as:

$$\Pi = P - B \times f(I) \tag{1}$$

and the PH's random wealth variable W as:

$$W = Y - [P - B \times f(I)] \tag{2}$$

In the absence of insurance the PH's utility is denoted by $\bar{u} > 0$. We denote the PH's utility as a function of realised wealth w as $u(w)$ where u satisfies strict non-satiation ($u' > 0$) and strict concavity ($u'' < 0$). The PH is therefore assumed to be risk averse over realised wealth, and as part (b) of Proposition 1 shows, in the absence of ambiguity would pay a positive amount for any simple insurance contract with payoffs not constant over the support of the uninsured portfolio Y . The FI is considered to be risk neutral.

Both PH and FI believe the cumulative distribution functions for Y and Z to be members of a two-parameter family of cdfs, $G(\theta, \phi)$, with support over some open subset of the real numbers S that is closed under addition in the following strong sense:¹⁴

$$\forall s, s' \in S, \exists s'' \in S \text{ s.t. } s + s'' = s' \quad (3)$$

$\phi \geq 0$ is the variance of the distribution and $\phi = 0$ corresponds to a constant random variable. $G(\theta, \phi)$ has full support over S except for $\phi = 0$. $\theta \in \mathbb{R}$ ranks distributions by strict first order stochastic dominance such that for any ϕ :

$$\begin{aligned} \theta_1 > \theta_2 &\Leftrightarrow G(\theta_1, \phi) \text{ strictly first order stochastically dominates } G(\theta_2, \phi) \\ &\Leftrightarrow G(\theta_1, \phi)(s) < G(\theta_2, \phi)(s), \text{ for all } s \in S \end{aligned}$$

Agents have common belief that the variance of Y is ϕ_Y and the variance of Z is ϕ_Z . However they may have different beliefs about the true θ parameters. We shall denote the potentially ambiguous beliefs of the PH and FI about Y and Z as Δ_Y^{PH} , Δ_Z^{PH} , Δ_Y^{FI} and Δ_Z^{FI} with the following parameterisation:

$$\begin{aligned} \Delta_Y^{PH} &= \{G(\theta, \phi_Y) | \underline{\theta}_Y \leq \theta \leq \bar{\theta}_Y\} && \text{where } \underline{\theta}_Y \leq \bar{\theta}_Y \\ \Delta_Z^{PH} &= \{G(\theta, \phi_Z) | \underline{\theta}_Z \leq \theta \leq \bar{\theta}_Z\} && \text{where } \underline{\theta}_Z \leq \bar{\theta}_Z \\ \Delta_Y^{FI} &= \{G(\eta, \phi_Y) | \underline{\eta}_Y \leq \eta \leq \bar{\eta}_Y\} && \text{where } \underline{\eta}_Y \leq \bar{\eta}_Y \\ \Delta_Z^{FI} &= \{G(\eta, \phi_Z) | \underline{\eta}_Z \leq \eta \leq \bar{\eta}_Z\} && \text{where } \underline{\eta}_Z \leq \bar{\eta}_Z \end{aligned}$$

We shall define the *ambiguity of belief of the PH over Y* to be $\mathcal{A}_Y^{PH} = \bar{\theta}_Y - \underline{\theta}_Y$ and refer to a belief containing a single measure as an *unambiguous belief* and a prior belief

¹⁴We assume that S is closed under addition in this strong sense to avoid the possibility of a specification for f which is able to separate the basis uncertainty from the risk to be insured. If this assumption is weakened and we allow the insurance product to separate the basis uncertainty from risk to be insured we find that the results still hold for some, but not all, simple insurance contracts on a specified index. For example, consider Y and Z both Bernoulli distributed with the PH preferring $Y = 1$ to $Y = 0$ and all other assumptions in the model satisfied. The PH would always buy a positive amount of an insurance product which paid out only in the state $\{I=0\}$ as the state $\{Y=0\}$ has occurred with certainty. Indeed, such an argument may be constructed for any closed state space and is why we separately restrict attention to an open state space.

containing more than one measure as an *ambiguous belief*. Beliefs of the FI and PH about $X \in \{Y, Z\}$ will be termed *consistent* if one agent's set of priors for X is a subset of the other agent's set of priors for X .

Given a functional form for f The FI chooses the simple insurance contract that maximises the Minmax Expectation of profits Π , provided that the premium is non-negative and satisfies the PH's participation constraint. Hence, the optimal insurance contract solves the program 4 below, where Π and W are given by 1 and 2, respectively:

$$\begin{aligned}
& \text{Max}_{P,B} && \min_{\mu \in \Delta_Y^{FI}, \nu \in \Delta_Z^{FI}} \mathbb{E}_{\mu, \nu}[\Pi] \\
& \text{s.t.} && (i) \quad \min_{\mu' \in \Delta_Y^{PH}, \nu' \in \Delta_Z^{PH}} \mathbb{E}_{\mu', \nu'}[u(W)] \geq \bar{u} \\
& && (ii) \quad B \geq 0
\end{aligned} \tag{4}$$

Now, since elements of Δ_Z^{PH} , Δ_Y^{FI} and Δ_Z^{FI} are ordered by first order stochastic dominance, Π is weakly increasing in the realisations of Y and Z and W is weakly decreasing in the realisation of Z , we may simplify this program to:

$$\begin{aligned}
& \text{Max}_{P,B} && \mathbb{E}_{G(\eta_Y, \phi_Y), G(\eta_Z, \phi_Z)}[\Pi] \\
& \text{s.t.} && (i) \quad \min_{\mu' \in \Delta_Y^{PH}} \mathbb{E}_{\mu', G(\bar{\theta}_Z, \phi_Z)}[u(W)] \geq \bar{u} \\
& && (ii) \quad B \geq 0
\end{aligned} \tag{5}$$

An analysis of the properties of the solution to 5 yields proposition 1. Part (a) of the proposition shows that the policyholder holding sufficiently unambiguous beliefs about Y and the volatility of Y being within some bound is sufficient for no trade in insurance contracts as long as either the PH or FI hold ambiguous beliefs about Z or the FI holds ambiguous beliefs about Y . Part (b) of the proposition shows that if there is a risk to be insured and beliefs are consistent then ambiguity of beliefs is necessary for no trade in insurance contracts.

Proposition 1 *For the optimization program posed in (5), the following holds:*

- (a.) *For a FI and PH holding consistent beliefs about Y and Z with at least one of $\bar{\theta}_Y > \underline{\eta}_Y$ and $\bar{\theta}_Z > \underline{\eta}_Z$ holding, there exists $\bar{\phi}_Y > 0$ such that if $\phi_Y \leq \bar{\phi}_Y$ then $B = 0$ at any solution to the optimization program. Further, if f is a continuous function then there exist $\bar{A} > 0$ and $\bar{\phi}_Y > 0$ such that if $\phi_Y \leq \bar{\phi}_Y$ and $\mathcal{A}_Y^{PH} \leq \bar{A}$ then $B = 0$ at any solution to the optimization program.*

(b.) Let P, B be a solution to the optimization program where $\phi_Y > 0$ and the FI and PH hold consistent and unambiguous beliefs about Y and Z . Then $B \neq 0$ at any solution to the optimization program.

Proof of Proposition 1 See Appendix

We now discuss two direct interpretations of this result. Firstly, consider an insurance market where policyholders hold unambiguous beliefs about the risk to be insured, but financial institutions price prudently, perhaps for reasons described in Sections 3.3 and 3.4. The above proposition shows that for any given insurance contract on the index, there will be no trade if the volatility of the risk to be insured is within some bound. The friction imposed here is not costs, but rather the prudence margins built into pricing perhaps to protect from adverse selection or to limit the probability of ruin.

Figure 1 demonstrates this interpretation diagrammatically. The PH's indifference curve corresponding to a utility of \bar{u} is shown, crossing the P-axis at $P = \bar{P}$. Profit lines are also shown for two FIs, which price the insurance product using differing degrees of prudence. A higher prudence margin may be caused by a FI holding more ambiguous beliefs about the payouts from the insurance product or being more averse to ambiguity. The slope of the profit lines are increasing in the prudence margins implicit in FI calculations.¹⁵ Higher degrees of prudence in FI pricing correspond to less insurance being purchased in equilibrium.

Secondly, consider an insurance market in which financial institutions do not price prudently but in which policyholders hold ambiguous beliefs about basis uncertainty. Provided that the policyholder's ambiguity about the risk to be insured and volatility of risk to be insured are within some bound, the above proposition shows there will be no trade in a given insurance product on the index. The policyholder prefers to keep the entire unambiguous risk rather than swapping some of it for an ambiguous risk.

Figure 2 demonstrates this interpretation diagrammatically. A profit line is shown for a FI holding unambiguous beliefs about the payoff from the insurance product. Indifference curves for two PHs are shown corresponding to a utility of \bar{u} . The slope of this indifference curve is the marginal willingness to pay an additional premium as the coverage increases. Intuitively, the higher the ambiguity over basis uncertainty, the smaller the additional premium an individual is willing to pay for greater coverage.¹⁶ Higher degrees

¹⁵The slope of the profit lines are given by $\mathbb{E}_{G(\underline{\eta}_Y, \phi_Y), G(\underline{\eta}_Z, \phi_Z)}[f(I)]$ and are decreasing both in $\underline{\eta}_Y$ and $\underline{\eta}_Z$. Lower values of $\underline{\eta}_Y$ or $\underline{\eta}_Z$ correspond to more inclusive sets of prior beliefs Δ_Y^{FI} and Δ_Z^{FI} , and may be considered as corresponding to FI calculations including higher *prudence margins*.

¹⁶The slope of the indifference curves when the PH holds unambiguous beliefs about Y , the risk to be insured, are given by:

$$\frac{\mathbb{E}_{G(\bar{\theta}_Y, \phi_Y), G(\bar{\theta}_Z, \phi_Z)}[f(I)u(W)]}{\mathbb{E}_{G(\bar{\theta}_Y, \phi_Y), G(\bar{\theta}_Z, \phi_Z)}[u(W)]}$$

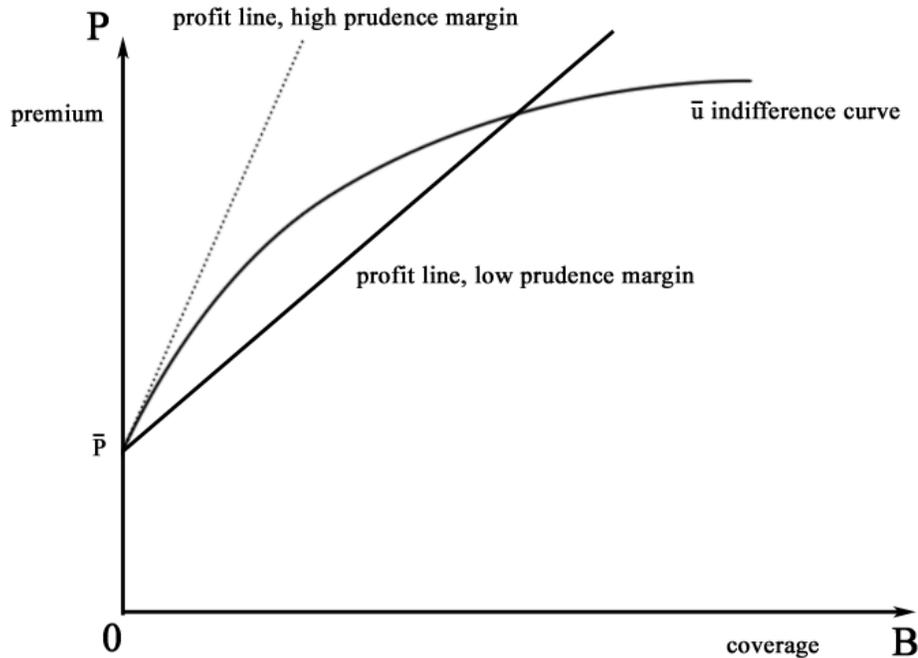


Figure 1: Prudence Margins and Equilibrium Insurance Purchase

of PH ambiguity over basis uncertainty correspond to less insurance being purchased in equilibrium.

In the context of rainfall insurance, ambiguity averse financial institutions may only offer products indexed to rainfall recorded at weather stations with suitably informative historic data. Any product sold on this index is likely to include basis uncertainty, and this uncertainty is likely to be greater the further away the farm from the station. Farmers are likely to have a very good understanding of the risks they face, but are likely to be much less certain about the idiosyncratic payoffs from an unknown insurance product. Indeed, many farmers in the developing world are reported to not understand the concept of insurance, interpretable within this model as holding a significantly ambiguous set of priors over the basis uncertainty in the product. If farmers employ an MEU decision rule under uncertainty, we might expect no trade in index insurance products, or only trade with those farmers who live close to a suitable rainfall station and have a good knowledge

This can be seen to be decreasing in $\bar{\theta}_Z$ as both $f(I)$ and $u(W)$ are strictly decreasing in the realisation of Z and therefore the numerator decreases at a quicker rate than the denominator. Higher values of $\bar{\theta}_Z$ correspond to a larger set of prior beliefs Δ_Z^{PH} and higher ambiguity over basis uncertainty and lead to a flatter indifference curve.

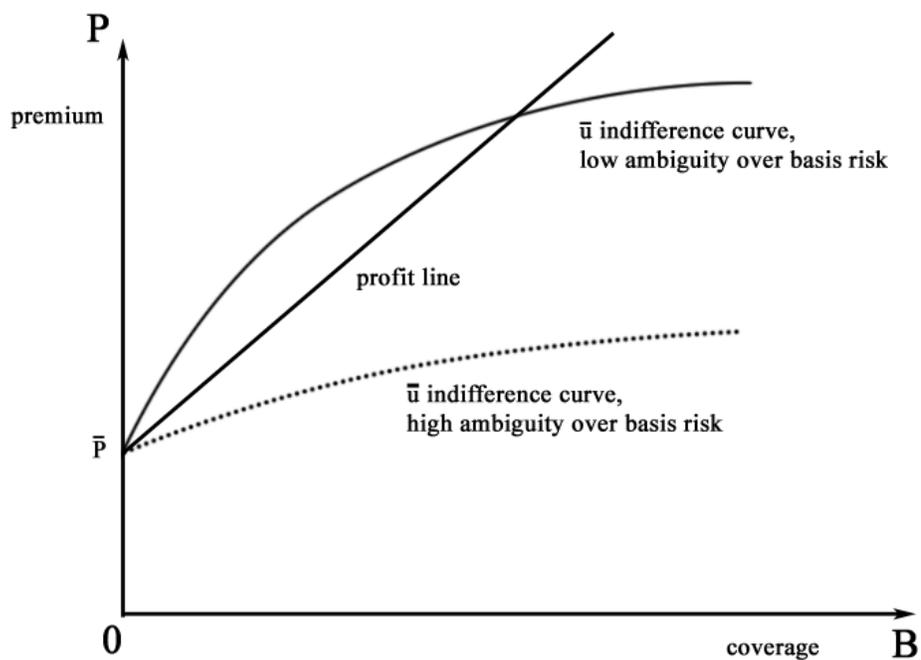


Figure 2: Policyholder Ambiguity over Basis Uncertainty and Equilibrium Insurance Purchase

of the idiosyncratic risks present in the product.

Similarly, the absence of livelihood index insurance may be attributed to ambiguity aversion of financial institutions or individuals. Financial institutions may include significant prudence margins when pricing livelihood index insurance products, making them unattractive to individuals. Alternatively, individuals may hold more precise beliefs about their salary progression than the behaviour of the index and this ambiguity may make livelihood index insurance less attractive.

5 Conclusions and future research

In this paper I have argued that prudential requirements and information asymmetries may cause financial institutions to be well modelled as ambiguity averse decision-makers. Indeed, any decision-making process which exhibits prudence under model or parameter uncertainty trivially exhibits ambiguity aversion and cannot be modelled as an expected profit maximiser. Aversion to ambiguity serves to reduce the desire to enter into financial contracts with ambiguous payoffs, and therefore serves to reduce the ability of the financial system to pool and manage uncertainty. The model presented also offers an explanation for other currently unexplained features of existing insurance markets, such as the almost exclusive use of traditional insurance policies which closely match the risk to be insured, and the absence of indexed products as suggested by Shiller (2003).

This analysis raises interesting questions for finance professionals and theorists. Prudence under ambiguity acts to decrease the amount of insurance in an economy. Would it be possible to devise robust financial regulation that allowed smaller prudence margins? How might insurance be best structured in developing countries, where limited data and information asymmetries cause much uncertainty to remain uninsured? Might different forms of ownership of financial institutions allow less ambiguity aversion in decision-making, thereby increasing the amount of profitable risk sharing? How valuable to society are the data held by large reinsurance companies, which allow trading on particular sources of uncertainty? Under what circumstances should governments purchase reinsurance company data for public use? Should developing country governments and donors fund accumulation of data for use by insurance companies in the developing world? The author intends to pursue these and related questions in future research.

References

- Akerlof, G.A.**, “The Market for “Lemons”: Quality Uncertainty and the Market Mechanism,” *The Quarterly Journal of Economics*, 1970, 84 (3), 488–500.
- Anscombe, FJ and RJ Aumann**, “A Definition of Subjective Probability,” *The Annals of Mathematical Statistics*, 1963, 34 (1), 199–205.
- Arrow, Kenneth J.**, “Uncertainty and the Welfare Economics of Medical Care,” *The American Economic Review*, 1963, 53 (5), 941–973.
- Besley, T.**, “Savings, Credit and Insurance,” in T.N. Srinivasan and J. Behrman, eds., *Handbook of Development Economics*, Vol. 3A, North-Holland, Amsterdam, 1993, pp. 2125–2207.
- Borch, K.**, “The Three Markets for Private Insurance,” *Geneva Papers on Risk and Insurance*, 1981, (20), 7–31.
- Bühlmann, H.**, *Mathematical Models in Risk Theory*, Springer Verlag, 1970.
- Carroll, C.D.**, “Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis,” *The Quarterly Journal of Economics*, 1997, 112 (1), 1–55.
- Casadesus-Masanell, R., P. Klibanoff, and E. Ozdenoren**, “Maxmin Expected Utility over Savage Acts with a Set of Priors,” *Journal of Economic Theory*, 2000, 92 (1), 35–65.
- Casella, G. and R.L. Berger**, *Statistical inference*, second ed., Duxbury Press, 2002.
- Crosby, E.U.**, “Fire Prevention,” *Annals of the American Academy of Political and Social Science*, 1905, 26, 224–238.
- Danthine, J.P. and J.B. Donaldson**, *Intermediate financial theory*, Prentice Hall, 2002.
- Deaton, A.**, “Household Saving in LDCs: Credit Markets, Insurance and Welfare,” *The Scandinavian Journal of Economics*, 1992, 94 (2), 253–273.
- Dercon, S.**, “Income Risk, Coping Strategies, and Safety Nets,” *The World Bank Research Observer*, 2002, 17 (2), 141–166.
- , **J. Hoddinott, and T. Woldehanna**, “Shocks and Consumption in 15 Ethiopian Villages, 1999–2004,” *Journal of African Economies*, 2005, 14 (4), 559–585.
- Dionne, G., ed.**, *Handbook of Insurance*, Kluwer Academic, 2000.
- Dow, J. and S.R.C. Werlang**, “Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio,” *Econometrica*, 1992, 60 (1), 197–204.
- Ellsberg, Daniel**, “Risk, Ambiguity, and the Savage Axioms,” *The Quarterly Journal of Economics*, 1961, 75 (4), 643–669.
- Faculty and Institute of Actuaries**, “ST2: Life Insurance Specialist Technical, Core reading,” 2006.

- Fafchamps, M. and S. Lund**, “Risk-sharing networks in rural Philippines,” *Journal of Development Economics*, 2003, *71*, 261–287.
- Frisch, Deborah and Jonathan Baron**, “Ambiguity and rationality,” *Journal of Behavioral Decision Making*, 1988, *1* (3), 149–157.
- Gärdenfors, P. and N.E. Sahlin**, “Unreliable probabilities, risk taking, and decision making,” *Synthese*, 1982, *53* (3), 361–386.
- Gilboa, Itzhak and David Schmeidler**, “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*, 1989, *18* (2), 141–153.
- Gine, X., HB Lilleor, RM Townsend, and J. Vickery**, “Weather Insurance in Semi-arid India,” 2005. Paper presented at the Annual Bank Conference on Development Economics, May 23-24, Amsterdam.
- Glauber, J.W.**, “Crop Insurance Reconsidered,” *American Journal of Agricultural Economics*, 2004, *86* (5), 1179–1195.
- Gollier, C.**, *The Economics of Risk and Time*, The MIT Press, 2001.
- , “To Insure or Not to Insure?: An Insurance Puzzle,” *The GENEVA Papers on Risk and Insurance-Theory*, 2003, *28* (1), 5–24.
- and **J.W. Pratt**, “Risk Vulnerability and the Tempering Effect of Background Risk,” *Econometrica*, 1996, *64* (5), 1109–1123.
- Gordon, T.J.**, “The Price of Actuarial Values,” Presented to the Staple Inn Actuarial Society 1999.
- Guiso, L. and T. Jappelli**, “Background Uncertainty and the Demand for Insurance Against Insurable Risks,” *The GENEVA Papers on Risk and Insurance-Theory*, 1998, *23* (1), 7–27.
- Hansen, L.P., T.J. Sargent, and T.D. Tallarini**, “Robust Permanent Income and Pricing,” *Review of Economic Studies*, 1999, *66* (4), 873–907.
- Heaton, J. and D. Lucas**, “Portfolio Choice in the Presence of Background Risk,” *The Economic Journal*, 2000, *110* (460), 1–26.
- Hellwig, M.**, “Some recent developments in the theory of competition in markets with adverse selection,” *European Economic Review*, 1987, *31* (1–2), 319–325.
- Hess, Ulrich, Jerry Skees, Barry Barnett, Andrea Stoppa, and John Nash**, “Managing Agricultural Production Risk: Innovations in Developing Countries,” *The World Bank, Agriculture and Rural Development Department, Report*, 2005.
- Hogarth, R.M. and H. Kunreuther**, “Pricing insurance and warranties: Ambiguity and correlated risks,” *The Geneva Papers on Risk and Insurance-Theory*, 1992, *17* (1), 35–60.
- Keynes, J.M.**, *A Treatise on Probability*, Macmillan, London, 1921.

- Klibanoff, P., M. Marinacci, and S. Mukerji**, “A Smooth Model of Decision Making Under Uncertainty,” *Econometrica*, 2005, 73 (6), 1849–1892.
- Knight, F.H.**, *Risk, Uncertainty and Profit*, Houghton Mifflin Company, 1921.
- Kunreuther, H., R. Hogarth, and J. Meszaros**, “Insurer ambiguity and market failure,” *Journal of Risk and Uncertainty*, 1993, 7 (1), 71–87.
- Ligon, E.**, “Risk Sharing and Information in Village Economies,” *Review of Economic Studies*, 1998, 65 (4), 847–864.
- Manuamorn, O.P.**, “Scaling-Up Micro Insurance,” *Washington DC, World Bank*, 2005.
- Mauro, C. Di and A. Maffioletti**, “The Valuation of Insurance under Uncertainty: Does Information about Probability Matter?,” *The GENEVA Papers on Risk and Insurance-Theory*, 2001, 26 (3), 195–224.
- Mayerson, A.L.**, “A Bayesian View of Credibility,” *Proceedings of the Casualty Actuarial Society*, 1964, 51, 85–104.
- Menon, Nidhiya**, “The Effect of Rainfall Uncertainty on Occupational Choice in Rural Nepal,” March 2006. Available at <http://ssrn.com/abstract=893194>.
- Morduch, J.**, “Consumption Smoothing Across Space: Testing Theories of Consumption Smoothing in the ICRISAT Study Region of South India,” in S. Dercon, ed., *Insurance against Poverty*, 2004, chapter 2.
- , “Why isn’t there more Financial Intermediation in Developing Countries?,” in A.V. Banerjee, R. Benabou, and D. Mookherjee, eds., *Understanding Poverty*, 2006, chapter 21.
- Mossin, J.**, “Aspects of Rational Insurance Purchasing,” *The Journal of Political Economy*, 1968, 76 (4), 553–568.
- Mukerji, S.**, “Understanding the nonadditive probability decision model,” *Economic Theory*, 1997, 9 (1), 23–46.
- and **J.M. Tallon**, “Ambiguity Aversion and Incompleteness of Financial Markets,” *Review of Economic Studies*, 2001, 68 (4), 883–904.
- and —, “Ellsberg’s two-color experiment, portfolio inertia and ambiguity,” *Journal of Mathematical Economics*, 2003, 39, 299–316.
- and —, “Ambiguity aversion and the absence of indexed debt,” *Economic Theory*, 2004, 24 (3), 665–685.
- and —, “Ambiguity aversion and the absence of wage indexation,” *Journal of Monetary Economics*, 2004, 51 (3).
- Pauly, M.V.**, “Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection,” *The Quarterly Journal of Economics*, 1974, 88 (1), 44–62.

- Pratt, J.W. and R.J. Zeckhauser**, “Proper Risk Aversion,” *Econometrica*, 1987, 55 (1), 143–154.
- Raviv, A.**, “The Design of an Optimal Insurance Policy,” *The American Economic Review*, 1979, 69 (1), 84–96.
- Raynes, H.E.**, *A History of British Insurance*, Pitman & Sons, 1948.
- Riley, J.G.**, “Silver Signals: Twenty-Five Years of Screening and Signaling,” *Journal of Economic Literature*, 2001, 39 (2), 432–478.
- Robbins, H.**, “The Empirical Bayes Approach to Statistics,” in J. Neyman, ed., *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability*, Vol. 1 University of California Press 1956, p. 157163.
- Rothschild, M. and J. Stiglitz**, “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *The Quarterly Journal of Economics*, 1976, 90 (4), 629–649.
- Savage, L.J.**, *The Foundation of Statistics*, John Wiley and Sons, Inc., 1954.
- Schmeidler, D.**, “Subjective Probability and Expected Utility without Additivity,” *Econometrica*, 1989, 57 (3), 571–587.
- Sen, A.K.**, *Poverty and famines: An Essay on Entitlement and Deprivation.*, Oxford: Clarendon Press New York, 1981.
- Shiller, R.J.**, *Macro Markets: Creating Institutions for Managing Society’s Largest Economic Risks*, Clarendon Pr, 1993.
- , *The new financial order: risk in the 21st century*, Princeton University Press, 2003.
- Skees, JR, P. Hazell, and M. Miranda**, “New Approaches to Public/Private Crop Yield Insurance,” *Agricultural Insurance Seminar in Zaragoza, Spain*, 2002.
- Spence, M.**, “Job Market Signaling,” *The Quarterly Journal of Economics*, 1973, 87 (3), 355–374.
- Stiglitz, J.E.**, “Risk, Incentives and Insurance: The Pure Theory of Moral Hazard,” *The Geneva Papers on Risk and Insurance*, 1983, 8 (26), 4–33.
- Townsend, Robert M.**, “Risk and Insurance in Village India,” *Econometrica*, 1994, 62 (3), 539–591.
- Udry, C.**, “Risk and Insurance in a Rural Credit Market: An Empirical Investigation in Northern Nigeria,” *The Review of Economic Studies*, 1994, 61 (3), 495–526.
- Wald, A.**, *Statistical decision functions*, Wiley, New York, 1950.
- Whitney, A.W.**, “The theory of experience rating,” *Proceedings of the Casualty Actuarial Society*, 1918, 4, 275–293.

A Proofs

To prove Proposition 1 we will need the following lemma.

Lemma 1 (Covariance inequalities)¹⁷ *Let X be any random variable with support over the real numbers and $g(x)$ and $h(x)$ any functions such that $\mathbb{E}[g(X)]$, $\mathbb{E}[h(X)]$ and $\mathbb{E}[g(X)h(X)]$ exist.*

(a.) *If $g(x)$ is a nondecreasing function and $h(x)$ is a non-increasing function, then:*

$$\mathbb{E}[g(X)h(X)] \leq (\mathbb{E}[g(X)])(\mathbb{E}[h(X)])$$

(b.) *If $g(x)$ and $h(x)$ are either both increasing or both decreasing, where $g(x)$ is strictly monotonic, $h(x)$ is non-constant over the smallest support of X , and X is non-constant then:*

$$\mathbb{E}[g(X)h(X)] > (\mathbb{E}[g(X)])(\mathbb{E}[h(X)])$$

Proof of Lemma 1 (a.) Let $\mathbb{E}[h(X)] = \mu_h$ and $x_h^* = \inf\{x|h(x) \geq \mu_h\}$. Using the existence of expectations we have:

$$\begin{aligned} & \mathbb{E}[g(X)(h(X) - \mu_h)] \\ &= \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) < \mu_h)] + \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) \geq \mu_h)] \\ &\leq \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) < \mu_h)] + \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) \geq \mu_h)] \\ &= g(x_h^*)\mathbb{E}[(h(X) - \mu_h)] \\ &= 0 \end{aligned}$$

(b.) Again let $\mathbb{E}[h(X)] = \mu_h$ and $x_h^* = \inf\{x|h(x) \geq \mu_h\}$. $h(x)$ is non-constant over the smallest support of X so both $\{x|h(x) < \mu_h\}$ and $\{x|h(x) > \mu_h\}$ must be nonempty. g is strictly monotone and increasing (decreasing) when h is non-decreasing (non-increasing) and therefore

$$\begin{aligned} \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) < \mu_h)] &> \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) < \mu_h)] \\ \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) \geq \mu_h)] &> \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) \geq \mu_h)] \end{aligned}$$

So

$$\begin{aligned} & \mathbb{E}[g(X)(h(X) - \mu_h)] \\ &= \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) < \mu_h)] + \mathbb{E}[g(X)(h(X) - \mu_h)I(h(X) \geq \mu_h)] \\ &> \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) < \mu_h)] + \mathbb{E}[g(x_h^*)(h(X) - \mu_h)I(h(X) \geq \mu_h)] \end{aligned}$$

¹⁷Part (a.) may be found in Casella and Berger (2002, p. 192).

$$\begin{aligned}
&= g(x_h^*)\mathbb{E}[(h(X) - \mu_h)] \\
&= 0 \quad \blacksquare
\end{aligned}$$

Proof of Proposition 1 For ease of notation we will use the following shorthand:

$$\mathbb{E}_{\bullet_Y, \bullet_Z} \text{ instead of } \mathbb{E}_{G(\bullet_Y, \phi_Y), G(\bullet_Z, \phi_Z)}.$$

We consider the program when $\mathcal{A}_Y^{PH} = 0$ with $\underline{\theta}_Y = \bar{\theta}_Y$ and may reduce 5 to:

$$\begin{aligned}
&Max_{P,B} \quad \mathbb{E}_{\eta_Y, \eta_Z}[\Pi] \\
&s.t. \quad (i) \quad \mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u(W)] \geq \bar{u} \\
&\quad\quad (ii) \quad B \geq 0
\end{aligned} \tag{A1}$$

Let γ_u and γ_B be the Lagrange multipliers associated with the constraints (i) and (ii). In addition to the complementary slackness conditions, the first order conditions are:

$$P : \quad 1 - \gamma_u \times \mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W)] = 0 \tag{A2}$$

$$B : \quad -\mathbb{E}_{\eta_Y, \eta_Z}[f(I)] + \gamma_u \times \mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W) \times f(I)] + \gamma_B = 0 \tag{A3}$$

Equation (A2) and strict non-satiation of u gives us:

$$\gamma_u = \frac{1}{\mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W)]} > 0 \tag{A4}$$

Rearranging equation (A3) and substituting out γ_u using equation (A4) gives:

$$\begin{aligned}
\gamma_B &= \mathbb{E}_{\eta_Y, \eta_Z}[f(I)] - \gamma_u \times \mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W) \times f(I)] \\
&= \mathbb{E}_{\eta_Y, \eta_Z}[f(I)] - \frac{\mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W) \times f(I)]}{\mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W)]}
\end{aligned} \tag{A5}$$

(a.) In the extreme case in which $\phi_Y = 0$ we may use part (a) of Lemma 1 with $f(I)$ as the random variable of interest. Noting that W is a weakly increasing function of $f(I)$ and therefore $u'(W)$ is a weakly decreasing function of $f(I)$, we obtain:

$$\begin{aligned}
\gamma_B &= \mathbb{E}_{\eta_Y, \eta_Z}[f(I)] - \frac{\mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W) \times f(I)]}{\mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[u'(W)]} \\
&\geq \mathbb{E}_{\eta_Y, \eta_Z}[f(I)] - \mathbb{E}_{\bar{\theta}_Y, \bar{\theta}_Z}[f(I)]
\end{aligned}$$

Now f is non-constant over S and distributions are strictly ordered by θ in the sense of stochastic dominance. Beliefs of the PH and FI about Y and Z are consistent so $\eta_Y \leq \bar{\theta}_Y$ and $\eta_Z \leq \bar{\theta}_Z$ and we are given that at least one of the inequalities is strict. The first term in the above equation must therefore be strictly greater than the second, and:

$$\gamma_B > 0$$

In this extreme case in which $\phi_Y = 0$ we find the unique Lagrange multipliers for the two constraints are both strictly positive, implying that the participation constraint binds and $B = 0$. By continuity γ_B remains positive and $B = 0$ whenever $\phi_Y \leq \bar{\phi}_Y$ for $\bar{\phi}_Y$ small enough.

When f is a continuous function we note that for \mathcal{A}_Y^{PH} small enough W is an increasing function of the realisation of Y and the program in 5 again reduces to A1. Again, in the extreme case in which $\phi_Y = 0$ we find the unique Lagrange multipliers for the two constraints are both strictly positive, implying that the participation constraint binds and $B = 0$. By continuity γ_B remains positive and $B = 0$ whenever $\mathcal{A}_Y^{PH} \leq \bar{\mathcal{A}}$ and $\phi_Y \leq \bar{\phi}_Y$ for some $\bar{\mathcal{A}}$ and $\bar{\phi}_Y$ small enough.

(b.) We consider $\phi_Y > 0$, and assume that $B = 0$. Substituting $\bar{\eta}_Y = \underline{\theta}_Y = \theta_Y$ and $\bar{\eta}_Z = \underline{\theta}_Z = \theta_Z$ into equation (A5) gives:

$$\gamma_B = \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \frac{\mathbb{E}_{\theta_Y, \theta_Z}[u'(W) \times f(I)]}{\mathbb{E}_{\theta_Y, \theta_Z}[u'(W)]} \quad (\text{A6})$$

Using the law of iterated expectations and $B = 0$ gives:

$$\begin{aligned}
\gamma_B &= \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \frac{\mathbb{E}_{\theta_Y, \theta_Z}[u'(Y - P) \times f(Y + Z)]}{\mathbb{E}_{\theta_Y, \theta_Z}[u'(Y - P)]} \\
&= \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \frac{\mathbb{E}_{\theta_Z}[\mathbb{E}_{\theta_Y}[u'(Y - P) \times f(Y + Z)|Z]]}{\mathbb{E}_{\theta_Y, \theta_Z}[u'(Y - P)]}
\end{aligned}$$

$u'(y - P)$ is strictly decreasing in y and $f(y + z)$ is weakly decreasing in y . Y is a non-constant random variable with full support over S (as $\phi_Y > 0$), f is non-constant over S and S is closed under addition in the strong sense of condition (3), so $f(y + z)$ is non-constant over $y \in S$ for any $z \in S$. We can therefore apply part (b.) of Lemma 1 with Y as the random variable of interest to give:

$$\gamma_B = \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \frac{\mathbb{E}_{\theta_Z}[\mathbb{E}_{\theta_Y}[u'(Y - P) \times f(Y + Z)|Z]]}{\mathbb{E}_{\theta_Y, \theta_Z}[u'(Y - P)]}$$

$$\begin{aligned}
&< \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \frac{\mathbb{E}_{\theta_Z}[\mathbb{E}_{\theta_Y}[u'(Y - P)|Z] \times \mathbb{E}_Y[f(Y + Z)|Z]]}{\mathbb{E}_{\theta_Y, \theta_Z}[u'(Y - P)]} \\
&= \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] - \mathbb{E}_{\theta_Y, \theta_Z}[f(I)] \\
&= 0
\end{aligned}$$

$\gamma_B < 0$ provides us with a contradiction in the sense that $\phi_Y > 0$, all beliefs unambiguous, all beliefs consistent and $B = 0$ cannot all hold. ■