Continuous Mortality Investigation

User Guide

for

The CMI Mortality Projections Model

Model Name / Version: 'CMI_2010'

It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of the source of the projection.

The CMI has produced this Model in compliance with the principles in the Technical Actuarial Standard M: Modelling (April 2010) published by the Board for Actuarial Standards, regarding its construction, checking, and documentation. Documentation of the Model, in addition to that in the Model itself, is contained in this User Guide and further information is detailed in the Working Papers referenced herein.

Extensive checking has been undertaken during the development of the Model, and the CMI is confident that it correctly produces projections from the various inputs in the manner described in this User Guide. However, the Institute and Faculty of Actuaries and the CMI do not warrant the accuracy of the Model and do not accept or assume any liability for its use.

November 2010

Contents

1. Introduction

In November 2009 the CMI published a new mortality projections model, denoted 'CMI_2009'.

The Model was released in response to the continuation of significant year-on-year increases in life expectancy, and to concerns over the continued widespread use, albeit with modifications, of the Interim Cohort Projections which have inevitably become increasingly out-of-date.

The release of the CMI_2009 Model followed a consultation on the prototype Model (CPMv0.0), which was issued alongside two Working Papers: Working Paper 38 provided an overview of the Model and set out specific questions for the consultation; Working Paper 39 detailed further analysis to help inform the setting of parameter values for the Model.

Working Paper 41 summarised the responses to this consultation, outlined the changes between the prototype Model and CMI_2009, and provided a quantification of the effect of updating the default parameter values in the CMI_2009 Model to reflect the publication of England & Wales population data for calendar year 2008.

The Model has now been updated to reflect the publication of England & Wales population data for calendar year 2009. It has been denoted 'CMI_2010' and is described in this User Guide. The structure of CMI_2010 is identical to that of CMI_2009; the changes made between the CMI_2009 Model and the current version, CMI_2010, are detailed in section 6 of this User Guide.

This User Guide is intended to describe the mechanics of the updated Model. Much of the content of this User Guide is unchanged from that associated with CMI_2009.

Working Paper 49 has been published alongside CMI_2010 to illustrate the impact of incorporating data for 2009 and to discuss the issues arising from the update to the Model.

The CMI_2010 Model, Working Paper 49 and this User Guide (as well as the earlier material) may be accessed and downloaded from the CMI pages under "Research and resources", on the UK Actuarial Profession's website.

As the changes made in the latest version of the Model are limited, the CMI is not undertaking a consultation exercise on the CMI_2010 version of the Model.

Feedback on the CMI Mortality Projections Model is always welcome, though, and can be sent via e-mail to projections@cmib.org.uk or in writing to: CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW. Such comments will be considered for future reviews and updates of the Model.

2. Overview of the Structure of the Model

2.1. The purpose of the Model

The purpose of the CMI Mortality Projections Model is to allow users to produce projections of annual rates of mortality improvement, in particular in the context of UK pension and annuity portfolios. Specifically, the Model:

- reflects the latest experience on trends in mortality; and
- allows users the flexibility to modify projections tailored to their own views and purpose.

The CMI intends to update the Model regularly to reflect emerging experience.

2.2. Model Structure

The structure of the Model is based on the projection of annual rates of mortality improvement (i.e. the pace of change in mortality rates). Specifically, the Model assumes that 'current' (i.e. recently observed) rates of change blend over time into a 'long-term' rate of change specified by the user. This approach has been adopted by practitioners in a number of countries. In the UK the mortality projections that have formed part of the population projections – now produced by the Office for National Statistics (ONS) and formerly by the Government Actuary's Department (GAD) – have utilised this methodology for a number of years.

In contrast to mathematical models of mortality, fitted directly to relevant data and 'extrapolated' to form a projection, the Model requires the user to set parameter values which directly control the projection. The Model produces a single, deterministic, mortality projection for each set of user inputs.

The structure of the Model allows user input of:

- Base mortality rates, reflecting the estimated current or recent past position
- Initial rates of mortality improvement, reflecting the current estimate of rates of change
- Assumed ultimate / long-term rates of mortality improvement
- An assumed speed and pattern of convergence from 'initial' to 'long-term'.

'Initial' and 'long-term' rates of mortality improvement are each subdivided into two components: 'by age' and 'by cohort'. These components are projected separately, by age and by year-of-birth cohort respectively, and then recombined.

Convergence from 'initial' to 'long-term' rates of mortality improvement is defined (separately for 'by age' and 'by cohort' components) by user inputs for the convergence time-period and the proportion of the total change in rate remaining by the mid-point of that period.

Effectively this approach assumes that, in the very short-term, a good guide as to the likely pace of change in mortality rates is the most recently observed experience. In the long-term, the forces driving mortality change are likely to be very different from those currently influencing patterns of improvement. Therefore, the long-term rate is better informed by 'expert opinion' and analysis of long-term patterns of change and the causes driving them. Over time, the relative weight placed on the recently observed past, versus the more subjective longer-term view, can shift appropriately.

Such a model structure could be achieved through a suitably parameterised statistical model. However, at the heart of the Model design is the desire to produce a tool which is easy to understand,

intuitive in structure and capable of widespread application by users with varying degrees of expertise in this field of actuarial work.

2.3. Core and Advanced Parameter layers

The Model may be operated at different levels of complexity, reflecting the needs and resources of different users and uses.

By selecting the 'Advanced' parameter level for a group of inputs, users obtain unrestricted access with considerable flexibility to set each of the parameters as tables or vectors, varying if desired by individual age (or year-of-birth cohort where relevant), year and gender.

However, when the 'Core' parameter level is selected for a group of inputs, either a set of default parameter values is applied, or users are required to set the value for a single, simplified parameter (from which a full set of parameter values for the group is derived via a default mapping). In this way, users may choose to operate the Model at a much simpler level. The default values are intended to be reasonable assumptions but should not necessarily be interpreted as "best estimates". Users need to consider whether or not the default values included in the Model are appropriate to the purpose for which they are using the Model.

If the 'Core' parameter level is selected for all groups of inputs the user needs to set values for just two parameters, representing the most critical inputs:
- A single rate (x.xx% pa) to define long-term rates of mortality improvement
    - x.xx% for ages 20-90, falling linearly to 0% at age 120, 0% for age > 120
- A constant additional rate of mortality improvement (z.zz% pa)
    - this constant (which may take a negative value) applies to the entire projection.

At the 'Core' parameter level all other inputs are given default values or restricted choice:
- Initial rates of improvement default to rates derived from England & Wales population data
- Convergence parameters default to a pattern and values set by the CMI Working Party
- Base mortality rates are restricted to a selection of published tables.

Projections produced using the 'Core' parameter level for all groups of inputs are referred to as 'Core Projections' and are subject to a formal naming convention which is described in section 4.1.1.

2.4. Outputs

The main outputs provided by the Model for each projection are:
- A table of projected annual rates of mortality improvement by age and calendar year
- A table of projected cumulative mortality reduction factors by age and calendar year.

The Model also produces a variety of charts and tables to illustrate the projection. As well as providing information to facilitate comparison of projections, these outputs are also designed to support the user in reviewing the reasonableness of the projection. Specifically, the following outputs are generated:
- A heat map of annual rates of mortality improvement, by age attained and calendar year
- A heat map of cumulative mortality reduction factors, by age attained and calendar year
- Sample expectation of life and annuity values

- Charts showing projected rates of mortality improvement
  - by age attained and calendar year
  - by year-of-birth cohort and calendar year
- Charts showing projected mortality rates
  - $q_{x,t}$ by age (x) and calendar year (t)
  - $logit[m_{x,t}] = ln[m_{x,t} \div (1 - m_{x,t})]$, by age (x) and calendar year (t)
- Charts showing projected life expectancies, on both 'period' and 'cohort' calculation bases
  - life expectancy at selected ages, by calendar year
  - annual increase in life expectancies at selected ages, by calendar year
- Charts showing projected survival probabilities, on both 'period' and 'cohort' calculation bases
  - between selected ages, by calendar year.

2.5. Dimensions of the Model

The data and the projection range together cover 1992 to 2130 (in line with the CMI Library of Mortality Projections) and an age range of 20 to 150 (the Library range is 20 to 120).

2.6. Sensitivity Tests

This User Guide does not illustrate the sensitivity of results from the Model to the various parameters, however users can refer to:
- Appendix B of the User Guide to the CMI_2009 Model, which contains a large range of sensitivities, illustrated relative to the Medium Interim Cohort Projection.
- A spreadsheet containing the results of a wider range of sensitivity tests for the CMI_2009 Model.
- Section 4 of Working Paper 49, which illustrates the sensitivity of results to the Long-Term Rate of Mortality Improvement for CMI_2010.

3. Parameterisation of the Model

The Model parameters are listed, with some summary information, in Table I on page 10. Further description of the parameters is given in sections 3.1 to 3.7, together with notes on their use and a brief statement of the defaults provided through the 'Core' parameter level. Additional commentary on the rationale for the default parameter values is set out in Appendix A of the CMI_2009 User Guide, and notes on the derivation of updated parameter values for CMI_2010 are set out in Working Paper 49.

Working Papers 38, 39, 41 and 49 also provide further analysis to help inform the setting of parameter values for the Model - specific references are given where relevant in the following sections.

All user input cells are highlighted by a <mark>yellow background</mark> in the Model.

In this User Guide:
- references to worksheets in the Model are indicated by using square brackets: [worksheet name];
- references to user selections from dropdown lists are indicated by curly brackets: {selection}.

3.1. Gender

Although the Model is designed to store parameters for both male and female lives, each projection produced by the Model is gender specific. The user must therefore select gender - {Male} or {Female} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

The default parameter values, accessed through the 'Core' parameter level, are differentiated by gender for Base Rates of Mortality and for Initial Rates of Mortality Improvement, but not for the other parameter groups.

Complete mortality projection parameter sets for both genders may be stored in the Model simultaneously, but separate projections must be run for {Male} and {Female} in order to produce the outputs required to model, or value, a mixed gender dataset.

3.2. Base Rates of Mortality

3.2.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

3.2.2. Base Table of Mortality Rates

A base table of initial mortality rates (q_x), by age and gender, is required so that the Model can calculate projected mortality rates and summary statistics, such as life expectancies and annuity values. **The base rates of mortality do not influence the projected rates of mortality improvement generated by the Model, so their function is purely to aid the illustration of the projection by providing a base level from which future mortality rates may be calculated**.

Where {**Core**} has been selected for this group of parameters, the user may select a base table of mortality rates from the dropdown list on the [Inputs] worksheet. The options are:

- 100% PCxA00   (published by CMI; assumed timing: 01/07/2000)
- 100% S1PxA   (published by CMI; assumed timing: 01/09/2002)
- 100% Ax00 Ult   (published by CMI; assumed timing: 01/07/2000)
- 100% 'ILT07-09x[E&W]' (published by ONS; assumed timing: 01/01/2008)
  - This is unofficial shorthand for the Interim Life Tables 2007-2009 published by ONS, based on ONS population estimates and death counts for the population of England & Wales
  - Separate ILTs are published for male and female lives
  - The published ILT mortality rates are not graduated and only cover ages up to 100 - a simple extrapolation beyond age 100, by reference to the AxC00 tables, is built into the Model

[where x = 'M' or 'F' as appropriate to the gender selected.]

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of mortality rates to suit their purpose:

- Paste the mortality rates into the appropriate cells in the [Advanced Parameters] worksheet
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

3.2.3. Timing Definition for Base Rates of Mortality

For the purpose of calculating projected mortality rates, it is necessary to specify a precise time interval to which the Base Table of Mortality Rates is deemed to apply. The Model therefore requires input of the date, dd/mm/yyyy, on which the 1-year time interval starts such that q_x is the probability that a life attaining age x exact on dd/mm/yyyy dies before reaching age x+1 exact on dd/mm in year yyyy+1.

Under the {**Core**} level of parameters, the effective dates are defaulted to those specified in section 3.2.2; for mortality rate tables set by the user under the {**Advanced**} level of parameters, the user must enter the effective date in the [Inputs] worksheet.

If a user does not wish to apply the default timing definition assumed for mortality tables available under the {Core} level of parameters, the user may simply switch to the {Advanced} parameter level, enter the mortality rates for the required table, and set the timing definition as they wish.

Table I : Summary of Parameters within 'CMI_2010' Mortality Projections Model

Parameter	Dimensions & Format	Core	Advanced
Gender			
Gender for Mortality Projection	Choice List: {Male; Female}	User Selection	User Selection
Base Rates of Mortality			
Parameter Level	Choice List: {Core; Advanced}	User Selection	User Selection
Mortality Rates	Ages 20 to 150; x.xxxxxx in range [0,1]	Selection {sample tables}	User Input: Vector (Age)
Timing Definition	Single Date: dd/mm/yyyy [Year Y_0]	Default	User Input: Single Date
Initial Rates of Mortality Improvement			
Parameter Level	Choice List: {Core; Advanced}	User Selection	User Selection
Initial Rates of Improvement	Ages 20 to 150; Years Y_0 to Y_1; x.xx% pa	Default [1991 to 2007]	User Input: Table (Age; Year)
Timing Definition	Single Date: dd/mm/yyyy [Year Y_1]	Default [01/01/2007]	User Input: Single Date
Initial Age/Period Component	Ages 20 to 150; x.xx% pa [for Y_1]	Default [for 2007]	User Input: Vector (Age)
Initial Cohort Component	YoB [Y_1-20] to [Y_1-150]; x.xx% pa [Y_1]	Default [for 2007]	User Input: Vector (YoB)
Long-Term Rates of Mortality Improvement			
Parameter Level	Choice List: {Core; Advanced}	User Selection	User Selection
Long-term Age/Period Component	Ages 20 to 150; x.xx% pa	User Input: Single Value	User Input: Vector (Age)
Long-term Cohort Component	YoB [Y_1-20] to [Y_1-150]; x.xx% pa	Default [= zero]	User Input: Vector (YoB)
Convergence			
Parameter Level	Choice List: {Core; Advanced}	User Selection	User Selection
Period of Convergence:			
• Age/Period Component	Ages 20 to 150; x years	Default	User Input: Vector (Age)
• Cohort Components	YoB [Y_1-20] to [Y_1-150]; x years	Default	User Input: Vector (YoB)
% Change after mid-point of Period:			
• Age/Period Component	Ages 20 to 150; x%	Default [= 50%]	User Input: Vector (Age)
• Cohort Components	YoB [Y_1-20] to [Y_1-150]; x%	Default [= 50%]	User Input: Vector (YoB)
Constant Addition to Rates of Mortality Improvement			
Constant	Single Value: x.xx%	User Input: Single Value	User Input: Single Value

3.3. Initial Rates of Mortality Improvement

3.3.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

3.3.2. Table of Initial Rates of Mortality Improvement

The mortality improvement rate for age x and year t, $r_{x,t}$, is defined as:

$$r_{x,t} \quad = \quad 1 - \{ \ q_{x,t} \ \div \ q_{x,t-1} \ \}$$

A table of initial rates of mortality improvement, by age and gender, is required for the Foundation Year of the projection - that is, the last year for which improvement rates are known, estimated or assumed, and accordingly which forms the start-point for the projection. In addition, rates of mortality improvement for earlier years are required to cover any time interval between the effective dates for the Base Rates of Mortality and the Initial Rates of Mortality Improvement.

Where {**Core**} has been selected for this group of parameters, a default table of rates of mortality improvement is adopted. This default table contains estimated improvement rates for individual ages, separately for males and females, for calendar years 1991 to 2007, derived using an age-cohort P-Spline model fitted to ONS data for the population of England & Wales (for ages 18 to 102 and the period 1961 to 2009).

The default Initial Rates of Mortality Improvement are taken as those for calendar year 2007; the first year of the projection is therefore assumed to be 2008. This approach of 'stepping back' two years from the final year for which raw experience data is available (2009) is taken in order to derive sufficiently reliable estimates of the rates of mortality improvement as it avoids much of the distortion of 'edge effects' which are a common 'danger' with smoothing algorithms (including P-Spline models).

Further information on the derivation of the default table of rates of mortality improvement is set out in Appendix A (section A.1.1) of the CMI_2009 User Guide; updated charts reflecting the addition of data for 2009 are contained in section 2 of CMI Working Paper 49. A general commentary on the approach taken to reduce 'edge effects' is given in section 3.1.2 of CMI Working Paper 39.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of rates of mortality improvement to suit their purpose:
- Paste the improvement rates into the appropriate cells in the [Advanced Parameters] worksheet
  - Cell highlighting is used in the worksheet to indicate the years for which data input is mandatory; further data may optionally be entered for earlier years to give additional 'history' for the projection outputs (for example the 'heat map' of mortality improvements).
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

Additional research and analysis of observed rates of mortality improvement for population, insured lives and pensioner datasets is presented in sections 3.1 to 3.3 of CMI Working Paper 39.

3.3.3. Timing Definition for Rates of Mortality Improvement

For the projection, it is necessary to specify a precise time interval to which the Initial Rates of Mortality Improvement are deemed to apply. The Model therefore requires input of the date, dd/mm/'year t', on which the 1-year time interval for $q_{x,t}$ (using the notation set out in 3.3.2) starts. Note in particular that 'year t' specifies the Foundation Year for the projection of improvement rates.

Under the {**Core**} level of parameters, the effective date is defaulted to 01/01/2007 reflecting the data and methodology used to derive the rates. For mortality improvement rate tables set by the user under the {**Advanced**} level of parameters, the user must enter the effective date in the [Inputs] worksheet.

3.3.4. Split of Initial Rates into Age/Period Component and Cohort Component

The Initial Rates of Mortality Improvement must be split into two components: a component influenced by age and period (but not year-of-birth cohort), and a component which is solely influenced by year-of-birth cohort. This split is only required for the initial rates - that is for the Foundation Year - but not for any earlier years.

The component values may be either positive or negative and vary by age, or year-of-birth, and by gender. The two components sum to give the Initial Rates of Mortality Improvement at each age for the Foundation Year.

Where {**Core**} has been selected for this group of parameters, a default component split is adopted. The default component rates have been derived using an age-period-cohort model developed explicitly for this purpose. Further information on the methodology and resulting split is set out in Appendix A (section A.1.2) of the CMI_2009 User Guide; updated charts reflecting the addition of data for 2009 are contained in section 2 of CMI Working Paper 49.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of component rates to suit their purpose: paste the age/period and cohort component rates of mortality improvement into the appropriate cells in the [Advanced Parameters] worksheet. The worksheet contains a check that the two components do sum to the input aggregate Initial Rates of Mortality Improvement for the Foundation Year.

The subdivision of aggregate rates of mortality improvement into age/period and cohort components is, at least partly, subjective. Some insight may be gleaned from the study of patterns in the emerging observations of mortality improvement rates, and through study of recent changes in mortality rates by cause of death. Additional research and analysis on these topics is presented in sections 3.3, 5.1 and 5.2 of CMI Working Paper 39.

3.4. Long-Term Rates of Mortality Improvement

3.4.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

3.4.2. Long-Term Rates of Mortality Improvement

The Model requires a table of long-term rates of mortality improvement, by age and gender, split into age/period and year-of-birth cohort components.

Where {**Core**} has been selected for this group of parameters:
- The user is required to set a value for the age/period component of the Long-Term Rate
  - Enter the value in the [Inputs] worksheet
  - The input value applies uniformly for ages 20 to 90 and to both genders
  - For higher ages, the age/period component of the Long-Term Rate is assumed to reduce (linearly) from age 90, reaching zero at age 120, and to be zero for ages above 120.
- The cohort component of the Long-Term Rate is set to a default value of zero (so the influence of year-of-birth features is assumed to dissipate completely over the convergence period).

Further commentary on the rationale for this mapping of the single 'Core' level input parameter to the full parameter set is given in Appendix A (section A.2) of the CMI_2009 User Guide.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of component Long-Term Rates to suit their purpose:
- Paste the age/period and cohort component Long-Term Rates into the appropriate cells in the [Advanced Parameters] worksheet.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

Important Note: When the Model is first downloaded from the CMI pages on the UK Actuarial Profession's website, the {Core} Long-Term Rate parameter on the [Inputs] worksheet is set to "?" and no projection is produced. Similarly the vectors for the Long-Term Rate components in the [Advanced Parameters] worksheet are also set to "?". The user must therefore enter a numeric value (or vectors of values) in the appropriate cells in order to produce a projection.

The choice of a suitable long-term rate is clearly subjective but it is intended that the research presented in sections 4.1 to 4.3 of CMI Working Paper 39 may help inform the selection of suitable parameter values.

In addition, the Appendix to CMI Working Paper 38 contains a description of future scenarios (in terms of different causes of death) consistent with projections using alternative long-term rates of change. Mortality models that decompose trends into constituent causes (either cause-of-death models or disease-based approaches) may provide users with insights into appropriate future rates of change. Moreover, the views of experts in different fields are likely to be especially valuable and the CMI welcomes attempts to build links with other professions as evidenced, for example, by the 'Joining Forces on Mortality and Longevity' multidisciplinary conference held in October 2009.

3.5. Convergence

3.5.1. Parameter Level

The user must select the level for this group of parameters - {Core} or {Advanced} - from the dropdown list on the [Inputs] worksheet prior to producing a projection.

3.5.2. Period of Convergence

The Model assumes that Initial Rates of Mortality Improvement converge smoothly towards Long-Term Rates of Mortality Improvement over a specified period. The convergence process operates separately for Age/Period Components and Cohort Components of improvement rates, so that different convergence periods may be set, in whole years, for each age and each year-of-birth cohort, and also by gender.

Where {**Core**} has been selected for this group of parameters, a default table of convergence parameters is adopted:
- The default convergence period for Age/Period Components of improvement rates is:
    – fixed at 10 years for ages up to 50,
    – then increasing by one year for each year of age up to 60,
    – fixed at 20 years for all ages 60 to 80,
    – then decreasing by one year for each year of age up to 95,
    – and fixed at 5 years for all ages 95 and above.
- The default convergence period for Cohort Components of improvements is:
    – fixed at 5 years for year-of-birth cohorts 1912 and earlier,
    – then increasing by one year for each year-of-birth cohort up to 1947,
    – and fixed at 40 years for all year-of-birth cohorts 1947 and later;
    – that is the convergence periods for the Cohort Components run to age 100, subject to a minimum of 5 years and a maximum of 40 years.

The default convergence periods are not differentiated by gender.

The default parameter values for the periods of convergence for Age/Period Components are unchanged from CMI_2009. The approach taken to setting the default values for the periods of convergence for Cohort Components is also unchanged (i.e. they again run to age 100, subject to a minimum of 5 years and a maximum of 40 years); as a result, the default periods of convergence for the Cohort Components have reduced by one year compared to CMI_2009 for year-of-birth cohorts from 1912 to 1946. This is discussed in section 2.2 of Working Paper 49. Further commentary on the rationale for the original selection of the default values is set out in Appendix A (section A.3.1) of the CMI_2009 User Guide.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of convergence parameters to suit their purpose:
- Paste the Age/Period and Cohort Component Convergence Periods into the appropriate cells in the [Advanced Parameters] worksheet. Convergence Periods are limited to a maximum of 50 years.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

The choice of a suitable pattern of convergence periods is clearly subjective. Some insight may be gleaned from the study of features evident in past patterns of mortality improvement rates, and through study of mortality trends by cause of death. Additional research and analysis on these topics is presented in sections 5.1 and 5.2 of CMI Working Paper 39.

3.5.3. Proportion of Convergence Remaining at Mid-point of Convergence Period

The pattern of convergence for each Age/Period Component and each year-of-birth Cohort Component may be altered to influence the initial trajectory of the projected mortality improvement

rates over time. This is achieved by specifying the proportion of the change, between the Initial Rate and the Long-Term Rate, that remains to be covered at the mid-point of the Period of Convergence.

The convergence process operates separately for Age/Period Components and year-of-birth Cohort Components of improvement rates, so that different 'tracks' may be set for each age and each year-of-birth cohort, and also by gender, by varying the Proportion Remaining parameters.

Where {**Core**} has been selected for this group of parameters, a default table of convergence parameters is adopted:
- The default Proportion of Convergence Remaining at Mid-point of Convergence Period is set at 50% for both Age/Period and Cohort Components and for all ages and years-of-birth
- Therefore the projected rates of mortality improvement half-way through the periods of convergence will be the average of the relevant Initial and Long-Term Rates.

This default parameter value is unchanged from CMI_2009; further commentary on the rationale for the value is set out in Appendix A (section A.3.2) of the CMI_2009 User Guide.

Alternatively, by selecting the {**Advanced**} level for this group of parameters, the user may input any table of convergence parameters to suit their purpose:
- Paste the Age/Period and Cohort Component Proportions Remaining into the appropriate cells in the [Advanced Parameters] worksheet.
- Enter a short reference name for the user-input table in the [Inputs] worksheet.

A great variety of transitions may be modelled in this way. Input values are not restricted to the range 0% to 100% as other scenarios are plausible. For example, where the Initial Rate is above the Long-Term Rate, selecting a relatively high proportion (say in the range 75% to 125%) can generate projected rates of mortality improvement that initially increase before falling towards the assumed Long-Term Rates. Users therefore have the flexibility, for example, to generate scenarios in which the rate of change accelerates in the short-term, before decelerating in the longer-term.

Figure 1 illustrates how the shape of the convergence path can be altered in practice (with a 2.0% p.a. initial rate converging towards a 1.0% p.a. long-term rate in 40 years' time).
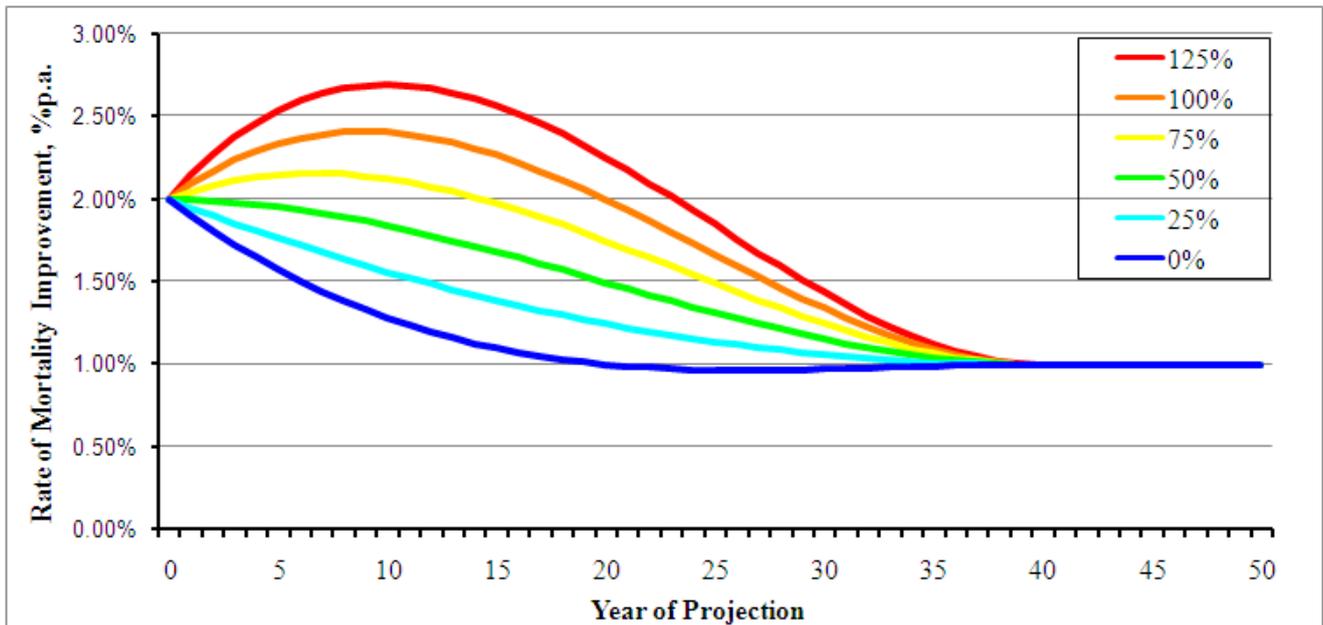
Figure 1: Illustration of the operation of the convergence algorithm over a 40-year period, with various Proportions of Convergence Remaining at Mid-Point

The research material noted in section 3.5.2 may also help inform views on likely patterns of convergence and, in turn, help in setting values for this parameter set.

3.6. Constant Additional Rate of Mortality Improvement

This parameter allows the user to incorporate a degree of prudence (or other appropriate adjustment) to a projection. The Constant Additional Rate of Mortality Improvement is applied uniformly to all ages and calendar years after the start of the projection.

The user may enter a value, if desired, in the [Inputs] worksheet. A negative value may be entered if the user wishes to reduce all projected rates of mortality improvement.

For the avoidance of doubt:
- The Constant Addition does not apply to the Foundation year, nor to earlier years
- In the first year of the projection (Foundation Year + 1), the annual rates of mortality improvement will be the Initial Rates, projected forward one year, plus the Constant Addition
- In the long-term, the annual rates of mortality improvement will be the Long-Term Rates plus the Constant Addition.

3.7. Further Parameters for Sample Expectation of Life and Annuity Values

Sample expectation of life and annuity values are calculated and presented on the [Sample EoL & Annuities] worksheet and these calculations require some additional parameters.

3.7.1. Calculation Date

The sample values apply as at the effective Calculation Date set by the user on the [Sample EoL & Annuities] worksheet.

3.7.2. Annuity Discount Rate

An interest / discount rate is required for the annuity value calculations. This may be set on the [Sample EoL & Annuities] worksheet. This single parameter is intended to reflect the net combined effect of pension / benefit escalation and allowance for future expected investment returns.

3.7.3. Model Points

The Model supports two independent sets of model points for sample expectation of life and annuity values.

A 'small' set of 6 model points may be selected via Table_01 on the [Sample EoL & Annuities] worksheet. The corresponding sample values are then also shown in Table_01. These model points are specified by setting:
- The age exact on the Calculation Date
  - that is age exact in whole years: there is no facility for part years of age
- The 'deferment' period
  - for annuities this specifies the period, in whole years, before the annuity commences payment
  - for expectation of life this specifies the delay, in whole years, before the remaining expectation of life is calculated - for example: to calculate e_x as at n years after the Calculation Date, enter age as x-n and deferment period as n years.

A larger 'grid' of model points may be selected via Table_02 on the [Sample EoL & Annuities] worksheet. The corresponding sample values are then shown in Table_02 and Table_03.

The 'grid' has 119 model points: 17 ages by 7 calendar years. For example this can be used to calculate sample expectation of life and annuity values for ages from 20 to 100 in 5-year intervals, for calculation dates in say 2010 to 2040 in 5-year intervals.

The 'grid' of model points is specified by setting:
- The 17 ages - age exact on the calculation dates
  - that is age exact in whole years: there is no facility for part years of age
- The 7 calculation dates   - specify 7 calendar years
  - The calculation dates take the day and month from the main Calculation Date (see 3.7.1)
- The minimum vesting age for the annuities
  - if the age specified for a model point is less than the minimum vesting age, annuity payment for that model point is deferred to the minimum vesting age; otherwise annuity payments are assumed to begin at the age specified for the model point.

4. Projection Naming Convention

4.1.1. Formal Naming Convention for Projections produced using 'Core' Parameters

It is recommended that 'Core Projections', that is those produced when the parameter level is set to {**Core**} for all parameter groups, should be referred to by using the following prescribed naming convention.

CMI_2010_x [a%] + c%
where:
- **CMI** refers to the CMI Mortality Projections Model
- '**_2010**' is the version 'number' of the Model and reflects the year of its publication
- '**_x**' specifies the gender and is either '**_M**' (male) or '**_F**' (female)
- **a%** is the Long-Term Rate of Mortality Improvement (value of Core parameter)
- **c%** is the Constant Additional Rate of Mortality Improvement (omitted if zero).

It is permissible to omit the gender extension where the 'Core Projections' being described cover both male and female lives, with identical Core Parameters (a% and c%) and the gender differentiated defaults applied - that is, it is sufficient to refer to 'CMI_2010 [a%]' instead of 'CMI_2010_M [a%] for males and CMI_2010_F [a%] for females'.

4.1.2. Informal Naming Convention for other Projections produced using the Model

The following informal naming convention is applied within the Model. It is hoped this will prove helpful for users, for example in their documentation, as a form of 'shorthand'.

CMI_2010_x [Advanced: IR(**tab_1**); LTR(**tab_2**); Conv(**tab_3**); Timing = **mm/dd/yyyy**] + **c%**
where:
- **CMI** refers to the CMI Mortality Projections Model
- '**_2010**' is the version 'number' of the Model and reflects the year of its publication
- '**_x**' specifies the gender and is either '**_M**' (male) or '**_F**' (female)
- '**tab_1**' is any short name adopted by the user to represent the tables used for the Initial Rates of Mortality Improvement group of parameters; '**default**' will be shown here if {**Core**} is selected for this parameter group
- '**tab_2**' is any short name adopted by the user to represent the tables used for the Long-Term Rates of Mortality Improvement group of parameters; the Core Long-Term Rate parameter (akin to **a%** in 4.1.1) will be shown here if {**Core**} is selected for this parameter group
- '**tab_3**' is any short name adopted by the user to represent the tables used for the Convergence group of parameters; '**default**' will be shown if {**Core**} is selected for this parameter group
- **Timing** shows the user input date for the timing definition for Initial Rates of Mortality Improvement
- **c%** is the Constant Additional Rate of Mortality Improvement (omitted if zero).

4.1.3. Description of Base Rates of Mortality

The naming conventions above only describe the basis for projection of mortality improvement rates; Base Rates of Mortality must be described separately. The following convention is applied within the Model and is recommended for adoption by users:

100% **TABLE (level)** for life aged x exact on **dd/mm/yyyy**
where:
- **TABLE** is a short name adopted to represent the table of mortality rates - this may be a published table name, for example 'PCMA00', or a name adopted by the user for a bespoke table
- **level** is {**Core**} or {**Advanced**} showing the parameter level selected for Base Rates of Mortality
- **dd/mm/yyyy** specifies the timing definition of the mortality rates described.

Important Note: The Model always uses 100% of the input mortality table, whether selected from the dropdown list of options available under the {Core} parameter level or directly entered as mortality rates under the {Advanced} parameter level. Therefore, to use a different percentage of a published table, the actual mortality rates required must be entered using the {Advanced} parameter level.

4.1.4. Changes to the Model's Calculations

Where a user chooses to modify the Model's calculations, the proposed naming conventions for projections should not be used without a clear statement of the changes made.

5. Mechanics of the Model

5.1. General

5.1.1. Software

The Model has been constructed as a Microsoft Office Excel 2007 Workbook but saved in Excel 97-2003 compatible file format.

It is intended and believed that the Model is fully compatible with Excel 97-2003 versions of the software although it is not practical for the CMI to undertake exhaustive testing in this regard.

5.1.2. Open Source Code

The Model is transparent in the sense that its mechanics are open for users to view (and indeed alter) if they wish.

Extensive checking has been undertaken during the development of the Model, and the CMI is confident that it correctly produces projections from the various inputs in the manner described in this User Guide. However, the Institute and Faculty of Actuaries and the CMI do not warrant the accuracy of the Model and do not accept or assume any liability for its use. Users must satisfy themselves that the parameters, calculations and outputs of the Model are suitable for whatever application they make of the Model.

The Model has been developed with a general philosophy of keeping the formulae relatively simple so that most actuarial users of Excel should be able to follow the calculations and methodology. It is acknowledged that there is some cost in this approach in terms of slower calculation speeds and larger file size than could have been obtained.

The worksheet structure of the Model is covered in section 5.2. With the intention of simplifying the presentation of the Model to users, especially those who prefer to operate it at the {Core} level of parameters, the worksheets are presented in three groups (rather than in their natural order of calculation):
- "Inputs" - the points for data / parameter entry and control of the projections
- "Outputs" - the presentation of the results of the projection
- "Mechanics" - the workings of the Model.

The "Inputs" and "Outputs" groups are the worksheets required for operation of the Model and so should be regarded as the 'Primary' worksheets. The "Mechanics" group should be regarded as 'Secondary' worksheets - users only need to study these sheets if they wish to follow through the detailed workings of the Model or view the full default parameter tables available through the {Core} parameter level.

If desired, the 'Secondary' worksheets could easily be hidden from view by using the commands:
- Excel 2007:   [Home] / [Format] / [Hide & Unhide] / [Hide / Unhide Sheet], or
- Excel 97-2003: [Format] / [Sheet] / [Hide / Unhide],
and selecting the required sheet if unhiding.

All the worksheets have the Excel worksheet protection facility enabled - apart from user input cells - in order to protect the Model from accidental change and corruption. However, this protection may easily be turned off if desired by using the commands:
- Excel 2007:   [Home] / [Format] / [Unprotect Sheet]   - no password is set or required
- Excel 97-2003: [Tools] / [Protection] / [Unprotect Sheet] - no password is set or required.

Where a user chooses to modify the Model's calculations the proposed naming conventions for projections should not be used without a clear statement of the changes made.

5.1.3. Excel Calculation Options

When the Model is run, to produce a projection and full set of outputs, the calculation time is typically of the order of 5 to 30 seconds depending on the speed of the computer processor used and the demands of any other software running simultaneously with the Model.

Excel Data Tables are used in the Model to populate some of the charts and also the sample Expectation of Life and Annuity Values. Some of these Data Tables have a heavy workload and so account for a very large proportion of the calculation time.

In the version of the Model available for download from the CMI pages on the UK Actuarial Profession's website, the Excel Calculation Option is set to "Automatic Except for Data Tables". This is intended to optimise the performance of the Model for most users and uses.

By setting the Excel Calculation Option to "Automatic Except for Data Tables":
- Most worksheets calculate automatically and quickly - in particular:
  - [Inputs] worksheet reacts quickly to choice of {Core} or {Advanced}
  - The output tables of Projected Annual Rates of Mortality Improvement and Projected Cumulative Mortality Reduction Factors update automatically and quickly
  - The Heat Maps, Input Charts and some of the Output Charts update automatically and quickly.
- However, those Charts and Tables fed by Excel Data Tables will not update until the user forces the Data Table calculations to be updated
  - Press the {F9} key to calculate the Excel Data Tables and wait for the outputs to update
  - Warning messages are in place for the Charts and Tables which need to be updated in this way.

Alternative Excel Calculations Options are:
- Automatic   -   All calculations and Data Tables will automatically update after any change to parameter values (or formulae). This removes the risk that Output Charts are out-of-line with the parameter values set, but will slow down the basic operation of the Model.
- Manual   -   This option is NOT recommended! It allows parameters to be set without any calculation-time delays, but the user must then force the calculations to run by pressing the {F9} key. *Important Note: The [Inputs] worksheet will not automatically update, for example for a change from {Core} to {Advanced} parameter level if this option is selected.*

The Excel Calculation Option may easily be changed if desired by using the commands:
- Excel 2007:   [Formulas] / [Calculations Options] and then selecting the desired option
- Excel 97-2003: [Tools] / [Options] / [Calculation] and then selecting the desired option.

5.1.4. Constraints & Reasonableness of Model Outputs

No constraints have been placed on parameter values other than basic input validation on the range of possible values - for example, Base Rates of Mortality must lie in the range [0,1].

Only a few constraints have been placed on the projection, notably:
- Projected mortality rates are restricted to the range [0,1]
- Projections only run up to age 150 - expectation of life and annuity values are truncated after age 150, effectively assuming all remaining lives die at the end of their 150th year
- Projections only run up to calendar year 2130 - expectation of life and annuity values are shown as "#N/A" where mortality rates would need to be calculated beyond 2130.

Users should therefore be aware that the Model may generate projections with unusual properties, or even errors, particularly if extreme parameter values are entered.

Accordingly, users are strongly advised to satisfy themselves of the reasonableness of any projections produced. As well as providing information to facilitate comparison of projections, the various Model outputs are also intended to support the user in reviewing the reasonableness of the projection.

It remains the responsibility of any actuary or other person using a projection of future mortality to ensure that it is appropriate for the particular purpose to which it is put, regardless of the source of the projection.

5.1.5. Saving Parameter Sets

The Model only holds a single set of Core parameters (defaults) and a single set of Advanced parameters (user input) for each gender. Therefore if, having completed a projection, the user wishes to save the parameter set for future reference, this is most easily achieved by saving a copy of the entire Model. Alternatively copies of the relevant parameter tables, or entire worksheets, could be made and stored as a 'library' of parameters, ready to be pasted back in to the Model for use when required.

5.2. Worksheet Structure of the Model

The Model contains a series of worksheets. These are listed, with some summary information, in Table II on page 24. A schematic of the Model's worksheet structure, outlining the major data and calculation flows between the worksheets is shown in Figure 2 on page 25. Further commentary on the worksheets is given in the following sections.

In the Model - for example, one of the worksheet names - and User Guide, the acronym 'CPM' is still used to refer generically to the CMI Mortality Projections Model. This acronym has been deliberately retained, rather than replaced by the current Model version name 'CMI_2010', in order to avoid the potential need for further updates to such references for each future version of the Model.

5.2.1. Model Inputs

The [Inputs] worksheet is the 'Home' sheet for users of the Model.

Using the input cells on this sheet, users:
- Select the level - {Core} or {Advanced} - for each of the four main parameter groups
    - Where {**Core**} is selected, additional parameter inputs on this sheet fully specify the projection
    - Where {**Advanced**} is selected, the user is required to input dates to specify the timing definition of the user-input tables of parameters, and is invited to input short names by which to refer to the tables
- Select gender for the projection - this determines whether the calculations in the Model work from the parameters stored for male lives or those for female lives
- Set any Constant Additional Rate of Mortality Improvement.

A shorthand description of the projection - using the naming conventions set out in section 4 and driven by the user's parameter choices - is also shown on this worksheet.

5.2.2. Core Parameters

The [Core Parameters] worksheet contains the full tables of default values which are applied when the {Core} level of parameters is selected. There are separate tables for each of the four main parameter groups, each covering the Model's full range of individual ages (or year-of-birth cohorts) and gender.

The colour coding applied to separately identify the parameters for males and females is followed through, depending on the user's selections, to the [Parameters Used by CPM] worksheet.

Table II : Summary of Worksheets within 'CMI_2010' Mortality Projections Model			
Worksheet Name	**Description**	**Type**	**Rank**
Inputs	User 'Home' page - primary control of the projection	User Input	Primary
Control	*Background spreadsheet mechanics*	*Mechanics*	*Secondary*
Core Parameters	*Full tables of default values for Core level of parameters*	*Information & Mechanics*	*Secondary*
Advanced Parameters	Full tables of user input values for Advanced level of parameters	User Input (when selected)	Primary
Parameters Used by CPM	*Full tables of final user-selection of parameters for the projection*	*Information & Mechanics*	*Secondary*
Convergence Fn by Age	*Detail calculation of convergence pattern for age/period component*	*Mechanics*	*Secondary*
Proj by Age	*Detail calculation of projected imp rates for age/period component*	*Mechanics*	*Secondary*
Convergence Fn by Cohort	*Detail calculation of convergence pattern for cohort component*	*Mechanics*	*Secondary*
Proj by Cohort	*Detail calculation of projected imp rates for cohort component*	*Mechanics*	*Secondary*
Proj Mort Imps	Full table of aggregate projected annual mortality improvement rates	Mechanics & Output	Primary
Mortality Reduction Factors	Full table of projected cumulative mortality reduction factors	Mechanics & Output	Primary
Sample q(x,t)	*Detail calculation of sample projected mortality rates*	*Mechanics*	*Secondary*
EoL & Annuity Calcs (1)	*Detail calculation of sample projected exp of life & annuity values*	*Mechanics*	*Secondary*
EoL & Annuity Calcs (2)	*Detail calculation of sample projected exp of life & annuity values*	*Mechanics*	*Secondary*
EoL & Survival Trends	*Detail calculation of projected exp of life and survival rates, by year*	*Mechanics*	*Secondary*
Input Tables	*Background spreadsheet mechanics for Input Charts*	*Mechanics*	*Secondary*
Input Charts	Charts to illustrate parameters selected	Output	Primary
Output Tables	*Background spreadsheet mechanics for Output Charts*	*Mechanics*	*Secondary*
Output Charts	Charts to illustrate projection results	Output	Primary
Heat Maps	Heat Maps to illustrate projection results	Output	Primary
Sample EoL & Annuities	Tables of expectation of life and annuity values for sample ages	Output	Primary

The worksheets [Inputs >>], [Outputs >>] and [Mechanics >>] are used purely as signposts and play no active part in the Model.
The worksheet [Notices] provides general information and plays no active part in the Model.

Figure 2: Schematic of Model Structure: Outline of major Data and Calculation flows between Worksheets
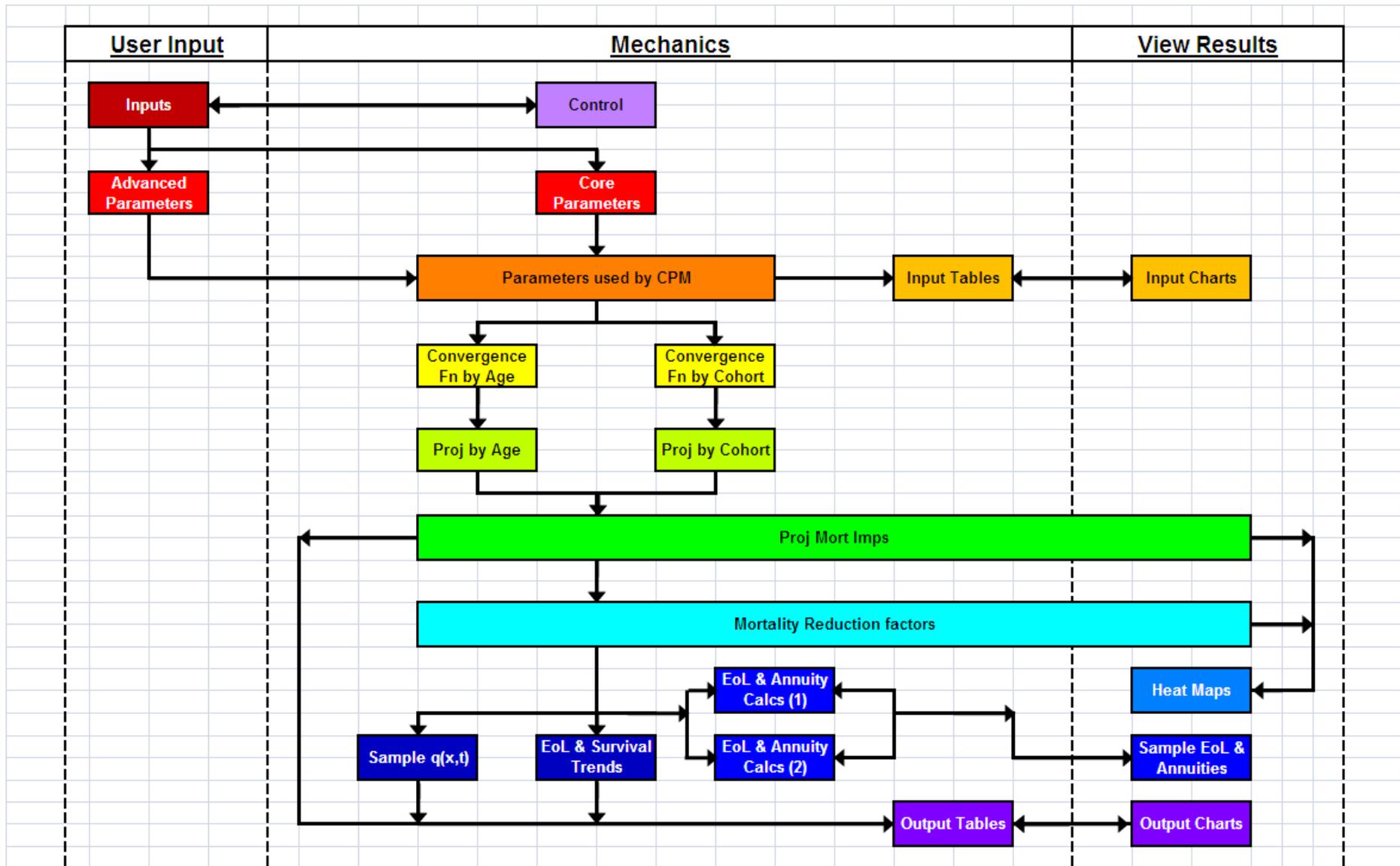
5.2.3. Advanced Parameters

The [Advanced Parameters] worksheet contains the cells into which user-input tables may be pasted when the {Advanced} level of parameters is selected. There are separate tables for each of the four main parameter groups, each covering the Model's full range of individual ages (or year-of-birth cohorts) and gender - that is, the layout is identical in most respects to that for the [Core Parameters] worksheet.

Cell highlighting - <mark>bright yellow</mark> for mandatory, <mark>pale yellow</mark> for optional, and driven by user selections for parameter levels - is used to indicate the areas of the [Advanced Parameters] worksheet open for user-input.

The colour coding applied to separately identify the parameters for males and females is followed through, depending on the user's selections, to the [Parameters Used by CPM] worksheet.

5.2.4. Parameters Used by CPM

The [Parameters Used by CPM] worksheet simply holds the final selected set of parameter tables which drive the projection. This sheet only contains parameters for the gender the user has chosen to model.

5.2.5. Calculation of Projection over Period of Convergence

The algorithm for the convergence process operates by calculating a weighted average of the Initial and Long-Term Rates of Mortality Improvement, with the weights varying over the Period of Convergence.

The worksheets [Convergence Fn by Age] and [Convergence Fn by Cohort] take the convergence group of parameters and create tables of weighting factors reflecting the selected pattern and duration of convergence, by age and by year-of-birth cohort for the respective components of mortality improvement rates.

The proportion of the weight to be placed on the Initial Rate is determined, independently for each individual age and year-of-birth, by fitting cubic polynomials in time, f(t), such that:
- $f(0) = 1$ where $t = 0$ at the start (Foundation Year) of the projection;
- $f(T) = 0$ where T is the length of the Period of Convergence (in whole years);
- $f(\frac{1}{2}T) = P$, the Proportion Remaining at the Mid-Point;
- The first derivative of f(t) at T is zero, that is $f'(T) = 0$.

Let $\tau = t \div T$, so that $\tau = 1$ at the end of the Period of Convergence.

and $f(\tau) = a\,\tau^3 + b\,\tau^2 + c\,\tau + d$

The four criteria above lead to four simultaneous equations:
- $d = 1$
- $a + b + c + d = 0$
- $\frac{1}{8}a + \frac{1}{4}b + \frac{1}{2}c + d = P$
- $3a + 2b + c = 0$.

These may be solved to give:
- a = 8P - 2
- b = 5 - 16P
- c = 8P - 4
- d = 1.

This gives full weight to the Initial Rates of Mortality Improvement at the start of the projection, and to the Long-Term Rates at the end of the Period(s) of Convergence. The convergence path is smooth, passing through the selected level at the mid-point of the convergence period and blending smoothly into the Long-Term Rate. However, the Model has no regard for the trend in rates of mortality improvement prior to the start of the projection, so although the transition from historic to projected rates is continuous, it may well not be smooth.

The worksheets [Proj by Age] and [Proj by Cohort] then apply these weights to the Initial and Long-Term Rates (for each age or year-of-birth cohort) to derive full tables of projected annual rates of mortality improvement for each component.

Note that, as convergence periods are limited to a maximum of 50 years, these 4 sheets only operate for the first 50 years of the projection.

5.2.6. Proj Mort Imps

The [Proj Mort Imps] worksheet combines the age/period and year-of-birth cohort components to produce a full projection of aggregate annual rates of mortality improvement.

The table covers the full age attained [20 to 150] and calendar year [1992 to 2130] range of the Model. The calculated values feed in to the output Heat Map of mortality improvement rates (on the [Heat Maps] worksheet) and may be exported for further use or analysis.

The timing definition for these improvement rates follows directly from that specified for the Initial Rates of Mortality Improvement. That is, if the effective date for Initial Rates is dd/mm/yyyy, then the projected rates all relate to movements in mortality rates ($q_{x,t}$) for exact ages on dd/mm of each year. For Core Projections produced using the 'CMI_2010' Model, the effective date for Initial Rates is 01/01/2007, so that the projected rates for Core Projections all relate to movements in mortality rates ($q_{x,t}$) for exact ages on 1st January of each year.

The choice of table for Base Rates of Mortality plays no part in these calculations. Accordingly the table of 'actual' and projected annual rates of mortality improvement is invariant to the assumed effective timing for Base Rates of Mortality.

5.2.7. Mortality Reduction Factors

The [Mortality Reduction Factors] worksheet shows cumulative mortality reduction factors. The timing definition for these factors matches that stated above for projected annual rates of mortality improvement. The factors start at 100% for the (mortality improvement rate) year which contains the effective date for the Base Rates of Mortality.

The table covers the full age attained [20 to 150] range of the Model and runs to 2130. The calculated values feed in to the output Heat Map of cumulative mortality reductions (on the [Heat Maps] worksheet) and may be exported for further use or analysis.

The choice of table for Base Rates of Mortality determines the start year for the table of 'actual' and projected mortality reduction factors. However, only the year element of the assumed effective timing for Base Rates of Mortality is taken into account here. Adjustments to take account of the assumed day and month elements of timing are only made when the mortality reduction factors are applied to calculate projected mortality rates (as described in the next section).

5.2.8. Calculation of Summary Projection Outputs

The worksheet [Sample q(x,t)] calculates projected mortality rates for selected calendar years, for all ages, via an Excel Data Table. The results feed into Chart_O_05 and Chart_O_06.

The worksheets [EoL & Annuity Calcs (1)] and [EoL & Annuity Calcs (2)] calculate projected expectation of life and annuity values for sample model points via Excel Data Tables. The results feed into the worksheet [Sample EoL & Annuities]. Results are produced on both 'period' and 'cohort' life projection bases.

The worksheet [EoL & Survival Trends] calculates projected expectation of life values and survival rates, by calendar year, for selected ages, again via Excel Data Tables. The results feed into charts Chart_O_07 to Chart_O_12. Results are produced on both 'period' and 'cohort' life projection bases.

In calculating projected mortality rates, $q_{x,t}$, as building blocks for each of the outputs noted above, the Model takes account of the timing definitions set for Base Rates of Mortality, Initial Rates of Mortality Improvement and the Calculation Date.

For this Model, timing definitions are specified in terms of the start date for the one-year time interval for which the relevant $q_{x,t}$ applies:
- Say 'Date A' is specified (user input or {Core} default) for Initial Rates of Mortality Improvement
- Say 'Date B' is specified (user input or {Core} default) for Base Rates of Mortality
- Say 'Date C' is specified (user input) as the Calculation Date
- The timing definition for cumulative Mortality Reduction Factors follows from 'Date A'
  - that is, it takes the exact day and month part of 'Date A'
- The timing definition for projected mortality rates follows from 'Date C'
  - that is, it takes the exact day and month part of 'Date C'.

Projected future mortality rates, $q_{x,t}$, are calculated as:

$$q_{x,t} = q_{x,0} \ x \ RF^*_{x,t} \div RF^*_{x,0}$$

where
- $q_{x,0}$ is the Base Rate of Mortality for age x
- $RF^*_{x,t}$ is the cumulative Mortality Reduction Factor for age x and timing consistent with the Calculation Date (Date C) rolled forwards / backwards to year t
- $RF^*_{x,0}$ is the cumulative Mortality Reduction Factor for age x and timing consistent with the Base Rates of Mortality (Date B)

- The RF* values are calculated from the table of cumulative Mortality Reduction Factors, in the [Mortality Reduction Factors] worksheet, using geometric interpolation to achieve the required timing definition.

For example using:
- the 'Core Projection' CMI_2010_M [1.00%],
- Base Mortality = 100% S1PMA (Core) for life aged x exact on 01/09/2002, and
- Calculation Date = 01/07/2010,

we have:
- 'Date A' = 01/01/2007
- 'Date B' = 01/09/2002
- 'Date C' = 01/07/2010.

$RF^*_{65,\,01/09/02}$, the Mortality Reduction Factor for age 65 and timing consistent with the Base Mortality Rate, is calculated by interpolating between $RF_{65,\,01/01/02}$ and $RF_{65,\,01/01/03}$
- $RF^*_{65,\,01/09/02}$ = $100.00\% \times (96.97\% \div 100.0\%)^{0.67}$ = 97.97%
- as 01/09/02 is 0.67 of a year (243 days) after 01/01/02, the effective date for the RF.

$RF^*_{65,\,01/07/2010}$, the Mortality Reduction Factor for age 65 and timing consistent with the Calculation Date, is calculated by interpolating between $RF_{65,\,01/01/10}$ and $RF_{65,\,01/01/11}$
- $RF^*_{65,\,01/07/10}$ = $78.46\% \times (75.76\% \div 78.46\%)^{0.50}$ = 77.11%
- as 01/07/10 is 0.50 of a year (181 days) after 01/01/10, the effective date for the RF.

So:

$q_{65,\,01/07/10}$ = $q_{65,\,01/09/02} \times RF^*_{65,\,01/07/10} \div RF^*_{65,\,01/09/02}$

= $0.011239 \times 77.11\% \div 97.97\%$ = 0.008846.

Important Notes:

The example calculation basis adopted in CMI Working Paper 37, and the CMI Library of Mortality Projections v1.1 User Guide, makes two simplifying assumptions:
- *First, that all the mortality reduction factor tables published in the Library may be applied to any base mortality table as if their effective timing were aligned*
  - *The reduction factors are applied as if their timing definition took the exact day and month part of 'Date B' (the timing definition for the base mortality rates), regardless of the 'natural' timing definition derived from the data or model underlying the reduction factors*
  - *If desired, users of the Model may follow these timing assumptions, rather than the Model's {Core} defaults, by using the {Advanced} parameter facility.*
- *Second, that projections only run up to age 120*
  - *Expectation of life and annuity values are truncated after age 120, effectively assuming all remaining lives die at the end of their 120th year*
  - *The Model allows projections to run up to age 150, but, if desired, users of the Model may apply this lower limit by ensuring that $q_{120} = 1$ in all years of the projection (using the {Advanced} parameter facility where necessary).*

The differences between the Model and the Library are discussed in section 6 of Working Paper 49.

5.2.9. Input Charts

The [Input Charts] worksheet contains a series of charts to illustrate the parameters set for the projection. Comparison is given, where relevant, against the defaults available for the {Core} level of parameters.

- Chart_I_01 : Initial Rates of Mortality Improvement - Aggregate
- Chart_I_02 : Initial Rates of Mortality Improvement - Cohort component
- Chart_I_03 : Initial Rates of Mortality Improvement - Age/Period component
- Chart_I_04 : Period of Convergence
- Chart_I_05 : Proportion of Convergence Remaining at Mid-point of Convergence Period
- Chart_I_06 : Base Mortality Rates - log q_x
- Chart_I_07 : Base Mortality Rates - as % of selected published tables.

A further chart, Chart_I_08, allows users to experiment with a model of convergence to see the effect varying the parameters has on the path of convergence.

The data underlying the charts is manipulated in the worksheet [Input Tables].

5.2.10. Output Charts

The [Output Charts] worksheet contains a series of charts to illustrate the results of the projection:

- Charts showing projected annual rates of mortality improvement
    - Chart_O_01 : by calendar year for selected ages
    - Chart_O_02 : by age attained for selected calendar years
    - Chart_O_03 : by calendar year for selected year-of-birth cohorts
    - Chart_O_04 : by year-of-birth cohort for selected calendar years.

- Charts showing projected mortality rates
    - Chart_O_05 : $q_{x,t}$ by age attained (x) for selected calendar years (t)
    - Chart_O_06 : logit[$m_{x,t}$] by age attained (x) for selected calendar years (t).

- Charts showing projected life expectancies
    - Chart_O_07 : Cohort expectation of life, by calendar year for selected ages
    - Chart_O_08 : Period expectation of life, by calendar year for selected ages.

- Further charts showing the projected annual increase in life expectancies, by calendar year for selected ages
    - Chart_O_09 : For cohort expectation of life, by calendar year for selected ages
    - Chart_O_10 : For period expectation of life, by calendar year for selected ages.

- Charts showing projected survival probabilities, between selected ages, by calendar year
    - Chart_O_11 : On a cohort life projection basis
    - Chart_O_12 : On a period life projection basis.

The user may select data series to show on the charts using the input cells (highlighted by a yellow background) to the left of each chart. On a number of charts the user may also select the range for the

x-axis using the start and end parameters (also highlighted by a <mark>yellow background</mark>) in the titles line at the top of the worksheet.

The data underlying the charts is manipulated in the worksheet [Output Tables]. Charts Chart_O_05 to Chart_O_12 are fed by Excel Data Tables - calculation of these Data Tables must be forced by pressing the {F9} key to update the charts.

5.2.11. Heat Maps

The [Heat Maps] worksheet shows the patterns of projected mortality across two dimensions - age attained and calendar years:
- Chart_H_01 : Projected Annual Rates of Mortality Improvement (% pa)
- Chart_H_02 : Projected Rates of Mortality, $q_{x,t}$, as % of Base Rates, $q_{x,0}$
  - that is, cumulative mortality reduction factors.

5.2.12. Sample EoL & Annuities

The [Sample EoL & Annuities] worksheet shows tables of expectation of life and annuity values calculated for sample model points (selected by the user).

Table_01 allows the user to select a 'small' set of 6 model points. Two sets of results are produced and shown within Table_01:
- The first is on the full projection basis specified by the user, showing values on both 'period' and 'cohort' life projection bases
- The second has certain elements of the basis fixed so as to be consistent with the basis adopted for calculation of the expectation of life and annuity values illustrated in CMI Working Paper 37 and the CMI Library of Mortality Projections v1.1 User Guide:
  - Base Mortality is set to 100% PCxA00 for life aged x exact on 01/07/2007 [not 2000]
  - The timing definition for Initial Rates of Mortality Improvement is deemed to be 01/07/yyyy
  - Calculation Date is set to 01/07/2007
  - Annuity discount rate is set to 5.0% pa.

These results tables are fed by two Excel Data Tables - calculation of the Data Tables must be forced by pressing the {F9} key to update the sample results table.

Table_02 allows the user to select a larger 'grid' of 119 model points. Results, calculated on the full projection basis specified by the user are shown in:
- Table_02, on a 'cohort' life projection basis; and
- Table_03, on a 'period' life projection basis.

These results tables are fed by four further Excel Data Tables - calculation of the Data Tables must be forced by pressing the {F9} key to update the sample results table.

6. Changes from the CMI_2009 Model

A relatively small number of changes have been made between the CMI_2009 Model and the current version, CMI_2010. These are detailed below.

The principal change in CMI_2010 has been to update the default parameter values supplied in the Core parameter layer for Initial Rates of Mortality Improvement to reflect the publication of England & Wales population data for calendar year 2009, supplied by the ONS, which became available on 30 September 2009 (the CMI_2009 Model used data up to 2008). A quantification of the effect of updating the default parameter values in the Model is presented in section 3 of Working Paper 49.

The methodology used for deriving smoothed estimates of mortality improvement rates for the CMI_2009 Model has been reapplied to the updated data; similarly the APC Model used to separate aggregate rates into Age/Period and Cohort Components has also been reapplied.

The following changes result in the Model:

- [Core Parameters] -   The aggregate Initial Rates of Mortality Improvement have been updated, based on the extended ONS dataset, giving revised estimates for 1991 to 2006, plus a first estimate for 2007
- [Core Parameters] - Similarly, the Component Split of Initial Rates of Mortality Improvement has been updated (and now refers to 2007 rather than 2006)
- [Inputs] - The timing definition for the Core parameter default Initial Rates of Mortality Improvement has been changed from 01/01/2006 to 01/01/2007
- [Advanced Parameters] - The initial values (that is those in place when the Model is first accessed via the CMI pages on the UK Actuarial Profession's website) have been updated in line with the changes in the [Core Parameters] worksheet.

The following minor changes have also been made:

- The naming convention for "Core Projections" has been updated to refer to CMI_2010
- The Interim Life Table option on the Core parameters layer for the Base Table of Mortality Rates has been updated from the 2005-2007 table to the 2007-2009 table
- The default Calculation Date on the [Sample EoL & Annuities] worksheet has been changed to 31/12/2010
- A correction has been made on the [EoL & Annuity Calcs (2)] worksheet to restore the link for Table_02 on the [Sample EoL & Annuities] worksheet between the user-input sample ages and the output values for cohort expectation of life and annuity values. In CMI_2009 the values in Table_02 related to default quinquennial ages from 20 to 100 and did not respond to user input changes to those defaults.

References

CMI Working Paper 38 "A Prototype Mortality Projections Model: Part One – An Outline of the Proposed Approach". (2009)

CMI Working Paper 39 "A Prototype Mortality Projections Model: Part Two – Detailed Analysis". (2009)

CMI Working Paper 41 "CMI Mortality Projections Model: Feedback on Consultation and Issue of 'CMI_2009' ". (2009)

CMI Working Paper 49 "The CMI Mortality Projections Model, CMI_2010". (2010)

User Guide for the CMI Mortality Projections Model - Model Name / Version: 'CMI_2009' (2009)

All of the above may be accessed and downloaded from the CMI pages, under "Research and resources", on the UK Actuarial Profession's website; in particular:
- CMI_2010 and its User Guide are located alongside Working Paper 49, and
- CMI_2009, its User Guide and the spreadsheet of parameter sensitivity tests are located alongside Working Paper 41.

Board for Actuarial Standards: "Technical Actuarial Standard M: Modelling (April 2010). This document can be found at: http://www.frc.org.uk/bas/standards/tas.cfm.