Continuous Mortality Investigation

User guide to version 1.3 of the CMI Library of Mortality Projections

November 2011

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It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of whether the projection is contained within the Library.

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1 Background

This User Guide is issued alongside version 1.3 of the CMI Library of Mortality Projections and seeks to provide an overview of the Library and the projections it contains.

The Library was originally released in response to the continuation of significant year-on-year increases in life expectancy, and to concerns over the continued widespread use, albeit with modifications, of the Interim Cohort Projections which inevitably became increasingly out-of-date. In particular the modifications to the Interim Cohort Projections were not being applied consistently between actuaries and there was a perceived need for consistency of use.

A draft "library" of mortality projections was published alongside CMI Working Paper 27 in July 2007. An extensive consultation exercise was undertaken. The results are summarised in CMI Working Paper 30 which was published simultaneously with version 1.0 of the Library in November 2007.

Version 1.0 contained 54 projections: the original "92" series and Interim Cohort Projections; sample Adjusted Cohort Projections; the 2004-based and 2006-based ONS National Population Projections; and a series of projections generated using three models – P-spline age-period, P-spline age-cohort and Lee Carter – applied to datasets covering time-series data running up to 2003, to 2004 and to 2005.

Version 1.1 of the Library was released in March 2009 and contained 15 additional projections, extending the series generated using the P-spline age-period, P-spline age-cohort and Lee Carter models by applying them to datasets running up to 2006 and to 2007.

Version 1.2 of the Library was released in June 2011 and contained 32 additional projections: further extending the series of P-spline age-cohort and Lee Carter projections, by applying the models to datasets running up to 2008 and to 2009; adding sample projections from the CMI Mortality Projections Model (both CMI_2009 and CMI_2010); and adding the ONS 2008-based National Population Projections.

Version 1.3 of the Library, released in November 2011, contains 22 additional projections: the ONS 2010-based National Population Projections; sample projections from the CMI Mortality Projections Model (CMI_2011); and further extending the series of P-spline age-cohort and Lee Carter projections, by applying the models to datasets running up to 2010.

The natural timing for each projection is given in Appendix A, for clarity, even though these are not recognised in the application of projections from the Library (see section 2).

Oversight of the Library within the CMI is combined with that of the CMI Mortality Projections Model by the CMI Mortality Projections Committee.

The Committee is not undertaking a consultation exercise on version 1.3 of the Library but feedback is always welcome, and can be sent via e-mail to projections@cmib.org.uk or in writing to: CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW. Such comments will be considered for future reviews and updates of the Library.

2 The Library of projections

The CMI published the Library of projections to provide a reference source for actuaries using projections of future mortality. It aims to establish a well-defined vocabulary for mortality projections and to ensure they are used consistently.

The CMI believes that each of the projections within the Library is sufficiently well-defined that it can be uniquely identified. In addition within this document we seek to indicate where divergences from these projections need to be disclosed, for clarity, and in some cases suggest how this should be done.

It is very important to note that none of the projections is recommended for any particular situation and their inclusion in the Library does not imply suitability.

Furthermore the fact that any particular projection is *not* included in the Library does not imply that it is unsuitable.

Provision of the Library does not take away the need for individual actuaries to use their judgement and make recommendations best suited to the firm or scheme.

Version 1.3 of the Library of projections is contained in a series of spreadsheets (referred to in this user guide as "volumes"). The projections in the Library are summarised in the table in Appendix A. This section seeks to explain how they can be used. More details on the derivation of the different projections are set out in subsequent sections of this paper.

Each of the projections contained within the Library is independent of any particular base table of mortality.

Each sheet within each spreadsheet contains a different projection (except the first page of each spreadsheet entitled "Notes"). The following points apply to all these projections:

- Each sheet contains a two-way table of cumulative mortality reduction factors, by age and calendar year.
- These cumulative reduction factors can be defined as:
  $RF(x,t) = q_{x,t} / q_{x,0}$
  where x is the age, t is the elapsed time from the Start Year.
- Thus each sheet starts from values of 100% in the Start Year and subsequent columns show the cumulative reduction factor to the year in question.
- The ONS National Population Projections commence in specific years as indicated by the title: for example, the 2004-based projections commence in 2004.
- All the other projections in the Library commence in 1992. The improvements between 1992 and 2010 in each sheet are a mixture of projected values and actual values, as follows:
  - For the Original "92" Series, all of the figures are projections.
  - The Interim Cohort Projections are an adaptation of the "92" Series projections that reflect smoothed actual improvements up to 1999 for one particular cohort only (see section 3 for more detail).
  - For other projections where the Base Year is later than 1992 (e.g. P-spline and Lee-Carter projections using data to, say, 2004) then the figures between 1992

and the Base Year (2004 in this example) are smoothed actual improvements, with the smoothing coming from the relevant model.
    - For the CMI Mortality Projections Model, the smoothed actual improvements are derived using a P-spline age-cohort model (see section 8 for more detail).
- Smoothed actual improvements for each projection are indicated by shading within the Library itself (the projected values are not shaded).
- In all cases, the projections in the Library are shown to 2130, regardless of the length of the projection period used to derive the projection.

Naming convention

One of the aims of the Library is to produce a standardised terminology for use between actuaries. The projections included in the Library are not intended to include every projection that an actuary might consider it appropriate to use, nor does it seek to prescribe methods by which projections should be derived. However it is intended that if the naming convention is used, as a form of shorthand descriptor, then the projection should be used as set out in the Library and in this document, or calculated in a consistent manner where indicated. Any departure from this should be specifically noted.

In an attempt to keep the proposed names brief, the names assigned to the P-spline and Lee-Carter projections intentionally do not include all aspects of the derivation of the projection. For example, the names of these projections do not currently state the age range that has been used; however it is intended that if projections are produced using a different age range to that indicated in the Library, this would need to be specifically disclosed.

Age and year definitions

For each projection, "age" is defined as "age exact" as in base tables of mortality produced by the CMI. There is no precise definition of the calendar period to which CMI base tables relate. The "00" Series tables, for example, are based on data from calendar years 1999 to 2002. The actual point to which mortality rates graduated from this dataset apply depends on how data volumes are spread over the quadrennium and how experience varies over the quadrennium. However in order that all the projections contained in the Library can be used consistently, we have assumed that the "00" Series tables apply to lives attaining each particular age x at 30 June 2000.

A consistent approach should be taken with earlier CMI-produced tables, such as the "92" Series.

As discussed in section 8 of Working Paper 35, the mortality rates in the "S1" Series SAPS tables are deemed to apply to a life attaining age x exact on 1 September 2002. In the context of the Library, the Committee considers it would be inappropriate to interpolate between the years of improvement in projections from the Library when combined with a "S1" Series base table, especially given the arbitrary nature of the designated date.

Consequently, application of a reduction factor from the Library from (say) year T to year $T+1$ should be applied identically whether to a "00" Series table (with a designated effective date of 30 June 2000) or a "S1" Series table (with a designated effective date of 1 September 2002), or disclose what has been done. This ignores the difference between the effective dates for these tables.

Examples are provided below to illustrate the precise application of projections from the Library to a "00" Series table and an "S1" Series table.

Where an actuary uses a base mortality table other than a CMI table, they will need to have due regard to the definition of that table with regard to age and calendar year. If it is then being projected using a projection from the Library, or based on one from the Library, the actuary should disclose what has been done if there is a need for absolute clarity over the application of the projection.

Where an actuary derives a base mortality assumption from an analysis of past experience, then they will need to have regard to the period of the experience data rather than the timing associated with the underlying table. Again, if it is then being projected using a projection from the Library, or based on one from the Library, the actuary should disclose what has been done if there is a need for absolute clarity over the application of the projection.

Example 1: "00" Series table

If one applies the medium cohort projection (sheet 4 of volume 1) to a base mortality assumption of 100% PNML00, then the generated mortality rates for a male aged 65 exact at 30/6/2000 would be:

Age	**Year**	**Derivation**	**Rate**
65	30/6/2000 – 30/6/2001	"00" Series tables based on age exact and assumed to relate to 30/6/2000, hence q_{65} at 30/6/2000 can be read from the table as q_{65} = 0.012853	0.012853
66	30/6/2001 – 30/6/2002	Base table value of q_{66} taken to be 0.014141; Improvement from 30/6/2000 to 30/6/2001 = 1 - 65.6255/68.4657 = 4.1484%; Adjusted value of q_{66} = 0.014141 * (1 – 0.041484)	0.013554
67	30/6/2002 – 30/6/2003	Base table value of q_{67} assumed to be 0.015689; Improvement from 30/6/2000 to 30/6/2002 = 1 - 62.2531/67.7614 = 8.1290%; Adjusted value of q_{67} = 0.015689 * (1 – 0.08129)	0.014414

(NB we have followed the CMI convention that mortality rates are rounded to 6 d.p throughout. Rounded values of the improvements from the Library are shown in these examples but, in practice, we would expect actuaries to use the numbers direct from the Library, i.e. in unrounded form.)

If mortality rates at age 65 are required as at 31 December 2000 using a "00" Series base table, for example, rather than at 30 June 2000 then (unless otherwise disclosed) it is necessary to incorporate an allowance for improvements during that half-year and the derivation of the rate at age 65 using the medium cohort projection will become:

- "00" Series tables based on age exact and assumed to relate to 30/6/2000;
- Need to allow for improvements for half-a-year between 30/6/2000 and 31/12/2000;
- Improvement from 30/6/2000 to 30/6/2001 at age 65 = 1 - 66.4489/69.1763 = 3.9427%;
- Improvement from 30/6/2000 to 31/12/2000 assumed to be 1 - [(1 – 0.039427) ^ (184 / 365)] = 2.0074%;
- Hence q_{65} at 31/12/2000 can be estimated as q_{65} * (1-0.020074) = 0.012595.

Example 2: "S1" Series table

If one applies the medium cohort projection with a 1% minimum (sheet 2 of volume 2) to a base mortality assumption of 100% S1PFL, then the generated mortality rates for a female aged 60 exact at 1/9/2002 would be:

Age	Year	Derivation	Rate
60	1/9/2002 – 1/9/2003	"S1" Series tables based on age exact and assumed to relate to 1/9/2002, hence q_{60} at 1/9/2002 can be read from the table as q_{60} = 0.006115	0.006115
61	1/9/2003 – 1/9/2004	Base table value of q_{61} taken to be 0.006422; Improvement from 1/9/2002 to 1/9/2003 = 1 - 65.5926/67.6713 = 3.0717%; Adjusted value of q_{61} = 0.006422 * (1 – 0.030717)	0.006225
62	1/9/2004 – 1/9/2005	Base table value of q_{62} assumed to be 0.006808; Improvement from 1/9/2002 to 1/9/2004 = 1 - 63.0536/66.9230 = 5.7819%; Adjusted value of q_{67} = 0. 006808 * (1 – 0.057819)	0.006414

If mortality rates at age 60 are required as at 31 December 2002 using a "S1" Series base table, for example, rather than at 1 September 2002 then (unless otherwise disclosed) it is necessary to incorporate an allowance for improvements during that part-year and the derivation of the rate at age 60 using the medium cohort projection with a 1% minimum will become:

- "S1" Series tables based on age exact and assumed to relate to 1/9/2002;
- Need to allow for improvements for four months between 1/9/2002 and 31/12/2002;
- Improvement from 1/9/2002 to 1/9/2003 at age 60 = 1 - 66.0063/68.1890 = 3.2009%;
- Improvement from 1/9/2002 to 31/12/2002 assumed to be 1 - [(1 – 0.032009) ^ (121 / 365)] = 1.0727%;
- Hence q_{60} at 31/12/2002 can be estimated as q_{60} * (1-0.010727) = 0.006049.

Natural Timing and Library Timing

A number of simplifying assumptions were adopted in developing the Library, in order to make the projections easier to use in practice; including ignoring the effective dates of the base mortality tables, as discussed above.

In addition, when a projection in the Library is used in accordance with this User Guide, no regard is paid to its "natural timing" which depends on the underlying data source and methodology. For clarity, the natural timing for each projection is given in Appendix A even though these are not recognised in the application of projections from the Library, as specified above.

Note that no such simplification is made within the CMI Mortality Projections Model; the differences in application of projections between the Library and the Model are discussed in section 6 of Working Paper 49. These differences necessitate a different naming convention where the projections from the Model are used in accordance with the Library (and this User Guide). The name "CMI_2011_ML [1.0%]" has been adopted, where L denotes "Library timing", to distinguish the projection from "CMI_2011_M [1.0%] used directly from the Model, for example.

The User Guide for the CMI_2011 Mortality Projections Model contains an example (on page 29) of the derivation of q_{65} at 01/07/2011 on the S1PMA tables with the CMI_2011_M [1.00%] projections.

If one applies the CMI_2011_ML projection, with a 1% Long-Term Rate (sheet 17 of volume 8) to a base mortality assumption of 100% S1PMA as at 01/09/2002, then the generated mortality rates for a male aged 65 exact at 1/7/2011 would be:

Age	Year	Derivation	Rate
65	01/07/2011 – 01/07/2012	"S1" Series tables based on age exact and assumed to relate to 01/09/2002, hence q_{65} at 01/09/2002 = 0.011239 The Library timing of the CMI_2011_ML projections can run from 01/09/yy to 01/09/yy+1. Using this, q_{65} at 01/09/2010 (i.e. improvement from 01/09/2002 to 01/09/2010) = 0.011239 * (54.7822%/69.5762%) = 0.008849 Improvement from 01/09/2010 to 01/07/2011 = 0.008849 * (52.9233%/54.7822%) ^ (0.8301) = 0.008599.	0.008599

Thus q_{65} using the "natural" timing of the CMI_2011 projection is 0.008584 whereas q_{65} using the "Library" timing of the CMI_2011 projection is 0.008599. Note that this particular example may not be representative in absolute value, but the differences between the two approaches are unlikely to be financially significant.

Limiting Age

All of the projections within the Library assume a limiting age of 120, i.e. that $q_{120} = 1$, throughout the period of the projection. This, and the assumptions at ages 90 to 119 more generally, are considered further in section 10.

Differential smoking or health status

It is common practice to differentiate between smokers and non-smokers for certain assurances and similar practice is now being applied to annuity pricing. All of the projections within the Library have been derived from data that is not differentiated by smoker status and actuaries will need to give additional consideration to whether modification is required for smoker-differentiated business. Similar considerations also apply in respect of substandard lives, especially if these constitute a significant part of the portfolio.

ONS classification of deaths

The ONS data used in version 1.0 of the Library (and earlier CMI research into P-spline and Lee-Carter models) classified deaths on an Occurrence basis for the years 1993-2005, with a Registration basis used for all earlier years.

The ONS has since moved to classification of deaths on a Registration basis for all years, so the 1961-2006 and subsequent datasets use Registration death data throughout. The projections included in the Library that use the ONS datasets to 2006, 2007, 2008, 2009 and

2010 are therefore inconsistent with those that were produced using data to 2003, 2004 and 2005. The Committee has not amended these earlier projections within the Library; however the impact of the data change is illustrated for the 1961-2005 dataset in Working Paper 37.

3 The "92" Series and Interim Cohort Projections

The original "92" Series

Full details of the projections that were incorporated in the "92" Series tables are contained in section 6 of CMI Report No. 17.

In brief, the Committee sought to reflect recent trends in observed experience, with particular attention to the period 1975-1994. Despite differences between the various CMI investigations, it was decided to use a single projection. In particular this applied to females as well as males, even though no clear pattern could be discerned in recent female improvements.

The model adopted to allow for mortality improvement was essentially the same as that used for the "80" Series tables (see section 4.3 of CMI Report No. 10) whereby at each age the rate of mortality is assumed to decrease exponentially to a limiting value. For the "92" Series, the speed of convergence to the limit depended on age (in contrast to the "80" Series).

The model assumed that the long-term rate of mortality at each age would be a percentage of the rate in 1992, with the percentage equal to 13% at ages up to and including 60, 100% at ages 110 and over, and increasing linearly between.

In addition, the model assumed that a fraction of the total fall in the rate of mortality at each age would occur in the first 20 years. This fraction was set to 0.55 for ages up to and including 60, 0.29 at age 110, and reducing linearly between.

These values were chosen as a 'best fit' to male experience over 1975-1994, although the choice of age 110, above which there were no mortality improvements, was arbitrary.

The Interim Cohort Projections

Full details of these projections are contained in CMI Working Paper 1, published in 2002.

The "92" Series projections were quickly found to understate the level of mortality improvements that were actually occurring in the CMI experience and evidence had emerged of a "cohort effect", present in both population and CMI data. The CMI responded by publishing Working Paper 1, containing the "interim cohort projections" late in 2002.

Based on improvements in mortality to 1999, these tables offered an ad hoc adjustment to the original "92" Series projections. Key points in these adjustments are:

- The adjustment was in respect of one cohort only, born either side of 1926.
- This cohort was assumed to exhibit a faster rate of improvement than the original "92" Series projections for an arbitrary period – to 2010 for the "Short Cohort" projection, 2020 for the "Medium Cohort" projection and 2040 for the "Long Cohort" projection.
- The annual rates of improvement from 1993-1999 were based on smoothed actual rates of improvement during that period.
- From 2001, the additional improvement rates were assumed to reduce linearly to zero at the end of the cohort period.
- The rates of improvement were subject to minimum values of the improvements in the original "92" Series.

- Initially the cohort was taken to include years of birth between 1910 and 1942. After 2000, the ‘width’ of the cohort effect was reduced so that by the end of the cohort period it included only one year, which relates to lives born in 1926.

4 Adjusted Interim Cohort Projections

As time has passed since the publication of the Interim Cohort Projections, some actuaries have modified these projections to make them more suitable for their use. One consequence of the informal application of such modifications is that they are not necessarily undertaken in a consistent manner. The Committee has therefore included some sample adjusted projections within the Library to try to establish consistency of practice. As with other projections within the Library, their inclusion should not be taken to infer that they are in any way recommended by the CMI.

Applying a minimum value

This modification seeks to apply a minimum improvement rate at all ages and calendar years to the mortality improvements in the Interim Cohort Projections.

Within the Library we have included one illustrative modification to an otherwise unadjusted cohort projection – based on applying a 1.00% minimum improvement rate to the q_x from the Medium Cohort projection. This should not be taken to imply that 1% is a recommended minimum. Other minima can be used, denoted by changing the value in the name of the projection, but should be calculated in a consistent manner to the example unless specifically noted otherwise.

Imposing a minimum value is relatively straightforward at most ages. From the cumulative reduction factors for the original projection, derive the annual rate of improvement for each age and calendar year. Any rates below the required minimum are replaced with the minimum value and the cumulative reduction factors are then re-calculated.

However the imposition of a minimum value to the cohort projections could be done in a variety of ways at older ages, although the overall financial impact of the different approaches is unlikely to be material. This arises because the original "92" Series projections (and, in most cases, the interim cohort projections) assume no improvements above age 110. Hence this assumption could be retained, even if the minimum improvement is applied elsewhere. If this is not done, then consideration of the limiting age is required. In many cases the underlying tables (and certainly those published recently by the CMI) use a limiting age of 120, as noted in section 2. Applying improvements to q_{120} will extend the table beyond that age and this may cause systems issues.

For the purposes of the illustrative projection in the Library, the Committee has assumed that the minimum value does apply above age 110 but that the limiting age of 120 is retained. If users state that they are applying a different minimum value to a cohort projection, they should either do so in a consistent manner or explicitly state the approach they have adopted.

It is also worth noting that a minimum rate of improvement can also be applied to any other projection in the Library, but it has only been illustrated with the specific projections described in this section.

Using a percentage of the cohort projections

This modification uses a percentage of the mortality improvements in the Interim Cohort Projections.

Within the Library we have included one illustrative projection – based on using 90% of the Medium Cohort projection. This should not be taken to imply that 90% is a recommended adjustment. Other figures can be used, to adjust the relevant cohort projection up or down, but should be applied in a consistent manner to the example and can be denoted by changing the value in the name of the projection.

For the purposes of the illustrative projection in the Library, the Committee has assumed that the approach to applying the percentage is as follows. From the original projection, derive the annual rate of improvement for each age and calendar year. Apply the required percentage and the cumulative reduction factors are then re-calculated.

Note that this approach applies the relevant percentage to all of the improvement rates within the projection, not just those rates that were uplifted by the Interim Cohort Projections from the original "92" Series projections.

Unlike the imposition of a minimum value to the cohort projections (see preceding section), the application of a percentage does not give rise to particular issues at older ages, as applying a percentage maintains the assumptions of no improvements above age 110 and the limiting age of 120.

Blending two cohort projections

This modification uses a mixture of the mortality improvements in two of the Interim Cohort Projections.

Within the Library we have included one illustrative projection – based on using an average of the Medium Cohort projection and the Long Cohort projection. Other mixtures can be used but should be applied in a consistent manner to the example and can be denoted by changing the name of the projection.

For the purposes of the illustrative projection in the Library, the Committee has assumed that this modification is applied by deriving the annual rate of improvement for each age and calendar year for each of the original projections, averaging these and then re-calculating the cumulative reduction factors.

Note that this approach (like the application of a percentage) does not give rise to particular issues at older ages.

Blending two cohort projections and applying a minimum value

To illustrate this combination of adjustments, the Library includes an example of a minimum value (1.5% p.a.) applied to an average of the Medium Cohort projection and the Long Cohort projection. This has been calculated assuming that the blending of the projections is undertaken BEFORE the minimum is applied. Any divergence from this practice should be specifically disclosed.

Using a percentage of the cohort projections and applying a minimum value

To illustrate this combination of adjustments, the Library includes an example of a minimum value (2.5% p.a.) applied to 120% of the Long Cohort projection. This has been calculated assuming that the percentage adjustment to the projection is undertaken BEFORE the minimum is applied. Any divergence from this practice should be specifically disclosed.

5 ONS National Population Projections

Acknowledgement
The CMI is grateful to the ONS for its permission to include these variants of the National Population Projections within the Library, and in particular for providing the additional information necessary to allow their publication in age- and year-specific form.

ONS 2004-based National Population Projections[1]
More details of these projections are contained in the report "National population projections 2004-based" available on the ONS / GAD websites.

Key points underlying the approach to future improvements in mortality within the 2004-based population projections are:

- It was assumed that the then current rates of improvements converge by age and tend to long-term "target" rates of improvement over the first 25 years of the projections (i.e. to 2029). The target rates were assumed to apply in 2029 and all years thereafter
- For the principal projections, this long-term target was 1% p.a. applicable to m_x for all ages, for both genders and the different countries of the UK; broadly equivalent to the average annual rate of improvement over the whole of the 20th century.
- The transition from the assumed rates of mortality improvement by age and gender for the first year of the projection to the target rate is more rapid at first for males, and less rapid for females. These transitions are illustrated in Table 7.2 of the "National population projections" paper and partially in the table overleaf.
- Note that for males, there are two sets of improvement factors; one applicable to England, Wales and Northern Ireland and one applicable to Scotland, since differing rates of improvement in the first year of the projection are assumed for males in Scotland, compared to the other countries, at some ages, and that different transition rates apply thereafter.
- For females, the same improvement factors and transition rates apply in each of the constituent countries and hence in the UK overall.
- Cohort effects were recognised in that the transitions for those born before 1960 (i.e. those shaded in the table below) were projected by cohort, that is, diagonally downwards in the projection.
- For generations born since 1960 (not shaded), there was little evidence of generation effects for these cohorts and the transitions in mortality rates were therefore projected by calendar year, that is, horizontally in the projection.
- The initial rates of mortality improvement by age and gender for 2004 were estimated by analysing past data. The initial rates of improvement for ages 90 and over should be regarded as less 'robust' than those for younger ages because:
  - official single year of age population estimates were not available for ages 90 and over so historical mortality rates at these oldest ages had to be estimated, and
  - the resulting estimated initial rates of improvement at ages 90 and over were further adjusted to ensure that the future mortality rates produced from them

[1] Following the Government's acceptance of the recommendations of the Morris review, responsibility for the production of the official population projections for the UK and its constituent countries was transferred from the Government Actuary's Department (GAD) to the Office for National Statistics (ONS) with effect from 31 January 2006.

looked plausible compared to those for younger ages, and between males and females.

- "Variant" projections were also prepared, where the long-term target is 2% p.a. or 0% p.a. These were referred to as "High life expectancy" and "Low life expectancy" projections. As the "National population projections" paper states "These are intended as plausible alternative scenarios and not to represent upper or lower limits…" Adjustments were also made to the assumed rates of improvement in 2004-5 for these variants to reflect uncertainty about the then current rates of improvement.

Assumed percentage reduction in central death rates, m_x, for selected ages between selected consecutive calendar years in the projection period and the total reduction in m_x over 25 years for the principal projections of the 2004-based National Population Projections.

Age	2004-05	2011-12	2021-22	2028-29	Reduction over 25 years
Males (England, Wales and Northern Ireland)					
22	**3.31**	**2.38**	**1.36**	**1.00**	**38.7**
32	**1.86**	**1.52**	**1.14**	**1.00**	**28.8**
42	**1.48**	**1.28**	**1.08**	**1.00**	**25.9**
52	**0.80**	**0.75**	**0.93**	**1.00**	**16.0**
62	**1.87**	**2.19**	**0.93**	**1.00**	**28.5**
72	**5.01**	**2.31**	**1.32**	**1.00**	**41.3**
82	**3.22**	**2.86**	**1.35**	**1.00**	**41.2**
92	**1.47**	**2.25**	**1.49**	**1.00**	**33.7**
Males (Scotland)					
22	**2.61**	**1.96**	**1.25**	**1.00**	**34.0**
32	**1.12**	**1.07**	**1.02**	**1.00**	**23.1**
42	**0.87**	**0.92**	**0.98**	**1.00**	**21.2**
52	**0.80**	**0.75**	**0.93**	**1.00**	**16.0**
62	**1.53**	**2.19**	**0.93**	**1.00**	**27.4**
72	**5.01**	**2.24**	**1.32**	**1.00**	**40.8**
82	**3.22**	**2.86**	**1.33**	**1.00**	**41.2**
92	**1.47**	**2.25**	**1.49**	**1.00**	**33.7**
Females (UK and constituent countries)					
22	**2.47**	**2.15**	**1.62**	**1.00**	**37.5**
32	**0.58**	**0.67**	**0.82**	**1.00**	**17.3**
42	**1.97**	**1.76**	**1.41**	**1.00**	**32.6**
52	**1.42**	**0.83**	**0.91**	**1.00**	**19.7**
62	**1.30**	**1.81**	**0.91**	**1.00**	**25.5**
72	**4.37**	**2.07**	**1.44**	**1.00**	**39.5**
82	**2.01**	**2.61**	**1.58**	**1.00**	**40.6**
92	**0.30**	**1.56**	**1.87**	**1.00**	**30.1**

The ONS 2004-based projections included in the Library relate to:

- Males (England, Wales and Northern Ireland only), Males (Scotland only) and Females (UK and constituent countries); and
- Principal, High life expectancy and Low life expectancy projections.

Note that the improvement factors were derived from an analysis of UK data; these were used as input data for the projections for each constituent country (with adjustment for Scottish

males). The improvement factors may differ from those derived from the published projected mortality rates at UK level since the latter were 'back calculated' from aggregated projected numbers of deaths and mid-year populations for each individual country and, as a result, are less smooth than the input assumptions which have been used to create the projections for the Library.

Note also that the target rates used in the 2004-based projections after 2029 apply to improvements in m_x whereas we have expressed improvements in the Library in the form of improvements in q_x. The improvements in the Library after 2029 are therefore slightly lower than the target rates, with the difference increasing with age.

A further point to note is that the ONS projections assumed that everyone dies when they reach age 120.5. Given that the Library uses an age definition of 'age exact', we have made the assumption that q_{120}=1 in incorporating the ONS 2004-based projections into the Library.

ONS 2006-based National Population Projections

More details of these projections are contained in the report "National population projections 2006-based" available on the ONS website.

Our understanding is that most of the key points outlined above in relation to the 2004-based projections apply also to future improvements in mortality within the 2006-based population projections, except of course replacing "2004" with "2006" and "2029" with "2031".

A key difference is that, although the long-term "target" rate of improvement after the first 25 years of the projections (i.e. in and after 2031) is 1% p.a. (in m_x, for the principal projections) at most ages, it is assumed that those born in the years 1923 to 1940 will continue to experience higher rates of mortality improvement in the future. The target rates of improvement in and after 2031 rise from 1% p.a. for those born before 1923 to a peak of 2.5% p.a. for those born in 1931 and then declining back to 1% p.a. for those born in 1941 or later.

A second difference is that the target rates for those born in 1911 and earlier were assumed to reduce from 1% p.a. for those born in 1911 to 0.1% p.a. for those born in 1902 and earlier.

A summary table of rates of improvement in the principal projections is given overleaf.

Note that for the variant projections, the long-term target rates of improvement were assumed to be 1% p.a. higher or 1% p.a. lower than those assumed for the principal projections. For the avoidance of doubt, this means that the prolonged "cohort effect" applied in the principal projection applies also to these variants with peak rates of improvement for those born in 1931 of 3.5% p.a. and 1.5% p.a. respectively. These variants are again referred to as "High life expectancy" and "Low life expectancy" projections.

Note that the ONS High life expectancy variants extend to age 124.5. This feature has NOT been replicated in the projections included within the Library where, as noted previously, we have retained the assumption of q_{120}=1 throughout. The significance of this assumption is discussed further in section 10.

Please also refer to the notes at the end of the description of the 2004-based projections for further detail; these also apply to the 2006-based projections except as noted above.

The ONS 2006-based projections included in the Library relate to:

- Males (England, Wales and Northern Ireland only), Males (Scotland only) and Females (UK); and
- Principal, High life expectancy and Low life expectancy projections.

Assumed percentage reduction in central death rates, m_x, for selected ages between selected consecutive calendar years in the projection period and the total reduction in m_x over 25 years for the principal projections of the 2006-based National Population Projections.

Age	2006-07	2011-12	2021-22	2030-31	Reduction over 25 years
Males (England, Wales and Northern Ireland)					
22	**5.12**	**3.90**	**1.95**	**1.00**	**49.2**
32	**3.04**	**2.44**	**1.47**	**1.00**	**36.9**
42	**1.64**	**1.45**	**1.15**	**1.00**	**27.1**
52	**0.58**	**1.00**	**1.00**	**1.00**	**20.7**
62	**2.61**	**3.03**	**1.00**	**1.00**	**33.1**
72	**5.40**	**2.79**	**1.66**	**1.00**	**41.7**
82	**3.32**	**3.90**	**1.65**	**1.00**	**46.9**
92	**1.93**	**2.78**	**2.89**	**1.20**	**44.8**
Males (Scotland)					
22	**5.12**	**3.90**	**1.95**	**1.00**	**49.2**
32	**2.05**	**1.74**	**1.24**	**1.00**	**30.2**
42	**1.37**	**1.26**	**1.09**	**1.00**	**25.1**
52	**0.58**	**0.82**	**0.94**	**1.00**	**19.3**
62	**2.34**	**3.03**	**0.94**	**1.00**	**32.5**
72	**5.19**	**2.66**	**1.66**	**1.00**	**40.6**
82	**3.32**	**3.78**	**1.61**	**1.00**	**46.3**
92	**1.93**	**2.78**	**2.85**	**1.20**	**44.7**
Females (UK and constituent countries)					
22	**2.62**	**2.38**	**1.82**	**1.00**	**38.9**
32	**1.41**	**1.35**	**1.21**	**1.00**	**26.8**
42	**2.62**	**2.38**	**1.82**	**1.00**	**38.8**
52	**1.48**	**1.20**	**1.12**	**1.00**	**24.8**
62	**2.07**	**2.27**	**1.12**	**1.00**	**31.6**
72	**4.95**	**2.11**	**1.75**	**1.00**	**39.9**
82	**2.34**	**3.16**	**1.70**	**1.00**	**45.4**
92	**0.99**	**2.14**	**2.85**	**1.20**	**42.7**

ONS 2008-based National Population Projections

More details of these projections are contained in the report "National population projections 2008-based" available on the ONS website.

Our understanding is that most of the key points outlined above in relation to the 2006-based projections apply also to future improvements in mortality within the 2008-based population projections, except of course replacing "2006" with "2008" and "2031" with "2033". In particular, this includes the "cohort enhancement" that was introduced in the 2006-based projections.

One difference from 2004- and 2006-based projections is that there are now two sets of improvement factors for females (as well as males); one applicable to England, Wales and Northern Ireland and one applicable to Scotland. The 2008-based projections applicable to Scotland have lower initial rates of improvement for females, compared to the projections applicable to England, Wales and Northern Ireland, at certain ages. Consequently there is no comparable 2008-based female projection to the earlier ONS projections for females, which were applicable to the whole of the UK.

A summary table of rates of improvement in the principal projections is given overleaf.

A second difference in the 2008-based projections is that all variants (Principal, High life expectancy and Low life expectancy) now extend to age 125.5. This feature has NOT been replicated in the projections included within the Library where, as noted previously, we have retained the assumption of q_{120}=1 throughout. The significance of this assumption is discussed further in section 10.

Please also refer to the notes at the end of the description of the 2004-based projections for further detail; these also apply to the 2008-based projections except as noted above.

The ONS 2008-based projections included in the Library relate to:

- Males (England, Wales and Northern Ireland only), Males (Scotland only), Females (England, Wales and Northern Ireland only) and Females (Scotland only); and
- Principal, High life expectancy and Low life expectancy projections.

Assumed percentage reduction in central death rates, m_x, for selected ages between selected consecutive calendar years in the projection period and the total reduction in m_x over 25 years for the principal projections of the 2008-based National Population Projections.

Age	2008-09	2012-13	2022-23	2032-33	Reduction over 25 years
Males (England, Wales and Northern Ireland)					
22	**5.89**	**4.72**	**2.32**	**1.00**	**53.2**
32	**1.64**	**1.49**	**1.17**	**1.00**	**27.2**
42	**0.91**	**0.93**	**0.98**	**1.00**	**21.5**
52	**1.14**	**1.82**	**1.29**	**1.00**	**29.3**
62	**3.37**	**2.58**	**1.29**	**1.00**	**32.9**
72	**5.00**	**2.49**	**1.56**	**1.00**	**38.4**
82	**3.23**	**4.20**	**1.53**	**1.00**	**46.6**
92	**2.26**	**3.35**	**3.10**	**1.00**	**46.4**
Males (Scotland)					
22	**5.89**	**4.72**	**2.32**	**1.00**	**53.2**
32	**0.44**	**0.57**	**0.85**	**1.00**	**17.6**
42	**-0.23**	**0.06**	**0.67**	**1.00**	**11.8**
52	**1.47**	**1.69**	**1.24**	**1.00**	**28.7**
62	**3.37**	**2.58**	**1.24**	**1.00**	**34.1**
72	**4.49**	**2.33**	**1.56**	**1.00**	**37.5**
82	**3.03**	**3.82**	**1.47**	**1.00**	**44.2**
92	**2.26**	**2.56**	**2.97**	**1.00**	**44.0**
Females (England, Wales and Northern Ireland)					
22	**4.69**	**4.25**	**3.00**	**1.00**	**55.2**
32	**1.76**	**1.66**	**1.41**	**1.00**	**30.4**
42	**1.87**	**1.77**	**1.47**	**1.00**	**31.6**
52	**1.44**	**1.63**	**1.39**	**1.00**	**29.7**
62	**2.42**	**2.37**	**1.39**	**1.00**	**33.1**
72	**4.22**	**2.01**	**1.84**	**1.00**	**36.9**
82	**2.62**	**3.68**	**1.62**	**1.00**	**46.0**
92	**1.83**	**2.96**	**3.23**	**1.00**	**47.3**
Females (Scotland)					
22	**4.69**	**4.25**	**3.00**	**1.00**	**55.2**
32	**1.66**	**1.58**	**1.36**	**1.00**	**29.4**
42	**1.87**	**1.77**	**1.47**	**1.00**	**31.6**
52	**1.44**	**1.63**	**1.39**	**1.00**	**29.7**
62	**2.42**	**2.37**	**1.39**	**1.00**	**33.1**
72	**3.87**	**1.81**	**1.84**	**1.00**	**35.9**
82	**2.51**	**3.49**	**1.50**	**1.00**	**44.5**
92	**1.83**	**2.75**	**3.11**	**1.00**	**45.8**

ONS 2010-based National Population Projections

More details of these projections are contained in the on-line report "Chapter 4 Mortality assumptions: 2010-based national population projections" on the ONS website.

Our understanding is that most of the key points outlined above in relation to the 2008-based projections apply also to future improvements in mortality within the 2010-based population projections, except of course replacing "2008" with "2010" and "2033" with "2035". In particular, this includes the "cohort enhancement" that was introduced in the 2006-based projections.

A key difference is that a long-term rate of improvement of 1.2% p.a. has been assumed compared to the long-term rate of 1% p.a. assumed in the 2008-based projections.

Note that for the variant projections, the long-term target rates of improvement were assumed to be 1.2% p.a. higher or 1.2% p.a. lower than those assumed for the principal projections. For the avoidance of doubt, this means that the prolonged "cohort effect" applied in the principal projection applies also to these variants with peak rates of improvement for those born in 1931 of 3.7% p.a. and 1.3% p.a. respectively. These variants are again referred to as "High life expectancy" and "Low life expectancy" projections.

As for the 2008-based projections, there are two sets of improvement factors for both males and females; one applicable to England, Wales and Northern Ireland and one applicable to Scotland. The 2010-based projections applicable to Scotland have lower initial rates of improvement for males and females, compared to the projections applicable to England, Wales and Northern Ireland, at certain ages (and at some ages higher rates of improvement for males).

A summary table of rates of improvement in the principal projections is given overleaf.

Also as for the 2008-based projections, all variants (Principal, High life expectancy and Low life expectancy) extend to age 125.5. This feature has NOT been replicated in the projections included within the Library where, as noted previously, we have retained the assumption of q_{120}=1 throughout. The significance of this assumption is discussed further in section 10.

Please also refer to the notes at the end of the description of the 2004-based projections for further detail; these also apply to the 2010-based projections except as noted above.

The ONS 2010-based projections included in the Library relate to:

- Males (England, Wales and Northern Ireland only), Males (Scotland only) and Females (England, Wales and Northern Ireland only) and Females (Scotland only); and
- Principal, High life expectancy and Low life expectancy projections.

Assumed percentage reduction in central death rates, m_x, for selected ages between selected consecutive calendar years in the projection period and the total reduction in m_x over 25 years for the principal projections of the 2010-based National Population Projections.

Age	2010-11	2014-15	2024-25	2034-35	Reduction over 25 years
Males (England, Wales and Northern Ireland)					
22	**3.75**	**3.14**	**1.89**	**1.20**	**43.2**
32	**1.35**	**1.31**	**1.24**	**1.20**	**27.2**
42	**0.01**	**0.30**	**0.88**	**1.20**	**16.5**
52	**2.58**	**2.25**	**1.57**	**1.20**	**35.9**
62	**1.91**	**0.95**	**1.57**	**1.20**	**30.6**
72	**2.74**	**2.58**	**1.11**	**1.20**	**35.5**
82	**3.92**	**3.59**	**1.69**	**1.20**	**42.8**
92	**2.87**	**1.86**	**2.82**	**1.20**	**42.4**
Males (Scotland)					
22	**3.75**	**3.14**	**1.89**	**1.20**	**43.2**
32	**-1.14**	**-0.58**	**0.57**	**1.20**	**6.2**
42	**-0.59**	**-0.16**	**0.72**	**1.20**	**11.3**
52	**2.81**	**2.43**	**1.63**	**1.20**	**37.3**
62	**1.91**	**1.38**	**1.63**	**1.20**	**32.7**
72	**2.14**	**2.33**	**1.26**	**1.20**	**34.5**
82	**3.49**	**3.27**	**1.60**	**1.20**	**40.2**
92	**2.34**	**1.51**	**2.71**	**1.20**	**39.7**
Females (England, Wales and Northern Ireland)					
22	**2.50**	**2.34**	**1.90**	**1.20**	**39.0**
32	**0.75**	**0.80**	**0.95**	**1.20**	**20.9**
42	**1.17**	**1.18**	**1.19**	**1.20**	**25.8**
52	**1.96**	**1.87**	**1.61**	**1.20**	**33.9**
62	**1.79**	**1.33**	**1.61**	**1.20**	**31.7**
72	**2.40**	**2.08**	**1.28**	**1.20**	**34.0**
82	**3.26**	**3.49**	**1.74**	**1.20**	**42.5**
92	**2.35**	**2.13**	**3.07**	**1.20**	**43.7**
Females (Scotland)					
22	**2.50**	**2.34**	**1.90**	**1.20**	**39.0**
32	**-0.21**	**-0.04**	**0.44**	**1.20**	**9.0**
42	**1.17**	**1.18**	**1.19**	**1.20**	**25.8**
52	**1.96**	**1.87**	**1.61**	**1.20**	**33.9**
62	**1.79**	**1.33**	**1.61**	**1.20**	**31.7**
72	**2.07**	**1.88**	**1.28**	**1.20**	**32.9**
82	**3.01**	**3.25**	**1.62**	**1.20**	**40.6**
92	**2.20**	**1.85**	**2.92**	**1.20**	**41.6**

6 P-spline projections

More details of the Penalised Spline (or P-spline) projection methodology are contained in Working Paper 15 and Working Paper 20:

- Working Paper 15 sets out the CMI Mortality Projections Working Party's work towards developing stochastic methodologies. Section 2.3 gives a brief description of the P-spline model.
- Working Paper 20 provides practical advice on using the P-spline model, gives examples based on the P-spline methodology and discusses various features of the model.

Both papers contain further useful references.

Key points to note regarding the P-spline model are summarised below:

- The P-spline model is an example of a non-parametric smoothing model. It is a local model that fits cubic splines to the data, and was used to model the CMI Permanent Assurances Lives dataset in CMI Working Paper 1 that introduced the Interim Cohort Projections.
- A 2-dimensional model can be fitted to mortality data using either the age and calendar year (age-period) dimensions or the age and year of birth (age-cohort) dimensions.
- Coefficients of the model are selected using a maximum likelihood approach subject to a penalty being imposed. The penalty acts to ensure that there is an appropriate balance between the level of smoothness and goodness of fit.
- The use of the penalty also enables the model to be used to generate projections, extrapolating recent trends in the data.
- P-spline age-period and age-cohort models are both able to identify cohort effects, if they exist, in the region of the data. However, the age-period model will only project the stronger cohort effects into the future. Examples of cohort features in projections using the age-period and age-cohort models are shown in Appendix E of Working Paper 20.
- The P-spline model generates standard deviations which can be used to generate percentiles to reflect parameter uncertainty. This is considered further below.

P-spline projections included in the Library

A number of applications of the P-spline model are included in the Library. These illustrate the impact of using:

- Age-period and age-cohort versions of the model.
- Different datasets: CMI Permanent Assurances Lives and ONS (England and Wales population) datasets for males; for females only the ONS (England and Wales population) dataset has been used.
- An additional year's data: data to 2003, 2004, 2005 and 2006 for all datasets; for the ONS datasets only, projections are also included using data to 2007 (for both versions of the model) and to 2008, 2009 and 2010 for the age-cohort version.

All of the projections have been generated using the CMI's illustrative software and in all cases the 50^{th} percentile projection has been included in the Library. This can be considered as a best estimate from the model.

As noted in section 2, the projections using the ONS datasets to 2006 onwards use a different classification of deaths to the projections in the Library using data to 2003, 2004 and 2005.

Further details of the method and parameters used to generate the projections are contained in Appendix B.

Potential issues with the CMI P-spline model

As noted above, no additional P-spline age-period projections have been included in the Library for the ONS datasets extending beyond 2007. During the work to update the CMI Mortality Projections Model to CMI_2010, the Committee identified two potential issues with the CMI P-spline model and population data. These issues are discussed in section 5 of Working Paper 49.

In the context of the Library, the Committee found that when the P-spline model was fitted to the ONS datasets including data to 2008 and to 2009, the age-cohort version showed a similar progression to previous projections in the series (in terms of fit and smoothing of the data).

However the age-period version gave very different results to those yielded by the datasets up to 2007. The Committee found this was because the fitted age-period models, for data to 2008 and to 2009, exhibited very little smoothing over time. This problem could be overcome by using alternative P-spline models that allow for overdispersion, and so smooth the population data to a greater extent than the CMI P-spline software, but the Committee considers that such models have not as yet been adequately exposed to the Profession.

The Committee concluded that it was reasonable to continue the series of P-Spline age-cohort projections, adding the variants based on data to 2008 and to 2009 into v1.2 of the Library, but that it would not be appropriate to extend the series of P-spline age-period projections in the Library beyond those previously published using data up to 2007.

When the P-spline model was fitted to the ONS dataset including data for 2010, the age-cohort variant again showed an apparently reasonable progression and has been included in v1.3 of the Library. The age-period variant showed a return towards the fit and smoothing seen in the series up to the 2007 dataset and, in particular, greater smoothing over time than for the models fitted to the data to 2008 and 2009. However, the Committee decided not to restart the series of P-spline age-period projections in the Library given the absence of the 2008 and 2009 variants and the apparent vulnerability of the age-period model.

Calculating percentiles from the Library

For each P-spline projection in the Library, as well as the 50^{th} percentile projection we have included two-way tables of the fitted log μ's and corresponding standard errors generated by the CMI's illustrative software. The figures have been provided for the age range used to generate the projections. Actuaries can use the log μ's and standard errors to calculate the improvements for alternative percentiles.

In order to calculate the projected improvements for any given percentile for a dataset:

- Calculate the percentile μ's by taking the exponential of the log μ's, adjusted to reflect the required percentile. The adjustment to the log μ's is as follows: log μ + Z x S.E. where Z is the standard normal value corresponding to the percentile and S.E. is the relevant standard error.

- Calculate the percentile q's:
  - For CMI data, the formula is: $q(x) = 1- \exp(- (\mu(x, t) + \mu(x+1, t)) / 2)$. Please note that the formula differs for $q(90)$, as we do not have a value of $\mu(91, t)$, so have assumed that $q(90,t) = 1-\exp(-\mu(90, t))$.
  - For ONS data, the formula is: $q(x, t) = 1- \exp(-\mu(x, t))$.
- The cumulative reduction factors in $q(x, t)$ can then be calculated.

Actuaries may find it helpful to check their calculations using the 50th percentile projection included in the Library and the annuity values provided below; however you should note that the Library has been generated using Office 2007. We understand that consistent figures are produced in Excel 2003, but that Excel 2002 (and earlier versions) returns different values for the normal variable, and may produce marginally different values (see e.g. http://www.louisepryor.com/2004/03/01/excelstats/).

The table below provides sample annuity values to allow users to check their application of percentiles to a P-spline projection. The projection and the percentiles chosen are purely illustrative. The basis is consistent with that used in section 9 below.

Males	**Annuity values for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCMA00 and interest at 5% p.a.**					
Projection	**${}_{20\|}\ddot{a}_{45}$**	**${}_{10\|}\ddot{a}_{55}$**	**$\ddot{a}_{60}$**	**$\ddot{a}_{65}$**	**$\ddot{a}_{70}$**	**$\ddot{a}_{80}$**
PSAC_Male_ONS_EW_2010_2.5	6.054	9.123	15.699	14.002	11.946	7.662
PSAC_Male_ONS_EW_2010_97.5	5.184	8.017	14.585	12.975	11.088	7.224

7 Lee-Carter projections

More details of the Lee-Carter projection methodology are contained in Working Paper 15 and Working Paper 25:

- Working Paper 15 sets out the CMI Mortality Projections Working Party's work towards developing stochastic methodologies. Section 2.2 gives a brief description of the Lee-Carter model.
- Working Paper 25 provides practical advice on using the Lee-Carter model, gives examples based on the Lee-Carter methodology and discusses various features of the model.

Both papers contain further useful references.

Key points to note regarding the Lee-Carter model are summarised below:

- The Lee-Carter model is a bilinear model in age (x) and time (t) of the following form:
  $$\log \mu(x, t) = a(x) + b(x)\, k(t) + z(x, t)$$
- The force of mortality, $\mu(x, t)$, in the region of the data is derived by fitting the model to the mortality data and obtaining estimates of the parameters. The components of the model describe:
  - the average level of mortality over time for a particular age, $a(x)$;
  - the overall change in mortality over time, $k(t)$;
  - the pattern of deviations by age from the overall level of changes in mortality, $b(x)$; and
  - the random errors (stochastic innovations), $z(x, t)$.
- The parameters are selected to fit the model to the data using a maximum likelihood approach. To achieve a unique choice of parameters, some constraints on the parameters are required. These are usually $\sum_x b(x) = 1$ and $\sum_t k(t) = 0$.
- Projected $\mu(x, t)$ are obtained by projecting $k(t)$ forward. If this is done by fitting a time-series model, such as an ARIMA (Auto-Regressive Integrated Moving Average) process, then stochastic projections are generated.
- If the stochastic error is excluded, then a unique central projection of the average projected $\mu(x, t)$ is generated. This is the method that has been used to generate the projections in the Library.
- Allowing for the stochastic error will generate sample paths for the projected $\mu(x, t)$. These are random unless the generation is controlled, by using a non-random seed. As the number of scenarios increases the mean of the projected mortality rates will tend towards the central projection.
- Generating $\mu(x, t)$ in this way has no regard for parameter risk. This can be introduced using a technique known as parametric bootstrapping (see Appendix C for a brief description) and generating a number of synthetic datasets. Each synthetic dataset is used as a basis for a simulation of $\mu(x, t)$.
- The Lee-Carter model does not smooth the volatility in mortality rates across calendar years to the same extent as the P-spline model. This may make it more difficult to identify features in the region of the data and the structure of the model means that cohort features are not projected into the future.

Lee-Carter projections included in the Library

A number of applications of the Lee-Carter model are included in the Library. As for the P-spline projections, these illustrate the impact of using:

- CMI Permanent Assurances Lives and ONS (England and Wales) datasets for males. For females, only the ONS (England and Wales) dataset has been used;
- Data to 2003, 2004, 2005 and 2006, thus illustrating the impact of adding an additional year's data. For the ONS datasets only, projections are also included using data to 2007, 2008, 2009 and 2010.

All of the projections have been generated using the CMI's illustrative software and in all cases the central projection has been included in the Library. This can be considered as a best estimate from the model and is generated without any allowance for uncertainty. This is considered further, along with illustrations of allowance for some of the uncertainty inherent in any projection of future mortality below.

As noted in section 2, the projections using the ONS datasets to 2006 onwards use a different classification of deaths to the projections in the Library using data to 2003, 2004 and 2005.

Further details of the method by which the projections included in the Library have been generated is summarised in Appendix C.

Illustrating Uncertainty

The Lee-Carter model generates sample paths, which may be considered advantageous if one wishes to incorporate these with economic scenarios in a combined model. These sample paths reflect both parameter uncertainty and stochastic uncertainty and can also be used to generate percentiles but, as explained in Appendix C, this can be done in different ways:

- The mortality rates at each age could be ranked to generate the required confidence interval but these rates would arise from different sample paths.
- Assumptions can be made as to base mortality and interest rates to calculate an annuity value for each sample path, which can then be ranked to generate confidence intervals. This approach produces much narrower confidence intervals than ranking mortality rates. This approach was adopted in Working Paper 25, except that the 50^{th} percentile values were based on the mean annuity value, not the ranking.

8 CMI Mortality Projections Model

More details of the CMI Mortality Projections Model are contained in CMI Working Papers 38, 39, 41, 49, 54 and 55 as well as the CMI_2009, CMI_2010 and CMI_2011 User Guides:

- Working Paper 38 set out an outline of the CMI Mortality Projections Working Party's proposed approach; including the Model's structure, core and advanced parameters and output, as well as research on mortality projections by cause of death. It also set out specific questions for consultation.
- Working Paper 39 provided more detail on the research carried out by the Working Party related to the structure of the Model and the choice of default parameter values for the Core parameter layer.
- Working Paper 41 summarised the responses to the consultation and the differences between the prototype Model and the CMI_2009 Model.
- Working Paper 49 predominantly discussed the changes in the updated CMI_2010 Model from the CMI_2009 Model, and the effect of adding data for calendar year 2009.
- Working Paper 54 summarises the methodology used by the CMI to mirror the calculation algorithm used by the ONS to produce high-age population estimates. The described methodology was used to allow earlier release of CMI_2011.
- Working Paper 55 predominantly discusses the changes in the updated CMI_2011 Model from the CMI_2010 Model, and the effect of adding data for calendar year 2010.
- The User Guides for CMI_2009, CMI_2010 and CMI_2011 describe the parameterisation and mechanics of the Models and changes from previous versions of the Model. For CMI_2009, the derivation of the default core values, and parameter sensitivities, were also covered in the User Guide; for CMI_2010 and CMI_2011, the corresponding information is presented in Working Papers 49 and 55 respectively.

These papers contain further useful references.

Key points to note regarding the CMI Mortality Projections Model are summarised below.

- The Model was developed to:
    - reflect the latest experience on trends in mortality;
    - be relatively straightforward to understand and describe;
    - allow users the flexibility to modify projections tailored to their own views and purpose; and
    - be regularly updated over time to reflect emerging experience.
- The structure of the Model allows user input of:
    - Initial rates of mortality improvement, reflecting the current estimate of rates of change;
    - Assumed long-term rates of mortality improvement; and
    - An assumed speed and pattern of convergence from 'initial' to 'long-term'.
- The Model then creates future rates of mortality improvement by age and calendar year (to 2130). In order to create mortality rates, expectations of life or annuities in the Model, users are also required to input base mortality rates to reflect the estimated current or recent past position.
- The Model operates at a 'Core' parameter level and an 'Advanced' parameter level. At the 'Core' level, the User is only required to enter:
    - Long-Term Rate of Mortality Improvement (this is the only parameter in the Model that is not given a default value); and

    - Constant Additional Rate of Mortality Improvement if required (this has a default value of 0%).
- At the 'Advanced' level, in addition to the parameters above the following inputs can also be entered:
    - Initial Rates of Mortality Improvement;
    - Period of Convergence; and
    - Proportion of Convergence remaining at Mid-Point (this has a default value of 50%).
- At the 'Advanced' level all parameters, except the Constant Additional Rate of Improvement, can be split by age/period and cohort components. The start year of the base mortality table can also be altered at the 'Advanced' level.
- The default initial rates of improvement in CMI_2009, CMI_2010 and CMI_2011 were derived using a P-spline age-cohort model fitted to ONS data for the population of England & Wales, for ages from 18 to 102, for the periods from 1961 to 2008, 2009 and 2010 respectively. The Model "steps back" two years from the final year of raw experience data in order to derive sufficiently reliable estimates of the rates of mortality improvement. Therefore the base year for the default Initial Rates of Mortality Improvement are taken as those for calendar years 2006, 2007 and 2008 respectively, i.e. the first year of the projections are assumed to be 2007, 2008 and 2009 for CMI_2009, CMI_2010 and CMI_2011, respectively.
- As well as providing estimates of "current" rates, this approach automatically also provides rates for earlier years on a consistent basis. Please note that only the improvements from 1992 onwards are contained within the Library.

Note that (although the Model uses a P-spline age-cohort model to smooth the ONS England & Wales dataset) the smoothed actual improvements in the projections from the Model are not identical to those in the P-spline age-cohort projections in the Library, described in section 6. For males, this is due to the different age range of the dataset:

- The Model used ONS data for ages 18 to 102.
- The P-spline projections in the Library used data up to and including age 89 only. The data at higher ages were not available when the first projections were produced and the more recent projections have been produced on a consistent basis. There is also a difference at younger ages, depending on the knot spacing adopted (see Appendix B).

In addition, for females, 4-year knot spacing was using for the smoothing in the Model whereas 5-year knot spacing has been used for the P-spline age-cohort projections in the Library.

CMI_2009, CMI_2010 and CMI_2011 projections included in the Library

The Committee decided that for each version of the Model, CMI_20xx, the following set of projections (for both males and females) would be most appropriate to be included in the Library:

- CMI_20xx with a Long-Term Rate = 1% per annum, Proportion of Convergence remaining at Mid-Point = 50%;
- CMI_20xx with a Long-Term Rate = 2% per annum, Proportion of Convergence remaining at Mid-Point = 50%; and
- CMI_20xx with a Long-Term Rate = 1% per annum, Proportion of Convergence remaining at Mid-Point = 75%.

The rationale behind the choice of these projections was that:

- The Long-Term Rates of 1% and 2% were chosen for inclusion within the Library because the Committee felt they were likely to represent current industry assumptions regarding long-term rates of mortality improvement.
- The Committee considered it useful to include projections illustrating the impact of varying the Proportion of Convergence remaining at the Mid-Point from the default value of 50%. This had been identified as a particularly difficult parameter to assign a default value to in the development of the Model. Consequently, the projections using a 1% long-term rate of improvement have also been included with a higher Proportion of Convergence remaining at the Mid-Point, 75%. This higher value alters the initial trajectory of the projected mortality improvement rates from the Model. It also demonstrates that there are alternative ways of strengthening a projection than through the two Core parameters.
- All the projections have a zero Constant Addition. This parameter was included in the Model as a simple means of including a prudential margin within a projection; given that no other projections in the Library were intended to include such a margin, the Committee chose not to illustrate the impact of varying this parameter in the Library.

Note that the Model allows projections to run up to age 150; however this does not apply to the particular projections from CMI_2009, CMI_2010 and CMI_2011 included in the Library where a limiting age of 120 (i.e. that $q_{120} = 1$ throughout the period of the projection) applies to the projections.

9 Illustrative values

Working Paper 27 contained a brief discussion on possible approaches to illustrating the choice of projection. Many other approaches were suggested in responses and these are documented in Working Paper 30. In this section we use just two approaches – annuity values and expectations of life – to illustrate the projections in the Library.

For earlier versions of this User Guide, the illustrative annuity and expectation of life values for each projection have almost all been calculated as at 01/07/2007. An exception noted in v1.2 is that the values for the ONS 2008-based projections were calculated as at 01/07/2008 as the factors for those projections only start in 2008. These values therefore were not directly comparable to the annuity and expectation of life values for the other projections. Inclusion of the ONS 2010-based projections in this new volume of the Library would have created a similar, but larger, comparability problem as the values would need to be calculated as at 01/07/2010 if a similar approach were to be followed.

In order to restore full comparability to the illustrative values, the calculation date for all projections has been rebased to 01/07/2012. Advancing the calculation also ensures the comparisons remain 'current' rather than historic. Note, in particular, that the relative position or ranking of the projections, by annuity or expectation of life values, is dependent on the effective date for the calculations.

Illustrative annuity due values over a range of ages for the year of use 2012 are contained in Appendix D. Values for complete expectation of life at various ages are also shown for 2012 and for age 65 in 2022 and 2032. In order to provide a comparison influenced purely by the future projection, all these values have been calculated using the same assumptions regarding base mortality, namely 100% of PCMA00 or PCFA00 for the year commencing 01/07/2012, for males and females respectively. An interest rate of 5% has been used in calculating all the annuity values. Note that the interest rate and base mortality have been chosen to illustrate the difference in the projections and should not be interpreted as representative assumptions.

Note that:

- The values assume a base mortality assumption of 100% of PCMA00 or PCFA00 as at 01/07/2012, with no allowance for improvements between 2000 and 2012.
- The PCMA00 and PCFA00 base tables in CMI Working Paper 22 only provide values of q_x for ages 50 and above. For the younger ages we used the age range extensions provided in CMI Working Paper 26.

In each case, a two-way table of q_x was produced by applying improvement factors from the Library. The values of q_x have been rounded to 6 decimal places, as is normal practice in the CMI Tables Program (STP).

For comparison purposes, values are also shown using just the base mortality and interest (and no projection) and also showing annual compound rates of improvement from 1% to 5% p.a. These projections are marked by an asterisk (*) to indicate that they are not included within the Library.

10 Alternative assumptions at the oldest ages

Assumptions at very old ages are hugely uncertain, as there is very limited data to assess current rates of mortality, let alone interpret rates of improvement. In Working Paper 27, we drew attention to the commonly-used approach of a limiting age of 120 and that many projections used a single assumption applying for ages 90 and above. With hindsight, the Committee recognised that the use of a single assumption (which, for many of the projections, was that the same improvement applies at these older ages as at the highest age within the projection) did not convey the range of approaches that could be legitimately taken in dealing with an area of extreme data shortage.

Whilst the Committee has retained a limiting age of 120 for all the projections in the Library, this section is intended to illustrate the uncertainty generated by these assumptions using alternative scenarios.

The Limiting Age

As noted in section 2, it has been the practice within recent CMI base mortality tables to assume a limiting age of 120, i.e. that $q_{120} = 1$. There is very little data (within either the CMI or ONS datasets) to justify this practice explicitly, although the rarity to date of survivors beyond that age is perhaps justification in itself for such base mortality assumptions.

This was a very convenient assumption, for practical purposes, adopted for version 1.0 of the Library. The Committee remains comfortable with this assumption at the current time and it has been retained for all the projections within version 1.3 of the Library. However it is important to recognise that there is less justification for this assumption when future mortality improvements are taken into account, especially for example if considering a high-improvement scenario within a stress test. Indeed, as noted previously:

- The ONS 2006-based projections use a higher limiting age in the "High Life Expectancy" variant;
- The ONS 2008 and 2010-based projections use a higher limiting age in all the variants; and
- The CMI Mortality Projections Model can be parameterised so that projections extend beyond age 120.

Actuaries should therefore consider whether it is appropriate to retain this assumption in their particular situation.

In particular, some of the projections in the Library imply quite significant rates of improvement in mortality at age 119, resulting in an unrealistic increase in mortality rates between ages 119 and 120. In order to illustrate this, consider the projection PSAC_Male_ONS_EW_2005_50, combined with a base mortality assumption (in 2007) of 100% of PCMA00. The base table contains a value of $q_{119} = 0.620322$ (and $q_{120} = 1$). By the end of the projection period in the Library (2130), the value of q_{119} has reduced to 0.000137 yet, within the Library, q_{120} still equals 1.

In order to illustrate the sensitivity of results to this assumption, let us consider alternative scenarios. First, let us retain our initial assumption that the value of q_{120} (and older ages) in 2007 is 1, but that in subsequent years the mortality rates improve at the same rates as at the oldest age within the projection (which we previously also applied up to age 119). Note that this has the impact of removing the assumption of a limiting age in the years after 2007 (although of course the proportion assumed to survive to such ages is minute initially). The

results of this alternative scenario (labelled "Scenario 1") are compared to the complete expectation of life and annuity values for the "Base Scenario" – that is, retaining $q_{120} = 1$ for all future years – in the table below.

A second alternative is to suppose that the value of q_{120} in 2007 is 0.65, which is much more reasonable in comparison with the graduated table at the immediately preceding ages. Further assume that $q_{121} = 0.70$, etc so that the limiting age in 2007 is 127 (i.e. $q_{127} = 1$) and that, as above, in subsequent years the mortality rates improve at the same rates as improvements at the oldest age within the projection. This is labelled "Scenario 2" in the table below.

Ages 90 to 119

As noted above, the assumption used in many of the projections in the Library that the improvements at ages 90 to 119 (or at 91 to 119) are the same as the improvements at age 89 (or 90) is also highly arbitrary.

An alternative scenario is to assume that the rates of improvement reduce linearly from those applicable to the highest age in the projection to zero at age 119 (NB we have assumed the reduction applies vertically down a calendar year rather than, say, diagonally down a cohort). An immediate corollary of this is that the "step" in mortality rates between ages only widens gradually and in particular the "step" between age 119 and age 120 remains constant throughout. The base table contains values of $q_{118} = 0.602053$, $q_{119} = 0.620322$ (and $q_{120} = 1$). By the end of the projection period in the Library (2130), under this alternative scenario, the value of q_{118} has only reduced to 0.581989, whilst the values of q_{119} and q_{120} remain equal to 0.620322 and 1, respectively. This is labelled "Scenario 3" in the table below.

Note that care may be required if an approach at older ages similar to that in Scenario 3 is used in conjunction with applying a minimum value to a projection, and that the order of the steps could have considerable impact.

All figures in the table below are for males and assume:

Base mortality = 100% PCMA00 for the year commencing 01/07/2007
Projection = PSAC_Male_ONS_EW_2005_50
Net interest = 5% p.a. (for the single life annuity due values)

[Note that a calculation date of 01/07/2007 has been retained from earlier versions of this User Guide for this section. Therefore the values shown serve to illustrate the sensitivity of results to the three scenarios but are not directly comparable with those in Appendix D.]

	Annuity values for a life aged *x* exact on 1 July 2007					
	$_{20\|}\ddot{a}_{45}$	$_{10\|}\ddot{a}_{55}$	$\ddot{a}_{60}$	$\ddot{a}_{65}$	$\ddot{a}_{70}$	$\ddot{a}_{80}$
Base Scenario	5.824	8.833	15.416	13.749	11.803	7.534
Scenario 1	6.023	8.994	15.535	13.823	11.842	7.539
Scenario 2	6.025	8.998	15.540	13.828	11.846	7.540
Scenario 3	5.405	8.415	15.037	13.428	11.549	7.395

	Complete expectation of life for a life aged 65 exact on 1 July		Complete expectation of life for a life aged *x* exact on 1 July 2007			
	2027	**2017**	$\mathbf{e_{60}}$	$\mathbf{e_{65}}$	$\mathbf{e_{70}}$	$\mathbf{e_{80}}$
Base Scenario	41.219	33.313	33.198	26.019	19.646	9.829
Scenario 1	63.546	45.377	39.967	29.376	20.975	9.925
Scenario 2	63.826	45.729	40.267	29.600	21.107	9.946
Scenario 3	30.839	27.278	29.267	23.485	18.130	9.368

The impact of allowing mortality improvements at ages 120 and above (Scenario 1) is significant, especially for the deferred annuity values illustrated. In contrast the incremental effect of varying the assumptions regarding the level of mortality in 2007 at ages above 120 (Scenario 2) is very small.

The significance of the assumptions regarding mortality improvements between ages 90 and 119 is illustrated by Scenario 3, which effectively assumes that the limiting age of 120 persists for the foreseeable future, with very little improvement in mortality at the immediately preceding ages, and hence illustrates the process of "rectangularisation" of survival curves that has been referred to by many commentators.

At the older ages shown in the table above, the effect of Scenario 3 is to reduce annuity values by around 2%, but again the more significant impact is on the deferred annuity values.

Note that we have intentionally chosen the projection PSAC_Male_ONS_EW_2005_50 to illustrate the impact of varying the assumptions at older ages for effect, as it is one of the projections within the Library illustrating the most rapid rates of future improvements. Other projections demonstrate much lower sensitivity to these assumptions. For example, the LC_Male_ONS_EW_2005_50 projection produces much lower rates of future improvements and demonstrates minimal variation under Scenarios 1 and 2 from the Base Scenario, and a very small reduction under Scenario 3. This is unsurprising when one considers that the value of q_{119} has reduced to 0.232780 by the end of the projection period in the Library (2130), in contrast to the value of 0.000137 noted earlier for the PSAC_Male_ONS_EW_2005_50 projection.

The Committee would like to emphasise that these alternative scenarios for mortality rates at the very old ages have been included to illustrate the significance of these assumptions. The Committee does not consider that the assumptions at these ages used in the Library itself are necessarily more likely to be borne out in practice than the alternative approaches outlined in this section. There is of course a plethora of other alternatives that could also be considered.

11 Future updates

The Committee is not committing to any specified review dates for the Library. However it will be appropriate to supplement the Library from time-to-time:

- To incorporate subsequent years' data, as has been done for version 1.3 of the Library;
- To incorporate experience from a new dataset, e.g. from the CMI SAPS investigation, when there is sufficient data;
- To incorporate new "intuitive" projections, in the light of likely or actual medical advances; or
- If future work on projection methodologies indicates that a new methodology is worthy of inclusion.

The criteria for the inclusion of projections within the Library should not be viewed as either prescriptive requirements or a complete set, however new projections should be:

- A worthwhile addition to what is already contained in the Library;
- Publicly available;
- Clearly described and documented;
- 'Road-tested' on different datasets and for different time-periods; and
- Adequately exposed to the Actuarial Profession for discussion.

It may of course be appropriate to revise this approach and these criteria over time.

The process by which the CMI supplements the Library may depend on the extent and impact of the new projections. For example:

- A minor change, such as adding projections based on subsequent data, may be incorporated without prior consultation;
- In contrast incorporating new projections generated from a "new" methodology is likely to only be done after consultation, perhaps by means of a Working Paper.

Whilst adding an additional year's data may be considered a routine update, comparison of the figures in Appendix D shows that it can have a substantial impact on Lee-Carter and, especially, P-spline projections. Actuaries making use of projections based on the latest year's data should not do so without due care, given the volatility of some projections to new data.

Note that as none of the projections in the Library is "recommended", there is unlikely to be a corresponding need to "withdraw" projections.

References

CMI Papers

CMI Report No. 10: Graduation of the 1979-82 Mortality Experience - the "80" Series (May 1990)

CMI Report No. 17: Graduation of the 1991-94 Mortality Experience - the "92" Series Standard Tables (June 1999)

CMI Working Paper 1: An interim basis for adjusting the "92" Series mortality projections for cohort effects (December 2002)

CMI Working Paper 15: Projecting Future Mortality: Towards a proposal for a stochastic methodology (July 2005)

CMI Working Paper 20: Stochastic Projection Methodologies: Further progress and P-Spline model features, example results and implication (Revised version, November 2007)

CMI Working Paper 22: The Graduation of the CMI 1999-2002 Mortality Experience: Final "00" Series Mortality Tables – Annuitants and Pensioners (August 2006)

CMI Working Paper 25: Stochastic projection methodologies: Lee-Carter model features, example results and implications (Revised version, November 2007)

CMI Working Paper 26: Extensions to Younger Ages of the "00" Series Pensioner Tables of Mortality (April 2007)

CMI Working Paper 27: The "Library" of Mortality Projections (July 2007)

CMI Working Paper 30: The CMI Library of Mortality Projections (November 2007)

"Errata to CMI Working Papers 20, 25 and 27 on Mortality Projections" CMI (November 2007)

CMI Working Paper 35: The graduations of the CMI Self-Administered Pension Schemes 2000-2006 mortality experience. Final "S1" Series of Mortality Tables (October 2008)

CMI Working Paper 37: Version 1.1 of the CMI Library of Mortality Projections (March 2009)

CMI Working Paper 38: A Prototype Mortality Projections Model: Part One – An Outline of the Proposed Approach (June 2009)

CMI Working Paper 39: A Prototype Mortality Projections Model: Part Two – Detailed Analysis (July 2009)

CMI Working Paper 41: CMI Mortality Projections Model: Feedback on Consultation and Issue of 'CMI_2009' (November 2009)

User Guide for The CMI Mortality Projections Model: Model Name/Version 'CMI_2009' (November 2009)

CMI Working Paper 49: The CMI Mortality Projections Model, CMI_2010 (November 2010)

User Guide for The CMI Mortality Projections Model: Model Name/Version 'CMI_2010' (November 2010)

CMI Working Paper 54: Advancing the release date of the CMI Mortality Projections Model (August 2011)

CMI Working Paper 55: The CMI mortality projections model, CMI_2011 (September 2011)

User Guide for The CMI Mortality Projections Model: Model Name/Version 'CMI_2011' (September 2011)

All the CMI documents (and the Library itself) are available from:
http://www.actuaries.org.uk/research-and-resources/pages/continuous-mortality-investigation

National population projections

National population projections 2004-based (GAD, Series PP2 No 25, 2006) available from:
http://www.gad.gov.uk/Documents/Demography/Projections/National_Population_Projections_2004_Based_Report.pdf

National Population Projections 2006-based (ONS, Series PP2 No 26, 2008) available from:
http://www.ons.gov.uk/ons/rel/npp/national-population-projections/2006-based-annual-reference-volume/index.html

National Population Projections 2008-based (ONS, Series PP2 No 27, 2010) available from:
http://www.ons.gov.uk/ons/rel/npp/national-population-projections/2008-based-reference-volume--series-pp2/index.html

National Population Projections 2010-based on-line documentation available from:
http://www.ons.gov.uk/ons/rel/npp/national-population-projections/2010-based-projections/index.html

Appendix A: Full list of projections in version 1.3 of the Library

The full list of projections included in version 1.3 of the Library is shown below:

Projection	Sheet in spreadsheet	Natural Timing (dd/mm)	Base Year
Volume 1: Previously-published Projections			
Original "92" Series	2	01/07	1992
Short Cohort	3	01/07	1992
Medium Cohort	4	01/07	1992
Long Cohort	5	01/07	1992
ONS_2004_Male_EWNI_Principal	6	01/01	2004
ONS_2004_Male_EWNI_High life expectancy	7	01/01	2004
ONS_2004_Male_EWNI_Low life expectancy	8	01/01	2004
ONS_2004_Male_S_Principal	9	01/01	2004
ONS_2004_Male_S_High life expectancy	10	01/01	2004
ONS_2004_Male_S_Low life expectancy	11	01/01	2004
ONS_2004_Female_UK_Principal	12	01/01	2004
ONS_2004_Female_UK_High life expectancy	13	01/01	2004
ONS_2004_Female_UK_Low life expectancy	14	01/01	2004
ONS_2006_Male_EWNI_Principal	15	01/01	2006
ONS_2006_Male_EWNI_High life expectancy	16	01/01	2006
ONS_2006_Male_EWNI_Low life expectancy	17	01/01	2006
ONS_2006_Male_S_Principal	18	01/01	2006
ONS_2006_Male_S_High life expectancy	19	01/01	2006
ONS_2006_Male_S_Low life expectancy	20	01/01	2006
ONS_2006_Female_UK_Principal	21	01/01	2006
ONS_2006_Female_UK_High life expectancy	22	01/01	2006
ONS_2006_Female_UK_Low life expectancy	23	01/01	2006
Volume 2: Adjusted Cohort Projections			
Medium Cohort_1% minimum	2	01/07	1992
90% Medium Cohort	3	01/07	1992
50% Medium Cohort_50% Long Cohort	4	01/07	1992
(50% Medium Cohort_50% Long Cohort)_1.5% minimum	5	01/07	1992
(120% Long Cohort)_2.5% minimum	6	01/07	1992
Volume 3: P-spline Age-Period Projections			
PSAP_Male_Ass_2003_50	2	01/01	2003
PSAP_Male_Ass_2004_50	3	01/01	2004
PSAP_Male_Ass_2005_50	4	01/01	2005
PSAP_Male_ONS_EW_2003_50	5	01/01	2003
PSAP_Male_ONS_EW_2004_50	6	01/01	2004
PSAP_Male_ONS_EW_2005_50	7	01/01	2005
PSAP_Female_ONS_EW_2003_50	8	01/01	2003
PSAP_Female_ONS_EW_2004_50	9	01/01	2004

Projection	Sheet in spreadsheet	Natural Timing (dd/mm)	Base Year
PSAP_Female_ONS_EW_2005_50	10	01/01	2005
Volume 4: P-spline Age-Cohort Projections			
PSAC_Male_Ass_2003_50	2	01/01	2003
PSAC_Male_Ass_2004_50	3	01/01	2004
PSAC_Male_Ass_2005_50	4	01/01	2005
PSAC_Male_ONS_EW_2003_50	5	01/01	2003
PSAC_Male_ONS_EW_2004_50	6	01/01	2004
PSAC_Male_ONS_EW_2005_50	7	01/01	2005
PSAC_Female_ONS_EW_2003_50	8	01/01	2003
PSAC_Female_ONS_EW_2004_50	9	01/01	2004
PSAC_Female_ONS_EW_2005_50	10	01/01	2005
Volume 5: Lee-Carter Projections			
LC_Male_Ass_2003_Central	2	01/01	2003
LC_Male_Ass_2004_Central	3	01/01	2004
LC_Male_Ass_2005_Central	4	01/01	2005
LC_Male_ONS_EW_2003_Central	5	01/01	2003
LC_Male_ONS_EW_2004_Central	6	01/01	2004
LC_Male_ONS_EW_2005_Central	7	01/01	2005
LC_Female_ONS_EW_2003_Central	8	01/01	2003
LC_Female_ONS_EW_2004_Central	9	01/01	2004
LC_Female_ONS_EW_2005_Central	10	01/01	2005
Volume 6: Additional Projections in version 1.1			
PSAP_Male_Ass_2006_50	2	01/01	2006
PSAP_Male_ONS_EW_2006_50	3	01/01	2006
PSAP_Male_ONS_EW_2007_50	4	01/01	2007
PSAP_Female_ONS_EW_2006_50	5	01/01	2006
PSAP_Female_ONS_EW_2007_50	6	01/01	2006
PSAC_Male_Ass_2006_50	7	01/01	2007
PSAC_Male_ONS_EW_2006_50	8	01/01	2006
PSAC_Male_ONS_EW_2007_50	9	01/01	2006
PSAC_Female_ONS_EW_2006_50	10	01/01	2007
PSAC_Female_ONS_EW_2007_50	11	01/01	2006
LC_Male_Ass_2006_Central	12	01/01	2006
LC_Male_ONS_EW_2006_Central	13	01/01	2007
LC_Male_ONS_EW_2007_Central	14	01/01	2006
LC_Female_ONS_EW_2006_Central	15	01/01	2006
LC_Female_ONS_EW_2007_Central	16	01/01	2007
Volume 7: Additional Projections in version 1.2			
PSAC_Male_ONS_EW_2008_50	2	01/01	2008
PSAC_Male_ONS_EW_2009_50	3	01/01	2009
PSAC_Female_ONS_EW_2008_50	4	01/01	2008
PSAC_Female_ONS_EW_2009_50	5	01/01	2009

Projection	Sheet in spreadsheet	Natural Timing (dd/mm)	Base Year
LC_Male_ONS_EW_2008_Central	6	01/01	2008
LC_Male_ONS_EW_2009_Central	7	01/01	2009
LC_Female_ONS_EW_2008_Central	8	01/01	2008
LC_Female_ONS_EW_2009_Central	9	01/01	2009
ONS_2008_Male_EWNI_Principal	10	01/01	2008
ONS_2008_Male_EWNI_High life expectancy	11	01/01	2008
ONS_2008_Male_EWNI_Low life expectancy	12	01/01	2008
ONS_2008_Male_S_Principal	13	01/01	2008
ONS_2008_Male_S_High life expectancy	14	01/01	2008
ONS_2008_Male_S_Low life expectancy	15	01/01	2008
ONS_2008_Female_EWNI_Principal	16	01/01	2008
ONS_2008_Female_EWNI_High life expectancy	17	01/01	2008
ONS_2008_Female_EWNI_Low life expectancy	18	01/01	2008
ONS_2008_Female_S_Principal	19	01/01	2008
ONS_2008_Female_S_High life expectancy	20	01/01	2008
ONS_2008_Female_S_Low life expectancy	21	01/01	2008
CMI_2009_ML [1.00%]_Midpoint50%	22	01/01	2006
CMI_2009_ML [2.00%]_Midpoint50%	23	01/01	2006
CMI_2009_ML [1.00%]_Midpoint75%	24	01/01	2006
CMI_2010_ML [1.00%]_Midpoint50%	25	01/01	2007
CMI_2010_ML [2.00%]_Midpoint50%	26	01/01	2007
CMI_2010_ML [1.00%]_Midpoint75%	27	01/01	2007
CMI_2009_FL [1.00%]_Midpoint50%	28	01/01	2006
CMI_2009_FL [2.00%]_Midpoint50%	29	01/01	2006
CMI_2009_FL [1.00%]_Midpoint75%	30	01/01	2006
CMI_2010_FL [1.00%]_Midpoint50%	31	01/01	2007
CMI_2010_FL [2.00%]_Midpoint50%	32	01/01	2007
CMI_2010_FL [1.00%]_Midpoint75%	33	01/01	2007
Volume 8: Additional Projections in version 1.3			
PSAC_Male_ONS_EW_2010_50	2	01/01	2010
PSAC_Female_ONS_EW_2010_50	3	01/01	2010
LC_Male_ONS_EW_2010_Central	4	01/01	2010
LC_Female_ONS_EW_2010_Central	5	01/01	2010
ONS_2010_Male_EWNI_Principal	6	01/01	2010
ONS_2010_Male_EWNI_High life expectancy	7	01/01	2010
ONS_2010_Male_EWNI_Low life expectancy	8	01/01	2010
ONS_2010_Male_S_Principal	9	01/01	2010
ONS_2010_Male_S_High life expectancy	10	01/01	2010
ONS_2010_Male_S_Low life expectancy	11	01/01	2010
ONS_2010_Female_EWNI_Principal	12	01/01	2010
ONS_2010_Female_EWNI_High life expectancy	13	01/01	2010
ONS_2010_Female_EWNI_Low life expectancy	14	01/01	2010
ONS_2010_Female_S_Principal	15	01/01	2010
ONS_2010_Female_S_High life expectancy	16	01/01	2010
ONS_2010_Female_S_Low life expectancy	17	01/01	2010

Projection	Sheet in spreadsheet	Natural Timing (dd/mm)	Base Year
CMI_2011_ML (1.0%), 50%	18	01/01	2008
CMI_2011_ML (2.0%), 50%	19	01/01	2008
CMI_2011_ML (1.0%), 75%	20	01/01	2008
CMI_2011_FL (1.0%), 50%	21	01/01	2008
CMI_2011_FL (2.0%), 50%	22	01/01	2008
CMI_2011_FL (1.0%), 75%	23	01/01	2008

Note that volumes 1 to 5 above are unchanged from version 1.0 of the Library, volume 6 is unchanged from version 1.1 of the Library and volume 7 is unchanged from version 1.2 of the Library. All the additional projections in version 1.3 are contained in volume 8.

Appendix B: Generating the P-spline projections in version 1.3 of the Library

Choice of dataset

- The P-spline model requires age-specific data for successive years; a minimum of 20 years was suggested in Working Paper 20. Additionally, for the age-ranges fitted, a large amount of data is required in each year of observation.
- The only UK datasets, available to the CMI, that satisfy these criteria are the ONS England and Wales population (males and females) dataset and the CMI Permanent Assurances Lives (males) dataset. These were the datasets used to illustrate the P-spline methodology in Working Paper 20.
- Datasets may be subject to retrospective adjustment. Ordinarily the projections in the Library use the original dataset. For example, the CMI dataset for the projections based on data to 2003 used in Working Paper 20 was based on data collected to 2003. The CMI Permanent Assurances Lives dataset has subsequently been amended reflecting revisions to the 1947-2003 data that arose during the processing of 2004 data but the projections in the Library using CMI data to 2003 all use the original 1947-2003 dataset. If projections are undertaken using a more recent dataset with the last year's/years' data removed, this should be specifically disclosed.
- As noted in section 2, the projections using the ONS datasets to 2006 onwards use a different classification of deaths to the projections that use data to 2003, 2004 and 2005.
- Note that whilst the CMI will be aware of such changes in its own datasets, it may not necessarily always have access to the first available ONS dataset.

Method of generating P-spline projections

- The P-spline model fits forces of mortality (i.e. μ_x) to the data. The age definition of the exposure and deaths for each of the datasets and the age (x) to which the fitted μ_x apply is as follows:

Dataset	**Age Definition**	**μ_x Estimate**
ONS	Age last birthday	$\mu_{x+½}$
CMI Permanent Assurances Lives	Age nearest	μ_x

- Mean values of $\mu_{x,t}$ are produced for each age x and year t within the fitted region of the dataset and in the region of the projection.
- The $\mu_{x,t}$ can be used to estimate the values of the $q_{x,t}$ and from these the calendar year improvements can be determined for each age.
- For ages above 90 for the CMI Permanent Assurances Lives data and above 89 for the ONS data, the improvements are assumed to equal the improvements at ages 90 and 89, respectively, whilst q_{120} is assumed to equal 1.
- The Library provides projected improvements to 2130. These have been derived from mean values of $\mu_{x,t}$ using the following approach:
    - For the CMI Permanent Assurances Lives data, values for $q_{x,t}$ were estimated as:
    $$q_{x,t} = 1- \exp \{- ½ (\mu_{x,t} + \mu_{x+1,t}) \}$$
    - For the ONS data, values for $q_{x,t}$ were estimated as:
    $$q_{x,t} = 1- \exp \{- \mu_{x+½,t} \}$$
    - The cumulative reduction for a particular year t has been calculated as $q_{x,t} / q_{x,0}$, where $q_{x,0}$ is the mortality rate for 1992.

- The parameters used to generate the projections are shown below.
- The positioning of knots has followed the convention outlined in Sections 7.9-7.10 of Working Paper 20. This explains that the knots have been positioned at both corners of the leading edge of the data. In practice, this means that:
    - For the age-period model, knots are positioned at the highest age in the age dimension and in the final year of the dataset in the period dimension. The data is curtailed at younger ages, if necessary, so that a knot is also positioned at the lowest age.
    - For the age-cohort model, knots are positioned at the highest age in the age dimension and, in the cohort dimension, on the cohort consistent with this age in the last year of the dataset. The data is again curtailed at younger ages, if necessary, so that a knot is also positioned at the lowest age.

Calculating percentiles for P-spline projections

- The P-spline model produces mean values for log $\mu_{x,t}$ and corresponding standard deviations for the log $\mu_{x,t}$, $\hat{s}_{x,t}$.
- A set of $\mu_{x,t}$ relating to a particular percentile can be calculated by applying the standard normal variable ($\not{Z}$), for the percentile in question, to the standard deviations and using this to adjust the mean $\mu_{x,t}$. This process is summarised by the following equation:

$$\mu_{x,t} = \exp\{\log(\mu_{x,t}) + \not{Z} \times \hat{s}_{x,t}\}$$

- These may be used to illustrate some of the uncertainty inherent in any projection of future mortality (see section 6 for more details).

Parameters used to generate the projections

We have used cubic B-splines and a penalty order of 2 for all our fits. In all cases we have produced projections for 130 years (Note that the models produced projections for 130 years, e.g. to 2133 for 2003 base year projections, but the projected improvements included in the Library are only provided up until 2130. Changing the length of the projection period may alter the fit produced.)

Age-Cohort model
For datasets fitted using the age-cohort model the following parameters were used:

	Permanent Assurances Lives Males	**ONS (E&W) Males**	**ONS (E&W) Females**
Order of B-spline	3	3	3
Penalty order	2	2	2
Calendar Year range	1947-2003 1947-2004 1947-2005 1947-2006	1961-2003 1961-2004 1961-2005 1961-2006 1961-2007 1961-2008 1961-2009 1961-2010	1961-2003 1961-2004 1961-2005 1961-2006 1961-2007 1961-2008 1961-2009 1961-2010
Age range	21-90	21-89	24-89
Knot spacing: - age dimension - cohort dimension	 Every 3 years Every 3 years	 Every 4 years Every 4 years	 Every 5 years Every 5 years
Fixed knot positions: - age dimension - cohort dimension	 90 Last year of data less 90	 89 Last year of data less 89	 89 Last year of data less 89
Minimum for penalty: - age dimension - cohort dimension	 0.0001 0.0001	 0.0001 0.0001	 0.0001 0.0001
Starting point for penalty: - age dimension - cohort dimension	 100 100	 100 100	 100 100
Projection Period	130 years	130 years	130 years
Classification of Deaths	Date of Death*	Projections to 2003/4/5 use : Date of Registration for 1961-1992 and Date of Occurrence for 1993-2003/4/5 Projections to 2006/7/8/9/10 use: Date of Registration for 1961-2006/7/8/9/10	

* Note that the "Classification of Deaths" for the CMI Permanent Assurances dataset was erroneously shown as "Date of Settlement" in versions 1.0 and 1.1 of this User Guide.

Age-period model

For datasets fitted using the age-period model the following parameters were used:

	Permanent Assurances Lives Males	**ONS (E&W) Males**		**ONS (E&W) Females**
Order of B-spline	3	3	3	3
Penalty order	2	2	2	2
Calendar Year range	1947-2003 1947-2004 1947-2005 1947-2006	1961-2003	1961-2004 1961-2005 1961-2006 1961-2007	1961-2003 1961-2004 1961-2005 1961-2006 1961-2007
Age range	22-90	23-89	24-89	23-89
Knot spacing: - age dimension - period dimension	 Every 4 years Every 4 years	 Every 6 years Every 6 years	 Every 5 years Every 5 years	 Every 6 years Every 6 years
Fixed knot positions: - age dimension - period dimension	 90 Last year of data	 89 Last year of data		 89 Last year of data
Minimum for penalty: - age dimension - period dimension	 0.0001 0.0001	 0.0001 0.0001		 0.0001 0.0001
Starting point for penalty: - age dimension - period dimension	 100 100	 100 100		 100 100
Projection Period	130 years	130 years		130 years
Classification of Deaths	Date of Death*	Projections to 2003/4/5 use : Date of Registration for 1961-1992 and Date of Occurrence for 1993-2003/4/5 Projections to 2006/7 use: Date of Registration for 1961-2006/7		

* Note that the "Classification of Deaths" for the CMI Permanent Assurances dataset was erroneously shown as "Date of Settlement" in versions 1.0 and 1.1 of this User Guide.

For the projection using male ONS data to 2004 generated using the age-period model it was not possible to use the same parameters as those used for the projections with data to 2003. A fit was obtained by altering the knot spacing (to every 5 years) but other ways of achieving this may be possible. The same parameterisation was used for projections using data to 2005, 2006 and 2007.

Please see section 6 of this paper for an explanation as to why the age-period models with ONS data to 2008 onwards have not been included in the Library.

Appendix C: Generating the Lee-Carter projections in version 1.3 of the Library

Choice of dataset

- The data requirements for the Lee-Carter model are similar to those for the P-spline model (described in Appendix B). However, the minimum number of successive calendar years covered by the data can be adjusted depending on the width of the age range being fitted. If a narrower age range is used then fewer than 20 calendar years of data are required.
- The same datasets have been used to illustrate the Lee-Carter methodology in Working Paper 25 and to generate the projections in the Library as were used for the P-spline projections.
- As noted in Appendix B for the P-spline projections, datasets may be subject to retrospective adjustment. Ordinarily the projections in the Library use the original dataset. For example, the CMI dataset for the projections in the Library and in Working Paper 25 does not reflect revisions to the 1947-2003 data that arose during the processing of 2004 data. If projections are undertaken using a more recent dataset with the last year's/years' data removed, this should be specifically disclosed.
- As noted in section 2, the projections using the ONS datasets to 2006 onwards use a different classification of deaths to the projections that use data to 2003, 2004 and 2005.
- Note that whilst the CMI will be aware of such changes in its own datasets, it may not necessarily always have access to the first available ONS dataset.

Method of generating Lee-Carter projections

- The Lee-Carter model fits forces of mortality (i.e. μ_x) to the data. The ages included in the datasets are specified below.
- Values of $\mu_{x,t}$ are produced for each age x and year t within the fitted region of the dataset and in the region of the projection.
- The $\mu_{x,t}$ can be used to estimate the values of the $q_{x,t}$ and from these the calendar year improvements can be determined for each age.
- For ages above 90 for the CMI Permanent Assurances Lives data and above 89 for the ONS data the improvements are assumed to equal the improvements at ages 90 and 89, respectively and q_{120} is assumed to equal 1.
- The Library provides cumulative reduction factors to 2130. These have been derived from the central projection of $\mu_{x,t}$.
- In addition to the central projections, it is possible to calculate projected improvements for particular percentiles, i.e. 97.5^{th} percentile (see section 7 for a brief explanation).

Parametric bootstrapping

The process of parametric bootstrapping generates each synthetic dataset using the following steps:

- Fit the Lee-Carter model to the data and calculate the $\mu(x, t)$.
- Use the $\mu(x, t)$ and the exposure data to determine the number of expected deaths, based on the Lee-Carter fit.
- Compare the actual deaths against the expected deaths to obtain deviance residuals for each age and year.
- For each age, randomly reallocate the deviance residuals across the years.

- Use the reassigned deviance residuals to simulate the number of deaths for each age and year.
- Re-fit the Lee-Carter model to the simulated deaths and the actual exposures and fit a time-series to the *k(t)* parameters.
- Use the fitted parameters to generate $\mu'(x, t)$ in the region of the dataset and the time-series to generate projected $\mu'(x, t)$. The $\mu'(x, t)$ form a simulation.

Calculating percentiles for Lee-Carter projections

- The percentiles for the Lee-Carter projections are determined from the scenarios generated.
- The $q_{x,t}$ can be calculated for each scenario. Percentiles could be generated by ordering the mortality rates from all the scenarios, for each age and year, and selecting those corresponding to a particular percentile. The volatility of the mortality rates projected using Lee-Carter means that confidence intervals around the mortality rates would be very wide.
- The approach used in Working Paper 25 was to assume a base table of $q_{x,0}$, reflecting actual experience in year zero [both "92" Series and "00" Series base tables were used] and an interest rate [4.5%] to calculate annuity values for each age and year, for each of 1,000 scenarios. The mean of these values is the figure shown in Working Paper 25 as the 50^{th} percentile value.
- Values for other percentiles were generated by ordering the annuity values from all the scenarios for each age and selecting the value corresponding to that particular percentile.
- The resulting confidence intervals are much narrower than those around the projected mortality rates.

It is important to note that using the method adopted for Working Paper 25 necessitates assumptions regarding interest rates and base mortality and different assumptions could result in a different ranking of the scenarios, and hence different confidence intervals. Furthermore the ranking of the scenarios will differ according to the start age of the annuity.

For these reasons we have not included projected mortality rates, other than the central projection, within the Library. Actuaries wishing to illustrate uncertainty by means of ranking scenarios using the Lee-Carter method will need to specify details of how these have been obtained if it is intended that another actuary should be able to reproduce them.

Parameters used to generate the projections

For all the Lee-Carter projections we have used an ARIMA(1,1,0) model to project the $k(t)$ parameters.

The following age ranges were used:

Permanent Assurances lives, males	20-90
ONS, males	20-89
ONS, females	20-89

For the projections using ONS datasets, the classification of deaths is as shown in Appendix B for P-spline projections.

Appendix D: Illustrative values for the projections in version 1.3 of the Library

See section 9 for a description of these values.

Males	**Single life annuity values for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCMA00 and net interest at 5% p.a.**					
Projection	**${}_{20\|}\ddot{a}_{45}$**	**${}_{10\|}\ddot{a}_{55}$**	**$\ddot{a}_{60}$**	**$\ddot{a}_{65}$**	**$\ddot{a}_{70}$**	**$\ddot{a}_{80}$**
0% p.a. improvement*	3.944	6.796	13.441	11.944	10.245	6.762
1% p.a. improvement*	4.325	7.207	13.842	12.311	10.560	6.946
2% p.a. improvement*	4.736	7.659	14.285	12.719	10.912	7.151
3% p.a. improvement*	5.171	8.154	14.778	13.177	11.307	7.380
4% p.a. improvement*	5.593	8.674	15.311	13.684	11.752	7.640
5% p.a. improvement*	5.962	9.181	15.856	14.223	12.240	7.932
“92” Series	4.357	7.218	13.828	12.269	10.497	6.872
Short Cohort	4.357	7.218	13.828	12.269	10.497	6.872
Medium Cohort	4.367	7.234	13.849	12.296	10.533	6.951
Long Cohort	4.448	7.366	14.019	12.517	10.829	7.327
Medium Cohort_1% minimum	4.457	7.325	13.937	12.378	10.607	6.997
90%_Medium Cohort	4.327	7.191	13.808	12.261	10.504	6.931
Average(MC_LC)	4.407	7.298	13.932	12.404	10.677	7.132
Average(MC_LC)_1.5% minimum	4.609	7.502	14.122	12.567	10.795	7.154
120% Long Cohort 2.5% minimum	5.020	7.980	14.610	13.042	11.234	7.489
ONS_2004_Male_EWNI_Principal	4.385	7.325	14.017	12.510	10.795	7.111
ONS_2004_Male_EWNI_HLE	4.731	7.688	14.364	12.818	11.049	7.242
ONS_2004_Male_EWNI_LLE	4.062	6.991	13.701	12.230	10.564	6.990
ONS_2004_Male_S_Principal	4.382	7.319	14.009	12.500	10.791	7.111
ONS_2004_Male_S_HLE	4.729	7.683	14.356	12.808	11.046	7.242
ONS_2004_Male_S_LLE	4.058	6.985	13.693	12.220	10.561	6.990
ONS_2006_Male_EWNI_Principal	4.481	7.479	14.206	12.717	11.036	7.296
ONS_2006_Male_EWNI_HLE	4.819	7.834	14.543	13.017	11.284	7.421
ONS_2006_Male_EWNI_LLE	4.163	7.153	13.897	12.444	10.811	7.181
ONS_2006_Male_S_Principal	4.471	7.468	14.190	12.696	11.018	7.290
ONS_2006_Male_S_HLE	4.810	7.823	14.528	12.996	11.266	7.415
ONS_2006_Male_S_LLE	4.154	7.142	13.882	12.424	10.793	7.175
ONS_2008_Male_EWNI_Principal	4.525	7.524	14.260	12.777	11.105	7.324
ONS_2008_Male_EWNI_HLE	4.845	7.853	14.571	13.049	11.327	7.431
ONS_2008_Male_EWNI_LLE	4.225	7.221	13.977	12.529	10.903	7.225
ONS_2008_Male_S_Principal	4.513	7.509	14.227	12.730	11.042	7.279
ONS_2008_Male_S_HLE	4.832	7.837	14.535	12.999	11.260	7.384
ONS_2008_Male_S_LLE	4.215	7.208	13.946	12.484	10.842	7.182
ONS_2010_Male_EWNI_Principal	4.572	7.535	14.227	12.779	11.025	7.307
ONS_2010_Male_EWNI_HLE	4.938	7.904	14.568	13.072	11.255	7.412
ONS_2010_Male_EWNI_LLE	4.235	7.204	13.924	12.518	10.820	7.213
ONS_2010_Male_S_Principal	4.560	7.511	14.179	12.710	10.939	7.253
ONS_2010_Male_S_HLE	4.925	7.878	14.516	12.999	11.165	7.354
ONS_2010_Male_S_LLE	4.226	7.182	13.878	12.453	10.738	7.161
PSAP_Male_Ass_2003_50	4.954	7.929	14.555	12.962	11.101	7.201
PSAP_Male_Ass_2004_50	4.931	7.898	14.522	12.930	11.072	7.186
PSAP_Male_Ass_2005_50	4.994	7.969	14.593	12.996	11.130	7.223
PSAP_Male_Ass_2006_50	5.225	8.249	14.879	13.269	11.369	7.357

Males	**Single life annuity values for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCMA00 and net interest at 5% p.a.**					
Projection	$_{20\|}\ddot{a}_{45}$	$_{10\|}\ddot{a}_{55}$	$\ddot{a}_{60}$	$\ddot{a}_{65}$	$\ddot{a}_{70}$	$\ddot{a}_{80}$
PSAP_Male_ONS_EW_2003_50	3.924	6.775	13.426	11.941	10.260	6.742
PSAP_Male_ONS_EW_2004_50	5.604	8.679	15.313	13.683	11.756	7.670
PSAP_Male_ONS_EW_2005_50	5.588	8.675	15.324	13.714	11.810	7.750
PSAP_Male_ONS_EW_2006_50	5.895	9.199	15.949	14.407	12.511	8.202
PSAP_Male_ONS_EW_2007_50	5.326	8.392	15.058	13.486	11.624	7.556
PSAC_Male_Ass_2003_50	5.100	8.039	15.106	13.409	11.473	7.389
PSAC_Male_Ass_2004_50	5.085	8.020	15.489	13.803	11.838	7.673
PSAC_Male_Ass_2005_50	5.122	8.063	15.585	13.878	11.897	7.700
PSAC_Male_Ass_2006_50	5.293	8.270	14.867	13.224	11.301	7.297
PSAC_Male_ONS_EW_2003_50	5.690	8.570	15.106	13.409	11.473	7.389
PSAC_Male_ONS_EW_2004_50	5.886	8.902	15.489	13.803	11.838	7.673
PSAC_Male_ONS_EW_2005_50	5.978	9.006	15.585	13.878	11.897	7.700
PSAC_Male_ONS_EW_2006_50	5.947	8.822	15.353	13.654	11.719	7.665
PSAC_Male_ONS_EW_2007_50	5.806	8.693	15.220	13.519	11.594	7.522
PSAC_Male_ONS_EW_2008_50	5.542	8.397	14.929	13.234	11.307	7.298
PSAC_Male_ONS_EW_2009_50	5.571	8.516	15.102	13.473	11.535	7.444
PSAC_Male_ONS_EW_2010_50	5.675	8.586	15.137	13.469	11.493	7.429
LC_Male_Ass_2003_Central	4.475	7.346	13.960	12.400	10.618	6.958
LC_Male_Ass_2004_Central	4.478	7.349	13.962	12.401	10.619	6.957
LC_Male_Ass_2005_Central	4.498	7.370	13.983	12.420	10.636	6.967
LC_Male_Ass_2006_Central	4.545	7.420	14.031	12.463	10.671	6.986
LC_Male_ONS_EW_2003_Central	4.442	7.307	13.917	12.351	10.567	6.912
LC_Male_ONS_EW_2004_Central	4.469	7.337	13.946	12.377	10.587	6.923
LC_Male_ONS_EW_2005_Central	4.483	7.353	13.961	12.391	10.598	6.926
LC_Male_ONS_EW_2006_Central	4.495	7.368	13.976	12.404	10.609	6.932
LC_Male_ONS_EW_2007_Central	4.506	7.381	13.989	12.416	10.619	6.937
LC_Male_ONS_EW_2008_Central	4.519	7.396	14.006	12.434	10.635	6.951
LC_Male_ONS_EW_2009_Central	4.533	7.414	14.023	12.449	10.648	6.957
LC_Male_ONS_EW_2010_Central	4.542	7.425	14.034	12.459	10.657	6.961
CMI_2009_ML [1.00%]_Midpoint50%	4.344	7.297	14.328	12.946	11.215	7.502
CMI_2009_ML [2.00%]_Midpoint50%	4.683	7.651	14.691	13.287	11.518	7.717
CMI_2009_ML [1.00%]_Midpoint75%	4.361	7.349	14.453	13.118	11.385	7.645
CMI_2010_ML [1.00%]_Midpoint50%	4.353	7.322	13.986	12.564	10.770	7.078
CMI_2010_ML [2.00%]_Midpoint50%	4.682	7.662	14.310	12.855	11.015	7.225
CMI_2010_ML [1.00%]_Midpoint75%	4.374	7.383	14.061	12.680	10.869	7.142
CMI_2011_ML [1.00%]_Midpoint50%	4.338	7.307	13.959	12.548	10.758	7.076
CMI_2011_ML [2.00%]_Midpoint50%	4.656	7.632	14.266	12.822	10.987	7.213
CMI_2011_ML [1.00%]_Midpoint75%	4.354	7.361	14.021	12.654	10.849	7.138

***** These projections are not included within version 1.3 of the Library

Males	**Complete expectation of life for a life aged 65 exact on 1 July**		**Complete expectation of life for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCMA00**			
Projection	**2032**	**2022**	**e_{60}**	**e_{65}**	**e_{70}**	**e_{80}**
0% p.a. improvement*	18.401	18.401	22.523	18.401	14.504	8.118
1% p.a. improvement*	21.356	20.456	24.090	19.584	15.351	8.479
2% p.a. improvement*	25.316	23.111	26.074	21.057	16.386	8.905
3% p.a. improvement*	30.535	26.612	28.662	22.951	17.691	9.416
4% p.a. improvement*	36.370	30.890	31.903	25.351	19.343	10.043
5% p.a. improvement*	41.729	35.441	35.565	28.185	21.351	10.811
"92" Series	20.516	19.988	23.781	19.299	15.101	8.317
Short Cohort	20.516	19.988	23.781	19.299	15.101	8.317
Medium Cohort	20.626	20.095	23.881	19.400	15.205	8.455
Long Cohort	21.547	20.981	24.711	20.240	16.079	9.210
Medium Cohort_1% minimum	21.723	20.766	24.362	19.757	15.461	8.567
90%_Medium Cohort	20.406	19.924	23.743	19.298	15.133	8.420
Average(MC_LC)	21.067	20.519	24.278	19.802	15.622	8.814
Average(MC_LC)_1.5% minimum	23.453	21.943	25.259	20.473	16.023	8.878
120% Long Cohort 2.5% minimum	28.313	25.238	27.737	22.370	17.419	9.586
ONS_2004_Male_EWNI_Principal	21.948	21.079	24.722	20.178	15.924	8.771
ONS_2004_Male_EWNI _HLE	25.314	23.269	26.312	21.323	16.698	9.051
ONS_2004_Male_EWNI _LLE	19.317	19.317	23.422	19.228	15.270	8.525
ONS_2004_Male_S_Principal	21.926	21.056	24.700	20.156	15.917	8.771
ONS_2004_Male_S _HLE	25.295	23.246	26.289	21.300	16.691	9.051
ONS_2004_Male_S _LLE	19.294	19.294	23.400	19.206	15.263	8.526
ONS_2006_Male_EWNI_Principal	22.884	21.962	25.547	20.929	16.631	9.136
ONS_2006_Male_EWNI _HLE	26.397	24.242	27.198	22.117	17.436	9.413
ONS_2006_Male_EWNI _LLE	20.131	20.128	24.198	19.943	15.950	8.891
ONS_2006_Male_S_Principal	22.828	21.905	25.492	20.871	16.588	9.126
ONS_2006_Male_S _HLE	26.343	24.184	27.142	22.057	17.392	9.403
ONS_2006_Male_S _LLE	20.076	20.073	24.145	19.888	15.910	8.881
ONS_2008_Male_EWNI_Principal	23.138	22.202	25.773	21.130	16.817	9.194
ONS_2008_Male_EWNI _HLE	26.517	24.360	27.318	22.228	17.551	9.434
ONS_2008_Male_EWNI _LLE	20.469	20.452	24.501	20.212	16.192	8.979
ONS_2008_Male_S_Principal	22.973	22.046	25.613	20.963	16.643	9.106
ONS_2008_Male_S _HLE	26.320	24.179	27.137	22.042	17.361	9.340
ONS_2008_Male_S _LLE	20.335	20.319	24.361	20.061	16.032	8.897
ONS_2010_Male_EWNI_Principal	23.448	22.285	25.713	21.126	16.631	9.168
ONS_2010_Male_EWNI_HLE	27.557	24.819	27.477	22.350	17.415	9.410
ONS_2010_Male_EWNI_LLE	20.367	20.327	24.326	20.143	15.987	8.960
ONS_2010_Male_S_Principal	23.216	22.063	25.490	20.890	16.401	9.059
ONS_2010_Male_S_HLE	27.271	24.553	27.216	22.081	17.159	9.293
ONS_2010_Male_S_LLE	20.186	20.142	24.133	19.933	15.778	8.858
PSAP_Male_Ass_2003_50	28.145	24.912	27.357	21.943	16.928	9.005
PSAP_Male_Ass_2004_50	27.746	24.651	27.167	21.804	16.834	8.971
PSAP_Male_Ass_2005_50	28.575	25.189	27.555	22.085	17.027	9.053
PSAP_Male_Ass_2006_50	31.906	27.461	29.225	23.303	17.856	9.358
PSAP_Male_ONS_EW_2003_50	17.967	18.189	22.426	18.363	14.508	8.072

Males	**Complete expectation of life for a life aged 65 exact on 1 July**		**Complete expectation of life for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCMA00**			
Projection	**2032**	**2022**	**e_{60}**	**e_{65}**	**e_{70}**	**e_{80}**
PSAP_Male_ONS_EW_2004_50	35.633	30.378	31.553	25.105	19.211	10.068
PSAP_Male_ONS_EW_2005_50	36.416	31.044	32.108	25.613	19.649	10.327
PSAP_Male_ONS_EW_2006_50	42.981	36.963	37.039	29.717	22.780	11.577
PSAP_Male_ONS_EW_2007_50	34.121	29.313	30.750	24.612	18.929	9.841
PSAC_Male_Ass_2003_50	29.583	25.501	27.660	22.053	16.929	8.966
PSAC_Male_Ass_2004_50	29.369	25.363	27.561	21.984	16.885	8.951
PSAC_Male_Ass_2005_50	29.876	25.702	27.809	22.165	17.012	9.010
PSAC_Male_Ass_2006_50	32.383	27.386	29.027	23.038	17.596	9.220
PSAC_Male_ONS_EW_2003_50	38.923	31.113	31.327	24.470	18.497	9.463
PSAC_Male_ONS_EW_2004_50	41.632	33.909	33.713	26.390	19.911	10.146
PSAC_Male_ONS_EW_2005_50	42.941	34.898	34.403	26.829	20.170	10.215
PSAC_Male_ONS_EW_2006_50	42.699	33.590	33.049	25.718	19.430	10.071
PSAC_Male_ONS_EW_2007_50	40.619	32.195	32.068	25.003	18.928	9.741
PSAC_Male_ONS_EW_2008_50	37.176	29.618	30.165	23.577	17.859	9.243
PSAC_Male_ONS_EW_2009_50	38.011	30.756	31.306	24.682	18.697	9.599
PSAC_Male_ONS_EW_2010_50	39.485	31.459	31.611	24.722	18.584	9.553
LC_Male_Ass_2003_Central	22.083	20.944	24.453	19.813	15.478	8.498
LC_Male_Ass_2004_Central	22.086	20.947	24.455	19.814	15.477	8.495
LC_Male_Ass_2005_Central	22.263	21.068	24.546	19.881	15.525	8.516
LC_Male_Ass_2006_Central	22.632	21.321	24.738	20.021	15.621	8.552
LC_Male_ONS_EW_2003_Central	21.616	20.621	24.204	19.606	15.310	8.404
LC_Male_ONS_EW_2004_Central	21.826	20.767	24.315	19.687	15.365	8.425
LC_Male_ONS_EW_2005_Central	21.925	20.838	24.367	19.725	15.389	8.430
LC_Male_ONS_EW_2006_Central	22.032	20.913	24.425	19.767	15.418	8.441
LC_Male_ONS_EW_2007_Central	22.125	20.978	22.523	18.401	14.504	8.118
LC_Male_ONS_EW_2008_Central	22.313	21.104	24.569	19.879	15.500	8.483
LC_Male_ONS_EW_2009_Central	22.434	21.189	24.633	19.926	15.532	8.493
LC_Male_ONS_EW_2010_Central	22.520	21.248	24.678	19.960	15.555	8.501
CMI_2009_ML [1.00%]_Midpoint50%	21.598	20.817	25.677	21.265	16.800	9.369
CMI_2009_ML [2.00%]_Midpoint50%	24.629	22.862	27.358	22.551	17.733	9.819
CMI_2009_ML [1.00%]_Midpoint75%	21.790	21.043	26.116	21.747	17.199	9.606
CMI_2010_ML [1.00%]_Midpoint50%	21.659	20.898	24.561	20.235	15.811	8.683
CMI_2010_ML [2.00%]_Midpoint50%	24.635	22.890	26.015	21.309	16.550	8.990
CMI_2010_ML [1.00%]_Midpoint75%	21.871	21.155	24.820	20.557	16.042	8.791
CMI_2011_ML [1.00%]_Midpoint50%	21.519	20.792	24.455	20.179	15.776	8.679
CMI_2011_ML [2.00%]_Midpoint50%	24.431	22.716	25.845	21.201	16.474	8.967
CMI_2011_ML [1.00%]_Midpoint75%	21.662	20.997	24.663	20.467	15.984	8.781

***** These projections are not included within version 1.3 of the Library

Females	**Single life annuity values for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCFA00 and net interest at 5% p.a.**					
Projection	$_{20\|}\ddot{a}_{45}$	$_{10\|}\ddot{a}_{55}$	$\ddot{a}_{60}$	$\ddot{a}_{65}$	$\ddot{a}_{70}$	$\ddot{a}_{80}$
0% p.a. improvement*	4.466	7.565	14.359	12.903	11.240	7.716
1% p.a. improvement*	4.830	7.972	14.766	13.287	11.580	7.930
2% p.a. improvement*	5.218	8.414	15.211	13.710	11.956	8.168
3% p.a. improvement*	5.620	8.891	15.699	14.179	12.378	8.434
4% p.a. improvement*	5.997	9.379	16.217	14.690	12.845	8.734
5% p.a. improvement*	6.314	9.841	16.731	15.221	13.349	9.070
“92” Series	4.812	7.932	14.709	13.210	11.488	7.835
Short Cohort	4.812	7.932	14.709	13.210	11.488	7.835
Medium Cohort	4.823	7.950	14.732	13.240	11.528	7.915
Long Cohort	4.914	8.100	14.924	13.488	11.854	8.340
Medium Cohort_1% minimum	4.929	8.062	14.842	13.345	11.624	7.981
90%_Medium Cohort	4.790	7.913	14.695	13.207	11.499	7.895
Average(MC_LC)	4.868	8.023	14.826	13.361	11.687	8.121
Average(MC_LC)_1.5% minimum	5.085	8.249	15.040	13.549	11.829	8.155
120% Long Cohort 2.5% minimum	5.476	8.721	15.533	14.042	12.297	8.530
ONS_2004_Female_UK_Principal	4.907	8.116	14.962	13.518	11.874	8.103
ONS_2004_Female_UK_HLE	5.189	8.409	15.239	13.761	12.073	8.202
ONS_2004_Female_UK_LLE	4.641	7.844	14.707	13.295	11.692	8.010
ONS_2006_Female_UK_Principal	4.999	8.256	15.126	13.701	12.070	8.253
ONS_2006_Female_UK_HLE	5.269	8.535	15.388	13.931	12.256	8.343
ONS_2006_Female_UK_LLE	4.743	7.998	14.886	13.491	11.898	8.169
ONS_2008_Female_EWNI_Principal	5.039	8.306	15.186	13.756	12.147	8.311
ONS_2008_Female_EWNI_HLE	5.293	8.562	15.423	13.960	12.310	8.387
ONS_2008_Female_EWNI_LLE	4.800	8.069	14.968	13.568	11.996	8.239
ONS_2008_Female_S_Principal	5.026	8.284	15.158	13.718	12.099	8.284
ONS_2008_Female_S_HLE	5.280	8.539	15.394	13.922	12.261	8.360
ONS_2008_Female_S_LLE	4.787	8.048	14.940	13.532	11.949	8.214
ONS_2010_Female_EWNI_Principal	5.061	8.293	15.149	13.735	12.060	8.300
ONS_2010_Female_EWNI_HLE	5.350	8.578	15.407	13.952	12.228	8.374
ONS_2010_Female_EWNI_LLE	4.793	8.035	14.916	13.540	11.909	8.233
ONS_2010_Female_S_Principal	5.042	8.261	15.107	13.680	11.996	8.258
ONS_2010_Female_S_HLE	5.332	8.545	15.364	13.895	12.161	8.330
ONS_2010_Female_S_LLE	4.775	8.004	14.876	13.487	11.847	8.192
PSAP_Female_ONS_EW_2003_50	4.052	7.091	13.886	12.459	10.846	7.437
PSAP_Female_ONS_EW_2004_50	5.356	8.594	15.409	13.917	12.163	8.306
PSAP_Female_ONS_EW_2005_50	5.299	8.515	15.323	13.835	12.097	8.278
PSAP_Female_ONS_EW_2006_50	6.492	10.104	17.027	15.538	13.687	9.403
PSAP_Female_ONS_EW_2007_50	5.860	9.220	16.073	14.610	12.882	8.968
PSAC_Female_ONS_EW_2003_50	5.406	8.643	15.460	13.966	12.198	8.251
PSAC_Female_ONS_EW_2004_50	5.691	9.020	15.868	14.382	12.598	8.557
PSAC_Female_ONS_EW_2005_50	5.770	9.103	15.955	14.467	12.679	8.607
PSAC_Female_ONS_EW_2006_50	5.827	9.196	16.067	14.596	12.820	8.729
PSAC_Female_ONS_EW_2007_50	5.714	9.040	15.887	14.402	12.622	8.574
PSAC_Female_ONS_EW_2008_50	5.458	8.713	15.538	14.049	12.285	8.340
PSAC_Female_ONS_EW_2009_50	5.642	8.971	15.832	14.368	12.604	8.577
PSAC_Female_ONS_EW_2010_50	5.615	8.943	15.809	14.343	12.558	8.509
LC_Female_ONS_EW_2003_Central	4.906	8.051	14.842	13.354	11.631	7.933

Females	**Single life annuity values for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCFA00 and net interest at 5% p.a.**					
Projection	**$_{20\|}\ddot{a}_{45}$**	**$_{10\|}\ddot{a}_{55}$**	**$\ddot{a}_{60}$**	**$\ddot{a}_{65}$**	**$\ddot{a}_{70}$**	**$\ddot{a}_{80}$**
LC_Female_ONS_EW_2004_Central	4.928	8.076	14.867	13.376	11.649	7.943
LC_Female_ONS_EW_2005_Central	4.938	8.087	14.877	13.385	11.655	7.945
LC_Female_ONS_EW_2006_Central	4.952	8.102	14.892	13.398	11.666	7.951
LC_Female_ONS_EW_2007_Central	4.957	8.108	14.897	13.402	11.669	7.952
LC_Female_ONS_EW_2008_Central	4.965	8.118	14.908	13.414	11.681	7.965
LC_Female_ONS_EW_2009_Central	4.984	8.140	14.930	13.434	11.698	7.975
LC_Female_ONS_EW_2010_Central	4.991	8.148	14.937	13.440	11.702	7.975
CMI_2009_FL [1.00%]_Midpoint50%	4.890	8.074	15.152	13.756	12.086	8.327
CMI_2009_FL [2.00%]_Midpoint50%	5.200	8.416	15.510	14.101	12.403	8.564
CMI_2009_FL [1.00%]_Midpoint75%	4.924	8.127	15.251	13.877	12.213	8.417
CMI_2010_FL [1.00%]_Midpoint50%	4.901	8.092	14.916	13.476	11.766	8.028
CMI_2010_FL [2.00%]_Midpoint50%	5.202	8.423	15.242	13.780	12.031	8.200
CMI_2010_FL [1.00%]_Midpoint75%	4.939	8.153	14.990	13.568	11.855	8.081
CMI_2011_FL [1.00%]_Midpoint50%	4.908	8.103	14.932	13.498	11.783	8.036
CMI_2011_FL [2.00%]_Midpoint50%	5.201	8.422	15.244	13.786	12.035	8.197
CMI_2011_FL [1.00%]_Midpoint75%	4.948	8.167	15.010	13.594	11.876	8.089

***** These projections are not included within version 1.3 of the Library

Females	**Complete expectation of life for a life aged 65 exact on 1 July**		**Complete expectation of life for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCFA00**			
Projection	**2032**	**2022**	**e_{60}**	**e_{65}**	**e_{70}**	**e_{80}**
0% p.a. improvement*	20.853	20.853	25.264	20.853	16.677	9.675
1% p.a. improvement*	23.989	23.088	27.026	22.211	17.672	10.123
2% p.a. improvement*	28.123	25.946	29.240	23.892	18.885	10.651
3% p.a. improvement*	33.400	29.642	32.089	26.033	20.406	11.287
4% p.a. improvement*	39.039	33.998	35.557	28.689	22.305	12.064
5% p.a. improvement*	43.981	38.436	39.323	31.725	24.555	13.008
"92" Series	22.840	22.377	26.504	21.768	17.306	9.901
Short Cohort	22.840	22.377	26.504	21.768	17.306	9.901
Medium Cohort	22.964	22.497	26.620	21.885	17.426	10.048
Long Cohort	24.031	23.538	27.619	22.889	18.455	10.943
Medium Cohort_1% minimum	24.296	23.355	27.264	22.373	17.783	10.213
90%_Medium Cohort	22.758	22.334	26.484	21.781	17.349	10.010
Average(MC_LC)	23.478	22.999	27.101	22.368	17.921	10.477
Average(MC_LC)_1.5% minimum	26.160	24.667	28.302	23.208	18.442	10.580
120% Long Cohort 2.5% minimum	31.153	28.181	31.053	25.355	20.056	11.422
ONS_2004_Female_UK_Principal	24.714	23.862	27.809	22.965	18.436	10.437
ONS_2004_Female_UK_HLE	27.784	25.830	29.244	23.982	19.111	10.664
ONS_2004_Female_UK_LLE	22.244	22.240	26.611	22.103	17.854	10.232
ONS_2006_Female_UK_Principal	25.629	24.727	28.612	23.697	19.086	10.762
ONS_2006_Female_UK_HLE	28.778	26.729	30.064	24.720	19.763	10.978
ONS_2006_Female_UK_LLE	23.098	23.083	27.404	22.832	18.504	10.567
ONS_2008_Female_EWNI_Principal	25.968	25.041	28.907	23.934	19.334	10.893
ONS_2008_Female_EWNI_HLE	28.992	26.927	30.256	24.870	19.944	11.079
ONS_2008_Female_EWNI_LLE	23.521	23.482	27.777	23.137	18.806	10.723
ONS_2008_Female_S_Principal	25.824	24.900	28.771	23.797	19.196	10.838
ONS_2008_Female_S_HLE	28.834	26.773	30.108	24.722	19.797	11.021
ONS_2008_Female_S_LLE	23.392	23.353	27.651	23.009	18.676	10.671
ONS_2010_Female_EWNI_Principal	26.191	25.041	28.806	23.862	19.112	10.876
ONS_2010_Female_EWNI_HLE	29.827	27.227	30.327	24.888	19.752	11.059
ONS_2010_Female_EWNI_LLE	23.383	23.310	27.581	23.019	18.575	10.714
ONS_2010_Female_S_Principal	25.976	24.832	28.605	23.660	18.924	10.786
ONS_2010_Female_S_HLE	29.586	26.994	30.105	24.667	19.550	10.966
ONS_2010_Female_S_LLE	23.195	23.123	27.399	22.833	18.399	10.629
PSAP_Female_ONS_EW_2003_50	17.589	18.502	23.418	19.414	15.603	9.122
PSAP_Female_ONS_EW_2004_50	30.689	27.745	30.635	24.975	19.687	10.989
PSAP_Female_ONS_EW_2005_50	29.837	27.139	30.165	24.630	19.464	10.921
PSAP_Female_ONS_EW_2006_50	46.737	41.215	41.752	33.784	26.221	13.939
PSAP_Female_ONS_EW_2007_50	38.517	33.607	35.267	28.704	22.641	12.648
PSAC_Female_ONS_EW_2003_50	31.752	28.369	31.055	25.250	19.822	10.874
PSAC_Female_ONS_EW_2004_50	35.802	31.550	33.646	27.316	21.384	11.623
PSAC_Female_ONS_EW_2005_50	36.881	32.320	34.246	27.778	21.729	11.751
PSAC_Female_ONS_EW_2006_50	37.686	33.098	34.955	28.414	22.269	12.050
PSAC_Female_ONS_EW_2007_50	35.973	31.644	33.716	27.376	21.446	11.650
PSAC_Female_ONS_EW_2008_50	32.292	28.829	31.456	25.598	20.122	11.065
PSAC_Female_ONS_EW_2009_50	34.988	31.117	33.401	27.224	21.390	11.656

Females	**Complete expectation of life for a life aged 65 exact on 1 July**		**Complete expectation of life for a life aged *x* exact on 1 July 2012 assuming base mortality of 100% PCFA00**			
Projection	**2032**	**2022**	**e_{60}**	**e_{65}**	**e_{70}**	**e_{80}**
PSAC_Female_ONS_EW_2010_50	34.587	30.871	33.220	27.062	21.201	11.490
LC_Female_ONS_EW_2003_Central	24.434	23.405	27.268	22.380	17.771	10.117
LC_Female_ONS_EW_2004_Central	24.623	23.538	27.371	22.456	17.824	10.138
LC_Female_ONS_EW_2005_Central	24.683	23.581	27.405	22.480	17.838	10.141
LC_Female_ONS_EW_2006_Central	24.803	23.665	27.470	22.528	17.872	10.155
LC_Female_ONS_EW_2007_Central	24.835	23.687	27.487	22.539	17.879	10.157
LC_Female_ONS_EW_2008_Central	24.978	23.784	27.563	22.600	17.927	10.188
LC_Female_ONS_EW_2009_Central	25.130	23.895	27.651	22.667	17.975	10.208
LC_Female_ONS_EW_2010_Central	25.200	23.941	27.684	22.689	17.987	10.209
CMI_2009_FL [1.00%]_Midpoint50%	24.287	23.464	28.325	23.497	18.819	10.768
CMI_2009_FL [2.00%]_Midpoint50%	27.296	25.584	30.110	24.887	19.856	11.286
CMI_2009_FL [1.00%]_Midpoint75%	24.542	23.716	28.688	23.853	19.123	10.918
CMI_2010_FL [1.00%]_Midpoint50%	24.358	23.539	27.502	22.708	18.079	10.267
CMI_2010_FL [2.00%]_Midpoint50%	27.321	25.613	29.093	23.908	18.937	10.644
CMI_2010_FL [1.00%]_Midpoint75%	24.640	23.820	27.778	22.979	18.299	10.357
CMI_2011_FL [1.00%]_Midpoint50%	24.407	23.590	27.557	22.767	18.120	10.280
CMI_2011_FL [2.00%]_Midpoint50%	27.318	25.611	29.097	23.924	18.942	10.638
CMI_2011_FL [1.00%]_Midpoint75%	24.700	23.882	27.845	23.052	18.347	10.372

***** These projections are not included within version 1.3 of the Library