

**Continuous Mortality Investigation**

**User Guide**

**for**

**The CMI Mortality Projections Model**

**Model Name / Version: ‘CMI\_2009’**

**It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of the source of the projection.**

Extensive checking has been undertaken during the development of the Model, and the CMI is confident that it correctly produces projections from the various inputs in the manner described in this User Guide. However, the Institute of Actuaries, the Faculty of Actuaries and the CMI do not warrant the accuracy of the Model and do not accept or assume any liability for its use.

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## 1. Background and Introduction

In response to the continuation of significant year-on-year increases in life expectancy, and to concerns over the continued widespread use, albeit with modifications, of the Interim Cohort Projections which have inevitably become increasingly out-of-date, the CMI established a Working Party to develop a mortality projection model which:

- reflects the latest experience on trends in mortality;
- is relatively straightforward to understand and describe;
- allows users the flexibility to modify projections tailored to their own views and purpose; and
- can be regularly updated over time to reflect emerging experience.

The CMI published a prototype Mortality Projections Model (CPMv0.0) in June 2009. The prototype Model was issued for consultation and was accompanied by its User Guide and two Working Papers: Working Paper 38 provided an overview of the Model and set out specific questions for the consultation; Working Paper 39 detailed further analysis to help inform the setting of parameter values for the Model.

Consultation meetings were held in Edinburgh and London in July 2009 and over 30 written responses to the consultation were received. The CMI is grateful to all those who contributed.

CMI Working Paper 41 summarises the responses to each of the questions posed in Working Paper 38, together with the Working Party's comments and details of the actions taken.

The Model has been updated and the new version, which has been denoted 'CMI\_2009', is described in this User Guide. The structure of the Model has been left broadly unchanged from the prototype version. However, the default parameters contained in the Model have been updated to reflect the publication of England & Wales population data for calendar year 2008.

As a result, default Initial Rates of Mortality Improvement have slightly reduced in general from those published in the prototype version and accordingly Core Projections generated by the new version of the Model produce life expectancies and annuity values which are marginally lower than those produced by the prototype Model (with other parameters held constant). Additional background information on the changes incorporated in the 'CMI\_2009' version of the Model is contained in Working Paper 41.

The Model 'CMI\_2009', the Working Papers, this User Guide, and a set of parameter sensitivity test results may be accessed and downloaded from the CMI pages on the UK Actuarial Profession's website.

Please note that as the changes made from the prototype version of the Model are relatively limited, the CMI is not undertaking a consultation exercise on the 'CMI\_2009' version of the Model.

Any further feedback received on the CMI Mortality Projections Model will be considered for future reviews and updates of the Model. Please send comments:

- via e-mail to: [projections@cmib.org.uk](mailto:projections@cmib.org.uk),
- or in writing to: Neil Robjohns, CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW.

## **2. Overview of the Structure of the Model**

### **2.1. Model Structure**

The structure of the Model is based on the projection of annual rates of mortality improvement (i.e. the pace of change in mortality rates). Specifically, the Model assumes that ‘current’ (i.e. recently observed) rates of change blend over time into a ‘long-term’ rate of change specified by the user. This approach has been adopted by practitioners in a number of countries. In the UK the mortality projections that have formed part of the population projections - now produced by the Office for National Statistics (ONS) and formerly by the Government Actuary’s Department (GAD) – have utilised this methodology for a number of years.

In contrast to mathematical models of mortality, fitted directly to relevant data and ‘extrapolated’ to form a projection, the Model requires the user to set parameter values which directly control the projection. The Model produces a single, deterministic, mortality projection for each set of user inputs.

The structure of the Model allows user input of:

- Base mortality rates, reflecting the estimated current or recent past position
- Initial rates of mortality improvement, reflecting the current estimate of rates of change
- Assumed ultimate / long-term rates of mortality improvement
- An assumed speed and pattern of convergence from ‘initial’ to ‘long-term’.

‘Initial’ and ‘long-term’ rates of mortality improvement are each subdivided into two components: ‘by age’ and ‘by cohort’. These components are projected separately, by age and by year-of-birth cohort respectively, and then recombined.

Convergence from ‘initial’ to ‘long-term’ rates of mortality improvement is defined (separately for ‘by age’ and ‘by cohort’ components) by user inputs for the convergence time-period and the proportion of the total change in rate remaining by the mid-point of that period.

Effectively this approach assumes that, in the very short-term, a good guide as to the likely pace of change in mortality rates is the most recently observed experience. In the long-term, the forces driving mortality change are likely to be very different from those currently influencing patterns of improvement. Therefore, the long-term rate is better informed by ‘expert opinion’ and analysis of long-term patterns of change and the causes driving them. Over time, the relative weight placed on the recently observed past, versus the more subjective longer-term view, can shift appropriately.

Such a model structure could be achieved through a suitably parameterised statistical model. However, at the heart of the Model design is the desire to produce a tool which is easy to understand, intuitive in structure and capable of widespread application by users with varying degrees of expertise in this field of actuarial work.

### **2.2. Core and Advanced Parameter layers**

The Model may be operated at different levels of complexity, reflecting the needs and resources of different users and uses.

By selecting the ‘Advanced’ parameter level for a group of inputs, users obtain unrestricted access with considerable flexibility to set each of the parameters as tables or vectors, varying if desired by individual age (or year-of-birth cohort where relevant), year and gender.

However, when the ‘Core’ parameter level is selected for a group of inputs, either a set of default parameter values is applied, or users are required to set the value for a single, simplified parameter (from which a full set of parameter values for the group is derived via a default mapping). In this way, users may choose to operate the Model at a much simpler level.

If the ‘Core’ parameter level is selected for all groups of inputs the user needs to set values for just two parameters, representing the most critical inputs:

- A single rate (x.xx% pa) to define long-term rates of mortality improvement
  - x.xx% for ages 20-90, falling linearly to 0% at age 120, 0% for age > 120
- A constant additional rate of mortality improvement (z.zz% pa)
  - this constant (which may take a negative value) applies to the entire projection.

At the ‘Core’ parameter level all other inputs are given default values or restricted choice:

- Initial rates of improvement default to rates derived from England & Wales population data
- Convergence parameters default to a pattern and values set by the Working Party
- Base mortality rates are restricted to a selection of published tables.

Projections produced using the ‘Core’ parameter level for all groups of inputs are referred to as ‘Core Projections’ and are subject to a formal naming convention which is described in section 4.1.1.

### 2.3. Outputs

The main outputs provided by the Model for each projection are:

- A table of projected annual rates of mortality improvement by age and calendar year
- A table of projected cumulative mortality reduction factors by age and calendar year.

The Model also produces a variety of charts and tables to illustrate the projection. As well as providing information to facilitate comparison of projections, these outputs are also designed to support the user in reviewing the reasonableness of the projection. Specifically, the following outputs are generated:

- A heat map of annual rates of mortality improvement, by age attained and calendar year
- A heat map of cumulative mortality reduction factors, by age attained and calendar year
- Sample expectation of life and annuity values
- Charts showing projected rates of mortality improvement
  - by age attained and calendar year
  - by year-of-birth cohort and calendar year
- Charts showing projected mortality rates
  - $q_{x,t}$  by age (x) and calendar year (t)
  - $\text{logit}[m_{x,t}] = \ln[m_{x,t} \div (1 - m_{x,t})]$ , by age (x) and calendar year (t)
- Charts showing projected life expectancies, on both ‘period’ and ‘cohort’ calculation bases
  - life expectancy at selected ages, by calendar year
  - annual increase in life expectancies at selected ages, by calendar year
- Charts showing projected survival probabilities, on both ‘period’ and ‘cohort’ calculation bases
  - between selected ages, by calendar year.

## **2.4. Dimensions of the Model**

The data and the projection range together cover 1992 to 2130 (in line with the CMI Library of Mortality Projections) and an age range of 20 to 150 (the Library range is 20 to 120).

### 3. Parameterisation of the Model

The Model parameters are listed, with some summary information, in Table I on page 10. Further description of the parameters is given in sections 3.1 to 3.7, together with notes on their use and a brief statement of the defaults provided through the 'Core' parameter level. Additional commentary on the rationale for the default parameter values is set out in Appendix A.

Working Papers 38, 39 and 41 also provide further analysis to help inform the setting of parameter values for the Model - specific references are given where relevant in the following sections.

All user input cells are highlighted by a **yellow background** in the Model.

In this User Guide:

- references to worksheets in the Model are indicated by using square brackets: [worksheet name];
- references to user selections from dropdown lists are indicated by curly brackets: {selection}.

#### 3.1. Gender

Although the Model is designed to store parameters for both male and female lives, each projection produced by the Model is gender specific. The user must therefore select gender - {Male} or {Female} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

The default parameter values, accessed through the 'Core' parameter level, are differentiated by gender for Base Rates of Mortality and for Initial Rates of Mortality Improvement, but not for the other parameter groups.

Complete mortality projection parameter sets for both genders may be stored in the Model simultaneously, but separate projections must be run for {Male} and {Female} in order to produce the outputs required to model, or value, a mixed gender dataset.

#### 3.2. Base Rates of Mortality

##### 3.2.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

##### 3.2.2. Base Table of Mortality Rates

A base table of initial mortality rates ( $q_x$ ), by age and gender, is required so that the Model can calculate projected mortality rates and summary statistics, such as life expectancies and annuity values. **The base rates of mortality do not influence the projected rates of mortality improvement generated by the Model, so their function is purely to aid the illustration of the projection by providing a base level from which future mortality rates may be calculated.**

Where {**Core**} has been selected for this group of parameters, the user may select a base table of mortality rates from the dropdown list on the [Inputs] worksheet. The options are:

- 100% PCxA00 (published by CMI; assumed timing: 01/07/2000)
- 100% S1PxA (published by CMI; assumed timing: 01/09/2002)
- 100% Ax00 Ult (published by CMI; assumed timing: 01/07/2000)
- 100% 'ILT05-07x[E&W]' (published by ONS; assumed timing: 01/01/2006)
  - This is unofficial shorthand for the Interim Life Tables 2005-2007 published by ONS, based on ONS population estimates and death counts for the population of England & Wales
  - Separate ILTs are published for male and female lives
  - The published ILT mortality rates are not graduated and only cover ages up to 100 - a simple extrapolation beyond age 100, by reference to the PCxA00 tables, is built into the Model [where x = 'M' or 'F' as appropriate to the gender selected.]

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of mortality rates to suit their purpose:

- Paste the mortality rates into the appropriate cells in the [Advanced Parameters] worksheet
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

### 3.2.3. Timing Definition for Base Rates of Mortality

For the purpose of calculating projected mortality rates, it is necessary to specify a precise time interval to which the Base Table of Mortality Rates is deemed to apply. The Model therefore requires input of the date, dd/mm/yyyy, on which the 1-year time interval starts such that  $q_x$  is the probability that a life attaining age  $x$  exact on dd/mm/yyyy dies before reaching age  $x+1$  exact on dd/mm in year  $yyyy+1$ .

Under the {**Core**} level of parameters, the effective dates are defaulted to those specified in section 3.2.2; for mortality rate tables set by the user under the {**Advanced**} level of parameters, the user must enter the effective date in the [Inputs] worksheet.

If a user does not wish to apply the default timing definition assumed for mortality tables available under the {**Core**} level of parameters, the user may simply switch to the {**Advanced**} parameter level, enter the mortality rates for the required table, and set the timing definition as they wish.

**Table I : Summary of Parameters within ‘CMI\_2009’ Mortality Projections Model**

<b>Parameter</b>	<b>Dimensions &amp; Format</b>	<b>Core</b>	<b>Advanced</b>
<b>Gender</b>			
Gender for Mortality Projection	Choice List: {Male; Female}	User Selection	User Selection
<b>Base Rates of Mortality</b>			
Parameter Level Mortality Rates Timing Definition	Choice List: {Core; Advanced} Ages 20 to 150; x.xxxxxx in range [0,1] Single Date: dd/mm/yyyy [Year Y <sub>0</sub> ]	User Selection Selection {sample tables} Default	User Selection User Input: Vector (Age) User Input: Single Date
<b>Initial Rates of Mortality Improvement</b>			
Parameter Level Initial Rates of Improvement Timing Definition Initial Age/Period Component Initial Cohort Component	Choice List: {Core; Advanced} Ages 20 to 150; Years Y <sub>0</sub> to Y <sub>1</sub> ; x.xx% pa Single Date: dd/mm/yyyy [Year Y <sub>1</sub> ] Ages 20 to 150; x.xx% pa [for Y <sub>1</sub> ] YoB [Y <sub>1</sub> -20] to [Y <sub>1</sub> -150]; x.xx% pa [ Y <sub>1</sub> ]	User Selection Default [1991 to 2006] Default [01/01/2006] Default [for 2006] Default [for 2006]	User Selection User Input: Table (Age; Year) User Input: Single Date User Input: Vector (Age) User Input: Vector (YoB)
<b>Long-Term Rates of Mortality Improvement</b>			
Parameter Level Long-term Age/Period Component Long-term Cohort Component	Choice List: {Core; Advanced} Ages 20 to 150; x.xx% pa YoB [Y <sub>1</sub> -20] to [Y <sub>1</sub> -150]; x.xx% pa	User Selection User Input: Single Value Default [= zero]	User Selection User Input: Vector (Age) User Input: Vector (YoB)
<b>Convergence</b>			
Parameter Level Period of Convergence: • Age/Period Component • Cohort Components % Change after mid-point of Period: • Age/Period Component • Cohort Components	Choice List: {Core; Advanced}  Ages 20 to 150; x years YoB [Y <sub>1</sub> -20] to [Y <sub>1</sub> -150]; x years  Ages 20 to 150; x% YoB [Y <sub>1</sub> -20] to [Y <sub>1</sub> -150]; x%	User Selection  Default Default  Default [= 50%] Default [= 50%]	User Selection  User Input: Vector (Age) User Input: Vector (YoB)  User Input: Vector (Age) User Input: Vector (YoB)
<b>Constant Addition to Rates of Mortality Improvement</b>			
Constant	Single Value: x.xx%	User Input: Single Value	User Input: Single Value

### 3.3. Initial Rates of Mortality Improvement

#### 3.3.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

#### 3.3.2. Table of Initial Rates of Mortality Improvement

The mortality improvement rate for age  $x$  and year  $t$ ,  $r_{x,t}$ , is defined as:

$$r_{x,t} = 1 - \{ q_{x,t} \div q_{x,t-1} \}$$

A table of initial rates of mortality improvement, by age and gender, is required for the Foundation Year of the projection - that is, the last year for which improvement rates are known, estimated or assumed, and accordingly which forms the start-point for the projection. In addition, rates of mortality improvement for earlier years are required to cover any time interval between the effective dates for the Base Rates of Mortality and the Initial Rates of Mortality Improvement.

Where {Core} has been selected for this group of parameters, a default table of rates of mortality improvement is adopted. This default table contains estimated improvement rates for individual ages, separately for males and females, for calendar years 1991 to 2006, derived using an age-cohort P-Spline model fitted to ONS data for the population of England & Wales (for ages 18 to 102 and the period 1961 to 2008).

The default Initial Rates of Mortality Improvement are taken as those for calendar year 2006; the first year of the projection is therefore assumed to be 2007. The Working Party's analysis strongly supports this approach of 'stepping back' two years from the final year for which raw experience data is available (2008) in order to derive sufficiently reliable estimates of the rates of mortality improvement - for example this avoids much of the distortion of 'edge effects' which are a common 'danger' with smoothing algorithms (including P-Spline models).

Further information on the derivation of the default table of rates of mortality improvement is set out in section A.1.1, and a general commentary on the approach taken to reduce 'edge effects' is given in section 3.1.2 of CMI Working Paper 39.

Alternatively, by selecting the {Advanced} level for this group of parameters, the user may input any table of rates of mortality improvement to suit their purpose:

- Paste the improvement rates into the appropriate cells in the [Advanced Parameters] worksheet
  - Cell highlighting is used in the worksheet to indicate the years for which data input is mandatory; further data may optionally be entered for earlier years to give additional 'history' for the projection outputs (for example the 'heat map' of mortality improvements).
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

Additional research and analysis of observed rates of mortality improvement for population, insured lives and pensioner datasets is presented in sections 3.1 to 3.3 of CMI Working Paper 39.

### 3.3.3. Timing Definition for Rates of Mortality Improvement

For the projection, it is necessary to specify a precise time interval to which the Initial Rates of Mortality Improvement are deemed to apply. The Model therefore requires input of the date, dd/mm/'year t', on which the 1-year time interval for  $q_{x,t}$  (using the notation set out in 3.3.2) starts. Note in particular that 'year t' specifies the Foundation Year for the projection of improvement rates.

Under the {**Core**} level of parameters, the effective date is defaulted to 01/01/2006 reflecting the data and methodology used to derive the rates. For mortality improvement rate tables set by the user under the {**Advanced**} level of parameters, the user must enter the effective date in the [Inputs] worksheet.

### 3.3.4. Split of Initial Rates into Age/Period Component and Cohort Component

The Initial Rates of Mortality Improvement must be split into two components: a component influenced by age and period (but not year-of-birth cohort), and a component which is solely influenced by year-of-birth cohort. This split is only required for the initial rates - that is for the Foundation Year - but not for any earlier years.

The component values may be either positive or negative and vary by age, or year-of-birth, and by gender. The two components sum to give the Initial Rates of Mortality Improvement at each age for the Foundation Year.

Where {**Core**} has been selected for this group of parameters, a default component split is adopted. The default component rates have been derived using an age-period-cohort model developed explicitly for this purpose. Further information on the methodology and resulting split is set out in section A.1.2.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of component rates to suit their purpose: paste the age/period and cohort component rates of mortality improvement into the appropriate cells in the [Advanced Parameters] worksheet. The worksheet contains a check that the two components do sum to the input aggregate Initial Rates of Mortality Improvement for the Foundation Year.

The subdivision of aggregate rates of mortality improvement into age/period and cohort components is, at least partly, subjective. Some insight may be gleaned from the study of patterns in the emerging observations of mortality improvement rates, and through study of recent changes in mortality rates by cause of death. Additional research and analysis on these topics is presented in sections 3.3, 5.1 and 5.2 of CMI Working Paper 39.

## 3.4. Long-Term Rates of Mortality Improvement

### 3.4.1. Parameter Level

The user must select the level for this group of parameters - {**Core**} or {**Advanced**} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

### 3.4.2. Long-Term Rates of Mortality Improvement

The Model requires a table of long-term rates of mortality improvement, by age and gender, split into age/period and year-of-birth cohort components.

Where {**Core**} has been selected for this group of parameters:

- The user is required to set a value for the age/period component of the Long-Term Rate
  - Enter the value in the [Inputs] worksheet
  - The input value applies uniformly for ages 20 to 90
  - For higher ages, the age/period component of the Long-Term Rate is assumed to reduce (linearly) from age 90, reaching zero at age 120, and to be zero for ages above 120.
- The cohort component of the Long-Term Rate is set to a default value of zero (so the influence of year-of-birth features is assumed to dissipate completely over the convergence period).

Further commentary on the rationale for this mapping of the single ‘Core’ level input parameter to the full parameter set is given in section A.2.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of component Long-Term Rates to suit their purpose:

- Paste the age/period and cohort component Long-Term Rates into the appropriate cells in the [Advanced Parameters] worksheet.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

*Important Note: When the Model is first downloaded from the CMI pages on the UK Actuarial Profession’s website, the {Core} Long-Term Rate parameter on the [Inputs] worksheet is set to “?” and no projection is produced. Similarly the vectors for the Long-Term Rate components in the [Advanced Parameters] worksheet are also set to “?”. The user must therefore enter a numeric value (or vectors of values) in the appropriate cells in order to produce a projection.*

The choice of a suitable long-term rate is clearly subjective but it is intended that the research presented in sections 4.1 to 4.3 of CMI Working Paper 39 may help inform the selection of suitable parameter values.

In addition, the Appendix to CMI Working Paper 38 contains a description of future scenarios (in terms of different causes of death) consistent with projections using alternative long-term rates of change. Mortality models that decompose trends into constituent causes (either cause-of-death models or disease-based approaches) may provide users with insights into appropriate future rates of change. Moreover, the views of experts in different fields are likely to be especially valuable and the CMI welcomes attempts to build links with other professions as evidenced, for example, by the ‘Joining Forces on Mortality and Longevity’ multidisciplinary conference held in October 2009.

## 3.5. Convergence

### 3.5.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

### 3.5.2. Period of Convergence

The Model assumes that Initial Rates of Mortality Improvement converge smoothly towards Long-Term Rates of Mortality Improvement over a specified period. The convergence process operates separately for Age/Period Components and Cohort Components of improvement rates, so that different convergence periods may be set, in whole years, for each age and each year-of-birth cohort, and also by gender.

Where {**Core**} has been selected for this group of parameters, a default table of convergence parameters is adopted:

- The default convergence period for Age/Period Components of improvement rates is:
  - fixed at 10 years for ages up to 50,
  - then increasing by one year for each year of age up to 60,
  - fixed at 20 years for all ages 60 to 80,
  - then decreasing by one year for each year of age up to 95,
  - and fixed at 5 years for all ages 95 and above.
- The default convergence period for Cohort Components of improvements is:
  - fixed at 5 years for year-of-birth cohorts 1911 and earlier,
  - then increasing by one year for each year-of-birth cohort up to 1946,
  - and fixed at 40 years for all year-of-birth cohorts 1946 and later;
  - that is the convergence periods for the Cohort Components run to age 100, subject to a minimum of 5 years and a maximum of 40 years.

The default convergence periods are not differentiated by gender.

Further commentary on the rationale for these default parameter values is set out in section A.3.1.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of convergence parameters to suit their purpose:

- Paste the Age/Period and Cohort Component Convergence Periods into the appropriate cells in the [Advanced Parameters] worksheet. Convergence Periods are limited to a maximum of 50 years.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

The choice of a suitable pattern of convergence periods is clearly subjective. Some insight may be gleaned from the study of features evident in past patterns of mortality improvement rates, and through study of mortality trends by cause of death. Additional research and analysis on these topics is presented in sections 5.1 and 5.2 of CMI Working Paper 39.

### 3.5.3. Proportion of Convergence Remaining at Mid-point of Convergence Period

The pattern of convergence for each Age/Period Component and each year-of-birth Cohort Component may be altered to influence the initial trajectory of the projected mortality improvement rates over time. This is achieved by specifying the proportion of the change, between the Initial Rate and the Long-Term Rate, that remains to be covered at the mid-point of the Period of Convergence.

The convergence process operates separately for Age/Period Components and year-of-birth Cohort Components of improvement rates, so that different ‘tracks’ may be set for each age and each year-of-birth cohort, and also by gender, by varying the Proportion Remaining parameters.

Where {**Core**} has been selected for this group of parameters, a default table of convergence parameters is adopted:

- The default Proportion of Convergence Remaining at Mid-point of Convergence Period is set at 50% for both age/period and cohort components and for all ages and years-of-birth
- Therefore the projected rates of mortality improvement half-way through the periods of convergence will be the average of the relevant Initial and Long-Term Rates.

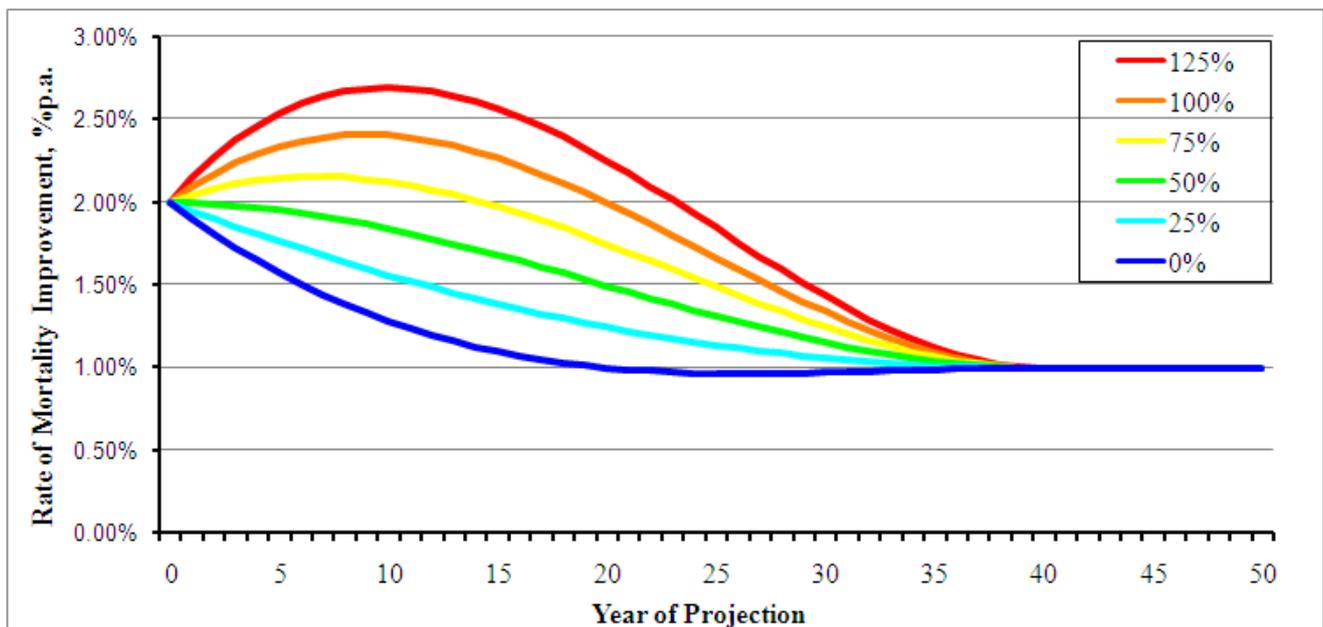
Further commentary on the rationale for this simple default parameter value is set out in section A.3.2.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of convergence parameters to suit their purpose:

- Paste the Age/Period and Cohort Component Proportions Remaining into the appropriate cells in the [Advanced Parameters] worksheet.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

A great variety of transitions may be modelled in this way. Input values are not restricted to the range 0% to 100% as other scenarios are plausible. For example, where the Initial Rate is above the Long-Term Rate, selecting a relatively high proportion (say in the range 75% to 125%) can generate projected rates of mortality improvement that initially increase before falling towards the assumed Long-Term Rates. Users therefore have the flexibility, for example, to generate scenarios in which the rate of change accelerates in the short-term, before decelerating in the longer-term.

Figure 1 illustrates how the shape of the convergence path can be altered in practice (with a 2.0% p.a. initial rate converging towards a 1.0% p.a. long-term rate in 40 years' time).



**Figure 1: Illustration of the operation of the convergence algorithm over a 40-year period, with various Proportions of Convergence Remaining at Mid-Point**

The research material noted in section 3.5.2 may also help inform views on likely patterns of convergence and, in turn, help in setting values for this parameter set.

### **3.6. Constant Additional Rate of Mortality Improvement**

This parameter allows the user to incorporate a degree of prudence (or other appropriate adjustment) to a projection. The Constant Additional Rate of Mortality Improvement is applied uniformly to all ages and calendar years after the start of the projection.

The user may enter a value, if desired, in the [Inputs] worksheet. A negative value may be entered if the user wishes to reduce all projected rates of mortality improvement.

For the avoidance of doubt:

- The Constant Addition does not apply to the Foundation year, nor to earlier years
- In the first year of the projection (Foundation Year + 1), the annual rates of mortality improvement will be the Initial Rates, projected forward one year, plus the Constant Addition
- In the long-term, the annual rates of mortality improvement will be the Long-Term Rates plus the Constant Addition.

### **3.7. Further Parameters for Sample Expectation of Life and Annuity Values**

Sample expectation of life and annuity values are calculated and presented on the [Sample EoL & Annuities] worksheet and these calculations require some additional parameters.

#### **3.7.1. Calculation Date**

The sample values apply as at the effective Calculation Date set by the user on the [Sample EoL & Annuities] worksheet.

#### **3.7.2. Annuity Discount Rate**

An interest / discount rate is required for the annuity value calculations. This may be set on the [Sample EoL & Annuities] worksheet. This single parameter is intended to reflect the net combined effect of pension / benefit escalation and allowance for future expected investment returns.

#### **3.7.3. Model Points**

The Model supports two independent sets of model points for sample expectation of life and annuity values.

A 'small' set of 6 model points may be selected via Table\_01 on the [Sample EoL & Annuities] worksheet. The corresponding sample values are then also shown in Table\_01. These model points are specified by setting:

- The age exact on the Calculation Date
  - that is age exact in whole years: there is no facility for part years of age
- The 'deferment' period
  - for annuities this specifies the period, in whole years, before the annuity commences payment
  - for expectation of life this specifies the delay, in whole years, before the remaining expectation of life is calculated - for example: to calculate  $e_x$  as at  $n$  years after the Calculation Date, enter age as  $x-n$  and deferment period as  $n$  years.

A larger 'grid' of model points may be selected via Table\_02 on the [Sample EoL & Annuities] worksheet. The corresponding sample values are then shown in Table\_02 and Table\_03.

The 'grid' has 119 model points: 17 ages by 7 calendar years. For example this can be used to calculate sample expectation of life and annuity values for ages from 20 to 100 in 5-year intervals, for calculation dates in say 2010 to 2035 in 5-year intervals.

The 'grid' of model points is specified by setting:

- The 17 ages - age exact on the calculation dates
  - that is age exact in whole years: there is no facility for part years of age
- The 7 calculation dates - specify 7 calendar years
  - The calculation dates take the day and month from the main Calculation Date (see 3.7.1)
- The minimum vesting age for the annuities
  - if the age specified for a model point is less than the minimum vesting age, annuity payment for that model point is deferred to the minimum vesting age; otherwise annuity payments are assumed to begin at the age specified for the model point.

## 4. Projection Naming Convention

### 4.1.1. Formal Naming Convention for Projections produced using ‘Core’ Parameters

It is recommended that ‘Core Projections’, that is those produced when the parameter level is set to {**Core**} for all parameter groups, should be referred to by using the following prescribed naming convention.

**CMI\_2009\_x [a%] + c%**

where:

- **CMI** refers to the CMI Mortality Projections Model
- ‘\_2009’ is the version ‘number’ of the Model and reflects the year of its publication
- ‘\_x’ specifies the gender and is either ‘\_M’ (male) or ‘\_F’ (female)
- **a%** is the Long-Term Rate of Mortality Improvement (value of Core parameter)
- **c%** is the Constant Additional Rate of Mortality Improvement (omitted if zero).

It is permissible to omit the gender extension where the ‘Core Projections’ being described cover both male and female lives, with identical Core Parameters (a% and c%) and the gender differentiated defaults applied - that is, it is sufficient to refer to ‘CMI\_2009 [a%]’ instead of ‘CMI\_2009\_M [a%]’ for males and CMI\_2009\_F [a%] for females’.

### 4.1.2. Informal Naming Convention for other Projections produced using the Model

The following informal naming convention is applied within the Model. It is hoped this will prove helpful for users, for example in their documentation, as a form of ‘shorthand’.

**CMI\_2009\_x [Advanced: IR(tab\_1); LTR(tab\_2); Conv(tab\_3); Timing = mm/dd/yyyy] + c%**

where:

- **CMI** refers to the CMI Mortality Projections Model
- ‘\_2009’ is the version ‘number’ of the Model and reflects the year of its publication
- ‘\_x’ specifies the gender and is either ‘\_M’ (male) or ‘\_F’ (female)
- ‘**tab\_1**’ is any short name adopted by the user to represent the tables used for the Initial Rates of Mortality Improvement group of parameters; ‘**default**’ will be shown here if {**Core**} is selected for this parameter group
- ‘**tab\_2**’ is any short name adopted by the user to represent the tables used for the Long-Term Rates of Mortality Improvement group of parameters; the Core Long-Term Rate parameter (akin to **a%** in 4.1.1) will be shown here if {**Core**} is selected for this parameter group
- ‘**tab\_3**’ is any short name adopted by the user to represent the tables used for the Convergence group of parameters; ‘**default**’ will be shown if {**Core**} is selected for this parameter group
- **Timing** shows the user input date for the timing definition for Initial Rates of Mortality Improvement
- **c%** is the Constant Additional Rate of Mortality Improvement (omitted if zero).

### 4.1.3. Description of Base Rates of Mortality

The naming conventions above only describe the basis for projection of mortality improvement rates; Base Rates of Mortality must be described separately. The following convention is applied within the Model and is recommended for adoption by users:

100% **TABLE (level)** for life aged x exact on **dd/mm/yyyy**  
where:

- **TABLE** is a short name adopted to represent the table of mortality rates - this may be a published table name, for example 'PCMA00', or a name adopted by the user for a bespoke table
- **level** is {**Core**} or {**Advanced**} showing the parameter level selected for Base Rates of Mortality
- **dd/mm/yyyy** specifies the timing definition of the mortality rates described.

*Important Note: The Model always uses 100% of the input mortality table, whether selected from the dropdown list of options available under the {Core} parameter level or directly entered as mortality rates under the {Advanced} parameter level. Therefore, to use a different percentage of a published table, the actual mortality rates required must be entered using the {Advanced} parameter level.*

### 4.1.4. Changes to the Model's Calculations

Where a user chooses to modify the Model's calculations, the proposed naming conventions for projections should not be used without a clear statement of the changes made.

## 5. Mechanics of the Model

### 5.1. General

#### 5.1.1. Software

The Model has been constructed as a Microsoft Office Excel 2007 Workbook but saved in Excel 97-2003 compatible file format.

It is intended and believed that the Model is fully compatible with Excel 97-2003 versions of the software although it is not practical for the CMI to undertake exhaustive testing in this regard.

#### 5.1.2. Open Source Code

The Model is transparent in the sense that its mechanics are open for users to view (and indeed alter) if they wish.

**Extensive checking has been undertaken during the development of the Model, and the CMI is confident that it correctly produces projections from the various inputs in the manner described in this User Guide. However, the Institute of Actuaries, the Faculty of Actuaries and the CMI do not warrant the accuracy of the Model and do not accept or assume any liability for its use. Users must satisfy themselves that the parameters, calculations and outputs of the Model are suitable for whatever application they make of the Model.**

The Model has been developed with a general philosophy of keeping the formulae relatively simple so that most actuarial users of Excel should be able to follow the calculations and methodology. It is acknowledged that there is some cost in this approach in terms of slower calculation speeds and larger file size than could have been obtained.

The worksheet structure of the Model is covered in section 5.2. With the intention of simplifying the presentation of the Model to users, especially those who prefer to operate it at the {Core} level of parameters, the worksheets are presented in three groups (rather than in their natural order of calculation):

- “Inputs” - the points for data / parameter entry and control of the projections
- “Outputs” - the presentation of the results of the projection
- “Mechanics” - the workings of the Model.

The “Inputs” and “Outputs” groups are the worksheets required for operation of the Model and so should be regarded as the ‘Primary’ worksheets. The “Mechanics” group should be regarded as ‘Secondary’ worksheets - users only need to study these sheets if they wish to follow through the detailed workings of the Model or view the full default parameter tables available through the {Core} parameter level.

If desired, the ‘Secondary’ worksheets could easily be hidden from view by using the commands:

- Excel 2007: [Home] / [Format] / [Hide & Unhide] / [Hide / Unhide Sheet], or
- Excel 97-2003: [Format] / [Sheet] / [Hide / Unhide],

and selecting the required sheet if unhiding.

All the worksheets have the Excel worksheet protection facility enabled - apart from user input cells - in order to protect the Model from accidental change and corruption. However, this protection may easily be turned off if desired by using the commands:

- Excel 2007: [Home] / [Format] / [Unprotect Sheet] - no password is set or required
- Excel 97-2003: [Tools] / [Protection] / [Unprotect Sheet] - no password is set or required.

Where a user chooses to modify the Model's calculations the proposed naming conventions for projections should not be used without a clear statement of the changes made.

### 5.1.3. Excel Calculation Options

When the Model is run, to produce a projection and full set of outputs, the calculation time is typically of the order of 5 to 30 seconds depending on the speed of the computer processor used and the demands of any other software running simultaneously with the Model.

Excel Data Tables are used in the Model to populate some of the charts and also the sample Expectation of Life and Annuity Values. Some of these Data Tables have a heavy workload and so account for a very large proportion of the calculation time.

In the version of the Model available for download from the CMI pages on the UK Actuarial Profession's website, the Excel Calculation Option is set to "Automatic Except for Data Tables". This is intended to optimise the performance of the Model for most users and uses.

By setting the Excel Calculation Option to "Automatic Except for Data Tables":

- Most worksheets calculate automatically and quickly - in particular:
  - [Inputs] worksheet reacts quickly to choice of {Core} or {Advanced}
  - The output tables of Projected Annual Rates of Mortality Improvement and Projected Cumulative Mortality Reduction Factors update automatically and quickly
  - The Heat Maps, Input Charts and some of the Output Charts update automatically and quickly.
- However, those Charts and Tables fed by Excel Data Tables will not update until the user forces the Data Table calculations to be updated
  - Press the {F9} key to calculate the Excel Data Tables and wait for the outputs to update
  - Warning messages are in place for the Charts and Tables which need to be updated in this way.

Alternative Excel Calculations Options are:

- Automatic - All calculations and Data Tables will automatically update after any change to parameter values (or formulae). This removes the risk that Output Charts are out-of-line with the parameter values set, but will slow down the basic operation of the Model.
- Manual - This option is NOT recommended! It allows parameters to be set without any calculation-time delays, but the user must then force the calculations to run by pressing the {F9} key. *Important Note: The [Inputs] worksheet will not automatically update, for example for a change from {Core} to {Advanced} parameter level if this option is selected.*

The Excel Calculation Option may easily be changed if desired by using the commands:

- Excel 2007: [Formulas] / [Calculations Options] and then selecting the desired option
- Excel 97-2003: [Tools] / [Options] / [Calculation] and then selecting the desired option.

#### 5.1.4. Constraints & Reasonableness of Model Outputs

No constraints have been placed on parameter values other than basic input validation on the range of possible values - for example, Base Rates of Mortality must lie in the range [0,1].

Only a few constraints have been placed on the projection, notably:

- Projected mortality rates are restricted to the range [0,1]
- Projections only run up to age 150 - expectation of life and annuity values are truncated after age 150, effectively assuming all remaining lives die at the end of their 150<sup>th</sup> year
- Projections only run up to calendar year 2130 - expectation of life and annuity values are shown as “#N/A” where mortality rates would need to be calculated beyond 2130.

**Users should therefore be aware that the Model may generate projections with unusual properties, or even errors, particularly if extreme parameter values are entered.**

Accordingly, users are strongly advised to satisfy themselves of the reasonableness of any projections produced. As well as providing information to facilitate comparison of projections, the various Model outputs are also intended to support the user in reviewing the reasonableness of the projection.

**It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of the source of the projection.**

#### 5.1.5. Saving Parameter Sets

The Model only holds a single set of Core parameters (defaults) and a single set of Advanced parameters (user input) for each gender. Therefore if, having completed a projection, the user wishes to save the parameter set for future reference, this is most easily achieved by saving a copy of the entire Model. Alternatively copies of the relevant parameter tables, or entire worksheets, could be made and stored as a ‘library’ of parameters, ready to be pasted back in to the Model for use when required.

## 5.2. Worksheet Structure of the Model

The Model contains a series of worksheets. These are listed, with some summary information, in Table II on page 24. A schematic of the Model's worksheet structure, outlining the major data and calculation flows between the worksheets is shown in Figure 2 on page 25. Further commentary on the worksheets is given in the following sections.

In the Model - for example, one of the worksheet names - and User Guide, the acronym 'CPM' is still used to refer generically to the CMI Mortality Projections Model. This acronym has been deliberately retained, rather than replaced by the current Model version name 'CMI\_2009', in order to avoid the potential need for further updates to such references for each future version of the

### 5.2.1. Model Inputs

The [Inputs] worksheet is the 'Home' sheet for users of the Model.

Using the input cells on this sheet, users:

- Select the level - {Core} or {Advanced} - for each of the four main parameter groups
  - Where {Core} is selected, additional parameter inputs on this sheet fully specify the projection
  - Where {Advanced} is selected, the user is required to input dates to specify the timing definition of the user-input tables of parameters, and is invited to input short names by which to refer to the tables
- Select gender for the projection - this determines whether the calculations in the Model work from the parameters stored for male lives or those for female lives
- Set any Constant Additional Rate of Mortality Improvement.

A shorthand description of the projection - using the naming conventions set out in section 4 and driven by the user's parameter choices - is also shown on this worksheet.

### 5.2.2. Core Parameters

The [Core Parameters] worksheet contains the full tables of default values which are applied when the {Core} level of parameters is selected. There are separate tables for each of the four main parameter groups, each covering the Model's full range of individual ages (or year-of-birth cohorts) and gender.

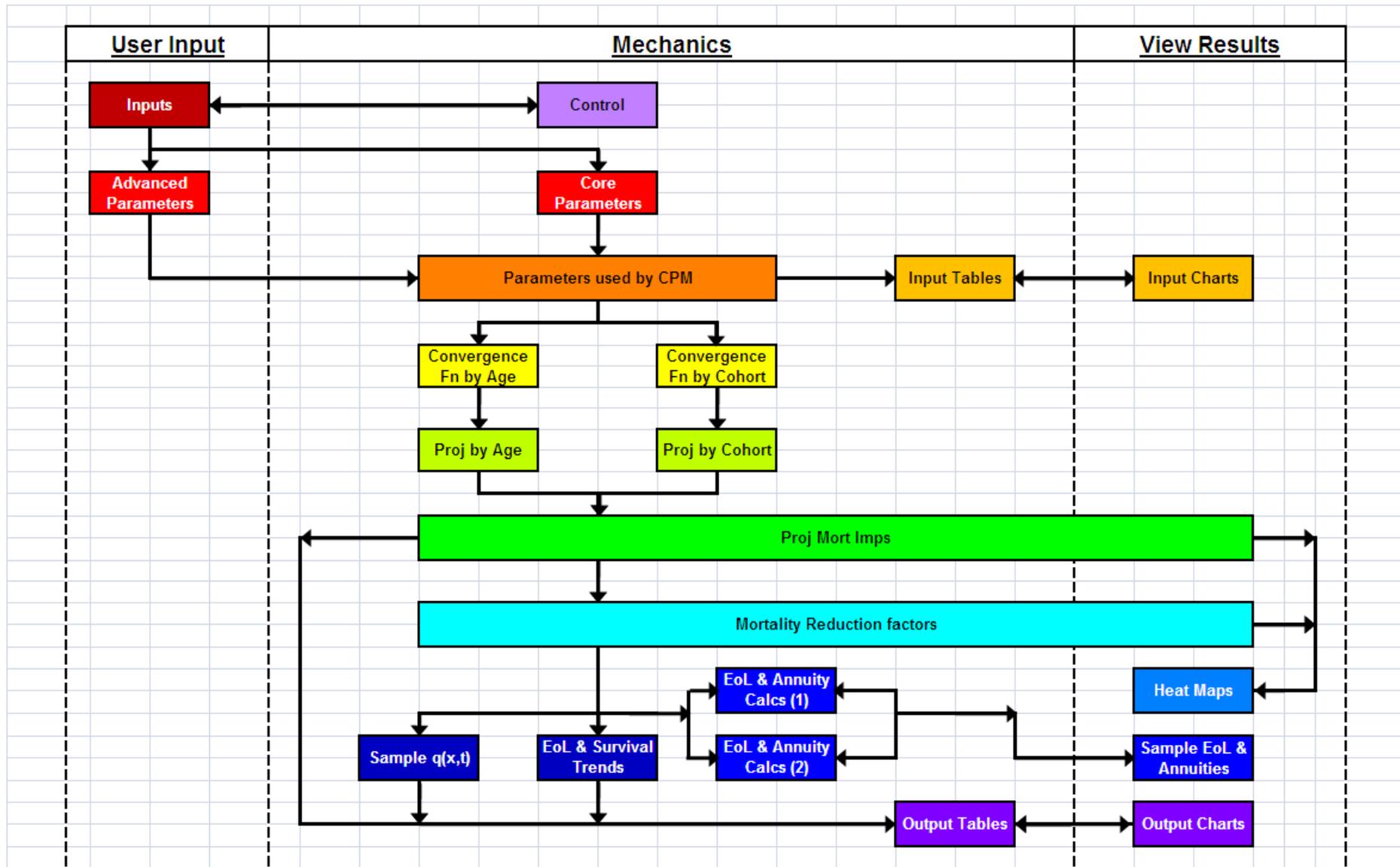
The colour coding applied to separately identify the parameters for males and females is followed through, depending on the user's selections, to the [Parameters Used by CPM] worksheet.

**Table II : Summary of Worksheets within ‘CMI\_2009’ Mortality Projections Model**

<b>Worksheet Name</b>	<b>Description</b>	<b>Type</b>	<b>Rank</b>
Inputs	User ‘Home’ page - primary control of the projection	User Input	Primary
<i>Control</i>	<i>Background spreadsheet mechanics</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>Core Parameters</i>	<i>Full tables of default values for Core level of parameters</i>	<i>Information &amp; Mechanics</i>	<i>Secondary</i>
Advanced Parameters	Full tables of user input values for Advanced level of parameters	User Input (when selected)	Primary
<i>Parameters Used by CPM</i>	<i>Full tables of final user-selection of parameters for the projection</i>	<i>Information &amp; Mechanics</i>	<i>Secondary</i>
<i>Convergence Fn by Age</i>	<i>Detail calculation of convergence pattern for age/period component</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>Proj by Age</i>	<i>Detail calculation of projected imp rates for age/period component</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>Convergence Fn by Cohort</i>	<i>Detail calculation of convergence pattern for cohort component</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>Proj by Cohort</i>	<i>Detail calculation of projected imp rates for cohort component</i>	<i>Mechanics</i>	<i>Secondary</i>
Proj Mort Imps	Full table of aggregate projected annual mortality improvement rates	Mechanics & Output	Primary
Mortality Reduction Factors	Full table of projected cumulative mortality reduction factors	Mechanics & Output	Primary
<i>Sample q(x,t)</i>	<i>Detail calculation of sample projected mortality rates</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>EoL &amp; Annuity Calcs (1)</i>	<i>Detail calculation of sample projected exp of life &amp; annuity values</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>EoL &amp; Annuity Calcs (2)</i>	<i>Detail calculation of sample projected exp of life &amp; annuity values</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>EoL &amp; Survival Trends</i>	<i>Detail calculation of projected exp of life and survival rates, by year</i>	<i>Mechanics</i>	<i>Secondary</i>
<i>Input Tables</i>	<i>Background spreadsheet mechanics for Input Charts</i>	<i>Mechanics</i>	<i>Secondary</i>
Input Charts	Charts to illustrate parameters selected	Output	Primary
<i>Output Tables</i>	<i>Background spreadsheet mechanics for Output Charts</i>	<i>Mechanics</i>	<i>Secondary</i>
Output Charts	Charts to illustrate projection results	Output	Primary
Heat Maps	Heat Maps to illustrate projection results	Output	Primary
Sample EoL & Annuities	Tables of expectation of life and annuity values for sample ages	Output	Primary

The worksheets [Inputs >>], [Outputs >>] and [Mechanics >>] are used purely as signposts and play no active part in the Model. The worksheet [Notices] provides general information and plays no active part in the Model.

**Figure 2: Schematic of Model Structure: Outline of major Data and Calculation flows between Worksheets**



### 5.2.3. Advanced Parameters

The [Advanced Parameters] worksheet contains the cells into which user-input tables may be pasted when the {Advanced} level of parameters is selected. There are separate tables for each of the four main parameter groups, each covering the Model's full range of individual ages (or year-of-birth cohorts) and gender - that is, the layout is identical in most respects to that for the [Core Parameters] worksheet.

Cell highlighting - **bright yellow** for mandatory, **pale yellow** for optional, and driven by user selections for parameter levels - is used to indicate the areas of the [Advanced Parameters] worksheet open for user-input.

The colour coding applied to separately identify the parameters for males and females is followed through, depending on the user's selections, to the [Parameters Used by CPM] worksheet.

### 5.2.4. Parameters Used by CPM

The [Parameters Used by CPM] worksheet simply holds the final selected set of parameter tables which drive the projection. This sheet only contains parameters for the gender the user has chosen to model.

### 5.2.5. Calculation of Projection over Period of Convergence

The algorithm for the convergence process operates by calculating a weighted average of the Initial and Long-Term Rates of Mortality Improvement, with the weights varying over the Period of Convergence.

The worksheets [Convergence Fn by Age] and [Convergence Fn by Cohort] take the convergence group of parameters and create tables of weighting factors reflecting the selected pattern and duration of convergence, by age and by year-of-birth cohort for the respective components of mortality improvement rates.

The proportion of the weight to be placed on the Initial Rate is determined, independently for each individual age and year-of-birth, by fitting cubic polynomials in time,  $f(t)$ , such that:

- $f(0) = 1$  where  $t = 0$  at the start (Foundation Year) of the projection;
- $f(T) = 0$  where  $T$  is the length of the Period of Convergence (in whole years);
- $f(\frac{1}{2}T) = P$ , the Proportion Remaining at the Mid-Point;
- The first derivative of  $f(t)$  at  $T$  is zero, that is  $f'(T) = 0$ .

Let  $\tau = t \div T$ , so that  $\tau = 1$  at the end of the Period of Convergence.

and  $f(\tau) = a\tau^3 + b\tau^2 + c\tau + d$

The four criteria above lead to four simultaneous equations:

- $d = 1$
- $a + b + c + d = 0$
- $\frac{1}{8}a + \frac{1}{4}b + \frac{1}{2}c + d = P$
- $3a + 2b + c = 0$ .

These may be solved to give:

- $a = 8P - 2$
- $b = 5 - 16P$
- $c = 8P - 4$
- $d = 1.$

This gives full weight to the Initial Rates of Mortality Improvement at the start of the projection, and to the Long-Term Rates at the end of the Period(s) of Convergence. The convergence path is smooth, passing through the selected level at the mid-point of the convergence period and blending smoothly into the Long-Term Rate. However, the Model has no regard for the trend in rates of mortality improvement prior to the start of the projection, so although the transition from historic to projected rates is continuous, it may well not be smooth.

The worksheets [Proj by Age] and [Proj by Cohort] then apply these weights, to the Initial and Long-Term Rates (for each age or year-of-birth cohort), to derive full tables of projected annual rates of mortality improvement for each component.

Note that, as convergence periods are limited to a maximum of 50 years, these 4 sheets only operate for the first 50 years of the projection.

### **5.2.6. Proj Mort Imps**

The [Proj Mort Imps] worksheet combines the age/period and year-of-birth cohort components to produce a full projection of aggregate annual rates of mortality improvement.

The table covers the full age attained [20 to 150] and calendar year [1992 to 2130] range of the Model. The calculated values feed in to the output Heat Map of mortality improvement rates (on the [Heat Maps] worksheet) and may be exported for further use or analysis.

The timing definition for these improvement rates follows directly from that specified for the Initial Rates of Mortality Improvement. That is, if the effective date for Initial Rates is dd/mm/yyyy, then the projected rates all relate to movements in mortality rates ( $q_{x,t}$ ) for exact ages on dd/mm of each year. For Core Projections produced using the 'CMI\_2009' Model, the effective date for Initial Rates is 01/01/2006, so that the projected rates for Core Projections all relate to movements in mortality rates ( $q_{x,t}$ ) for exact ages on 1<sup>st</sup> January of each year.

The choice of table for Base Rates of Mortality plays no part in these calculations. Accordingly the table of 'actual' and projected annual rates of mortality improvement is invariant to the assumed effective timing for Base Rates of Mortality.

### **5.2.7. Mortality Reduction Factors**

The [Mortality Reduction Factors] worksheet shows cumulative mortality reduction factors. The timing definition for these factors matches that stated above for projected annual rates of mortality improvement. The factors start at 100% for the (mortality improvement rate) year which contains the effective date for the Base Rates of Mortality.

The table covers the full age attained [20 to 150] range of the Model and runs to 2130. The calculated values feed in to the output Heat Map of cumulative mortality reductions (on the [Heat Maps] worksheet) and may be exported for further use or analysis.

The choice of table for Base Rates of Mortality determines the start year for the table of ‘actual’ and projected mortality reduction factors. However, only the year element of the assumed effective timing for Base Rates of Mortality is taken into account here. Adjustments to take account of the assumed day and month elements of timing are only made when the mortality reduction factors are applied to calculate projected mortality rates (as described in the next section).

### 5.2.8. Calculation of Summary Projection Outputs

The worksheet [Sample  $q(x,t)$ ] calculates projected mortality rates for selected calendar years, for all ages, via an Excel Data Table. The results feed into Chart\_O\_05 and Chart\_O\_06.

The worksheets [EoL & Annuity Calcs (1)] and [EoL & Annuity Calcs (2)] calculate projected expectation of life and annuity values for sample model points via Excel Data Tables. The results feed into the worksheet [Sample EoL & Annuities]. Results are produced on both ‘period’ and ‘cohort’ life projection bases.

The worksheet [EoL & Survival Trends] calculates projected expectation of life values and survival rates, by calendar year, for selected ages, again via Excel Data Tables. The results feed into charts Chart\_O\_07 to Chart\_O\_12. Results are produced on both ‘period’ and ‘cohort’ life projection bases.

In calculating projected mortality rates,  $q_{x,t}$ , as building blocks for each of the outputs noted above, the Model takes account of the timing definitions set for Base Rates of Mortality, Initial Rates of Mortality Improvement and the Calculation Date.

For this Model, timing definitions are specified in terms of the start date for the one-year time interval for which the relevant  $q_{x,t}$  applies:

- Say ‘Date A’ is specified (user input or {Core} default) for Initial Rates of Mortality Improvement
- Say ‘Date B’ is specified (user input or {Core} default) for Base Rates of Mortality
- Say ‘Date C’ is specified (user input) as the Calculation Date
- The timing definition for cumulative Mortality Reduction Factors follows from ‘Date A’
  - that is, it takes the exact day and month part of ‘Date A’
- The timing definition for projected mortality rates follows from ‘Date C’
  - that is, it takes the exact day and month part of ‘Date C’.

Projected future mortality rates,  $q_{x,t}$ , are calculated as:

$$q_{x,t} = q_{x,0} \times RF^*_{x,t} \div RF^*_{x,0}$$

where

- $q_{x,0}$  is the Base Rate of Mortality for age  $x$
- $RF^*_{x,t}$  is the cumulative Mortality Reduction Factor for age  $x$  and timing consistent with the Calculation Date (Date C) rolled forwards / backwards to year  $t$
- $RF^*_{x,0}$  is the cumulative Mortality Reduction Factor for age  $x$  and timing consistent with the Base Rates of Mortality (Date B)

- The RF\* values are calculated from the table of cumulative Mortality Reduction Factors, in the [Mortality Reduction Factors] worksheet, using geometric interpolation to achieve the required timing definition.

For example using:

- the ‘Core Projection’ CMI\_2009\_M [1.00%],
- Base Mortality = 100% S1PMA (Core) for life aged x exact on 01/09/2002, and
- Calculation Date = 01/07/2009,

we have:

- ‘Date A’ = 01/01/2006
- ‘Date B’ = 01/09/2002
- ‘Date C’ = 01/07/2009.

RF\*<sub>65, 01/09/02</sub>, the Mortality Reduction Factor for age 65 and timing consistent with the Base Mortality Rate, is calculated by interpolating between RF<sub>65, 01/01/02</sub> and RF<sub>65, 01/01/03</sub>

- $RF^*_{65, 01/09/02} = 100.00\% \times (96.90\% \div 100.0\%)^{0.67} = 97.93\%$
- as 01/09/02 is 0.67 of a year (243 days) after 01/01/02, the effective date for the RF.

RF\*<sub>65, 01/07/09</sub>, the Mortality Reduction Factor for age 65 and timing consistent with the Calculation Date, is calculated by interpolating between RF<sub>65, 01/01/09</sub> and RF<sub>65, 01/01/10</sub>

- $RF^*_{65, 01/07/09} = 82.32\% \times (79.59\% \div 82.32\%)^{0.50} = 80.96\%$
- as 01/07/09 is 0.50 of a year (181 days) after 01/01/09, the effective date for the RF.

So:

$$q_{65, 01/07/09} = q_{65, 01/09/02} \times RF^*_{65, 01/07/09} \div RF^*_{65, 01/09/02}$$

$$= 0.011239 \times 80.96\% \div 97.93\% = 0.009291.$$

#### Important Notes:

*The example calculation basis adopted in CMI Working Paper 37, and the CMI Library of Mortality Projections v1.1 User Guide, makes two simplifying assumptions:*

- *First, that all the mortality reduction factor tables published in the Library may be applied to any base mortality table as if their effective timing were aligned*
  - *The reduction factors are applied as if their timing definition took the exact day and month part of ‘Date B’ (the timing definition for the base mortality rates), regardless of the ‘natural’ timing definition derived from the data or model underlying the reduction factors*
  - *If desired, users of the Model may follow these timing assumptions, rather than the Model’s {Core} defaults, by using the {Advanced} parameter facility.*
- *Second, that projections only run up to age 120*
  - *Expectation of life and annuity values are truncated after age 120, effectively assuming all remaining lives die at the end of their 120<sup>th</sup> year*
  - *The Model allows projections to run up to age 150, but, if desired, users of the Model may apply this lower limit by ensuring that  $q_{120} = 1$  in all years of the projection (using the {Advanced} parameter facility where necessary).*

### 5.2.9. Input Charts

The [Input Charts] worksheet contains a series of charts to illustrate the parameters set for the projection. Comparison is given, where relevant, against the defaults available for the {Core} level of parameters.

- Chart\_I\_01 : Initial Rates of Mortality Improvement - Aggregate
- Chart\_I\_02 : Initial Rates of Mortality Improvement - Cohort component
- Chart\_I\_03 : Initial Rates of Mortality Improvement - Age/Period component
- Chart\_I\_04 : Period of Convergence
- Chart\_I\_05 : Proportion of Convergence Remaining at Mid-point of Convergence Period
- Chart\_I\_06 : Base Mortality Rates -  $\log q_x$
- Chart\_I\_07 : Base Mortality Rates - as % of selected published tables.

A further chart, Chart\_I\_08, allows users to experiment with a model of convergence to see the effect varying the parameters has on the path of convergence.

The data underlying the charts is manipulated in the worksheet [Input Tables].

### 5.2.10. Output Charts

The [Output Charts] worksheet contains a series of charts to illustrate the results of the projection:

- Charts showing projected annual rates of mortality improvement
  - Chart\_O\_01 : by calendar year for selected ages
  - Chart\_O\_02 : by age attained for selected calendar years
  - Chart\_O\_03 : by calendar year for selected year-of-birth cohorts
  - Chart\_O\_04 : by year-of-birth cohort for selected calendar years.
- Charts showing projected mortality rates
  - Chart\_O\_05 :  $q_{x,t}$  by age attained (x) for selected calendar years (t)
  - Chart\_O\_06 :  $\text{logit}[m_{x,t}]$  by age attained (x) for selected calendar years (t).
- Charts showing projected life expectancies
  - Chart\_O\_07 : Cohort expectation of life, by calendar year for selected ages
  - Chart\_O\_08 : Period expectation of life, by calendar year for selected ages.
- Further charts showing the projected annual increase in life expectancies, by calendar year for selected ages
  - Chart\_O\_09 : For cohort expectation of life, by calendar year for selected ages
  - Chart\_O\_10 : For period expectation of life, by calendar year for selected ages.
- Charts showing projected survival probabilities, between selected ages, by calendar year
  - Chart\_O\_11 : On a cohort life projection basis
  - Chart\_O\_12 : On a period life projection basis.

The user may select data series to show on the charts using the input cells (highlighted by a **yellow background**) to the left of each chart. On a number of charts the user may also select the range for the

x-axis using the start and end parameters (also highlighted by a **yellow background**) in the titles line at the top of the worksheet.

The data underlying the charts is manipulated in the worksheet [Output Tables]. Charts Chart\_O\_05 to Chart\_O\_12 are fed by Excel Data Tables - calculation of these Data Tables must be forced by pressing the {F9} key to update the charts.

### 5.2.11. Heat Maps

The [Heat Maps] worksheet shows the patterns of projected mortality across two dimensions - age attained and calendar years:

- Chart\_H\_01 : Projected Annual Rates of Mortality Improvement (% pa)
- Chart\_H\_02 : Projected Rates of Mortality,  $q_{x,t}$ , as % of Base Rates,  $q_{x,0}$ 
  - that is, cumulative mortality reduction factors.

### 5.2.12. Sample EoL & Annuities

The [Sample EoL & Annuities] worksheet shows tables of expectation of life and annuity values calculated for sample model points (selected by the user).

Table\_01 allows the user to select a 'small' set of 6 model points. Two sets of results are produced and shown within Table\_01:

- The first is on the full projection basis specified by the user, showing values on both 'period' and 'cohort' life projection bases
- The second has certain elements of the basis fixed so as to be consistent with the basis adopted for calculation of the expectation of life and annuity values illustrated in CMI Working Paper 37 and the CMI Library of Mortality Projections v1.1 User Guide:
  - Base Mortality is set to 100% PCxA00 for life aged x exact on 01/07/2007 [not 2000]
  - The timing definition for Initial Rates of Mortality Improvement is deemed to be 01/07/yyyy
  - Calculation Date is set to 01/07/2007
  - Annuity discount rate is set to 5.0% pa.

These results tables are fed by two Excel Data Tables - calculation of the Data Tables must be forced by pressing the {F9} key to update the sample results table.

Table\_02 allows the user to select a larger 'grid' of 119 model points. Results, calculated on the full projection basis specified by the user are shown in:

- Table\_02, on a 'cohort' life projection basis; and
- Table\_03, on a 'period' life projection basis.

These results tables are fed by four further Excel Data Tables - calculation of the Data Tables must be forced by pressing the {F9} key to update the sample results table.

## **A. Appendix A: Derivation of Default Values for the Core Parameter Layer**

### **A.1 Initial Rates of Mortality Improvement**

The Model structure allows Initial Rates of Mortality Improvement to be set by individual age and year-of-birth for Age/Period and Cohort Components respectively. In addition, further years of past mortality improvement rates may be entered to bridge any gap between the Base Mortality Rates table used and the time point for the Initial Rates.

Users of the Advanced parameter layer may set the Initial Rates of Mortality Improvement at this granular level, varying by component, gender, and age or year-of-birth, to reflect whatever level and pattern of rates they view as appropriate. However, tables of default values are applied if the user selects the Core parameter layer for Initial Rates of Mortality Improvement. These default values are discussed further in the following sections.

#### **A.1.1 Initial Aggregate Rates of Mortality Improvement**

In principle, this parameter group is the one best supported by research and analysis, and is likely to be one of the parameters where the flexibility of the Model's parameter sets is used most.

Whilst Users are encouraged to undertake their own analysis, the Working Party considered that the research it has undertaken to date does give a reasonable basis on which to establish a set of default values for the Core parameter layer.

The default tables for Initial Rates of Mortality Improvement cover calendar years 1991 to 2006 and contain values for individual ages, separately for males and females. These rates represent the total rate of improvement by age, year and gender before any split into Age/Period and Cohort Components and are referred to as 'aggregate' rates in papers relating to the Model.

These rates of improvement were derived using an age-cohort P-Spline model fitted to ONS data for the population of England & Wales, for ages from 18 to 102, for the period 1961 to 2008. As well as providing the smoothed data from which to estimate 'current' rates, this approach automatically also provides rates for earlier years on a consistent basis.

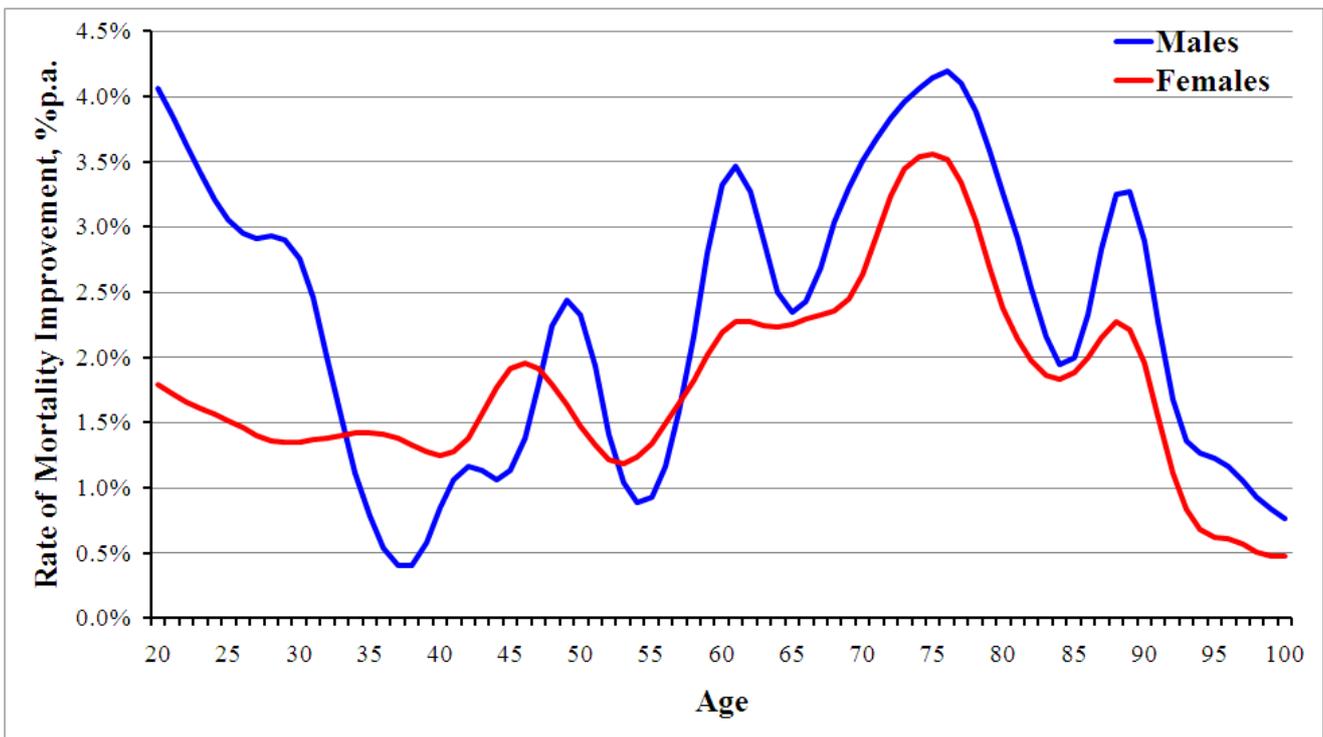
The 'current' (initial) rates of mortality improvement are taken as those for calendar year 2006, that is:  $r_{x,2006} = 1 - \{ q_{x,2006} \div q_{x,2005} \}$ . The first year of the projection is therefore assumed to be 2007. This reflects the Working Party's view that 2006, i.e. 2 years inside the edge of the available data, is the latest year for which sufficiently robust estimates of rates of mortality improvement may be made at present.

The Working Party carefully considered the merits of population, insured lives and pensioner datasets in terms of credibility and relevance of the data. Data at the level of the England & Wales population has been used for the Core parameter level, primarily because the Working Party considers the estimation errors on sub-population datasets to be too great to support robust conclusions about patterns of mortality improvement rates by age and by cohort. In addition, there is no single alternative dataset that would meet the needs of all users. Further discussion on the choice of dataset is presented in section 2.7.1 of CMI Working Paper 41.

Whilst it is clear that only the population-level data could support an assumption set with strong features by age and cohort, it was initially expected that comparison of the smaller, but arguably more relevant, insured lives and pensioner datasets against the population dataset might provide evidence for a high-level modification of the population-based table of rates of mortality improvement. However, the results of this analysis were inconclusive particularly for the most recent decade or so. Further information, and analysis of the observed mortality improvement rates, by dataset, is presented in sections 3.1 to 3.3 of CMI Working Paper 39.

An age-cohort P-Spline model with 4-year knot spacing was selected for deriving the default rates. For both males and females, this version of the P-Spline family proved to be the best model based on statistical criteria. More generally, basing its conclusions on the work carried out comparing different P-Spline models and a number of alternative methodologies for smoothing the data, the Working Party consider that this model provides a better representation of the features of the data. In addition, tests were carried out to check, for example, that the resulting projections are not overly sensitive to the choice of smoothing model or methodology.

The derived estimates of aggregate Initial Rates of Mortality Improvement for calendar year 2006 are shown in Figure A.1.



**Figure A.1: Estimated aggregate Initial Rates of Mortality Improvement; 2006**

Given the nature and definition of the underlying population data, the rates of mortality improvement derived apply from one calendar year to the next. So, the Initial Rates reflect the falls in mortality rates from  $q_{x,2005}$  to  $q_{x,2006}$  for lives aged  $x$  exact on 01/01/2005 and 01/01/2006 respectively.

The P-Spline model was fitted to data for ages 18 to 102, but the derived rates of improvement were only used for ages 20 to 100 to reduce the possible distortion of ‘edge effects’ by age.

A very simple run-off was applied above age 100: the aggregate rate of mortality improvement was extrapolated by age (within each calendar year) towards zero at the rate of 0.1% p.a. each year. For

example, if the rate at age 100 were 0.55%, the extrapolation would give 0.45% at age 101, 0.35% at age 102, and so on until 0.05% at age 105 and 0.0% for ages 106 and above.

### **A.1.2 Split of Initial Rates into Age/Period Component and Cohort Component**

The Model design incorporates splitting rates of mortality improvement into Age/Period and Cohort Components. This concept is well-supported by the Working Party's research and has some simple parallels with the 'base projection + cohort' structure of the CMI Interim Cohort Projections, and also with the approach taken by the GAD / ONS for the mortality element of the most recent National Population Projections. This section describes the method the Working Party developed to estimate the split of aggregate Initial Rates of Mortality Improvement into the two components.

Although the task is far from straightforward, the Working Party consider that estimating this split - so that the rate for an individual age might be allocated say 40% to Cohort and 60% to Age/Period Components - does lead to a simpler model in concept, as it removes the 'missing triangles' and the old-age complications which occur in models which assume, or permit, only a sudden switch from by-age to by-cohort projection - that is models in which the rate for each age is allocated either 0% or 100% to the Cohort Component.

A simple additive age-period-cohort (APC) model was created specifically for this purpose. Its inputs were the aggregate Initial Rates of Mortality Improvement, that is smoothed aggregate rates of mortality improvement taken from the age-cohort P-Spline model fitted to the ONS dataset for the population of England & Wales.

Two constraints were applied to the APC model to ensure a unique solution. These were:

- The sum over all ages of the age components must be zero; and
- The sum over the full age-year data grid of the cohort components must be zero.

Basis-splines with 4-year knot spacing (consistent with the knot-spacing in the P-Spline models applied to the 'raw' population data) were used to ensure reasonable smoothness in the derived APC components (but without imposing any particular shape).

The APC model was fitted to the smoothed aggregate rates to estimate the age, period and cohort components separately (utilising 'solver' in Excel to minimise the sum of squares of residual errors, having checked the solution produced was unique and stable).

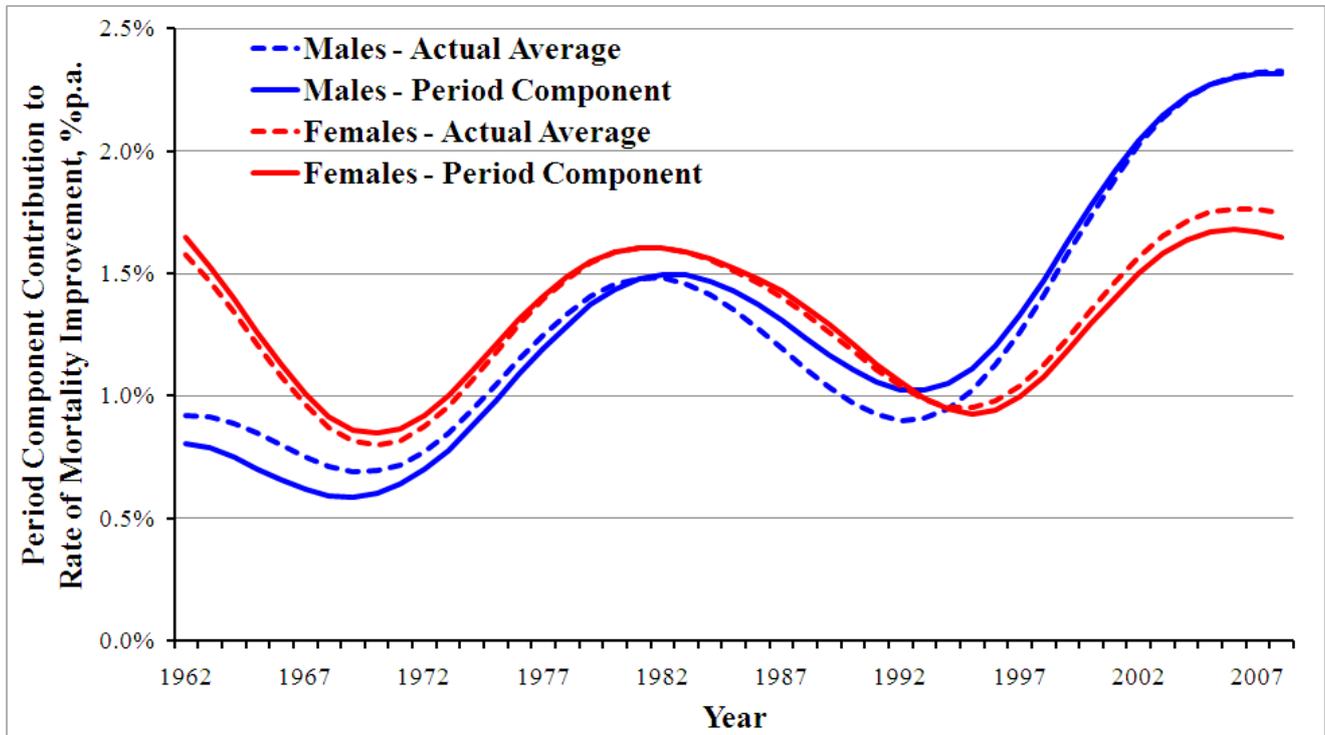
The age and period elements could be combined to give the Age/Period Component, and the cohort element mapped directly to the Cohort Component. However, it is also necessary to assess and allocate the residual error - that is the difference between the aggregate rates from the P-Spline model and the sum of the APC model elements.

Figure A.2 shows the period components derived from the APC model and compares them against the actual average (across ages 18 to 102) rate of improvement by calendar year. The close match between the APC model period component and the actual averages reflects the constraints applied.

Figures A.3 and A.4 show the age, period and cohort components from the fitted APC models, for males and females respectively. Also shown are the residual errors.

Reasonableness checks were performed on the age, period and cohort components, for example by comparison against the Working Party's previous analysis of the underlying population data. The

Working Party consider that the peaks and troughs of the derived cohort component reasonably reflect those noted in CMI Working Paper 39, and that the raised age component for age 60 to 80 reasonably reflects the feature of generally high rates of mortality improvement for those ages over the past two decades (most likely driven by reductions in mortality rates from circulatory disease).

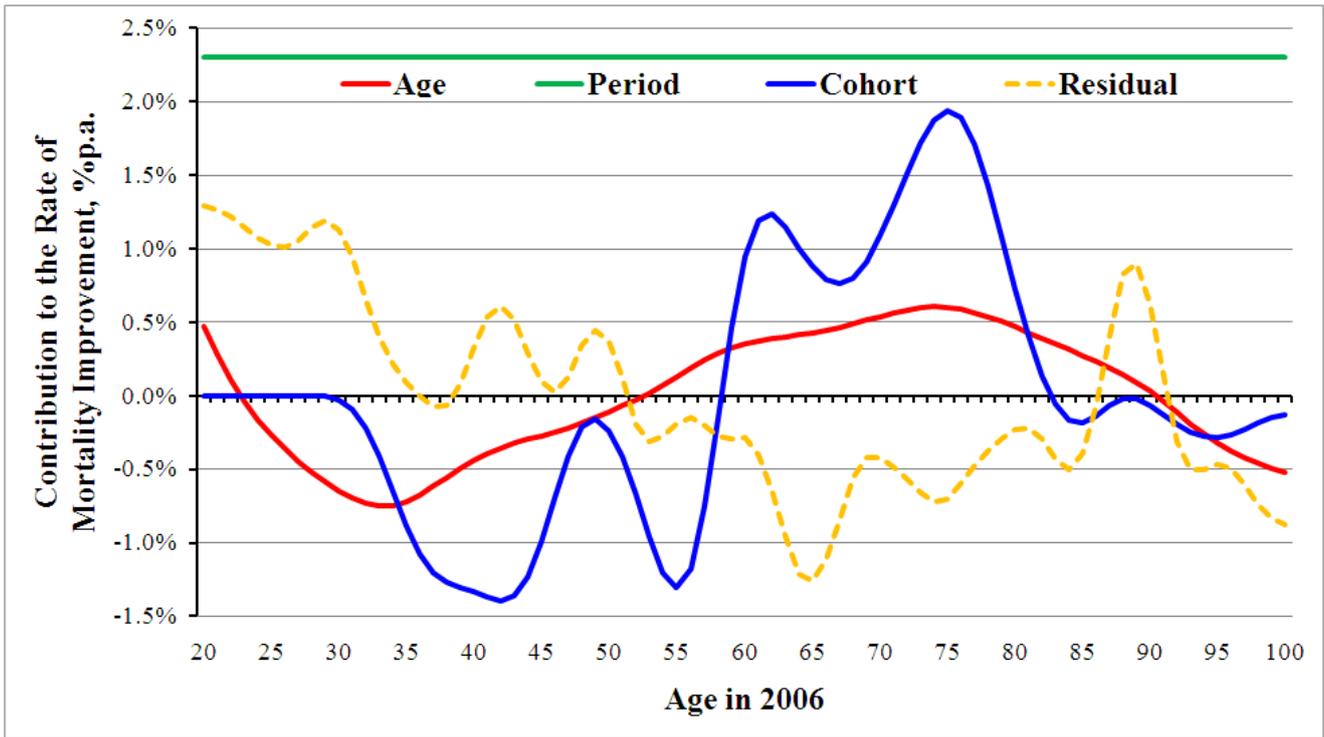


**Figure A.2: Comparison of Period Components derived in APC Model against All Age Average Rates of Improvement from Input Table**

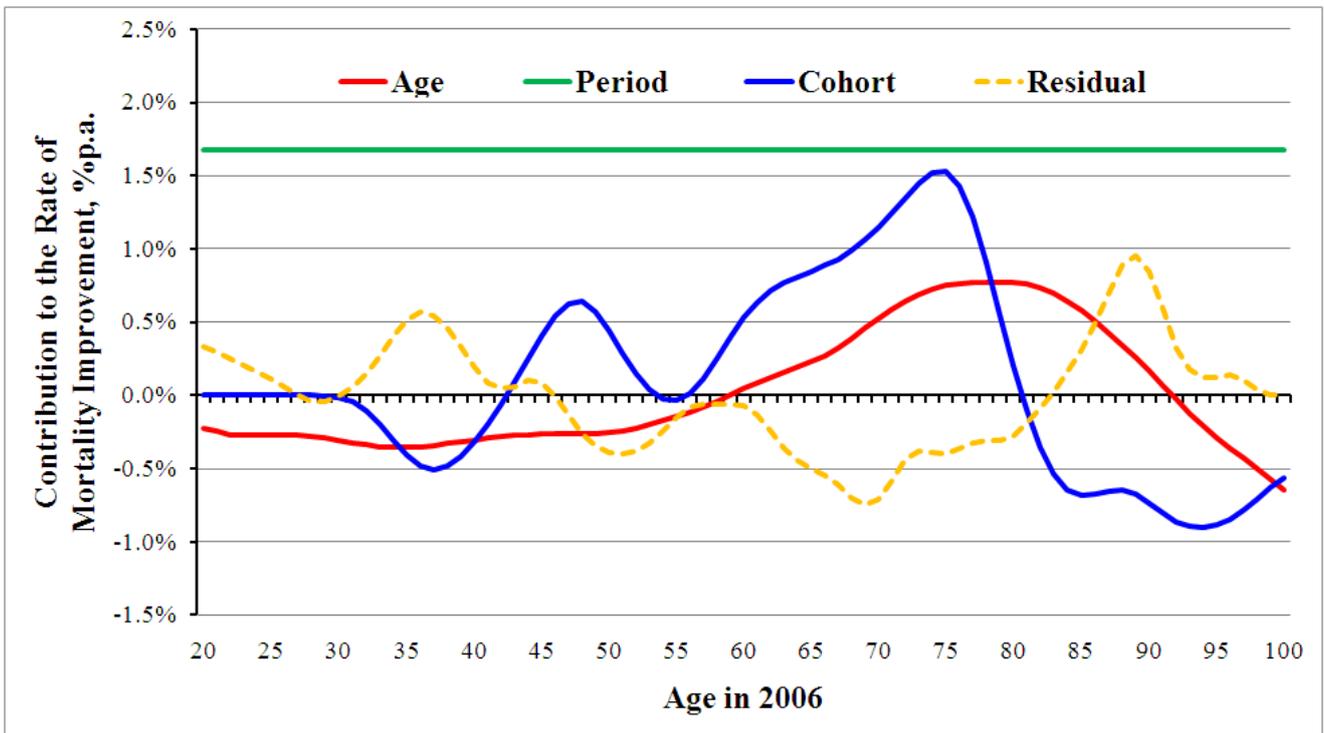
The Working Party initially considered allocating all the residual errors to the Age/Period Component on the grounds that these were likely to be short-term deviations from the underlying model and so should be run-off relatively quickly. However, this was subsequently amended at the middle and higher ages for the following reasons:

- At the higher ages, the default Period of Convergence parameters in the Model run-off the Age/Period Component more slowly than the Cohort Component
- Residual differences seen in the fit against the underlying P-Spline Model appeared as short-term cohorts, making the Age/Period allocation look somewhat inconsistent
- Allocation to the Age/Period Component produced a visible spike at certain high ages, and this was not considered to be a genuine feature of the data and so was deemed inappropriate
- It is generally accepted that for middle and higher ages, the mix of causes of deaths changes slowly with age, leading to the conclusion that this spike, and any other narrow-age-range fluctuations, should be assigned to the Cohort Component rather than the Age/Period Component for middle and higher ages.

The Working Party decided to allocate the residual errors below age 30 to the Age/Period Component, and errors above age 60 to the Cohort Component, with a linear transition in between. In addition, the fitted Cohort Component was constrained to be zero near the edges of the data and in particular up to age 30 (as there are too few years' data, and too much 'noise' in the data at young ages, to form a safe conclusion on cohort components).

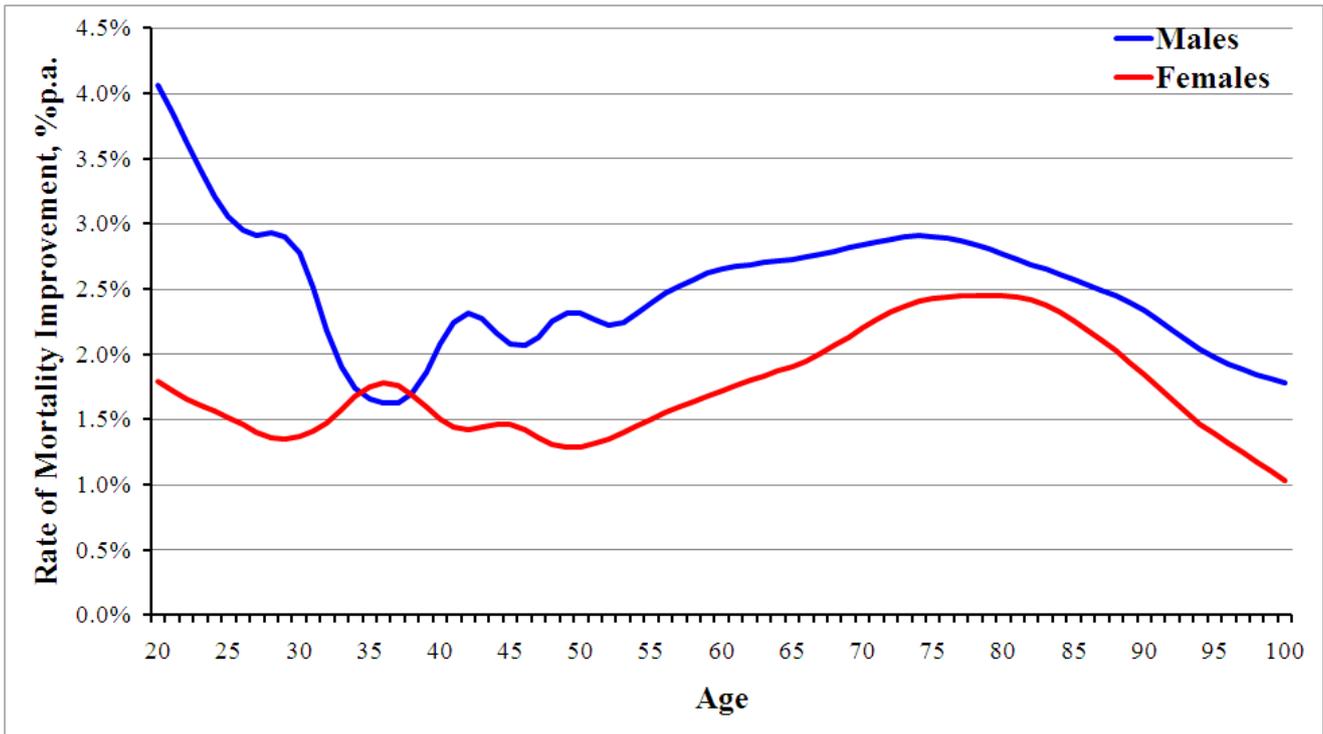


**Figure A.3: Results Derived from Fitted APC Model, Males; 2006  
Age, Period and Cohort Components, plus Residual Errors**

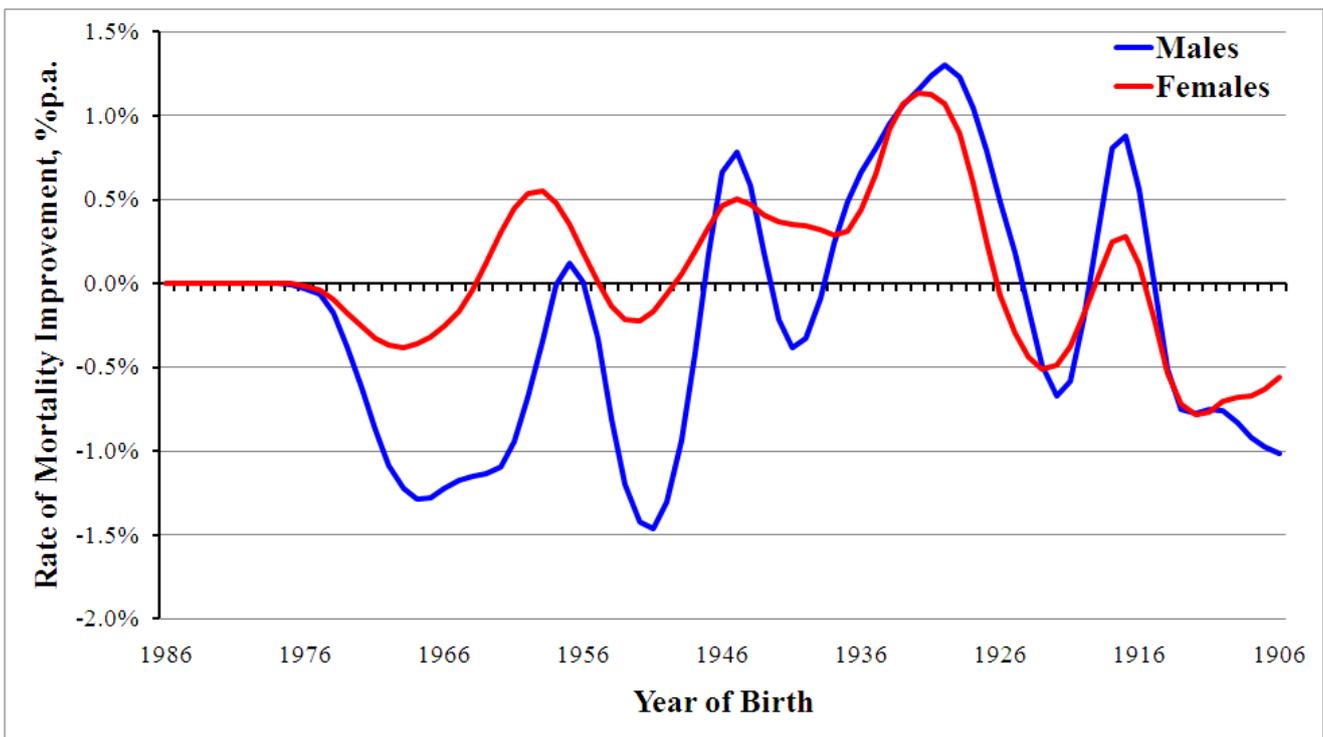


**Figure A.4: Results Derived from Fitted APC Model, Females; 2006  
Age, Period and Cohort Components, plus Residual Errors**

Figures A.5 and A.6 show the final derived Age/Period and Cohort Components of the Initial Rates of Mortality Improvement.



**Figure A.5: Estimated Age/Period Component of Initial Rates of Improvement; 2006**



**Figure A.6: Estimated Cohort Component of Initial Rates of Improvement; 2006**

The Working Party noted the negative strength of the Cohort Component for males between ages 30 and 40 (e.g. -1.3% at age 38) but satisfied itself through a review of the evidence, including heat maps, that this is a genuine cohort feature and consider it not unreasonable to project it on a cohort basis.

## **A.2 Long-Term Rate of Mortality Improvement**

The Working Party considers the Long-Term Rate assumption to be the single most important parameter for Users to set for each projection. Therefore, there is no default parameter table or overall value set for this parameter group.

The structure of the Model allows for the Long-Term Rate of Mortality Improvement to be set by individual age and year-of-birth for Age/Period and Cohort Components respectively.

Users of the Advanced parameter layer may set the Initial Rates of Mortality Improvement at this granular level, varying by component, gender, and age or year-of-birth, to reflect whatever level and pattern of rates they view as appropriate. However, if the user selects the Core parameter layer for Long-Term Rates of Mortality Improvement, a default structure is applied so that only a single input value is required. This default structure, mapping the single input value to the full parameter set, is discussed further in the following sections.

### **A.2.1 Long-Term Rate of Improvement: Pattern at High Ages**

The Core parameter layer requires only a single input value for the Long-Term Rate. The default mapping to the full parameter set makes the Long-Term Rate equal to the single input value for all ages up to and including age 90. For higher ages, the Long-Term Rate for each calendar year is assumed to reduce linearly from age 90, reaching zero at age 120, and to be zero for ages above 120.

This pattern at high ages reflects the observation that rates of mortality improvement have generally run off towards zero at high ages, but that the age-point of reaching zero has been increasing and currently appears to be in the range 100 to 110.

### **A.2.2 Long-Term Rate of Improvement: Age/Period and Cohort Components**

For the Core parameter layer, the whole of the Long-Term Rate of Mortality Improvement is assigned to the Age/Period Component, whilst the Cohort Component of the Long-Term Rate is set to zero.

The Initial Rates of Mortality Improvement are structured so that the Cohort Components average zero across all years-of-birth. The Cohort Component of the Long-Term Rate should therefore be zero if it is assumed that the influence of current year-of-birth features on patterns of improvement will dissipate completely over the Period of Convergence.

Accordingly, for the Core parameter layer, the Long-Term Rate parameter (single value input) simply sets a general level for the long-term rate of mortality improvement, with no particular features by cohort or age, other than a run-off to zero at high-ages.

### A.3 Convergence

The Model assumes that Initial Rates of Mortality Improvement converge towards Long-Term Rates of Improvement. The convergence path is controlled by two sets of parameters:

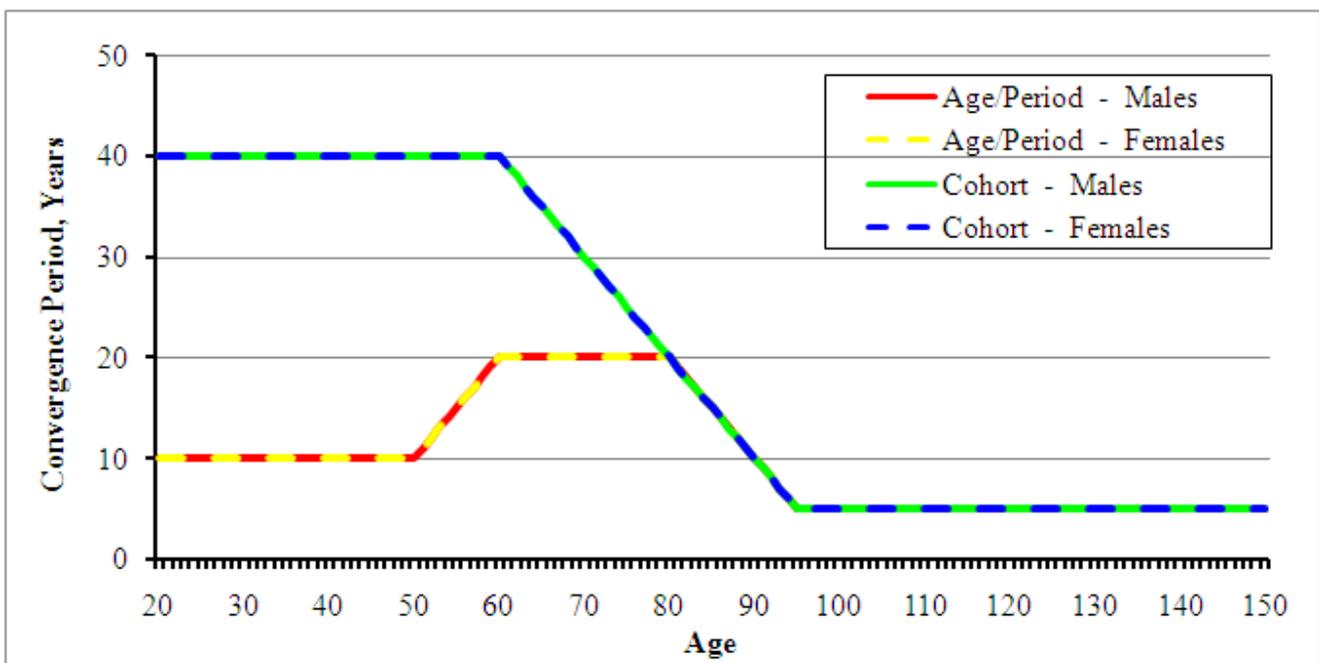
- The Period of Convergence
- The Proportion of Convergence Remaining at the Mid-Point of the Convergence Period.

The convergence process is operated separately for Age/Period and Cohort components, and both sets of parameters may be varied by age and by year-of-birth cohort respectively for the two components.

Users of the Advanced parameter layer may set the Period of Convergence, and the Proportion Remaining at Mid-Point, at this granular level to reflect whatever pattern they view as appropriate. However, tables of default convergence parameters are applied if the user selects the Core parameter layer for this group of parameters. These default values are discussed further in the following sections.

#### A.3.1 Period of Convergence

The Working Party considers that the research undertaken to date does justify some variation in the Period of Convergence, by age and by year-of-birth cohort, but only supports a relatively simple, and somewhat subjective, pattern. Having noted that the sensitivity of projection results to changes across the likely range for these parameters is relatively low, the Working Party concluded that it is reasonable to offer a set of default parameter values for Period of Convergence under the Core parameter layer. The default values are shown in Figure A.7.



**Figure A.7: Core parameter layer default values for the Period of Convergence (for the Cohort Component, horizontal axis shows age attained in 2006)**

The default Period of Convergence for the Age/Period Component is:

- fixed at 10 years for ages up to 50,
- then increasing by one year for each year of age up to 60,
- fixed at 20 years for all ages 60 to 80,
- then decreasing by one year for each year of age up to 95,
- and fixed at 5 years for all ages 95 and above.

This recognises that age/period features in mortality improvements for younger adults have tended to change from decade to decade. However, for ages 60 to 80, where a few major causes of death dominate, there is greater scope for longer-running features and there appears to be no clear end in sight yet for the high rate of mortality improvements driven by advances in relation to circulatory diseases. For the oldest ages it seems appropriate to use a short Period of Convergence, in keeping with the general concept of mortality improvement rates running to zero relatively quickly after age 100.

The default Period of Convergence for the Cohort Component is:

- fixed at 5 years for year-of-birth cohorts 1911 and earlier,
- then increasing by one year for each year-of-birth cohort up to 1946,
- and fixed at 40 years for all year-of-birth cohorts 1946 and later.

This recognises that cohort features tend to be long-running, but assumes little continuation beyond age 100. The Periods have been capped at 40 years partly due to concern over the problems of recognising cohort features at younger ages, and therefore the possible inappropriateness of projecting a weakly-based cohort feature far into the future.

The high-level arguments supporting the default parameter values are not gender specific. Accordingly, as can be seen from Figure A.7, the default Periods of Convergence are not differentiated by gender.

### **A.3.2 Proportion of Convergence Remaining at Mid-Point of the Convergence Period**

The Working Party recognised that features noted in past mortality data come in many shapes and sizes, and that there is little evidence to support any one particular pattern of convergence. Therefore, in designing the Model, the Working Party sought to offer a mechanism for convergence which would be flexible and yet relatively simple, whilst offering a smooth transition into the Long-Term Rates.

Flexibility to influence the initial trajectory of the projected mortality improvements over time, has been achieved in the Model by allowing Users to specify the proportion of the change, between the Initial and Long-Term Rates, that remains at the mid-point of the Period of Convergence.

For the Core parameter level, the Proportion of Convergence Remaining by the Mid-Point is defaulted to 50% for all ages and year-of-birth cohorts. The Working Party considered what inference to draw, for the future track of convergence, from the observation that rates of mortality improvement have increased in general across a wide range of ages over the last 15 years. However, noting that the emerging picture (see Figure A.2 on page 35) provides possible evidence of mortality improvement rates appearing to level-off, the complexity of trends by age and cohort, and the large degree of uncertainty in estimating such trends, the Working Party favoured a simple approach giving equal weight at the mid-point to the Initial and Long-Term views. Further discussion on the choice of values for this parameter is presented in sections 2.4.2 and 2.7.3 of CMI Working Paper 41.

## **B. Appendix B: Parameter Sensitivities**

### **B.1 Sensitivity of Projections to Changes in the Parameter Values - Basis of Illustration**

Sensitivity is illustrated in this User Guide by reference to a selection of annuity and expectation of life values, for male lives, calculated on the following basis:

- The values are calculated as at 31/12/2009
- Annuities are assumed to be payable yearly in advance, and allow for a discount rate of 5.0% p.a.
- The expectation of life values calculated are complete rather than curtate
- The underlying mortality projections are on a cohort, rather than period, basis and have been derived using the PCMA00 base mortality table, projected to 2006 using the past rates of mortality improvement contained in the Core parameter layer of the Model, and forward from 2006 using the rates of mortality improvement given by the various projections illustrated.

In all cases the result is expressed by showing the value calculated on the illustrated projection basis as a percentage of the corresponding value calculated using the Medium Interim Cohort Projection (from 2006).

Sensitivity of results to the Long-Term Rate of Mortality Improvement is illustrated in section B.2.1 for the full selection of annuity and expectation of life values.

Sensitivity of results to all the other parameters is illustrated in sections B.2.2 and B.3 by reference to the annuity value at age 65. For each of the charts in these sections, the different colour bars show the relative values of  $\ddot{a}_{65}$  for males calculated on the various projections tested, whilst the 4 blocks of bars show the interaction of these results with changes in the assumed Long-Term Rate of Mortality Improvement.

Additional results for the sensitivity tests presented are available in a spreadsheet which may be downloaded from the CMI pages on the UK Actuarial Profession's website. The full selection of calculated annuity and expectation of life values for male lives are presented for each of the parameter sensitivity tests described in this User Guide.

Although results are only presented in this User Guide for male lives, similar conclusions may be drawn from the parameter sensitivity results produced by running the Model for female lives. The results of a full parallel set of sensitivity tests on values for female lives (using PCFA00) are shown in the sensitivity test results spreadsheet.

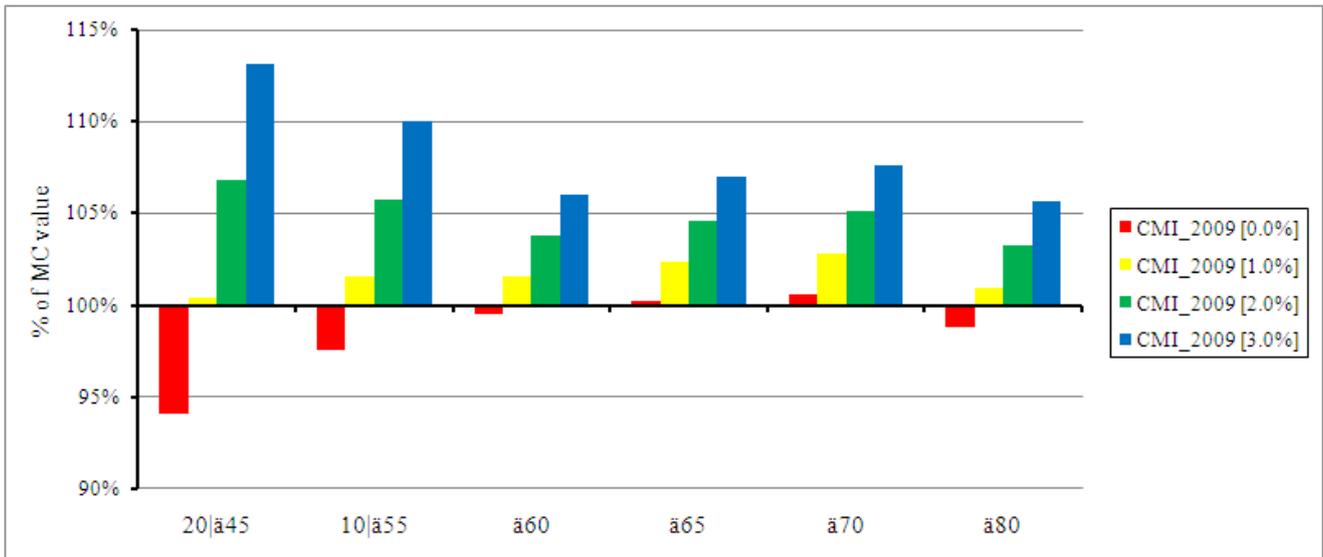
### **B.2 Sensitivity of Projections to Changes in the Core Parameters**

Core Projections are produced when the Core parameter level is selected for all inputs with the user setting values for just two simplified parameters:

- The Long-Term Rate of Mortality Improvement
- A Constant Additional Rate of Mortality Improvement.

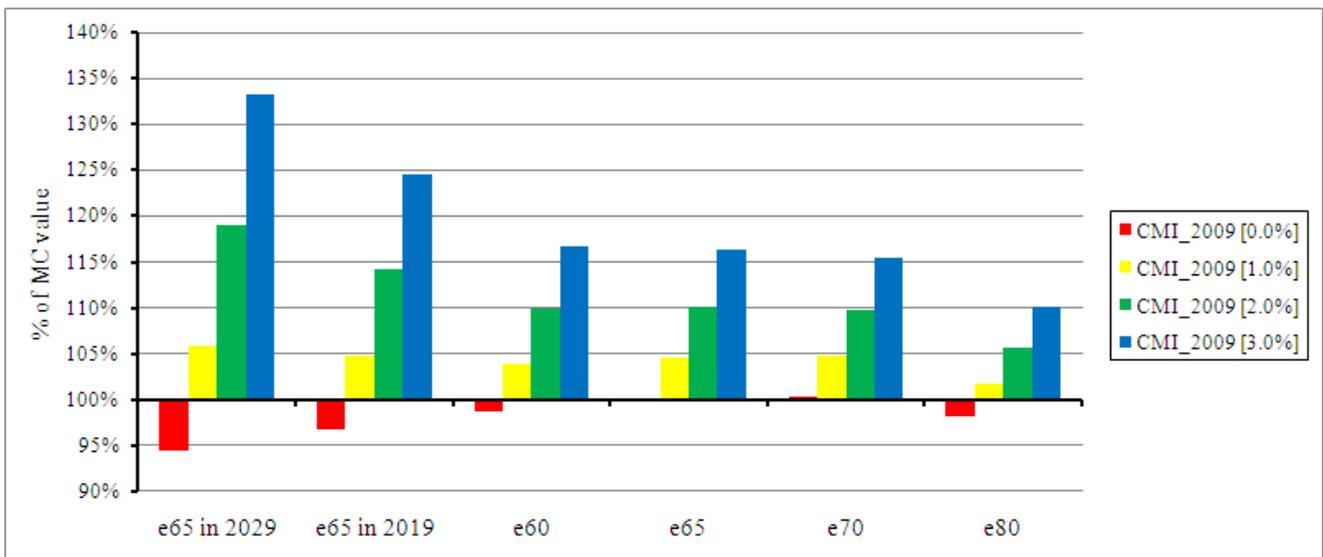
#### **B.2.1 Long-Term Rates of Mortality Improvement**

Figures B.1 and B.2 show the sensitivity of selected expectation of life and annuity values for males to the value set for the Long-Term Rate of Mortality Improvement.



**Figure B.1: Variation in selected annuity values for males, as % of the values resulting from the Medium Interim Cohort Projection, for changes in assumed Long-Term Rate of Mortality Improvement**

Figure B.1 shows that immediate annuity values, calculated using a discount rate of 5%, for males aged 60 to 80, increase by approximately 2% for each 1.0% p.a. added to the Long-Term Rate of Mortality Improvement. The equivalent increase is substantially greater for deferred annuities.



**Figure B.2: Variation in selected cohort expectation of life values for males, as % of the values resulting from the Medium Interim Cohort Projection, for changes in assumed Long-Term Rate of Mortality Improvement**

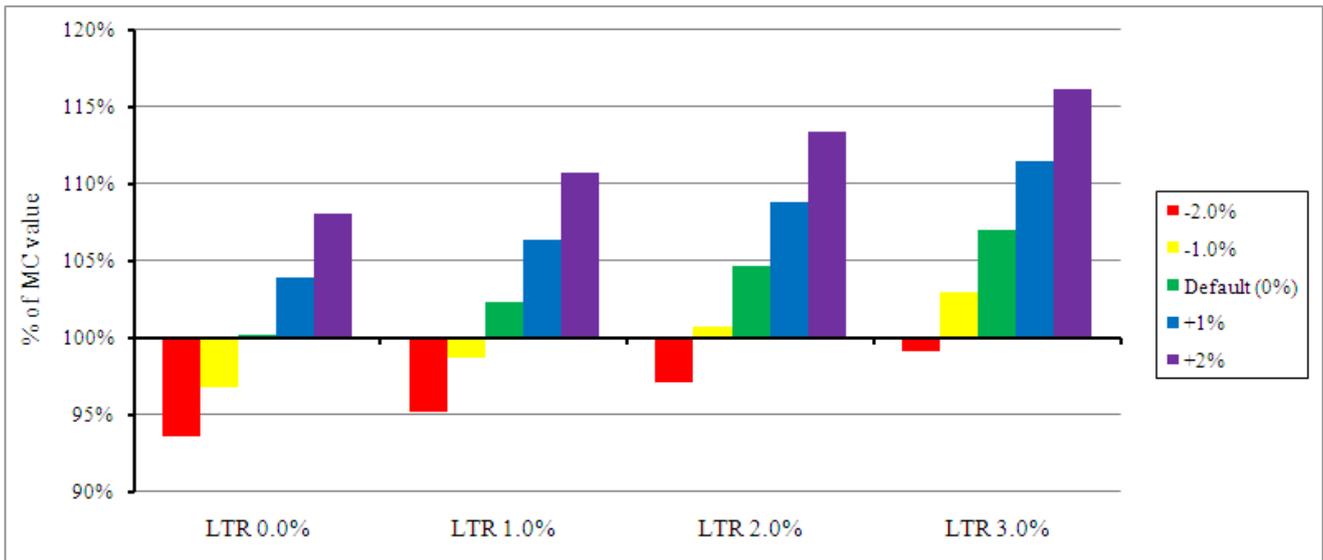
Figure B.2 shows the cohort expectation of life, for males aged 60 to 80, increases by approximately 5% for each 1.0% p.a. added to the Long-Term Rate of Mortality Improvement. Again the impact is considerably more marked for expectations-of-life calculated in respect of future years.

### B.2.2 Constant Additional Rate of Mortality Improvement

The other parameter available at the Core level is the Constant Additional Rate of Mortality Improvement. Sensitivity to this parameter is illustrated in Figure B.3 by reference to  $\ddot{a}_{65}$  for males.

The interaction of these results with changes in the assumed Long-Term Rate of Mortality Improvement is also shown.

Figure B.3 shows that the illustrated annuity values increases by approximately 4% to 5% for each 1.0% p.a. increase in the Constant Additional Rate of Mortality Improvement.



**Figure B.3: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the Constant Additional Rate of Mortality Improvement and in the assumed Long-Term Rate of Mortality Improvement**

The two parameters used for Core Projections represent the most critical inputs to the Model. This is confirmed by comparing the sensitivity of projection results to these two parameters, as shown above, with the sensitivity of results to changes to values set in the extensive Advanced parameter layer, as illustrated in the following sections.

### B.3 Sensitivity of Projections to Changes in the Advanced Parameters

Throughout this section, sensitivity to the relevant parameter(s) is illustrated by reference to  $\ddot{a}_{65}$  for males. The interaction of results with changes in the assumed Long-Term Rate is also shown.

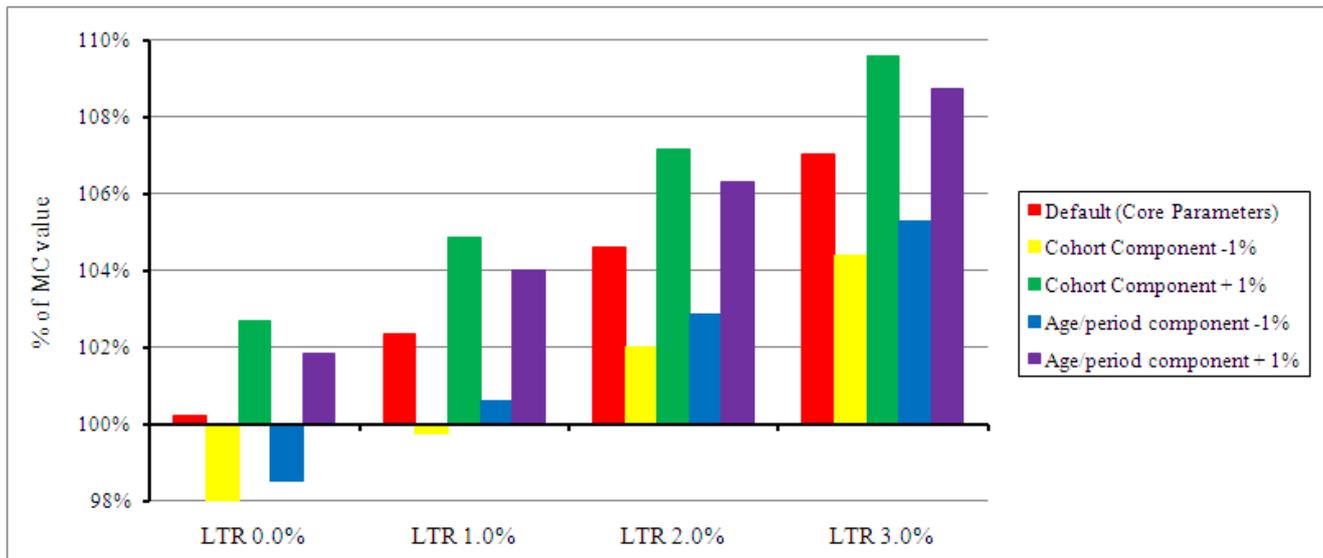
#### B.3.1 Initial Rates of Mortality Improvement

The default values, applied when the Core Parameter level is selected for this group of inputs, are a set of estimated mortality improvement rates for 2006 derived using an age-cohort P-Spline model applied to England & Wales population mortality data. The split between Age/Period and Cohort Components is estimated using a relatively simple age-period-cohort model applied to the mortality improvement rates derived from the fitted P-Spline surface - see section A.1 for further details.

Sensitivity to the overall level of each of the components of Initial Rates of Mortality Improvement is illustrated by the following scenarios:

- Add 1.0% p.a. to the Age/Period Component (only)
- Add 1.0% p.a. to the Cohort Component (only)
- Subtract 1.0% p.a. from the Age/Period Component (only)
- Subtract 1.0% p.a. from the Cohort Component (only).

In all cases, the addition / subtraction applies uniformly by age or year-of-birth cohort as appropriate.



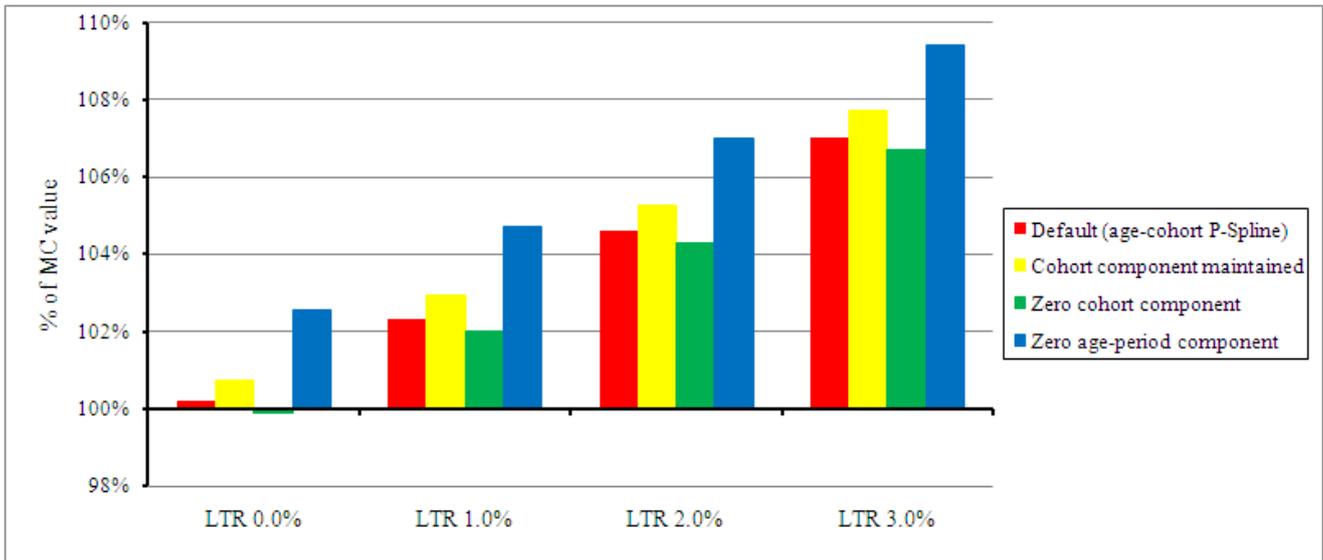
**Figure B.4: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the level of estimated Initial Rates of Mortality Improvement and in the assumed Long-Term Rate of Mortality Improvement**

Figure B.4 shows that a 1% p.a. change in Initial Rates of Mortality Improvement results in a 2% - 3% change in  $\ddot{a}_{65}$ , and that the effect is slightly larger for changes to the Cohort Component of initial rates than for changes in the Age/Period Component.

Figure B.4 also suggests that a combination of say +1% p.a. on the Cohort Component and -1% p.a. on the Age/Period Component would result in a net increase of under 0.5% in the sample annuity values. This demonstrates that the choice of constraints in the APC model - in particular that the sum over the full age-year data grid of the cohort components must be zero (see A.1.2) - has relatively low significance.

To consider further the sensitivity of results to the split of the aggregate Initial Rates of Mortality Improvement into the two component parts, the following additional sensitivity tests are explored:

- All Cohort Components set to zero - this sensitivity test is consistent with the scenario that cohort features will not have an impact on future mortality improvements and the pattern of future mortality change will be determined solely by age and period elements
- All Age/Period Components set to zero - this sensitivity test is consistent with the scenario that the pattern of future mortality change will be determined solely by year-of-birth cohort elements
- Maintaining the Cohort Component at its current level for ever, rather than the default assumption of running it to a long-term rate of zero for all year-of-birth cohorts.



**Figure B.5: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the component split for estimated Initial Rates of Mortality Improvement and in the assumed Long-Term Rate of Mortality Improvement**

Figure B.5 shows that the selected annuity value is relatively insensitive to whether the Cohort Component is assumed to run off to zero or to be maintained as a long-term rate (in the context of the other default parameter values).

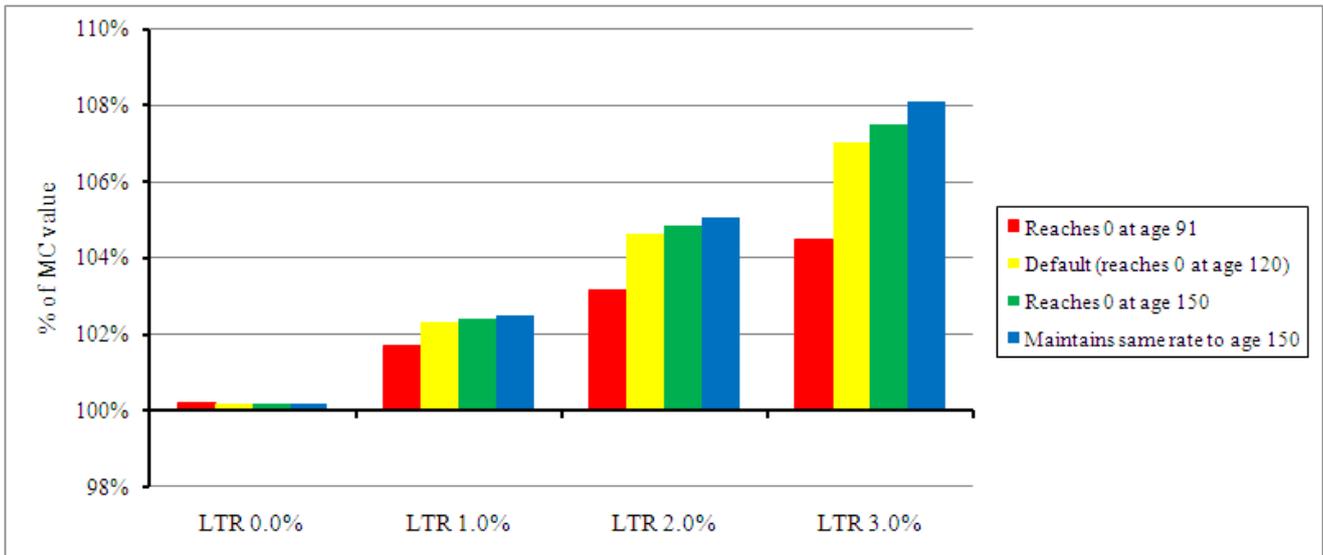
However, Figure B.5 also shows that the results are more sensitive to the split of Initial Mortality Improvement Rates between Age/Period and Cohort Components, although the scenarios tested do represent the two extremes of the parameter range. However, even across this pair of scenarios, the change in annuity values is only similar in magnitude to that generated by a 1.0% addition to the Long-Term Rate of Mortality Improvement. Note also that the results for these two particular sensitivity tests are highly dependent on the pattern of initial values by age.

### B.3.2 Long-Term Rates of Mortality Improvement at High Ages

The default mapping, applied when the Core Parameter level is selected for the Long-Term Rate, is for the user-input Long-Term Rate to apply to all ages up to 90, and then for the Long-Term Rate of Mortality Improvement to run, linearly with age, from its assumed value at age 90 to zero at age 120, and to be zero for ages above 120.

The sensitivity tests explored are as follows:

- Long-Term Rate drops from its assumed value at age 90 to zero for ages 91 and above.
- Long-Term Rate runs linearly from its assumed value at age 90 to zero at age 150.
- Long-Term Rate holds its assumed value for all ages.



**Figure B.6: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the assumed Long-Term Rate of Mortality Improvement and its pattern at high ages**

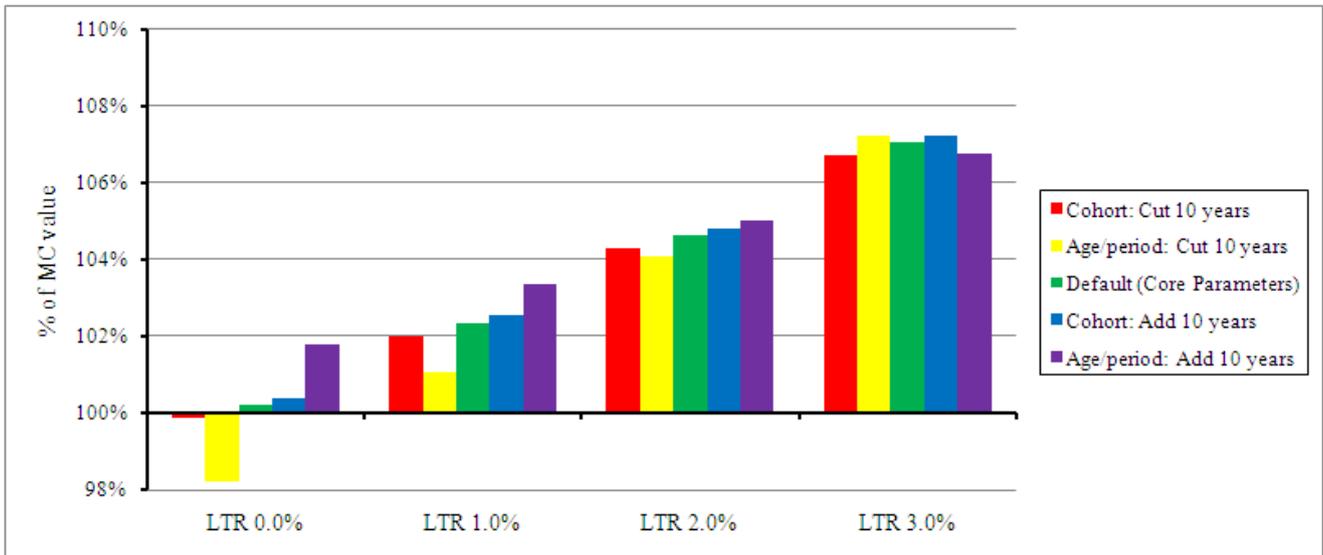
Figure B.6 shows that the results are relatively insensitive to the assumed pattern of long-term rates of mortality improvement at high ages. These tests are set in the broad context of assuming that improvement rates are likely to run to zero at some high age, and therefore the results are very dependent on the assumed value for the Long-Term Rate below age 90.

For Core Projections, the default mapping applies the total Long-Term Rate to the Age/Period Component and sets the Cohort Component to zero - that is, it assumes the influence of year-of-birth features dissipates completely over the Period of Convergence. See section B.3.1 for the results of a sensitivity test in which the Cohort Component of the Long-Term Rates is set equal to the corresponding component of the Initial Rates of Mortality Improvement - that is a scenario assuming year-of-birth cohort features continue on indefinitely.

### B.3.3 Period of Convergence from Initial to Long-Term Rates of Mortality Improvement

The default values, applied when the Core Parameter level is selected for the Convergence parameter group, vary by age and by year-of-birth cohort: see section A.3 for details.

The sensitivity tests explored cover increasing Periods of Convergence by 10 years, at all ages, and reducing them by 10 years (subject to a minimum period of 5 years). The tests are applied separately to Age/Period and Cohort Components of mortality improvement.



**Figure B.7: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the Period of Convergence and in the assumed Long-Term Rate of Mortality Improvement**

Figure B.7 shows that the results are relatively insensitive to changes in the assumed Periods of Convergence. Naturally, the sensitivity increases with the size of the gap between Initial and Long-Term Rates, as reflected by the greater variation in results in the left-hand block (Long-Term Rate = 0%) than in the right-hand block (Long-Term Rate = 3%). For the same reason the variation in the left-hand block is much more marked for the Age/Period Component (for which the Core parameter defaults for males aged 65 give 2.73% for the Initial Rate converging to the assumed Long-Term Rate) than the Cohort Component (for which the Core parameter defaults give -0.38% for the Initial Rate converging to zero, regardless of the assumed Long-Term Rate).

However, even for the Age/Period scenarios, the change in annuity values is generally of lesser magnitude than that generated by a 1.0% p.a. addition to the Long-Term Rate of Mortality Improvement until the Long-Term Rate assumption is reduced below 1% p.a..

### B.3.4 Pattern of Convergence from Initial to Long-Term Rates of Mortality Improvement

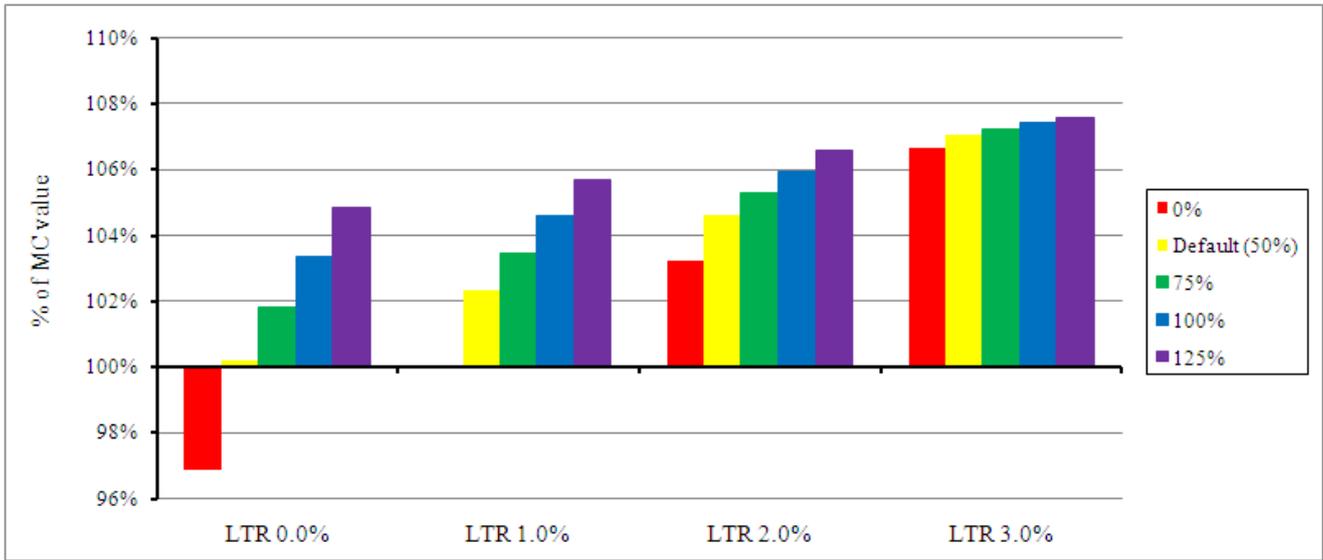
The default pattern, applied when the Core Parameter level is selected for the Convergence parameter group, is that 50% of the change from Initial to Long-Term Rates remains to be achieved after the mid-point of the Periods of Convergence.

The sensitivity tests explored are as follows:

- Proportion Remaining at mid-point is zero - that is, the mid-point value is equal to the assumed Long-Term Rate, so that the projected rate converges rapidly to reach its target by the mid-point (although then over-shooting to some degree before regaining its target)
- Proportion Remaining at mid-point is 75% - that is, the mid-point value is equal to a 3:1 weighted average of the Initial and Long-Term Rates, so that the projected rate moves away from its target initially before changing direction to converge to the Long-Term Rate (see sample trajectories in 3.5.3): for example, this could reflect a scenario where rates of improvement have been increasing and are projected to increase further in the short-term before falling back to a lower Long-Term Rate

- Proportion Remaining at mid-point is 100% - that is, the mid-point value is equal to the Initial Rate, so that rate diverges initially and then moves back to its start level at the mid-point, and then converges to the Long-Term Rate over the second half of the Period of Convergence
- Proportion Remaining at mid-point is 125% - that is, a more extreme version of the ‘100% scenario’, in which the rate diverges initially and is further away from its target at the mid-point than at the beginning.

Examples of the patterns of convergence resulting from these tests are illustrated in Figure 1 on page 15. In all cases the test parameter value is applied to both Age/Period and Cohort Components.



**Figure B.8: Variation in  $\ddot{a}_{65}$  for males, as % of the value resulting from the Medium Interim Cohort Projection, for changes in the pattern of Convergence and in the assumed Long-Term Rate of Mortality Improvement**

Comparing Figures B.7 and B.8 shows the results are somewhat more sensitive to the pattern of convergence than to changes in the Period of Convergence, although the scenarios tested may be thought of as being towards the extremes of realistic. In particular, given these results and the general trend of increasing mortality improvement rates across a wide age-range over the past 20 years, it seems appropriate for users of the Model to prioritise further consideration of using this parameter to reflect their view of the likely short-term path of rates of mortality improvement, and how this path may blend into convergence to their Long-Term rate assumption over a longer period.

## **C. Appendix C: Changes from Previous Version of the Model**

### **C.1 Changes from the Prototype Model CPMv0.0**

A relatively small number of changes have been made between the prototype Model 'CPMv0.0' and the current version 'CMI\_2009'.

#### **C.1.1 Updating the Initial Rates of Mortality Improvement**

The default parameters supplied in the Core parameter layer for Initial Rates of Mortality Improvement have been updated to reflect the publication of England & Wales population data for calendar year 2008, which became available on 17 September 2009.

The population dataset, supplied by the ONS, has been extended to include calendar year 2008 (the prototype Model used only data up to 2007). The methodology used for deriving smoothed estimates of mortality improvement rates for the prototype Model has been reapplied to the updated data; similarly the APC Model used to separate aggregate rates into age/period and cohort components has also been reapplied.

The following changes result in the Model:

- [Core Parameters] - The aggregate Initial Rates of Mortality Improvement have been updated, based on the extended ONS dataset, giving revised estimates for 1991 to 2005, plus a first estimate for 2006
- [Core Parameters] - Similarly, the Component Split of Initial Rates of Mortality Improvement has been updated (and now refers to 2006 rather than 2005)
- [Inputs] - The timing definition for the Core parameter default Initial Rates of Mortality Improvement has been changed from 01/01/2005 to 01/01/2006
- [Advanced Parameters] - The initial values (that is those in place when the Model is first accessed via the CMI pages on the UK Actuarial Profession's website) have been updated in line with the changes in the [Core Parameters] worksheet.

Further background information and analysis of the effect of adding ONS data for calendar year 2008 is given in section 3 of CMI Working Paper 41. A quantification of the resulting change in expectation of life and annuity values is given in section C.2 of this User Guide.

#### **C.1.2 Expand the Facility to Calculate Sample Expectation of Life and Annuity Values**

The outputs worksheet [Sample EoL & Annuities] has been expanded by adding the facility to set a 'grid' of model points (17 ages by 7 calendar year calculation dates) for which expectation of life and annuity values are calculated (on both 'cohort' and 'period' life projection bases) - see section 3.7.3.

This change includes:

- [Sample EoL & Annuities] - The original 6 model point sample has been retained and is now labelled 'Table\_01'
- [Sample EoL & Annuities] - 'Table\_02' and 'Table\_03' have been added to allow both the selection of the model point 'grid' and the presentation of results
- [EoL & Annuity Calcs] has been renamed [EoL & Annuity Calcs (1)] but is otherwise unchanged and performs the calculations for the original 'small' sample of model points linked to 'Table\_01'

- A new worksheet [EoL & Annuity Calcs (2)] has been added to perform the calculations for the new ‘grid’ of model points linked to ‘Table\_02’ and ‘Table\_03’.

This change extends the range of results shown but does not alter the values calculated.

### **C.1.3 Miscellaneous**

The following minor changes have also been made

- The naming convention for ‘Core Projections’ has been revised - see section 4
- The schematic diagram of the Model structure has been revised, mainly to reflect the addition of worksheet [EoL & Annuity Calcs (2)]
- A warning has been added to the [Inputs] worksheet to prompt the user to enter a value for the Long-Term Rate on first use of the Model
- Minor corrections have been made to column headings and table titles where required.

These changes do not alter either the range of results shown or the values calculated.

## **C.2 Effect of Changes from the Prototype Model CPMv0.0**

As stated in section C.1 the only Model change which leads to altered values for the Core Projections is the updating of the Initial Rates of Mortality Improvement, by taking into account the extra year of data (calendar 2008) which became available between the publication of the prototype Model and the release of this ‘CMI\_2009’ version of the Model.

The effect of the change is illustrated by comparing cohort expectation of life values, for males and females, for a large grid of model points on the following basis:

- Like-for-like Core Projections using a Long-Term Rate of 1.5% p.a.
- Base Rates of Mortality are the published S1Px tables (for lives aged x exact on 01/09/2002)
- Calculation Date(s) are 01/07/year.

Tables C1 and C3 show the sample expectation of life values produced by the ‘CMI\_2009’ version of the model, for males and females respectively; Tables C2 and C4 show the percentage change measured against the equivalent values produced by ‘CPMv0.0’, for males and females respectively.

Tables C2 and C4 show that updating the Core parameters for Initial Rates of Mortality Improvement, to reflect the addition of a further year’s observations on England & Wales population mortality experience, results in a small reduction in cohort expectation of life values. The change is of the order of -0.5%, and shows a slightly greater reduction for the values for females than for males.

**Table C1: Cohort Expectation of Life for Age x exact on 01/07/Year  
Males ; Base Rates of Mortality = 100% S1PMA as at 01/09/2002  
Core Projection: CMI\_ 2009\_M [1.5%]**

Age, x	Year						
	2009	2014	2019	2024	2029	2034	2039
20	69.773	70.529	71.269	71.994	72.705	#N/A	#N/A
25	64.112	64.874	65.622	66.356	67.076	67.781	#N/A
30	58.461	59.229	59.983	60.724	61.452	62.166	62.865
35	52.853	53.621	54.378	55.123	55.855	56.575	57.281
40	47.369	48.096	48.850	49.593	50.325	51.046	51.754
45	42.073	42.721	43.433	44.168	44.893	45.607	46.311
50	36.911	37.542	38.185	38.881	39.590	40.291	40.983
55	31.790	32.499	33.118	33.754	34.427	35.106	35.778
60	26.952	27.485	28.145	28.749	29.372	30.017	30.665
65	22.100	22.767	23.268	23.881	24.466	25.068	25.682
70	17.534	18.196	18.780	19.255	19.824	20.380	20.949
75	13.325	13.978	14.524	15.034	15.486	16.003	16.518
80	9.537	10.208	10.706	11.156	11.602	12.021	12.477
85	6.601	7.088	7.541	7.923	8.293	8.667	9.033
90	4.522	4.762	5.067	5.376	5.662	5.945	6.235
95	3.007	3.197	3.370	3.562	3.759	3.955	4.154
100	2.162	2.275	2.392	2.511	2.634	2.760	2.889

**Table C2: % Change in Cohort Expectation of Life for Age x exact on 01/07/Year  
Males ; Base Rates of Mortality = 100% S1PMA as at 01/09/2002  
Core Projections: CMI\_ 2009\_M [1.5%] against CPMv0.0 [1.5%] {males}**

Age, x	Year						
	2009	2014	2019	2024	2029	2034	2039
20	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%		
25	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%	
30	-0.2%	-0.2%	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%
35	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.1%	-0.1%
40	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.1%
45	-0.1%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%
50	-0.1%	-0.1%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%
55	-0.1%	-0.1%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%
60	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%	-0.2%
65	-0.4%	-0.3%	-0.2%	-0.2%	-0.2%	-0.3%	-0.2%
70	-0.5%	-0.5%	-0.4%	-0.3%	-0.3%	-0.3%	-0.3%
75	-0.6%	-0.6%	-0.5%	-0.4%	-0.4%	-0.4%	-0.4%
80	-0.5%	-0.6%	-0.5%	-0.5%	-0.4%	-0.4%	-0.4%
85	-0.4%	-0.5%	-0.5%	-0.4%	-0.4%	-0.4%	-0.4%
90	0.0%	0.0%	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%
95	-0.6%	-0.4%	-0.4%	-0.4%	-0.4%	-0.3%	-0.3%
100	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

**Table C3: Cohort Expectation of Life for Age x exact on 01/07/Year  
Females ; Base Rates of Mortality = 100% S1PFA as at 01/09/2002  
Core Projection: CMI\_ 2009\_F [1.5%]**

Age, x	Year						
	2009	2014	2019	2024	2029	2034	2039
20	71.424	72.158	72.876	73.576	74.259	#N/A	#N/A
25	65.757	66.503	67.233	67.945	68.641	69.320	#N/A
30	60.132	60.887	61.625	62.347	63.053	63.742	64.414
35	54.600	55.356	56.097	56.823	57.532	58.226	58.905
40	49.241	49.970	50.704	51.424	52.130	52.821	53.498
45	44.091	44.789	45.491	46.194	46.885	47.564	48.229
50	39.053	39.770	40.438	41.108	41.775	42.432	43.078
55	34.100	34.804	35.475	36.110	36.745	37.376	37.998
60	29.199	29.821	30.474	31.103	31.708	32.312	32.911
65	24.256	24.917	25.500	26.107	26.698	27.275	27.849
70	19.459	20.122	20.720	21.262	21.824	22.375	22.919
75	14.984	15.602	16.173	16.704	17.203	17.714	18.220
80	10.909	11.544	12.042	12.522	12.986	13.432	13.886
85	7.675	8.103	8.555	8.948	9.340	9.727	10.107
90	5.229	5.475	5.769	6.085	6.380	6.679	6.977
95	3.429	3.615	3.796	3.992	4.196	4.399	4.605
100	2.365	2.482	2.603	2.727	2.854	2.983	3.116

**Table C4: % Change in Cohort Expectation of Life for Age x exact on 01/07/Year  
Females ; Base Rates of Mortality = 100% S1PFA as at 01/09/2002  
Core Projections: CMI\_ 2009\_F [1.5%] against CPMv0.0 [1.5%] {females}**

Age, x	Year						
	2009	2014	2019	2024	2029	2034	2039
20	-0.3%	-0.3%	-0.3%	-0.3%	-0.2%		
25	-0.3%	-0.3%	-0.3%	-0.3%	-0.3%	-0.3%	
30	-0.4%	-0.3%	-0.3%	-0.3%	-0.3%	-0.3%	-0.3%
35	-0.4%	-0.4%	-0.4%	-0.4%	-0.3%	-0.3%	-0.3%
40	-0.4%	-0.4%	-0.4%	-0.4%	-0.4%	-0.4%	-0.4%
45	-0.4%	-0.4%	-0.5%	-0.4%	-0.4%	-0.4%	-0.4%
50	-0.5%	-0.5%	-0.5%	-0.5%	-0.5%	-0.4%	-0.4%
55	-0.5%	-0.5%	-0.5%	-0.5%	-0.5%	-0.5%	-0.5%
60	-0.5%	-0.6%	-0.5%	-0.5%	-0.5%	-0.5%	-0.5%
65	-0.6%	-0.6%	-0.6%	-0.6%	-0.6%	-0.6%	-0.6%
70	-0.7%	-0.7%	-0.7%	-0.7%	-0.7%	-0.7%	-0.6%
75	-0.8%	-0.8%	-0.8%	-0.8%	-0.8%	-0.8%	-0.7%
80	-0.8%	-0.9%	-0.9%	-0.9%	-0.9%	-0.9%	-0.8%
85	-0.9%	-0.9%	-0.9%	-1.0%	-1.0%	-0.9%	-0.9%
90	-1.1%	-1.1%	-1.1%	-1.2%	-1.2%	-1.1%	-1.1%
95	-1.6%	-1.5%	-1.5%	-1.5%	-1.5%	-1.4%	-1.4%
100	-1.4%	-1.4%	-1.4%	-1.3%	-1.3%	-1.3%	-1.3%