

Continuous Mortality Investigation
Working Paper 49
The CMI Mortality Projections Model, CMI_2010

November 2010

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Executive Summary

In November 2009 the CMI published a new mortality projections model, denoted 'CMI_2009'. The Model has now been updated and CMI_2010 has been published alongside this paper together with its User Guide.

The structure of the CMI_2010 Model is identical to that of the CMI_2009 version; however, the default parameters contained in the Model have been updated to reflect the publication of England & Wales population mortality data for calendar year 2009.

This Working Paper illustrates the impact of incorporating data for 2009 and discusses the issues arising from the update to the Model.

The default Initial Rates of Mortality Improvement have, in general, increased from those published in CMI_2009. Accordingly Core Projections generated by the CMI_2010 Model produce expectations of life which are generally slightly higher than those produced by CMI_2009 (with other parameters held constant).

The smoothed P-Spline estimates of the annual rates of mortality improvement using the datasets to 2009 and to 2008 are compared in the paper. It remains difficult to draw conclusions regarding the current trajectory of rates of mortality improvement. The period components of these rates using data to calendar year 2008 appeared to show a slowing down however this trend has been partially reversed by the 2009 data for males, and wholly negated for females.

Sample annuity values and expectations of life are included in the paper to illustrate the sensitivity of CMI_2010 to the key parameter, the Long-Term Rate of Mortality Improvement. The relative significance of the sensitivity of CMI_2010 to other parameters is unchanged from CMI_2009, but such tests are not illustrated in this paper.

Overall, the CMI believes that the CMI_2010 Model has shown a smooth evolution from the CMI_2009 Model. The paper illustrates that the degree of change between the two versions is small compared to the sensitivity to the Long-Term Rate of Mortality Improvement.

The CMI has become aware of two potential issues that can arise with the CMI P-Spline model which is used to smooth the data to produce the default Initial Rates of Mortality Improvement in the Model. The first is that the software sometimes locates a local minimum rather than the global minimum; the second is that the ONS datasets show considerable overdispersion that can result in the P-Spline model under-smoothing the data. These two issues are discussed in this paper although the Committee is confident that neither materially impacts on CMI_2010.

The final section of the paper clarifies the differences between projections from the Model and those in the CMI Library of Mortality Projections.

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1. Introduction

1.1. Background

In November 2009 the CMI published a new mortality projections model, denoted ‘CMI_2009’. The Model was released in response to the continuation of significant year-on-year increases in life expectancy, and to concerns over the continued widespread use, albeit with modifications, of the Interim Cohort Projections which have inevitably become increasingly out-of-date.

In producing the Mortality Projections Model, the CMI sought to develop a model which:

- reflects the latest experience on trends in mortality;
- is relatively straightforward to understand and describe;
- allows users the flexibility to modify projections tailored to their own views and purpose; and
- can be regularly updated over time to reflect emerging experience.

The release of the CMI_2009 Model followed a consultation on the prototype Model, which was issued alongside two Working Papers: Working Paper 38 provided an overview of the Model and set out specific questions for the consultation; Working Paper 39 detailed further analysis to help inform the setting of parameter values for the Model.

Working Paper 41 summarised the responses to this consultation and also provided details of the updated default parameters in the CMI_2009 Model to reflect the publication of England & Wales population data for calendar year 2008.

The Model has now been updated and CMI_2010 has been published alongside this paper together with its User Guide.

Oversight of the Model within the CMI is now combined with that of the Library of Mortality Projections by a small group, comprising: Gordon Sharp (Chair), Kevin Armstrong (CMI Life Office Mortality Committee) and Brian Wilson (CMI SAPS Mortality Committee), with Dave Grimshaw as Secretary.

1.2. Changes from CMI_2009

The structure of the CMI_2010 Model is identical to that of the CMI_2009 version.

The principal purpose of this annual update of the Model is to incorporate 2009 population mortality data into the default Core parameters for the Initial Rates of Mortality Improvement. Other changes have been kept to a minimum. The Committee will undertake a wider review when this is felt necessary, in particular to take account of how the Model is being used in practice. It envisages that a general review will be carried out every five years at the latest; this will include areas such as the end-year of the projections which has been retained as 2130 in CMI_2010, but will clearly need to be updated in due course.

In addition to updating the default parameters for 2009 population mortality data, the following minor changes have been made to CMI_2010:

- The naming convention for “Core Projections” has been updated to refer to CMI_2010

- The Interim Life Table option on the Core parameters layer for the Base Table of Mortality Rates has been updated from the 2005-2007 table to the 2007-2009 table. (This is one of the base mortality tables contained in the Model to produce illustrative annuity and expectation of life values.)
- The default Calculation Date on the [Sample EoL & Annuities] worksheet has been changed to 31/12/2010
- A correction has been made on the [EoL & Annuity Calcs (2)] worksheet.

These changes are described in more detail in section 6 of the CMI_2010 User Guide, whilst the impact of the updated ONS data is discussed in section 3 of this paper.

1.3. The Scope of this Working Paper

This paper documents a number of areas arising from the updated Model.

Section 2 describes the shape of the Initial Rates of Mortality Improvement in 2007 for the core parameter layer of CMI_2010. This provides an update to the research detailed in Appendix A to the CMI_2009 User Guide.

Section 3 discusses the effect of adding data for calendar year 2009 whilst the impact on sample expectation of life values is shown in Appendix A.

Section 4 illustrates the sensitivity of CMI_2010 to the key parameter, the Long-Term Rate of Mortality Improvement.

Section 5 discusses issues arising from the use of the CMI's current P-Spline software.

Section 6 discusses the key differences between projections from the Model and those in the CMI Library of Mortality Projections.

1.4. Feedback on this paper

As the changes made in the latest version of the Model are limited, the CMI is not undertaking a consultation exercise on these revisions.

Feedback is always welcome, though, and any comments on this Working Paper and the Model can be sent via e-mail to projections@cmib.org.uk or in writing to: CMI, Cheapside House, 138 Cheapside, London, EC2V 6BW. Such comments will be considered for future work.

2. Derivation of Default Values for the Core Parameter Layer

The approach used to derive the default values for CMI_2010 is essentially unchanged from that used for CMI_2009, which was described in Appendix A of the User Guide for that version of the Model.

The research undertaken by the Mortality Projections Model Working Party for the initial version of the Model has not been repeated for this update; this section comprises updated figures and charts for the new dataset with a brief accompanying commentary. (NB Figure 1 below corresponds to Figure A.1 of the CMI_2009 User Guide, etc.)

Note that the User Guide for CMI_2009 discussed two other assumptions in the Model that are not considered further below:

- The Long-Term Rate of Mortality Improvement. This assumption is the key parameter for users to set for each projection and – as for CMI_2009 – there is no default parameter table or overall value set for this parameter group.
- The Proportion of Convergence Remaining at Mid-Point of the Convergence Period. As noted in the User Guide for CMI_2009, there is little evidence to support any one particular pattern of convergence. For the Core parameter level, this assumption defaulted to 50% for all ages and year-of-birth cohorts in CMI_2009, an approach that has been retained for CMI_2010.

2.1. Initial Rates of Mortality Improvement

2.1.1. Initial Aggregate Rates of Mortality Improvement

The default tables for Initial Rates of Mortality Improvement cover calendar years 1991 to 2007 and contain values for individual ages, separately for males and females. These rates represent the total rate of improvement by age, year and gender before any split into Age/Period and Cohort Components and are referred to as ‘aggregate’ rates in papers relating to the Model.

These rates of improvement were derived using an age-cohort P-Spline model fitted to ONS data for the population of England & Wales, for ages from 18 to 102, for the period 1961 to 2009. As well as providing the smoothed data from which to estimate ‘current’ rates, this approach automatically also provides rates for earlier years on a consistent basis.

The ‘current’ (initial) rates of mortality improvement are taken as those for calendar year 2007, that is: $r_{x,2007} = 1 - \{ q_{x,2007} \div q_{x,2006} \}$. The first year of the projection is therefore assumed to be 2008. This reflects the view that 2007, i.e. 2 years inside the edge of the available data, is the latest year for which sufficiently robust estimates of rates of mortality improvement may be made at present. This approach has been retained for CMI_2010.

The derived estimates of aggregate Initial Rates of Mortality Improvement for calendar year 2007 are shown in Figure 1.

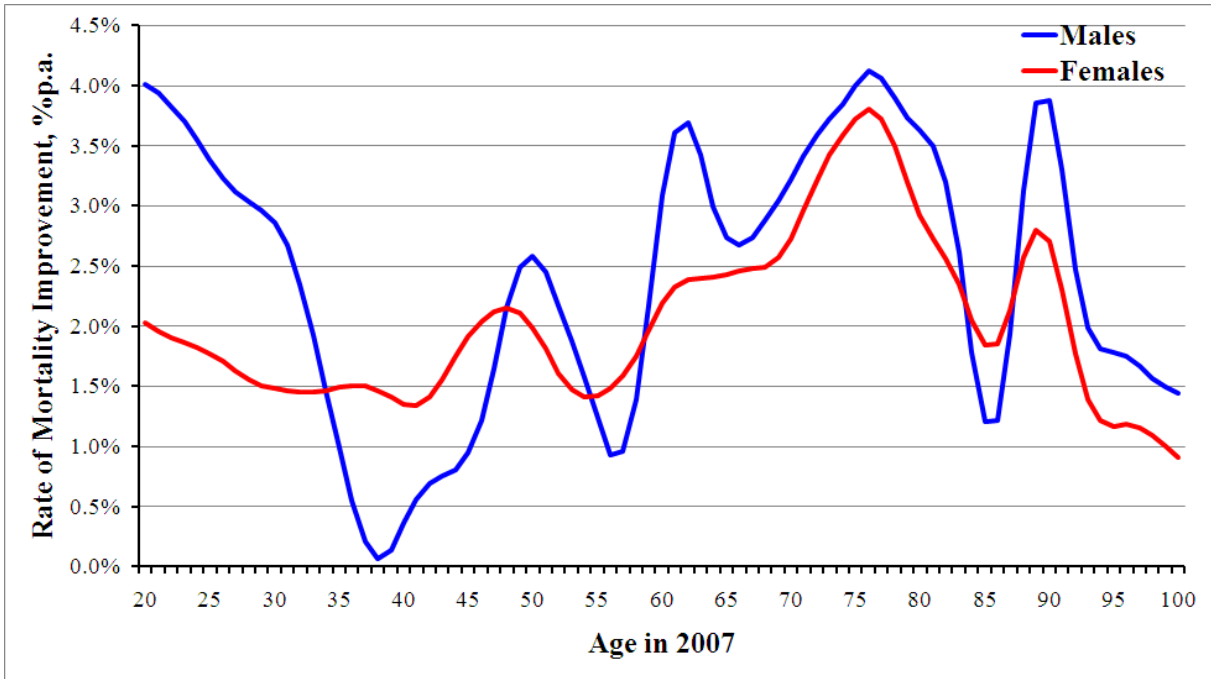


Figure 1: Estimated aggregate Initial Rates of Mortality Improvement; 2007

2.1.2. Split of Initial Rates into Age/Period Component and Cohort Component

The Model design incorporates splitting Initial Rates of Mortality Improvement into two components; Age/Period and Cohort Components. This approach has been retained for CMI_2010. This section illustrates the split of aggregate Initial Rates of Mortality Improvement into the two components for CMI_2010.

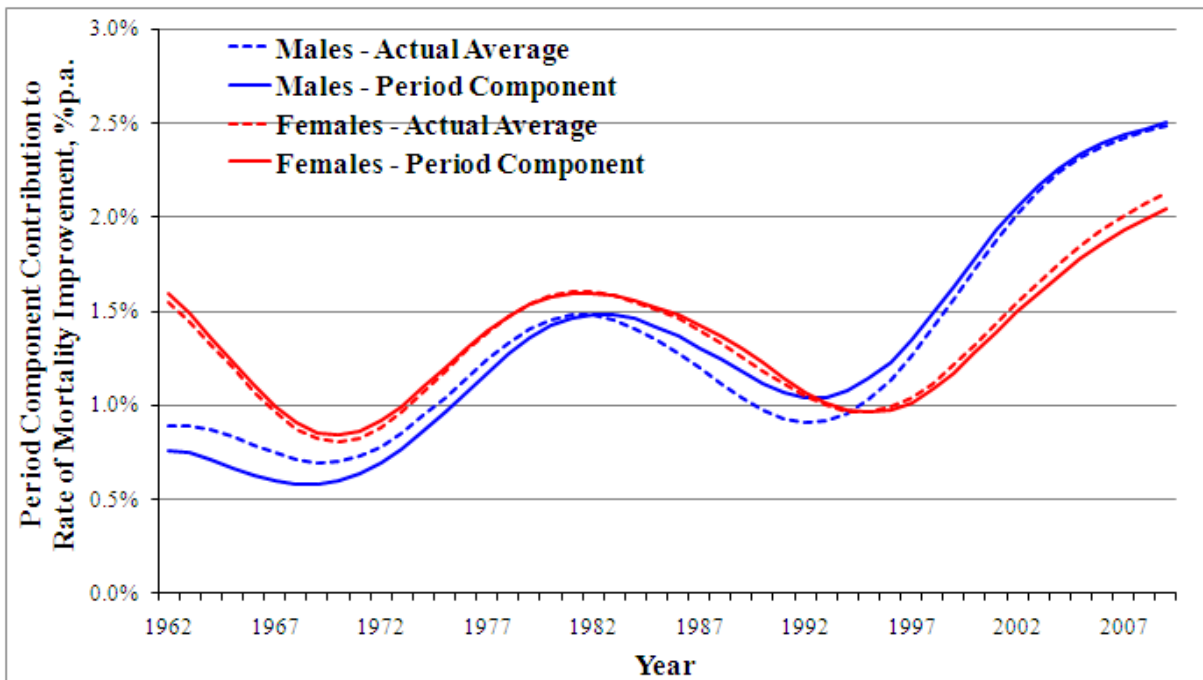
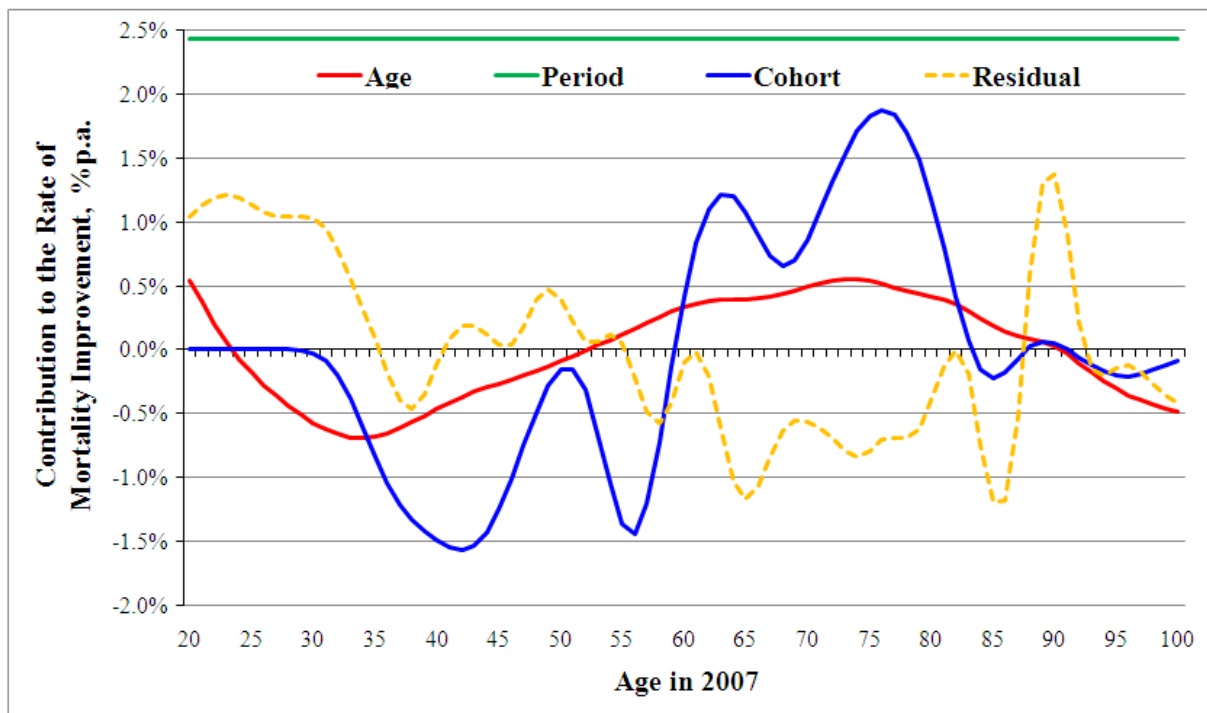


Figure 2: Comparison of Period Components derived in APC Model against All Age Average Rates of Improvement from Input Table

Figure 2 shows the period components derived from the APC model and compares them with the actual average (across ages 18 to 102) rate of improvement by calendar year. The close match between the APC model period component and the actual averages reflects the constraints applied (which are unchanged from CMI_2009; see section A1.2 of the CMI_2009 User Guide).

Figures 3 and 4 show the age, period and cohort components from the fitted APC models, for males and females respectively. Also shown are the residual errors.

Reasonableness checks were performed on the age, period and cohort components, for example by comparison with analysis of the underlying population data and against those in the CMI_2009 Model. The residual errors were allocated in an identical manner to CMI_2009 – the errors below age 30 were allocated to the Age/Period Component and those above age 60 to the Cohort Component, with a linear transition in between. In addition, the fitted Cohort Component was constrained to be zero near the edges of the data and in particular up to age 30 (as there are too few years’ data, and too much ‘noise’ in the data at young ages, to form a safe conclusion on cohort components).



**Figure 3: Results Derived from Fitted APC Model, Males; 2007
Age, Period and Cohort Components, plus Residual Errors**

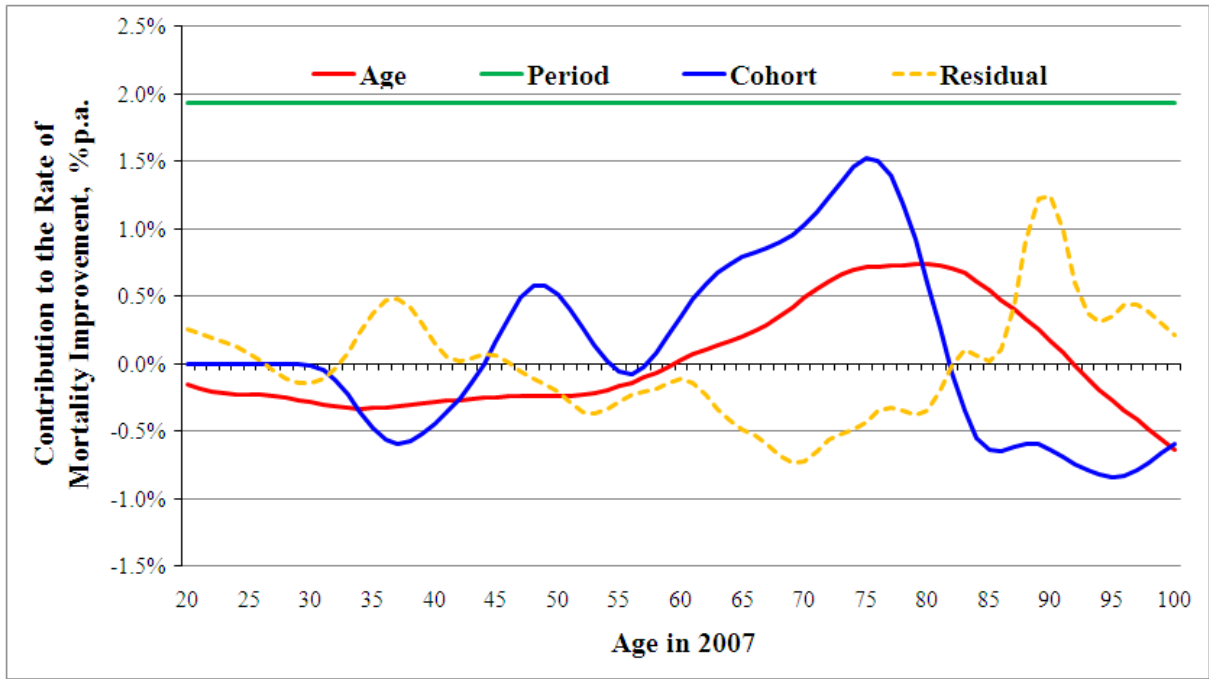


Figure 4: Results Derived from Fitted APC Model, Females; 2007
Age, Period and Cohort Components, plus Residual Errors

Figures 5 and 6 show the final derived Age/Period and Cohort Components of the Initial Rates of Mortality Improvement.

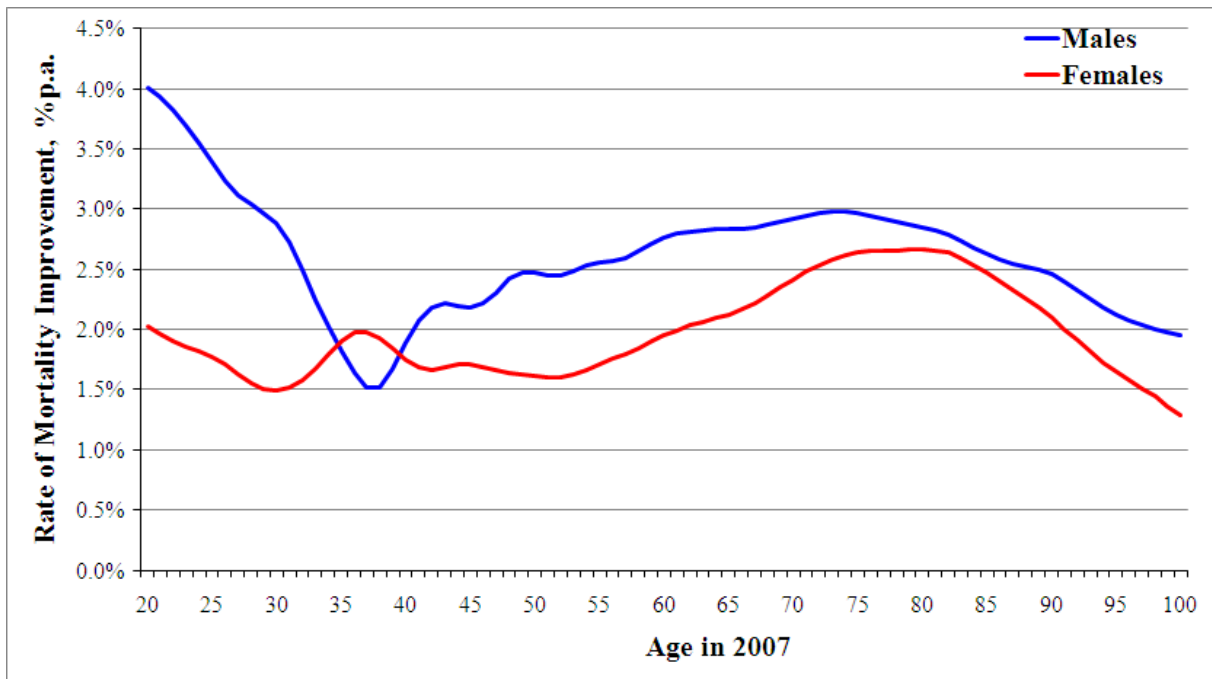


Figure 5: Estimated Age/Period Component of Initial Rates of Improvement; 2007

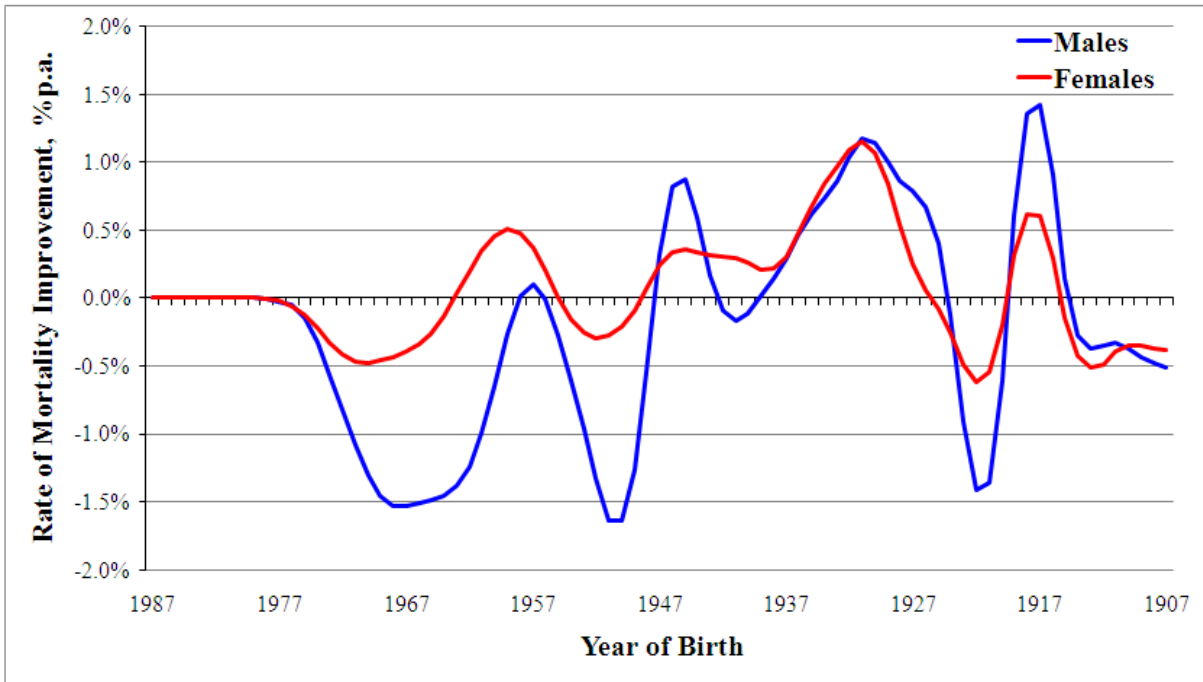


Figure 6: Estimated Cohort Component of Initial Rates of Improvement; 2007

2.2. Convergence

The Model assumes that Initial Rates of Mortality Improvement converge towards Long-Term Rates of Improvement. The convergence path is controlled by two sets of parameters:

- The Period of Convergence
- The Proportion of Convergence Remaining at the Mid-Point of the Convergence Period.

The convergence process is operated separately for Age/Period and Cohort components, and both sets of parameters may be varied by age and by year-of-birth cohort respectively for the two components.

As noted earlier, the default value of 50% for the Proportion of Convergence Remaining at the Mid-Point of the Convergence Period has been retained in CMI_2010.

For the CMI_2010 Model the Committee considered the relative merits of whether to maintain the length of the convergence periods, moving the end-point on by one year, or to fix the end-point from which the Long-Term Rates of Improvement apply. This question was discussed in the context of CMI_2009 – see section 2.11 of Working Paper 41.

The Committee decided to maintain:

- the default parameter values for the periods of convergence for Age/Period Components in the CMI_2010 Model, and
- the approach taken to setting the default values for the periods of convergence for Cohort Components in the CMI_2010 Model. This approach assumes that the periods run to age 100, subject to a minimum of 5 years and a maximum of 40 years; as a

result, the default periods of convergence for the Cohort Components have reduced by one year compared to CMI_2009 for year-of-birth cohorts from 1912 to 1946.

The default values are therefore unchanged from those shown in Figure A.7 of the User Guide for the CMI_2009 Model, which illustrated the convergence periods by current age.

Users can change these periods using the Advanced parameters within the Model if they wish.

3. The Effect of Adding Data for Calendar Year 2009

3.1. Changes to the Population Dataset for England & Wales

Default values for Initial Rates of Mortality Improvement in the Core parameter layer of the CMI_2010 Model, issued alongside this paper, were derived from an ONS dataset for the population of England & Wales, covering calendar years from 1961 to 2009. The base year for the projection in CMI_2010 is 2007. The corresponding dataset to 2008 was used in the CMI_2009 Model, with a base year of 2006.

Aside from the addition of data for 2009, the ONS made some minor changes to mid-year population estimates for earlier years. These affected the data at all ages for recent years, but with some ‘rippling back’ to much earlier years for ages 90+. Some changes at these ages were expected as they occur naturally in the iterative method used to derive population estimates for the very elderly; however these changes do not have a significant impact on derived mortality rates, with the majority of changes to $m_{x,t}$ typically being within 1% (of $m_{x,t}$).

The remainder of this section discusses the impact of these data changes on the Model. A similar discussion on the impact of adding 2008 data is contained in section 3 of Working Paper 41. The impact on selected cohort expectation of life values is illustrated in Appendix A to this paper.

3.2. Observed Rates of Mortality Improvement to 2009

At an aggregate level, mortality rates fell substantially in 2009. This is illustrated in Table 1, which shows crude annual mortality improvement rates for all-age mortality (ages 18-102, age-standardised using 2001 population estimates) for recent years.

**Table 1: Observed Crude Annual Mortality Improvement Rates
England & Wales Population, ages 18-102**

Year	Males	Females
2001	+3.0%	+1.8%
2002	+1.5%	+0.1%
2003	+1.7%	-0.5%
2004	+5.4%	+6.2%
2005	+2.7%	+0.9%
2006	+3.5%	+4.0%
2007	+2.5%	+0.9%
2008	+1.5%	+0.3%
2009	+4.4%	+6.2%

For the avoidance of doubt, a positive value in Table 1 (and for the mean difference in Table 2) means that mortality rates have fallen.

As there is considerable natural variation in mortality from year from year, it is necessary to apply some form of smoothing mechanism over time in order to try to detect time trends. The 2007 and 2008 data, in particular, had shown a slower pace of improvement than the

preceding years, and it is not clear what the higher improvements in 2009 mean for the general pace of mortality change.

Note that the headline improvement rate figures for England and Wales in 2009, published by the ONS, of 4.1% for males and 5.5% for females are age-standardised to the European Standard Population and are for **all** ages. The figures in the above table are age-standardised using England & Wales population estimates for 2001 and show the improvement rates for ages 18-102.

3.3. Changes in Estimates of Current Aggregate Rates of Mortality Improvement

In order to calculate default values for initial rates of mortality improvement, age-cohort P-Spline models were again fitted to the population dataset. As expected, the addition of a year's data does affect the fitted surface for earlier years; this is especially true given the scale of the change in the 2009 data. Table 2 shows two measures of the difference in estimated mortality improvement rates for recent years – the all-age mean difference, and the all-age mean absolute difference (calculated over the age range 18-102).

**Table 2: Mean Difference and Mean Absolute Difference
in fitted P-Spline model Estimates of Annual Rates of Mortality Improvement (%)
for the 1961-2009 dataset minus the 1961-2008 dataset
England & Wales Population, ages 18-102**

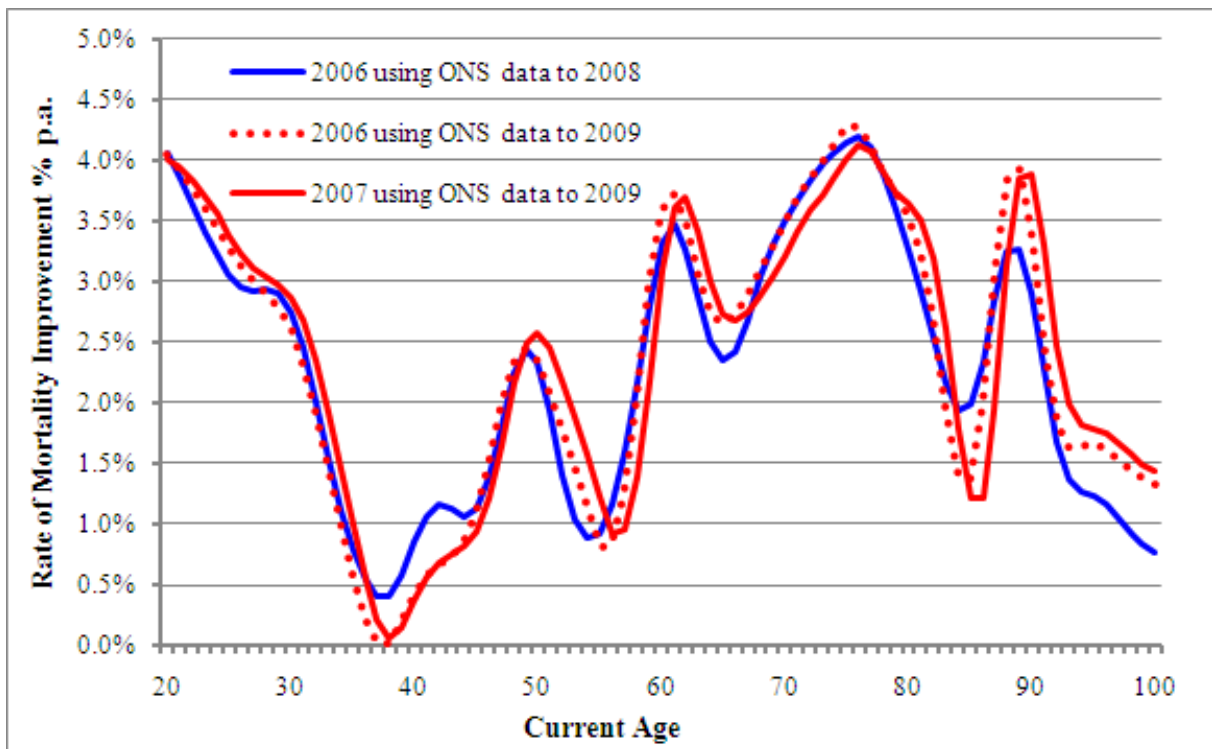
Year	Mean Diff		Mean Absolute Diff	
	Males	Females	Males	Females
2001	-0.01	-0.03	0.13	0.07
2002	-0.01	+0.02	0.13	0.07
2003	+0.00	+0.00	0.15	0.07
2004	+0.02	+0.04	0.17	0.08
2005	+0.04	+0.09	0.20	0.12
2006	+0.07	+0.16	0.24	0.18
2007	+0.10	+0.24	0.30	0.25
2008	+0.13	+0.32	0.35	0.33

For the avoidance of doubt, the value of +0.10 for the Mean Difference in Table 2 for males in 2007, for example, corresponds to an increase in the mean mortality improvement rate for that year from 2.32% p.a. using the 1961-2008 dataset to 2.42% p.a. using the 1961-2009 dataset.

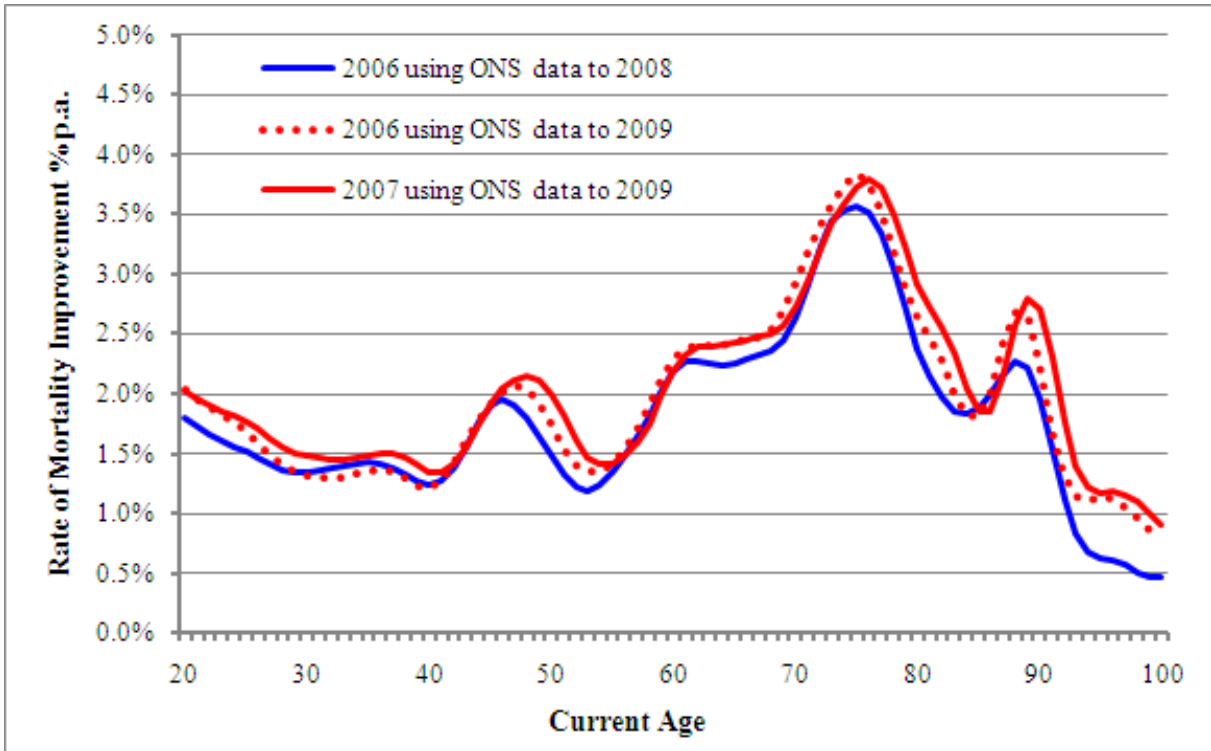
For females, the differences are of similar magnitude to those illustrated in the corresponding table last year (see Table 2 of Working Paper 41), whilst those for males are smaller. For both they are smaller than the differences derived in the back-testing of the prototype model (see section 3.1.2 of Working Paper 39) and suggest there is nothing unusual about the development of the time-series data.

Figures 7 and 8 compare the estimates for mortality improvement rates in 2006 (in CMI_2009, based on ONS data to 2008) with revised estimates (based on data to 2009). The estimates for 2007 (based on ONS data to 2009) are also shown. The revised estimates show some modest shape changes and are generally a little higher, consistent with the high-level data and results shown in Tables 1 and 2. The improvement rates at the oldest ages, for both males and females, are noticeably higher in the latest dataset. The Committee has made no

changes to the run-off patterns used at the oldest ages in CMI_2010, although the absolute values will be different and the higher values at age 100 will mean improvement rates now reach zero at a higher age than in CMI_2009.



**Figure 7: Estimated Aggregate Rates of Mortality Improvement, by age and dataset
Males, England & Wales Population
Estimates derived by fitting age-cohort P-Spline models**



**Figure 8: Estimated Aggregate Rates of Mortality Improvement, by age and dataset
Females, England & Wales Population
Estimates derived by fitting age-cohort P-Spline models**

Note that the x-axis for both Figure 7 and Figure 8 shows “current age”, hence there is a tendency for the improvements for 2007 to move one year to the right. (This applies also to the Cohort Components in Figures 9 and 10, below.)

3.4. Changes in Estimates of Components of Current Rates of Mortality Improvement

The Age/Period and Cohort Components for the 2007 Initial Rates of Mortality Improvement were again determined by fitting the Age-Period-Cohort (APC) model. The results are shown in Figures 9 and 10, together with the 2006 components used in CMI_2009.

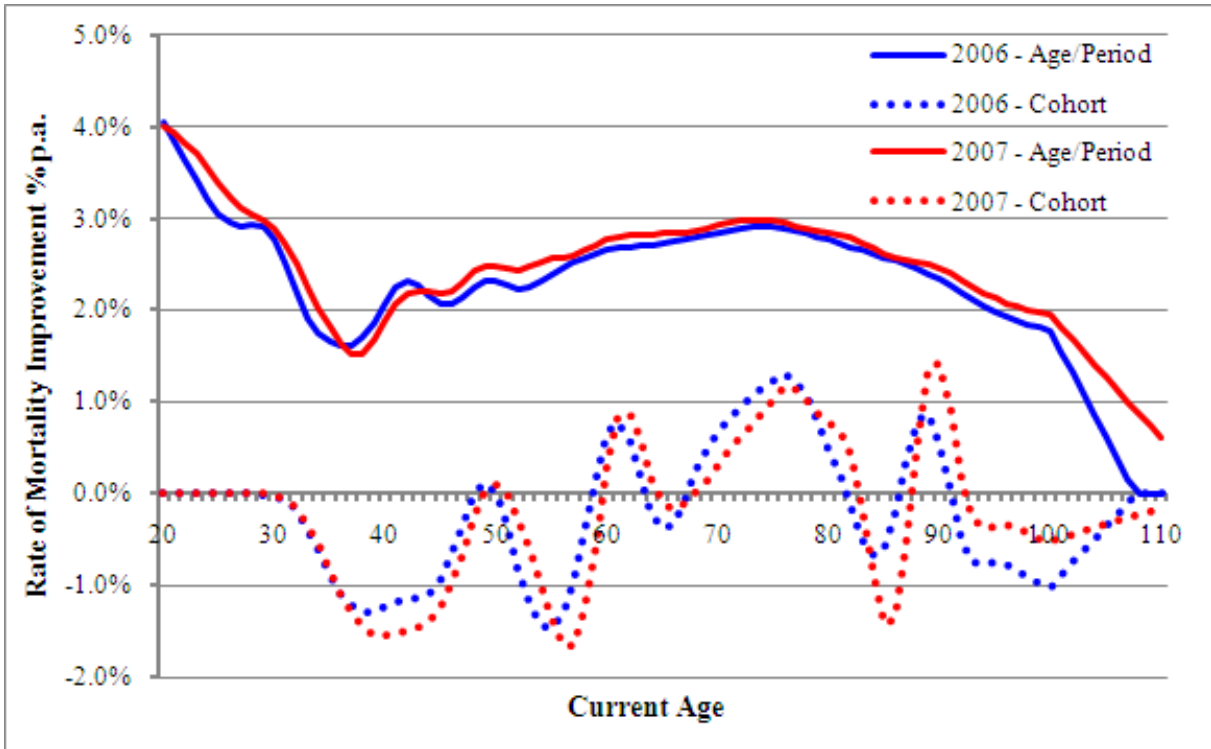


Figure 9: Estimated Age/Period and Cohort Components of Mortality Improvement, by age and dataset; Males, England & Wales Population
Estimates derived by fitting APC models to smoothed mortality improvement rates

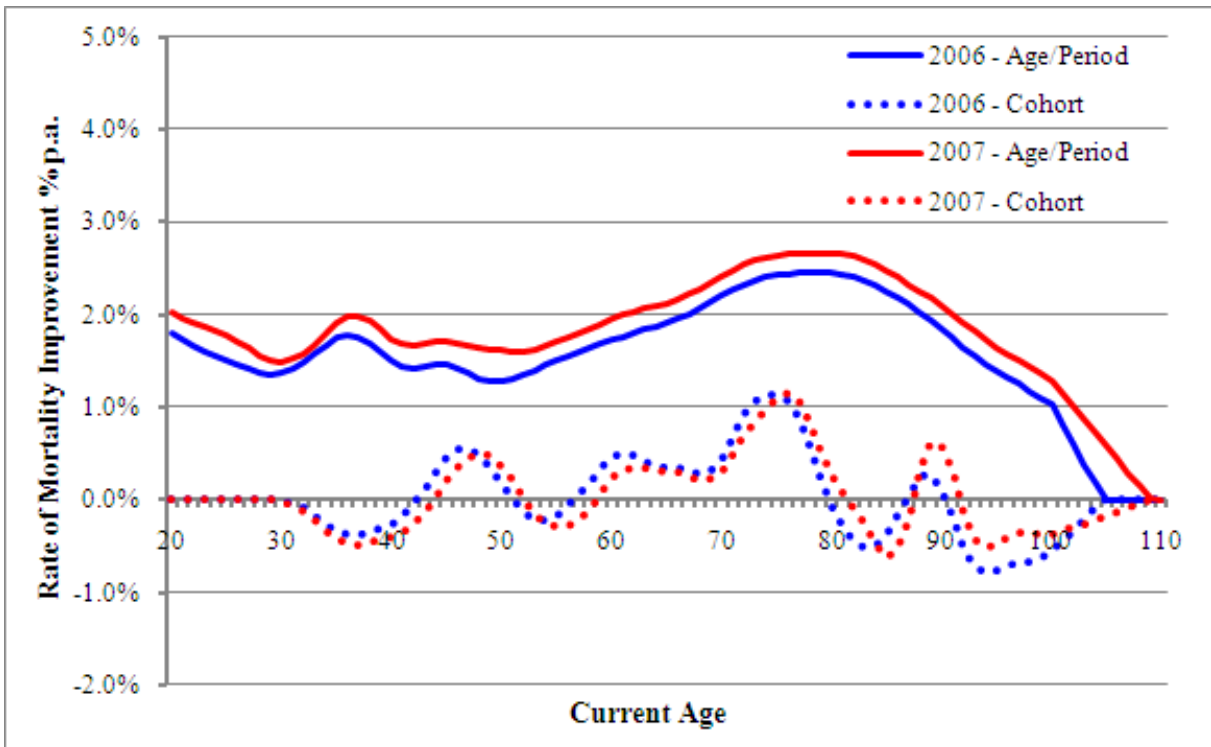


Figure 10: Estimated Age/Period and Cohort Components of Mortality Improvement, by age and dataset; Females, England & Wales Population
Estimates derived by fitting APC models to smoothed mortality improvement rates

Figure 11 illustrates how the Period Components have changed across the two “updates” to the Model to date. This degree of change, resulting from the addition of each year’s data emphasises the difficulty in drawing sound conclusions regarding the current trajectory of rates of mortality improvement. Indeed, the lower trajectory apparent from including the 2008 data has been partially reversed by the 2009 data for males, and wholly negated for females.

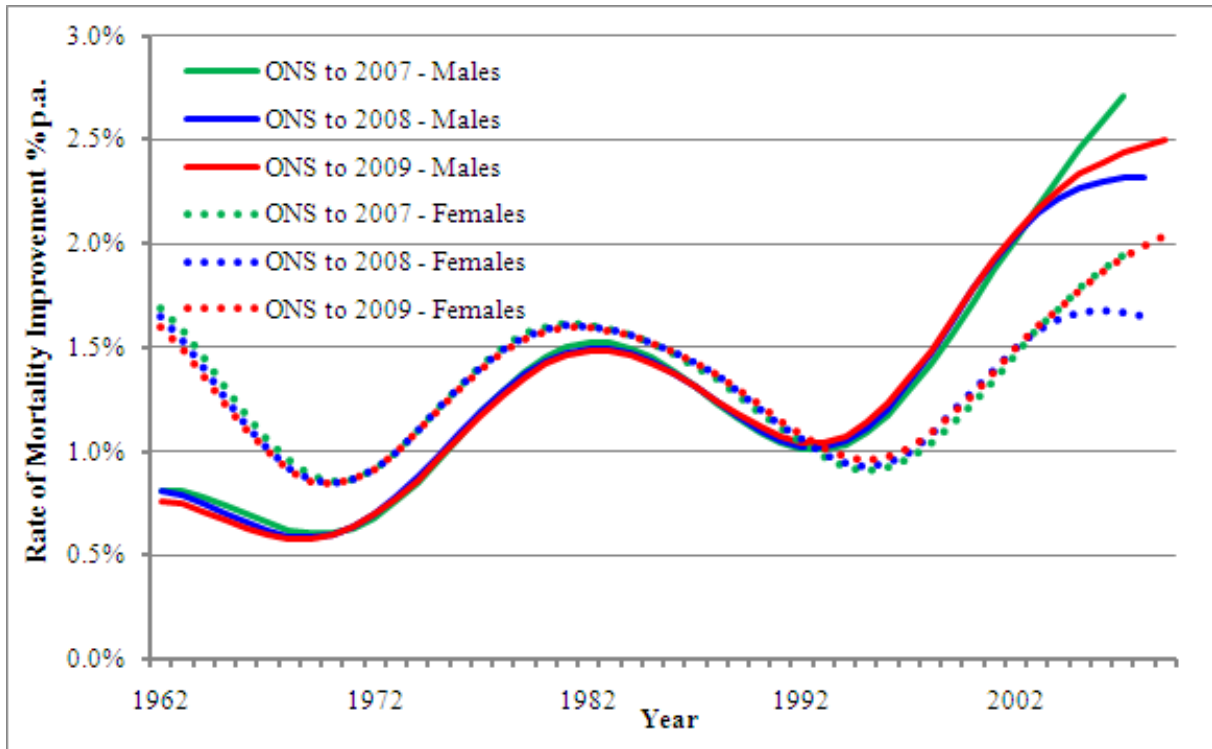


Figure 11: Period Component of the Rate of Mortality Improvement, by year, dataset, and gender; England & Wales Population
Estimates derived by fitting APC models to smoothed mortality improvement rates

3.5. Quantification of the Effect of Reflecting Data for 2009 in the Model

The CMI_2010 version of the Model published alongside this paper includes Core parameter default values for Initial Rates of Mortality Improvement as at 2007, derived using population data up to 2009 – that is, taking account of an extra year’s observations in the underlying dataset compared with that used for the CMI_2009 Model.

The effect of this change is illustrated in Appendix A by comparing cohort expectation of life values at a range of ages, for males and females, using a Long-Term Rate of Mortality Improvement of 1.5% p.a. In summary, cohort life expectancies have risen by around 0.3% to 0.7% for a typical spread of ages in a portfolio of male lives and by around 0.5% to 1.5% for female lives, depending on the type of business under consideration. To put this in context, increasing the Long-term Rate of Mortality Improvement by 1% p.a. adds around 5% to cohort life expectancies at age 65.

4. Parameter Sensitivities

When CMI_2009 was issued, it was important that the sensitivity of the new Model to the various parameters was widely understood. In particular:

- Appendix B of the User Guide to the CMI_2009 Model contains a large range of sensitivities, illustrated relative to the Medium Interim Cohort Projection.
- A spreadsheet containing the results of a wider range of sensitivity tests was made available for download from the CMI pages of the Actuarial Profession's website.

These sources remain available to users who can, of course, also investigate sensitivities using the Model itself.

The relative significance of the sensitivity of the Model to the various parameters is unchanged, so the Committee decided that it would be appropriate to illustrate explicitly only the sensitivity of results to the Long-Term Rate of Mortality Improvement for CMI_2010.

Figures 12 and 13 show sample single life annuity values and expectations of life for males for various Long-Term Rates of Mortality Improvement in CMI_2010. All other parameter values are unchanged from their Core values in the Model. The annuity and expectation of life values are calculated on the following basis:

- The values are calculated as at 31/12/2010
- Annuities, based on payments of 1 p.a, are assumed to be payable yearly in advance using a net discount rate of 3.0% p.a.
- The expectation of life values are complete rather than curtate
- The values have been derived using S1PMA base mortality table at 01/09/2002 projected to 2007 using the past rates of mortality improvement contained in the Core parameter layer of the Model, and forward from 2007 using the rates of mortality improvement given by the various projections illustrated.

Note that the y-axes for Figures 12 and 13 do not start at zero.

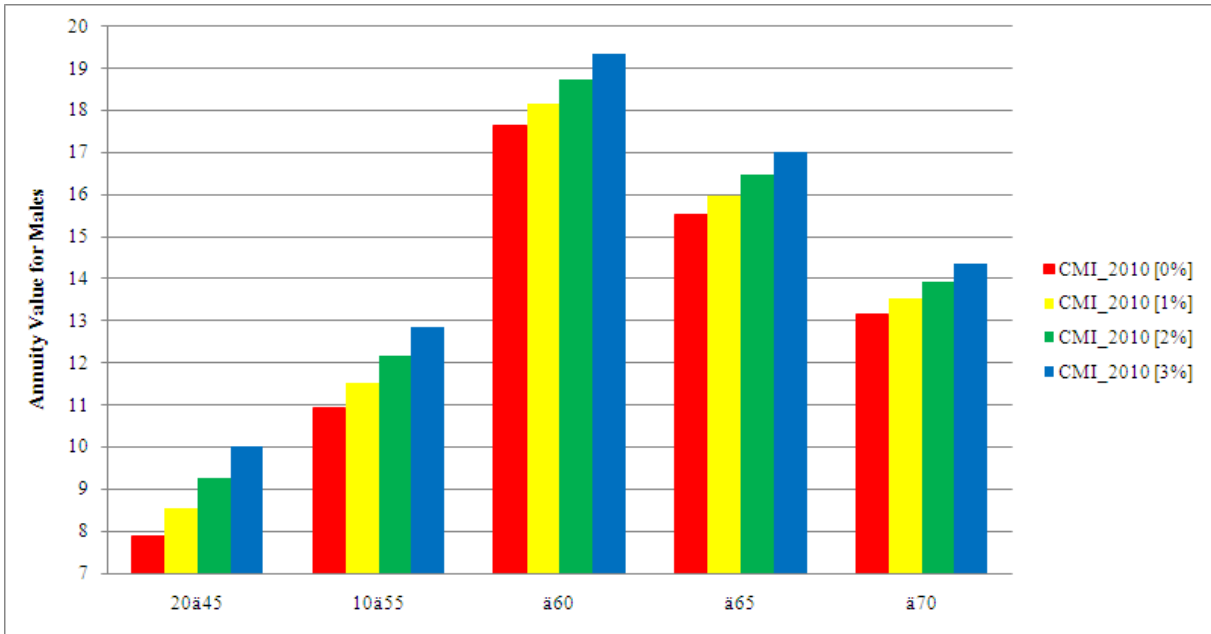


Figure 12: Variation in selected single life annuity values for males, for changes in assumed Long-Term Rate of Mortality Improvement.

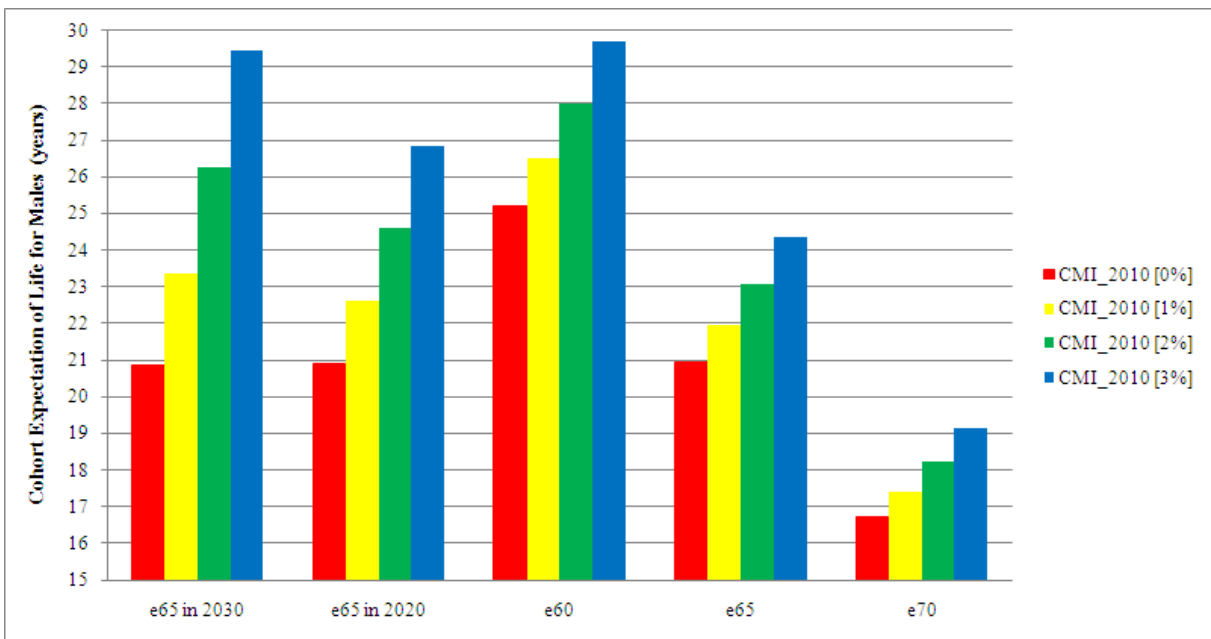


Figure 13: Variation in selected cohort expectation of life values for males, for changes in assumed Long-Term Rate of Mortality Improvement.

5. Potential Issues with the CMI P-Spline model

The default Initial Rates of Mortality Improvement in CMI_2010 were again derived by fitting an age-cohort P-Spline model to ONS population mortality data. In updating the Model to CMI_2010, the Committee identified two potential issues with the CMI P-Spline model and population data:

- The CMI's P-Spline software sometimes fails to reach an optimal solution (as it may locate a local minimum rather than the global minimum for the model test statistic).
- Any overdispersion in the data causes the CMI P-Spline model to under-smooth by year and by age. This also makes the software more vulnerable to the "local minimum" issue.

Investigation of the first of these issues has shown that this has not affected the results of the age-cohort models used in deriving the default parameter values in CMI_2010, but it may impact other projections produced using the software.

The second issue is more complex. The CMI P-Spline model assumes an underlying Poisson distribution for deaths. If that assumption holds true then the variance of the residual differences after fitting the P-Spline surface, measured in terms of standard errors across all points on the surface, would be close to 1.0.

However the ONS population datasets used for the CMI_2010 Model show considerable overdispersion with a higher variance of approximately 3.1 for both males and females. Two particular features of these datasets appear to contribute significantly to these high values:

- The 1919 birth-year cohort shows crude mortality rates significantly out of line with adjacent cohorts particularly over 2000-2009. One known contributory factor is that births in 1919 were not evenly distributed over the year which leads to distorted mortality rates due to the use of mid-year population estimates for the exposure.
- There is high volatility from year to year in crude mortality rates for ages between 60 and 90 over the decade from 1961 to 1971. The 1886 birth-year cohort shows particularly high residual differences. This latter feature appears to arise from variations in the population estimates rather than the death counts.

The Committee estimates that the variance would fall to around 1.8 if the areas of the data that give rise to these features were omitted.

Alternative P-Spline models are available that allow for overdispersion and so smooth the population mortality data to a greater extent than the CMI P-Spline software. However it is not clear to the Committee at this stage the extent to which the specific features noted above arise from weaknesses in the methodology used to derive the underlying population data or from genuine variations in mortality, and therefore to what extent it is appropriate to reflect the observed overdispersion in the modelling.

The Committee has investigated the impact of allowing for overdispersion in estimating the initial rates of mortality improvement required for the CMI_2010 Model and found it to be within the range of previously-reported model uncertainty (arising from different knot-spacing, age-period versus age-cohort P-Spline models, or alternative smoothing approaches). As a result, the Committee is confident that even though the age-cohort P-Spline model actually used does not allow for overdispersion, it remains suitable for the purpose of smoothing initial rates of mortality improvement in the Model.

6. The Model and the Library

The Committee intends to incorporate projections from CMI_2010 (and CMI_2009) into the CMI Library of Mortality Projections in the next update of the Library, but considered updating the Model to be a higher priority.

However at this stage, it may be helpful to clarify the key differences between projections from the Model and those in the Library.

6.1. Timing of Projections

The Model requires values for three specific dates and uses these precisely. The three dates are:

- The date at which the Initial Rates of Mortality Improvement apply (referred to as ‘Date A’ in the User Guide). This has a default value of 1/1/2007 in CMI_2010.
- The date at which the Base Rates of Mortality apply (‘Date B’). This does not affect the projected improvements derived from the Model, but will affect any projected mortality rates. The default value in CMI_2010 depends on the table specified.
- The Calculation Date (‘Date C’), which only affects annuity and expectation of life values derived from the Model.

In contrast, for simplicity, the Library assumes that each projection can be applied, without adjustment, to either a “00” Series table (with an assumed effective date of 30 June) or an “S1” Series table (with an assumed effective date of 1 September). This ignores the difference between the effective dates for these tables that is used explicitly in the Model.

In addition, no regard is paid to the “natural timing” of each projection contained in the Library. In the Core version of the Model, the default Initial Rates of Mortality Improvement are based on ONS population data and, as noted above, these apply in the year commencing 1/1/2007 in CMI_2010. The Library contains a variety of projections, based on different data sources, to which different “natural timings” apply, as shown below:

Natural Timing	Categories of projections in the Library
1 January	<ul style="list-style-type: none">• ONS National Population Projections• P-spline projections using CMI or ONS data• Lee-Carter projections using CMI or ONS data
1 July	<ul style="list-style-type: none">• “92” Series• Interim Cohort projections• Adjusted Cohort Projections

These timings and their application will be clarified in the updated User Guide accompanying the next version of the Library.

6.2. Limiting Age

In the Library, all the projections assume a limiting age of 120, i.e. that $q_{120} = 1$, throughout the period of the projection.

The Model allows projections to run up to age 150, but, if desired, users of the Model may apply this lower limit by ensuring that $q_{120} = 1$ in all years of the projection (using the Advanced parameter facility). Note that this constraint would only need to be applied to Core Projections from the Model if a positive value were assigned to the Constant Addition to Rates of Mortality Improvement.

6.3. Use of ONS Data at Older Ages

The initial rates of improvement in CMI_2010 were derived using an age-cohort P-Spline model fitted to ONS data for the population of England & Wales for ages 18 to 102. Single age data for population estimates for ages 90 – 104 were provided to the CMI by the ONS and are now in the public domain.

To date, projections based on ONS data in the Library have been derived from population estimates up to and including age 89 only. The data for a wider age range were not available when the work was undertaken. Improvements for all ages above age 89 were assumed to equal the improvements at ages 89 (whilst q_{120} is assumed to equal 1).

References

CMI Working Paper 38 “A Prototype Mortality Projections Model: Part One – An Outline of the Proposed Approach”. (2009)

CMI Working Paper 39 “A Prototype Mortality Projections Model: Part Two – Detailed Analysis”. (2009)

CMI Working Paper 41 “CMI Mortality Projections Model: Feedback on Consultation and Issue of ‘CMI_2009’ ”. (2009)

User Guide for the CMI Mortality Projections Model - Model Name / Version: ‘CMI_2009’ (2009)

User Guide for the CMI Mortality Projections Model - Model Name / Version: ‘CMI_2010’ (2010)

All of the above may be accessed and downloaded from the CMI pages, under “Research and resources”, on the UK Actuarial Profession’s website; in particular:

- CMI_2010 and its User Guide are located alongside Working Paper 49, and
- CMI_2009, its User Guide and the spreadsheet of parameter sensitivity tests are located alongside Working Paper 41.

Appendix A. Sample Expectations of Life from the CMI_2010 Model

A relatively small number of changes have been made between the CMI_2009 Model and the current version, CMI_2010. These are summarized in section 1.2 of this paper and detailed in section 6 of the accompanying User Guide. The impact of the updated data is illustrated in section 3 and this Appendix shows the impact of these changes on selected cohort expectation of life values.

The effect of updating of the Initial Rates of Mortality Improvement, by taking into account the extra year of data (calendar year 2009), is illustrated by comparing cohort expectation of life values, for males and females, for a large grid of model points on the following basis:

- Like-for-like Core Projections using a Long-Term Rate of 1.5% p.a.
- Base Rates of Mortality are the published S1PxA tables (for lives aged x exact on 01/09/2002)
- Calculation Date(s) are 31/12/year.

Tables 3 and 5 show the sample expectation of life values produced by CMI_2010, for males and females respectively; Tables 4 and 6 show the percentage change measured against the equivalent values produced by 'CMI_2009', for males and females respectively.

Tables 4 and 6 show that updating the Core parameters for Initial Rates of Mortality Improvement, to reflect the addition of a further year's observations on England & Wales population mortality experience, results in small increases in cohort expectation of life values.

**Table 3: Cohort Expectation of Life for Age x exact on 31/12/Year
Males ; Base Rates of Mortality = 100% S1PMA as at 01/09/2002
Core Projection: CMI_ 2010_M [1.5%]**

Age, x	Year						
	2010	2015	2020	2025	2030	2035	2040
20	70.13	70.88	71.62	72.35	73.06	#N/A	#N/A
25	64.46	65.22	65.97	66.70	67.42	68.13	#N/A
30	58.80	59.57	60.33	61.07	61.80	62.51	63.21
35	53.19	53.96	54.72	55.46	56.20	56.92	57.62
40	47.69	48.43	49.19	49.93	50.67	51.39	52.10
45	42.41	43.05	43.77	44.51	45.24	45.95	46.66
50	37.28	37.88	38.52	39.22	39.93	40.63	41.33
55	32.16	32.85	33.45	34.08	34.76	35.44	36.12
60	27.22	27.83	28.48	29.07	29.70	30.35	31.00
65	22.49	23.03	23.59	24.20	24.78	25.38	26.00
70	17.81	18.54	19.02	19.54	20.11	20.67	21.24
75	13.61	14.22	14.82	15.26	15.74	16.26	16.78
80	9.83	10.45	10.92	11.41	11.81	12.25	12.71
85	6.81	7.31	7.74	8.11	8.51	8.87	9.24
90	4.70	4.95	5.27	5.57	5.86	6.16	6.44
95	3.16	3.36	3.54	3.73	3.93	4.13	4.34
100	2.28	2.39	2.51	2.64	2.76	2.89	3.02

**Table 4: % Change in Cohort Expectation of Life for Age x exact on 31/12/Year
Males ; Base Rates of Mortality = 100% S1PMA as at 01/09/2002
Core Projections: CMI_ 2010_M [1.5%] against CMI_ 2009_M [1.5%]**

Age, x	Year						
	2010	2015	2020	2025	2030	2035	2040
20	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	#N/A	#N/A
25	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	#N/A
30	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%
35	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%
40	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.2%	+ 0.3%
45	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%
50	+ 0.5%	+ 0.4%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%
55	+ 0.5%	+ 0.5%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%
60	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.4%	+ 0.4%
65	+ 0.6%	+ 0.5%	+ 0.6%	+ 0.6%	+ 0.5%	+ 0.5%	+ 0.5%
70	+ 0.5%	+ 0.7%	+ 0.6%	+ 0.6%	+ 0.6%	+ 0.6%	+ 0.6%
75	+ 0.4%	+ 0.6%	+ 0.8%	+ 0.6%	+ 0.7%	+ 0.7%	+ 0.6%
80	+ 0.7%	+ 0.7%	+ 0.8%	+ 0.9%	+ 0.8%	+ 0.8%	+ 0.8%
85	+ 1.0%	+ 1.1%	+ 1.0%	+ 1.1%	+ 1.1%	+ 1.1%	+ 1.1%
90	+ 2.4%	+ 2.2%	+ 2.1%	+ 2.0%	+ 2.0%	+ 2.0%	+ 2.0%
95	+ 3.3%	+ 3.4%	+ 3.2%	+ 3.1%	+ 3.0%	+ 3.0%	+ 2.9%
100	+ 3.7%	+ 3.6%	+ 3.5%	+ 3.5%	+ 3.4%	+ 3.3%	+ 3.3%

**Table 5: Cohort Expectation of Life for Age x exact on 31/12/Year
Females ; Base Rates of Mortality = 100% S1PFA as at 01/09/2002
Core Projection: CMI_ 2010_F [1.5%]**

Age, x	Year						
	2010	2015	2020	2025	2030	2035	2040
20	71.85	72.57	73.28	73.97	74.65	#N/A	#N/A
25	66.18	66.92	67.64	68.35	69.04	69.71	#N/A
30	60.56	61.31	62.04	62.76	63.45	64.13	64.80
35	55.03	55.78	56.52	57.24	57.94	58.62	59.29
40	49.67	50.40	51.12	51.84	52.54	53.22	53.89
45	44.53	45.21	45.91	46.61	47.29	47.96	48.62
50	39.51	40.19	40.85	41.51	42.18	42.83	43.47
55	34.52	35.23	35.88	36.51	37.14	37.77	38.38
60	29.60	30.23	30.88	31.50	32.10	32.70	33.29
65	24.68	25.30	25.89	26.50	27.08	27.65	28.22
70	19.87	20.53	21.09	21.63	22.19	22.74	23.28
75	15.37	15.98	16.54	17.05	17.55	18.06	18.56
80	11.28	11.88	12.37	12.85	13.29	13.74	14.20
85	7.92	8.39	8.83	9.22	9.61	9.99	10.37
90	5.41	5.67	5.98	6.29	6.59	6.89	7.19
95	3.55	3.74	3.93	4.13	4.34	4.54	4.75
100	2.43	2.55	2.68	2.80	2.93	3.06	3.19

**Table 6: % Change in Cohort Expectation of Life for Age x exact on 31/12/Year
Females ; Base Rates of Mortality = 100% S1PFA as at 01/09/2002
Core Projections: CMI_ 2010_F [1.5%] against CMI_ 2009_F [1.5%]**

Age, x	Year						
	2010	2015	2020	2025	2030	2035	2040
20	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	#N/A	#N/A
25	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	#N/A
30	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%
35	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.3%	+ 0.3%	+ 0.3%	+ 0.3%
40	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%
45	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.4%	+ 0.4%	+ 0.4%	+ 0.4%
50	+ 0.6%	+ 0.6%	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.5%	+ 0.5%
55	+ 0.6%	+ 0.6%	+ 0.6%	+ 0.6%	+ 0.6%	+ 0.5%	+ 0.5%
60	+ 0.7%	+ 0.7%	+ 0.7%	+ 0.7%	+ 0.7%	+ 0.6%	+ 0.6%
65	+ 0.9%	+ 0.8%	+ 0.8%	+ 0.8%	+ 0.8%	+ 0.7%	+ 0.7%
70	+ 1.1%	+ 1.0%	+ 1.0%	+ 0.9%	+ 0.9%	+ 0.9%	+ 0.9%
75	+ 1.3%	+ 1.3%	+ 1.2%	+ 1.1%	+ 1.1%	+ 1.1%	+ 1.0%
80	+ 1.5%	+ 1.6%	+ 1.5%	+ 1.4%	+ 1.3%	+ 1.3%	+ 1.2%
85	+ 1.7%	+ 1.8%	+ 1.8%	+ 1.7%	+ 1.6%	+ 1.5%	+ 1.4%
90	+ 2.0%	+ 2.1%	+ 2.0%	+ 1.9%	+ 1.9%	+ 1.8%	+ 1.7%
95	+ 2.0%	+ 2.1%	+ 2.0%	+ 2.0%	+ 1.9%	+ 1.8%	+ 1.7%
100	+ 1.4%	+ 1.4%	+ 1.3%	+ 1.3%	+ 1.3%	+ 1.2%	+ 1.2%