

**Continuous Mortality Investigation  
Income Protection Sub-Committee**

**Working Paper 5**

**The Graduation of Claim Recovery and Mortality Intensities  
for the Individual Income Protection Experience for 1991-98 of  
Males, Occupation Class 1**

May 2004



# Continuous Mortality Investigation Income Protection Sub-Committee

## Working Paper 5

### THE GRADUATION OF CLAIM RECOVERY AND MORTALITY INTENSITIES FOR THE INDIVIDUAL IP EXPERIENCE FOR 1991-98 OF MALES, OCCUPATION CLASS 1

#### SUMMARY

The claims data used in the investigation and how it was compiled and classified are described in Section 1. Some considerations affecting the approach to the analysis of the data and its graduation are described in Section 2.

The investigation of the recovery intensities, reported in Section 3, showed broad similarities of pattern to those discovered in the comparable study of the Males Standard Experience, 1975-78, as described in *C.M.I.R.* **12** (1991), Part B as regards the trends of intensities by sickness duration and by age at sickness inception. However, the current investigation, in contrast to the earlier one, reveals significant differences in the overall level of recovery intensities between different deferred periods. It also shows a dissimilarity in the trend by sickness duration (up to roughly 16 weeks) between 1 week and 4 weeks deferred period policies, which is inconvenient since such diversity inhibits attempts to develop a simple and convincing unifying model for recovery intensities. Partly in consequence, the graduation of recovery intensities, described in Section 4, has proved more difficult than early promising but limited trials had suggested. To achieve a satisfactory graduation a complicated graduation formula has been developed incorporating various adjustments to cope with particular features of the data.

The investigation into the mortality of claimants is described in Section 5. The mortality intensities for 1 week deferred period policies were observed to be lighter than for the other deferred periods and this is reflected in the graduated rates, where a distinction is made between the rates for 1 week and for longer deferred periods. The graduation is described in Section 6. After some experimentation a reasonably satisfactory graduation formula was found.

Continuation tables for specimen ages at falling sick are discussed in Section 7.

Tables 1 and 2 give graduated recovery and mortality intensities for specimen ages and sickness durations, for different deferred periods. Tables 3 and 4 give summaries of the data together with a comparison of actual and expected recoveries and deaths respectively. Examples of continuation tables and claim survivorship probabilities are given in Table 5 and Table 6.

#### 1. INTRODUCTION

1.1 An investigation was made into the distributions of the length of claims under individual IP policies for male lives, classified as Occupation Class 1 risks, as reported by contributing offices to the C.M.I. Bureau for the eight years of

experience, 1991-98, with the intention of graduating the data to produce tables of recovery and mortality rates for claimants. The data analysed consisted of the Standard\* sub-set of the total (Aggregate) data, produced by eliminating non-UK policies, policies with identifiable underwriting exclusions and policies with special benefit types (e.g. lump sums). The methods of investigation and graduation used for this report were similar to those used for the corresponding investigations based on the Males Standard Experience, 1975-78, as described in *C.M.I.R.* 12, Part B.

1.2 In this report the symbols DP1, DP4, DP13, DP26 and DP52 are used as abbreviations for deferred periods of 1, 4, 13, 26 and 52 weeks respectively. The term Class 1 refers to Occupation Class 1.

1.3 Preliminary comparisons were made with the experiences for males of other occupational classes, establishing that the Class 1 experience differs sufficiently from the others to justify separate detailed investigation and graduation, though the differences are not extremely large. The graduated rates for Class 1 form a reference basis for evaluating the experiences for other occupational classes and other experiences (see the report “The Claim Termination Experience of Income Protection Business, 1991-98, for Other Male Occupations, Females and Group”, a draft version of which appears as Working Paper 7). Comparisons were also made between the experiences in individual years of the octennium, for evidence of any clear time trends or other distinctive features. Whilst some particular minor features were observed it was considered that the data for the octennium as a whole was sufficiently homogeneous to be combined for the investigations and graduations. The much larger volume of data encompassed by an eight-year investigation period, against the usual quadrennial periods for C.M.I. investigations, is a distinct advantage.

1.4 Data submitted by offices undergoes screening upon receipt by the Bureau to pick up possible errors of coding and there is also a procedure to try to identify and eliminate duplicate claims (where a claim on one life arises on more than one policy) which is believed to be effective. During preliminary investigations of data for this report an hitherto undetected type of error came to light in the form of false one-day claims. These are claims reported as starting and ending on the same day, nearly always the first day after the deferred period. They are thought almost certainly to relate to sicknesses which actually ended during the deferred period and did not result in a claim. They have been excluded from the experience.

Further comment on the subject of false one-day claims is to be found in Section 1.5 of the report “Date-Related Features of Income Protection Claims 1975-98”, a draft version of which appears as Working Paper 6. That report also describes certain other features which suggest that the length of sickness for which claims are settled may often not be exactly the same as the natural length of sickness.

It has to be accepted that it is impossible to ensure that the data is totally free of imperfections and some are suspected to remain in the data for recoveries at high ages, as discussed later in Section 3.

1.5 The following is a summary of the data under investigation, excluding any data during the deferred periods, but including data for all durations, however long, up to age 65 (for the 1975-78 investigations, data beyond 11 years duration were omitted).

## Individual IP 1991-98 Standard\* Experience, Males, Class 1

Deferred period	Exposed to risk (days)	Recoveries	Deaths	Expiries	Revivals
1 week	1,702,713	8,297	112	358	2
4 weeks	1,664,194	2,174	153	341	11
13 weeks	1,781,454	683	188	375	3
26 weeks	2,107,887	347	170	442	1
52 weeks	941,027	71	66	201	2
All	8,197,275	11,572	689	1,717	19

The exposed to risk was calculated in units of life-days of exposure and it is convenient to state it in those units in the tables in this report. Exposures are calculated using the supposition that a year always contains exactly 365 days (regardless of the calendar). As intensities of recovery and death are expressed as yearly rates, it should be remembered that an exposed to risk expressed in days must be translated into years by division by 365 before being multiplied by a recovery or claim rate to calculate expected recoveries or deaths.

1.6 From the original individual claim records summary records were compiled, separately for each deferred period, classifying the data (i) by age last birthday at date of falling sick and (ii) by sickness duration partitioned into specified intervals chosen as suitable for calculation purposes. The form in which date of birth is stated varies between offices but all offices give the exact date of falling sick. A long-standing error in the interpretation of age at falling sick was discovered and this has been thoroughly investigated. Full details are given in the Appendix, which includes an explanation of the method for calculating age for the data used in this report. Age last birthday at commencement of sickness can be calculated accurately in the large majority of cases and estimated sensibly for the remainder. For the purposes of these graduations, sickness duration has been partitioned as follows:

- (i) for the first 28 days of sickness, into single days;
- (ii) for the remainder of the first year of sickness, into weeks of 7 days, with a final long week of 8 days ending the year;
- (iii) for the next 4 years of sickness, into lunar months of 28 days, with a long month of 29 days at the end of each year;
- (iv) after 5 years of sickness, into single years of 365 days each.

The data is thereby classified in effect into a 2-dimensional array, comprising data cells for each possible age and sickness duration interval. For each such data cell, totals are held of the claim exposure (in units of life-days) and of the related numbers of claims terminating by recovery, death, or claim expiry, and of claim revivals. We are not normally interested in the numbers of expiries, i.e. claims terminated other than by recovery or death.

There were very few claim revivals (19 in all) and they have been treated in the investigation as equivalent to negative recoveries and their numbers have been netted off against numbers of recoveries in the various tables in this report. Whenever reference is made henceforth to numbers of recoveries, it should be understood that this means recoveries net of revivals.

Tabulations of the data, grouped by age and duration, may be seen in Tables 3 and 4.

1.7 The graduation formulae for the transition intensities of recovery and death, to be introduced later, are expressed in terms of the variables  $y$  and  $z$ , where  $y$  is exact age at falling sick and  $z$  is exact duration of sickness, measured in years. When calculating the expected numbers of recoveries or deaths for a given data cell, by multiplying the exposed to risk by a transition intensity, the value of the transition intensity for the mid-point of the duration interval for the cell is used. In principle it is being assumed that the transition intensity is constant over that sickness interval and that the decrements for the cell follow a Poisson distribution with mean equal to the exposed to risk multiplied by the graduated transition intensity. For a group of lives aged  $y$  last birthday at falling sick, the exact age to be used for calculating the graduated transition intensity is taken to be  $y + \frac{1}{2}$ . The scheme of sickness intervals described in Section 1.6 was chosen to balance the wish to use intervals which are not too long seriously to compromise the foregoing assumption, against the slowing down of graduation computations as the array of data is increased in size.

1.8 Whilst detailed sub-division of the data, as described above, is needed for accurately calculating expected decrements and doing other associated mathematical calculations, it is more appropriate for data analysis, for summarising graduation results and for their statistical appraisal etc., to adopt much broader data groupings. For instance, quinquennial age groups are generally more suitable than single ages. Data should be grouped by duration to produce cells as far as possible containing enough data to be meaningful. However, because the exposure volumes are unevenly spread by age and duration, any table of results will almost inevitably include some cells with very sparse data or which are empty.

The mean age last birthday for each quinquennial age group was calculated based on the data for all durations and all deferred periods combined. The sole purpose is for use in plotting graphs where age is the independent variable. The mean ages are as follows:

---

Age group	Under 25	25-29	30-34	34-39	40-44	45-49	50-54	55-59	60-64
Mean age	23.3	27.5	32.5	37.2	42.2	47.2	52.2	56.8	61.1

---

For the data investigations described in Section 3 (for recoveries) and Section 5 (for deaths), the data was grouped into single weeks for the first 30 weeks of sickness (with the mean duration falling in the centre of each week) and then into the following bands:

Sickness period	Estimated mean of band
30-39 weeks	34.4 weeks or 0.66 years
39 weeks-1 year	45.5 weeks or 0.87 years
1-2 years	77.5 weeks or 1.49 years
2-4 years	153.0 weeks or 2.93 years
4-8 years	296.8 weeks or 5.69 years
Over 8 years	568.6 weeks or 10.90 years

The mean durations are used only for plotting graphs where duration is the independent variable.

## 2. PRELIMINARY CONSIDERATIONS

2.1 The investigation into claim recovery and mortality rates for the Individual Males Standard Experience, 1975-78, reported in *C.M.I.R. 12*, Part B, establishes the groundwork for this current investigation into the experience for Individual Males, Class 1, 1991-98. Much the same considerations apply now as were set out in *C.M.I.R. 12*, Part B, Section 2.

2.2 The prime purpose of the investigation is to produce a basis for calculating intensities of transition from the state of sickness by recovery or death in the context of a multi-state model for sickness. A crucial problem is that the data collected for sickness claims under insurance policies does not track sickness throughout its course from its inception. The data records an occurrence of sickness only from the end of the policy deferred period, which may be as short as one week or as long as 52 weeks after the onset of sickness. For DP1, once transition intensities for sickness durations exceeding one week have been produced, it is a reasonable step to extrapolate to zero duration. In the 1975-78 investigation it was judged that the DP4, DP13 and DP26 experiences did not differ from one another or from DP1 to an extent requiring them to be separately distinguished, except for short (4-week) run-in periods immediately following the end of the respective deferred periods. (DP52 was not included in the investigation.) Bundling the deferred periods together enabled joint rates conveniently to be postulated for all deferred periods back to zero sickness duration. It would likewise be convenient if a similar coherence of the experiences for different deferred periods were found in the 1991-98 investigation.

2.3 From the way the data is compiled, dividing the number of recoveries or deaths by the exposed to risk for a data cell will give a central rate of recovery or death for the cell, which is taken to be an estimate of the transitional intensity of recovery or death at the sickness duration at the mid-point of the cell.

## 3. RECOVERY INTENSITIES – INVESTIGATION

3.1 Initial comparison of the observed recovery rates indicated differences in the overall level of recovery rates between deferred periods and recognition of these differences has been a feature of the investigations and graduations. Examination of

the rates cross-classified by age and duration for each deferred period suggested that the effects of age and duration as factors might be largely independent of each other and that multiplicative models should be explored of the form:

$$\rho(y+z, z) = F(y).G(z) \quad (1)$$

where  $\rho(y+z, z)$  is the recovery intensity at attained age  $y+z$  and sickness duration  $z$ ,  $y$  being the age at falling sick.  $F(y)$ ,  $G(z)$  are respectively functions of  $y$  and  $z$ .

Before speculating about possible mathematical forms for the two functions it is helpful to express  $F(y)$  and  $G(z)$  in terms of numerical values derived from marginal rates for the (age  $\times$  duration) grouped arrays into which the data has been formed for each deferred period, as explained in Section 1.8. The method which has been used for calculating maximum likelihood values for  $F(y)$  and  $G(z)$  is described in *C.M.I.R.* **12**, Part B, 3.2. The actual numerical values themselves are of less interest than the patterns they reveal when displayed graphically.

3.2 If the pair  $\{F(y)\}$ ,  $\{G(z)\}$  is a particular solution to the estimation of the required factors, then  $\{s.F(y)\}$ ,  $\{1/s.G(z)\}$ , where  $s$  is any scalar, is a general solution. In order to control both the stability of the iterative process of determining best estimates and the comparability of the factors as between different deferred periods, the formulae which were actually fitted took the form:

$$\rho(d, y+z, z) = S(d).F(d, y).G(d, z) \quad (2)$$

where  $d$  symbolises the deferred period (i.e.  $d \in \{DP1, DP4, DP13, DP26, DP52\}$ ) and where  $S(d)$  is a coefficient representing the relative overall level of rates for deferred period  $d$  whilst  $F(d, y)$  and  $G(d, z)$  are specific to deferred period  $d$ . The coefficients  $S(d)$  were determined separately as prior estimates, as explained later. Their actual values were:

---

DP1	DP4	DP13	DP26	DP52
1.000000	1.424520	1.203432	0.905404	0.639256

---

As a further stabilising control, the values of  $F(d, y)$  were scaled so that for each deferred period their weighted mean over all data (using exposed to risk as weights) is equal to 1.

3.3 Graphs of  $F(d, y)$  for each deferred period are shown in Figure 1. After age 30, DP1, DP4 and DP13 appear to show a similar trend and are joined after age 45 by DP26 and DP52. The disparity between the deferred periods at younger ages may be largely a result of scattering due to small numbers. For example, the outlier for DP26 for the youngest age group is based on only 4 recoveries; for DP52 there are only 13 recoveries for all ages under 40.

There is a fairly steady decline in the recovery factors over a major part of the age range but for the highest age group the graphs have levelled out and there is even an indication of a slight upturn, suggesting that recovery rates for 60-64 could be



higher than for 55-59. As this seems a dubious feature, the data for the highest age group was scrutinised carefully for signs of anything odd and enquiries were made as to the possibility of data errors. Although firm evidence is lacking and it is not possible to be definite, the outcome of these enquiries is that it is suspected that a number of claim terminations at high ages which should have been coded as expiries may have been miscoded as recoveries.

Given these doubts about its reliability, it seemed advisable that data for ages 60-64 should be excluded from the graduation. It was however retained provisionally, pending graduation trials to compare the effect of either including or excluding data for that age group, which might reveal more about the problem and possibly show if it was confined to certain areas of the data (e.g. particular deferred periods or sickness durations).

Discussion of the  $G(d, z)$  values is deferred to Section 3.6.

3.4 Age and duration factors for all deferred periods combined were found by fitting the following alternative model:

$$\rho(d, y + z, z) = S(d).F(y).G(z) \quad (3)$$

where the  $S(d)$  coefficients allow for differences between deferred periods, but  $F(y)$  and  $G(z)$  are to be common estimates for all deferred periods. Estimates of  $S(d)$  were based on the data for sickness after 30 weeks only (with DP52 contributing only after 52 weeks), partly because it is only after 30 weeks sickness that there are contributions of data from all of DP1, DP4, DP13 and DP26, allowing for a possible 4-week run-in period for DP26. The  $S(d)$  values were determined so as to equalise actual and expected recoveries after 30 weeks for each deferred period, conditional upon the fitted values for  $F(y)$  and  $G(z)$ . These values of  $S(d)$  were separately used as the pre-determined values for the model described in Section 3.2.

3.5 The  $F(y)$  factors for the combined model are plotted in Figure 2, showing a rise with increasing age from the youngest ages up to the 30-34 age group and then a closely linear decline over the 35-39 to 55-59 age groups and the questionable upturn for ages 60-64 already discussed.

3.6 When plotting  $G(d, z)$  and  $G(z)$  factors it was found best to plot the logarithm of  $G$  against a transform of the duration variable  $z$ . For the 1975-78 investigation it was found that plotting  $\ln\{G(z)\}$  against the square root of  $z$  gave a nearly linear trend for the first 12 months, though the square root transformation was less satisfactory for longer durations. In the present investigation, the transformation  $t = w/(1 + k.w)$  (where  $k$  is a constant and  $w$  is the duration in weeks) has been adopted. A value  $k = 0.025$  was found suitable for plotting the graphs in this Section. In graduations,  $k$  has been treated as an unknown parameter whose optimum value is to be determined. An advantage of this transformation is that, as  $z$  increases without limit,  $t$  tends towards an upper limit of  $1/k$ : e.g. to  $t = 40$  in the accompanying graphs. One can be reasonably sure that should rates for very long durations be calculated by extrapolating a graduation formula beyond the graduation range, the results should not be nonsensical.

3.7 Values of  $G(d, z)$  calculated separately for each deferred period are plotted in Figure 3. The graphs are close to one another in the right-hand half of the figure, i.e. for  $z > 30$  weeks. This is partly helped by the method of calculating the  $S(d)$ . Also, for  $z > 30$  weeks the data is grouped into wider bands than the single weeks of sickness used for the first 30 weeks, which reduces the apparent variability. The appearance of approximately 4-week run-in periods for DP4 and DP13, as also observed in the 1975-78 experience, is clear, though the evidence for a DP26 run-in period is less clear. The run-in phenomenon was discussed at some length in *C.M.I.R.* **12**, Part B, Section 3.3, when the IP Sub-Committee came to the conclusion that, on balance, it was probably due to policyholders not bothering to pursue short-lasting claims, so that sicknesses resulting in recovery very soon after the deferred period were under-represented in the data.

Quite apart from the run-in shown by the first 4 weeks of DP4 the graphs of DP4 and DP1 diverge for durations shorter than around 15 weeks. This is a troublesome feature. The attitude taken for the graduation has been to regard DP1 as the norm for the experience as a whole and DP4 as departing from the norm. This attitude is influenced by the wish to place confidence in DP1 for piloting the course of rates down to zero duration, as explained in Section 2.

There is a noticeable dip in the curve for DP13 from the 20th week until it climbs to a spike in the 27th week. A similar but rather more erratic feature can be seen for DP1 in the four weeks before the 27th week, when it also hits a peak. It has been suggested that this pattern may reflect a practice by offices of reviewing claims at certain durations, one obvious time being after 26 weeks' sickness, resulting in a heaping up of reported recoveries at such points. Indeed, an examination of Figure 3 may suggest an approximately monthly cyclical pattern to recoveries during the first 6 months.

It may be surmised that a straight line might be fitted for  $z > 30$ , and that this could well be extended back to around the 17th week, where the DP13 run-in segment joins the main curve. There seems however to be a change of slope between about the 6th and 17th week and again prior to the 6th week. Taken together with the DP4 divergence already mentioned, problems may be expected with graduating the experience at the shorter durations, where the bulk of the experience is concentrated.

3.8 The result of fitting  $G(z)$  to the combined data for all deferred periods combined (but segregating the data for the first 4 weeks of DP4, DP13 and DP26) is shown in Figure 4. The general picture is similar to what was found in the 1975-78 investigation as will be seen by reference to *C.M.I.R.* **12**. There is quite a lot of fluctuation in the main curve between the 6th and 17th weeks, partly reflecting the combination of DP4 with DP1 over a duration span when they are individually disparate.

Notwithstanding the foregoing observations, it was hoped that it might be possible to graduate the recovery rates on the basis that, subject to scaling the rates for each deferred period to reflect the overall differences in recovery rates between deferred periods, a common graduation formula could be found to fit all the data. It was recognised however that DP4 would pose special problems.

#### 4. RECOVERY INTENSITIES – GRADUATION

4.1 In the form in which the data was prepared for graduation, exposed to risk was recorded in units of life-days and sickness duration in units of days. This was the most

convenient way of storing the data. On the other hand, both mortality rates and recovery rates are expressed in the conventional manner as rates per annum; hence the expected number of deaths or recoveries for a given quantum of exposed to risk is to be calculated by multiplying the exposure in days by the mortality or recovery rate per annum and the result divided by 365.

Within the graduation formula itself it is convenient to work in duration units of weeks for the most part. The duration stated in years is given the symbol  $z$ . This may be translated into weeks of duration, given the symbol  $w$ , by the conversion formula  $w = 365z/7$ .

The variable  $t(z)$  is a function of duration defined as  $t = w/(1 + k.w)$  where  $w = 365z/7$ .

Although rates are defined in terms of natural age,  $y$ , (the exact age at sickness inception), it is convenient within the graduation formulae to measure age from an origin of 50, by the variable  $Y$ , where  $Y = (y - 50)$ . Some scaling is done within the formulae in order to keep the values of fitted parameters to broadly similar magnitudes.

For convenience in applying the graduation formula in the context of the multi-state model, it is assumed that after 5 years of sickness (i.e. for  $z > 5$ ), recovery rates depend only upon attained age  $y + z$ .

It is hoped that this preliminary explanation of symbols that appear later will help avoid confusion.

4.2 The graduation of recovery rates depends upon overcoming the problem of finding a suitable formula to fit the complicated patterns of the bivariate data, using the data investigations previously described for clues. Very many trial graduations were carried out. The initial stage was to research formulae of the form:

$$\rho(d, y + z, z) = S(d).F(y).\exp\{-b.t(z)\} \quad (4)$$

where  $\rho(d, y + z, z)$  is the recovery intensity for deferred period  $d$  for age  $y$  at falling sick and sickness duration  $z$ . The symbols  $S(d)$  and  $F(y)$  have been explained previously and the exponential term expresses what is believed to be the basic functional form of the duration factor  $G(z)$ .

In light of the findings in Section 3 it was thought that the data for durations over 30 weeks would be more amenable to graduation than the data for durations up to 30 weeks. An initial experimental graduation was conducted on the data for  $z > 30$  weeks but leaving out data for ages over 59, as the 60-64 age group was expected to be difficult. It was pleasing to discover that a good graduation was obtained with a very simple formula:

$$\rho(d, y + z, z) = S(d).(1 - a.y).\exp\{-b.t(z)\} \quad (5)$$

involving only eight parameters in total:  $a$ ,  $b$ ,  $k$  and five for the  $S(d)$  coefficients.

4.3 Further trial graduations were made, extending the duration range downwards by one week at a time, whilst omitting data for the 4-week run-in periods and continuing to exclude data for ages 60-64. At each successive stage, if it was found that the existing graduation formula did not give a good graduation result, various experimental modifications were made to the formula until it did. By this method, a

formula was steadily developed with more terms and parameters than at the start, which seemed satisfactory down to about 5 or 6 weeks duration. The original linear expression for  $F(y)$  had by then been replaced by:

$$F(y, z) = \exp\{a_1.Y + a_2.Y^2 + a_3.Y^3 + a_4.Y.t(z)\} \quad (6)$$

where the term  $a_4.Y.t(z)$  allows for the presence of an element of (age  $\times$  duration) interaction.

Both  $F(y, z)$  and  $G(z)$  are now in the form of exponential expressions, as will be certain adjustments to the graduation formula to be introduced later. It is therefore convenient to make a notational change at this point by substituting:

$$f(y, z) = \ln\{F(y, z)\}, g(z) = \ln\{G(z)\}, \text{ and } s(d) = \ln\{S(d)\}, \text{ so that}$$

$$\ln\{\rho(d, y + z, z)\} = s(d) + f(y, z) + g(z) \quad (7)$$

The original linear formula for  $g(z)$  needed adjustment for a change of slope in  $\ln(\rho)$  for shorter durations and  $w = 26$  weeks was found to be a suitable point at which to effect a change in the value of the parameter  $b$ . Thus

$$g(z) = -b_1.t(z) \text{ for } w \leq 26;$$

$$g(z) = -b_1.t(26) - b_2.(t(z) - t(26)) \text{ for } w > 26 \quad (8)$$

A further special adjustment to the value of  $b$  was also required for DP4 durations between 8 and 16 weeks. This was achieved by adding to (7) a term  $q(z)$ , where:

$$q(z) = -r_1.(16 - w)/8 \text{ for } 8 \leq w < 16; q(z) = 0 \text{ outside this range} \quad (9)$$

Graphs such as those presented in Section 3 were useful guides to formulating suitable formula adjustments.

4.4 A major difficulty was then encountered with trying to accommodate the data for the first few weeks of DP1 into the graduation. It was found that the pattern of rates with respect to age for the first few weeks for DP1 differed somewhat from the pattern applying at longer durations and there was much unevenness in the observed rates across the data cells during those early weeks of DP1 claims. As a substantial proportion of the total data is concentrated in those first few weeks of DP1 any marked deviations between actual and expected recoveries can be statistically very significant. To reflect the difference in the first 4 weeks of sickness an extra term  $h(y + z, z)$  was added to (7), where  $h$  is a polynomial evolved by trial and error, bringing in as few parameters as would suffice to give a reasonable fit to the data. The formula is:

$$h(y + z, z) = \{t(4) - t(z)\} \cdot \{h_0 + h_1.Y + h_2.t(z)\} \text{ for } w < 4;$$

$$h(y + z, z) = 0 \text{ for } w \geq 4 \quad (10)$$

The expression  $\{t(4) - t(z)\}$  reduces the value of  $h$  to zero as  $z$  increases from 0 to 4 weeks.

4.5 When the age range was enlarged to include data cells for ages 60-64, to pursue the question raised in Section 3.3, widespread deviations between actual and expected recoveries appeared. These were collectively too significant to be ascribed to random deviations from the trend set by the graduation, for rates at higher ages to decline with increasing age. Nor were they confined to particular areas of the data. It was therefore finally decided to exclude data for ages 60-64 from the graduation of recoveries entirely. This means that for graduated recovery rates for these top ages, reliance is being placed on extrapolating the graduation formula beyond age 59.

4.6 There remained the problem of the 4-week run-in periods. The picture given by Figure 4 is very similar to what is shown in *C.M.I.R.* 12 for the 1975-78 experience, when the graduation formula, using a single parameter, allowed for a linear run-in adjustment applying equally to DP4, DP13 and DP26 weeks, though the amount and clarity of the data supporting this form of adjustment was much less for DP13 and DP26 than for DP4.

A single parameter adjustment for DP4 again proved satisfactory for the 1991-98 experience (as Figure 4 would lead one to expect) and is tolerably satisfactory for DP13 taking the 13-17 week period as a whole. Although in the case of DP13 a closer fit could have been obtained by increased parameterisation of the DP13 run-in factor, the amount of data is rather small and the reason for its odd shape is not understood and there may be a risk of over-graduation. It was felt that a simple one-parameter adjustment factor sufficient to reach a close balance of actual and expected recoveries over the 4-week period is as much modelling as can be justified.

The adjustments to (7) for the DP4 and DP13 4-week run-in periods take the form:

$$\text{for DP4: } r(w) = -r_2 \cdot (8 - w)/4 - r_1 \text{ for } 4 \leq w < 8; \quad r(w) = 0 \text{ for } w \geq 8 \quad (11)$$

$$\text{for DP13: } r(w) = -r_3 \cdot (17 - w)/4 \text{ for } 13 \leq w < 17; \quad r(w) = 0 \text{ for } w \geq 17 \quad (12)$$

In the case of DP4 the run-in adjustment for the period 4-8 weeks needs to connect up with the adjustment  $q$  for 8-16 weeks mentioned earlier.

A simple linear run-in adjustment for DP26 did not make an appreciable improvement to the graduation for weeks 26-30 and as the data is thin and as evidence for a run-in period is unclear it was decided not to include a run-in factor in the graduation for DP26. Indeed, there is a reasonably good balance of actual and expected recoveries for the 26-30 weeks period as a whole without introducing a run-in adjustment.

4.7 The formula may be summarised in simple symbolic form as follows:

$$\ln\{\rho(d, y, z)\} = s + f + g + h + q + r \quad (13)$$

where  $s$  is a scalar varying by deferred period;

$f$  is a polynomial for the age-related dependency;

$g$  is a piecewise linear expression for the trend of  $\ln(\rho)$  as a function of the duration variable  $t$ , where  $t = w/(1 + k.w)$ ; the slope changes after 26 weeks;

$h$  is an adjustment for the first 4 weeks of sickness;

$q$  applies only to DP4 and is a linear expression over the duration range 8 to 16 weeks only; elsewhere its value is zero; and

$r$  is a linear expression for the 4-week run-in periods for DP4 and DP13 with different parameters for DP4 and DP13. Its value is zero outside the run-in periods.

4.8 Full details of the component terms of (13) are as follows:

$$s = s(d) \text{ and } d \in \{DP1, DP4, DP13, DP26, DP52\},$$

$$f = a_1.(Y/100) + a_2.(Y/100)^2 + a_3.(Y/100)^3 + a_4.(Y/100).t(Z),$$

$$g = -b_1.t(Z) \text{ for } w \leq 26;$$

$$g = -b_1.t(26) - b_2.\{t(Z) - t(26)\}$$

$$= -b_3 - b_2.t(Z) \text{ where } b_3 = (b_2 - b_1).t(26) \text{ for } w > 26,$$

$$h = \{t(4) - t(Z)\}.\{h_0 + h_1.Y/100 + h_2.t(Z)\} \text{ for } w < 4;$$

$$h = 0 \text{ for } w \geq 4,$$

$$q = -r_1.(16 - w)/8 \text{ for DP4 if } 8 \leq w < 16;$$

$$q = 0 \text{ otherwise,}$$

$$r = -r_2.(8 - w)/4 - r_1 \text{ for DP4 if } 4 \leq w < 8;$$

$$r = -r_3.(17 - w)/4 \text{ for DP13 if } 13 \leq w < 17;$$

$$r = 0 \text{ otherwise,}$$

In the above expressions:

$$Z = z \text{ for } z \leq 5;$$

$$Z = 5 \text{ for } z > 5, \text{ where } z \text{ is the exact duration in years,}$$

$$w = (365/7).Z, \text{ i.e. } Z \text{ translated to units of weeks,}$$

$$t = w/(1 + k.w),$$

$y$  = exact age (in years) at sickness inception;

$Y = y - 50$  for  $z \leq 5$ ;

$Y = y - 55 + z$  for  $z > 5$ ,

(It may be noted that for  $z > 5$ ,  $Y + Z = (y - 55 + z) + 5 = x - 50$ , i.e. attained age relative to an origin of 50.)

$s(d)$ ,  $k$ ,  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$ ,  $b_1$ ,  $b_2$ ,  $b_3$ ,  $h_0$ ,  $h_1$ ,  $h_2$ ,  $r_1$ ,  $r_2$ ,  $r_3$  are constants.

4.9 The parameter values and their estimated standard errors are:

$i$	Symbol	$c(i)$	$SE(i)$
1	$s(1)$	3.036467	0.029720
2	$s(2)$	3.316474	0.045410
3	$s(3)$	3.025743	0.048326
4	$s(4)$	2.856549	0.053863
5	$s(5)$	2.511347	0.077871
6	$k$	0.016000	0.001285
7	$a_1$	-3.080944	0.152614
8	$a_2$	-6.419924	1.445250
9	$a_3$	20.048953	6.486295
10	$a_4$	-0.113352	0.009593
11	$b_1$	0.195291	0.005383
12	$b_2$	0.108662	0.009402
13	$h_0$	0.198289	0.012853
14	$h_1$	-0.724805	0.069255
15	$h_2$	0.047682	0.009067
16	$r_1$	0.622543	0.039663
17	$r_2$	1.197880	0.056345
18	$r_3$	1.830356	0.120325

Leaving aside the various piecewise adjustments applying to short-term durations, the ratio of recovery intensities for a given deferred period to those for DP1 is given by the expression

$$\phi(d) = \exp\{s(d)\} / \exp\{s(1)\}$$

which evaluates to:

Deferred period	DP1	DP4	DP13	DP26	DP52
$d$	1	2	3	4	5
$\phi(d)$	1	1.323139	0.989333	0.835339	0.591484

These values may be compared with the trial values quoted in Section 3.2, which have a slightly different meaning and which were estimated on a different basis from the graduation formula parameters, though the relative order is similar. The value for  $\phi(3)$  is nearly unity and, apart from the 4-week run-in period for DP13 it may be concluded that there is no evidence of a difference in recovery rates between DP1 and DP13.

Although  $\phi(2)$  is high, it should be remembered that for durations under 16 weeks the rates for DP4 are reduced by the operation of the factor involving  $r_1$ .

Table 1 gives values for  $\rho$  for specimen ages and durations for each deferred period.

4.10 A comparison of actual recoveries with those expected by the graduated rates is given in Table 3. To avoid voluminous tables, the results for all deferred periods have been combined except for the first 4 claim weeks (the run-in periods) for DP4, DP13 and DP26. (Although the graduation formula has given no recognition to a distinct run-in period for DP26 it is nevertheless of interest to show the comparison of actual and expected recoveries for the putative run-in period.) The data is grouped by age and duration sufficiently to provide enough recoveries in a cell in most cases for meaningful appraisal.

In calculating the standardised deviations shown in the table, continuity corrections have been applied. For those cells where there are fewer than five expected deaths, the numerical value of the standardised deviation (being liable to wide random variation) is not stated but the sign of the deviation is indicated. Values of the  $\chi^2$  statistic were calculated by summing the squares of standardised deviations (incorporating the continuity corrections). This gives a value, based on the results grouped into the 197 cells shown in Table 3, of 220.3. Allowing for a loss of 18 degrees of freedom by parameter fitting, the result is statistically significant, with a probability value of about 0.025. (It could be held however that the extensive trials of alternative graduation formulae are equivalent to the absorption of a further number of degrees of freedom and that the result is actually more significant than this.)

The high  $\chi^2$  value is affected by a few outlier deviations. In total there are 14 standardised deviations with absolute value exceeding 2 (two of them being greater than 3), whereas about 10 would be expected for a Normal distribution. In weeks 21-26 and weeks 26-30 there are three of these high standardised deviations in a range of 14 cells, reflecting a deliberate decision not to attempt to track closely the crude rates over those weeks, in the belief that the reported recovery rates are to some degree distorted and not following a natural progression in the region of 26 weeks' duration.

In light of the foregoing observations it is considered that the graduation, though less than ideal, is acceptable.



4.11 Whilst tables of comparisons of actual and expected recoveries are not given for separate deferred periods, graphs based on the marginal rates by duration (for all ages combined) but using the finer durational groupings specified in Section 1.8 are given in Figure 5. They may be compared with Figure 2. One feature shown by the graphs is how the graduated curves cut through the waves in the crude rates around the spike in the 27th week.

Figure 6 provides a visual comparison of the graduated recovery rates for the different deferred periods as a function of sickness duration up to 10 years. The graphs relate to a life aged 40 at date of falling sick; the picture would be similar for other ages. The Figure illustrates the main graduation features, e.g. the run-in periods for DP4 and DP13, the effect of the formula adjustment for sickness up to 4 weeks for DP1, and the relative differences between deferred periods at longer durations. It may be noted that after the 4-week run-in period, the curves for DP1 and DP13 are effectively indistinguishable.

It may be seen from the standardised deviations for the totals of actual and expected recoveries for all deferred periods and all durations combined in Table 3(e) that the overall fit of the graduation by age is satisfactory. This is illustrated by Figure 7.

4.12 The IP Sub-Committee, when investigating the 1975-78 experience for *C.M.I.R.* 12, took the view that the 4-week run-in periods identified for DP4, DP13 and DP26 should be ascribed to a tendency for policyholders not to report very short periods of sickness which would give rise to claims for only small amounts of benefit and that the natural progression of recovery rates for the purpose of the multi-state model could be assumed to be the graduated rates calculated without the run-in adjustments. Furthermore it was assumed to be valid for all deferred periods to extrapolate rates from the graduation formula down to zero duration.

In the present investigation it was found appropriate to allow for differences in recovery rates between deferred periods and the extrapolation of rates to zero duration is more problematical. Once again we may start by disregarding the run-in adjustments for the first 4 claim weeks for DP4 and DP13 (there is no such adjustment for DP26). For DP13, DP26 and DP52 the graduated rates are then simple multiples  $\phi(d)$  of the DP1 rates for the sickness durations covered by the graduation data. It seems reasonable to assume, for the want of other evidence, that those same multiples apply at all durations down to zero. In the case of DP4 the question is complicated by the graduation formula adjustment for durations 8-16 weeks. If the latter adjustment were assumed to continue in its linear downward trend in the extrapolation of rates to zero duration, the resulting rates would appear far too low for common sense. It seems better to assume that the 8-16 weeks' adjustment remains at its 8 weeks value for all durations under 8 weeks. This is perhaps the best that can be done with the information available, as a basis for including DP4 in the full implementation of the multi-state model. No final conclusion is reached and the subject is not pursued further in this report but will be more appropriately considered in connection with the investigation of sickness inception rates for the full implementation of the multi-state model for sickness.

## 5. MORTALITY INTENSITIES – INVESTIGATION

5.1 A preliminary investigation of the experienced mortality rates was made, similar to that for recoveries described in Section 3. Sickness duration, rather than age, is the

dominant explanatory variable for mortality intensity except in the very long term. In the early weeks of sickness mortality rates in aggregate (i.e. without differentiating by cause of sickness) rise rapidly, reaching a peak after 4 or 5 months, and then show a steady decline until there is a residue of relatively long-term disabled survivors. There are not enough deaths in the experience for a clear and detailed examination of the pattern of mortality rates by duration for separate deferred periods and factors  $F(y)$ ,  $G(z)$  for a multiplicative model were fitted only for all deferred periods combined. Although a broad comparison of the levels of mortality between deferred periods had suggested possible differences, particularly that mortality for DP1 might be lighter than for other deferred periods, this question was left to be considered later, based on graduation results.

5.2 The multiplicative model  $\nu(y+z, z) = F(y).G(z)$ , when the values of  $F(y)$  and  $G(z)$  had been calculated, was judged from  $\chi^2$  test values to give a rather poor fit to the data. This implied that a suitable graduation formula may not take a multiplicative form. Nevertheless this method of analysing the data initially was felt to be useful.

5.3 The graph of age-related factors in Figure 8 appears to have a slight upwards curvature in the upper age range, suggesting that a Gompertz formula should be tried. It should be noted that most deaths are at ages of 40 and above, there being fewer than 50 deaths below age 40.

5.4 The duration-related factors in Figure 9 are plotted against  $\ln(z)$ . The two peaks, in the middle of the graph, relate to the 18th and 22nd weeks of sickness. After falling steadily, the graph eventually levels out; the three right-hand points on the graph are for sickness periods exceeding 2 years. The last point, which represents sickness exceeding 8 years, may be showing the upturn that we would expect at very long durations when, it is thought, mortality is more likely to be a function of increasing age attained than of sickness duration itself.

## 6. MORTALITY INTENSITIES – GRADUATION

6.1 Figure 9 gives a much more clearly defined shape for the mortality curve as a function of duration than was the case with the 1975-78 experience and various well-known mathematical curves could be considered as candidates for a graduation formula. However trials quickly showed that the curve which was derived from the Weibull distribution (*C.M.I.R.* **12**, Part B, 38) in graduating the 1975-78 experience appeared suitable for the current experience. Thus we postulate a formula  $g(z)$  to represent the main hump-backed feature of Figure 8:

$$g(z) = \frac{a \cdot \exp\{-b/(z+c)^n\}}{(z+c)^{n+1}} \quad \text{where } a, b \text{ and } c \text{ are constants} \quad (14)$$

To the extent that a multiplicative model is found appropriate, the coefficient  $a$  should be replaced by an age function  $f(y)$ , so that  $\nu(y+z, z) = f(y).g(z)$ . In the graduation of the 1975-78 experience,  $f(y)$  took the form of a quadratic expression in  $y$ .

By such a formula,  $\nu$  tends toward zero as  $z$  increases indefinitely and it was recognised that an additional component in the graduation formula would be needed to provide for a long-term trend towards mortality rising with attained age, as noted in

Section 5. It was surmised that, as for the 1975-78 graduation, a Gompertz term,  $r \cdot \exp(s \cdot x)$ , where  $x$  is attained age, might be suitable.

6.2 Trials of several formulae based on the approach described above were made, based on the data for all deferred periods combined. Two features became apparent: firstly, parameter estimates of the value of  $n$  in the Weibull formula were so close to 1 that it was possible to set  $n = 1$  in the graduation formula *ab initio* without noticeably affecting the resulting graduation result; secondly, the graduation fit was not sufficiently improved by replacing the coefficient  $a$  by a function  $f(y)$  in the Weibull formula to justify the additional parameters.

This led to the following graduation formula:

$$v(y+z, z) = \frac{a \cdot \exp\{-b/(Z+c)\}}{(Z+c)^2} + (r/100) \cdot \exp\{s \cdot (Y+Z)\} \quad (15)$$

where  $a$ ,  $b$ ,  $c$ ,  $r$  and  $s$  are constants. (In the formula,  $r$  has been divided by 100 in order to make the size of  $r$  commensurate with the other parameters.)  $Y$ ,  $Z$  are in accordance with the definitions in Section 4.8, i.e.:

$$Z = z \text{ for } z \leq 5;$$

$$Z = 5 \text{ for } z > 5, \text{ where } z \text{ is the true duration in years,}$$

$$Y = y - 50 \text{ for } z \leq 5;$$

$$Y = y - 55 + z \text{ for } z > 5, \text{ where } y \text{ is exact age at falling sick.}$$

6.3 For this preliminary graduation the parameter values and their estimated standard errors are:

$i$	Symbol	$c(i)$	$SE(i)$
1	$a$	0.183560	0.017062
2	$b$	1.096534	0.059442
3	$c$	0.126771	0.017961
4	$r$	0.230616	0.084904
5	$s$	0.152179	0.027756

6.4 Using this preliminary graduation basis to calculate expected deaths, it was found that for data for all deferred periods combined there was a good fit between actual and expected deaths, ranging over ages and sickness durations. The same basis was then applied to data for each deferred period separately, to ascertain the extent of differences in mortality levels between deferred periods, with the following results:

Deferred period	Actual deaths $A$	Expected deaths $E$	$A/E\%$	$(A-E)/\sqrt{E}$	$(A-E)^2/E$
1 week	112	141.3	79.3	-2.46	6.08
4 weeks	153	156.4	97.8	-0.27	0.07
13 weeks	188	161.2	116.6	+2.11	4.46
26 weeks	170	168.7	100.8	+0.10	0.01
52 weeks	66	61.3	107.7	+0.60	0.36
Total	689	689.0			10.98

The experience for DP1 is significantly lighter than for the other deferred periods. DP13 is heavy, but if DP1 were excluded from the comparison, it would not appear statistically extreme compared with the other deferred periods. The  $\chi^2$  total in the final column is significant by a 5% test criterion. Inspection of the results broken down into age and duration groups did not suggest that the differences in total between deferred periods could be ascribed to particular ranges of ages or durations. It was inferred that DP1 lives as a whole, for example, experience lighter mortality than lives insured for other deferred periods.

It was consequently considered that the graduation basis should be modified so as to allow for overall differences in mortality between deferred periods, such as was done for the graduation of recoveries.

6.5 The formula set out in Section 6.2 was accordingly modified to include scalar multipliers  $q(d)$ , where  $d$  identifies the deferred period. This ensures that total actual and expected deaths are approximately equal for each deferred period. The overall graduation was thereby greatly improved. However it was observed that most of the improvement came from the parameter differentiating DP1 from the rest; there was relatively little gain in maximum likelihood from including parameters to differentiate between DP4, DP13, DP26 and DP52. Increasing the total number of parameters from 5 to 9 had proved to be over-elaborate. It was therefore decided that it is sufficient to retain the formula set out in Section 6.2 providing that a scalar adjustment is included in respect of DP1 only.

6.6 The final graduation basis is thus as follows:

$$v(y+z, z) = \left( \frac{a \cdot \exp\{-b/(Z+c)\}}{(Z+c)^2} + (r/100) \cdot \exp\{s \cdot (Y+Z)\} \right) \cdot q(d) \quad (16)$$

where  $q(d) = q$  for DP1;  $q(d) = 1$  for other deferred periods;

and where  $a, b, c, r, s, q$  are constants;

and where  $Y, Z$  are, as stated before:

$$\begin{aligned} Z &= z \text{ for } z \leq 5; \\ Z &= 5 \text{ for } z > 5, \text{ where } z \text{ is the true duration in years,} \end{aligned}$$

$$Y = y - 50 \text{ for } z \leq 5;$$

$$Y = y - 55 + z \text{ for } z > 5, \text{ where } y \text{ is exact age at falling sick.}$$

6.7 For this final graduation the parameter values and their estimated standard errors are:

$i$	Symbol	$c(i)$	$SE(i)$
1	$a$	0.188906	0.017845
2	$b$	1.081708	0.060205
3	$c$	0.132474	0.019228
4	$r$	0.257331	0.089870
5	$s$	0.149466	0.026343
6	$q$	0.744739	0.039118

Corresponding to the table in Section 6.4, the comparison of total actual and expected deaths by deferred period is shown below and is considered satisfactory.

Deferred period	Actual deaths $A$	Expected deaths $E$	$A/E\%$	$(A-E)/\sqrt{E}$	$(A-E)^2/E$
1 week	112	112.0	100.0	0.00	0.00
4 weeks	153	165.9	92.2	-1.00	1.00
13 weeks	188	169.6	110.8	+1.41	2.00
26 weeks	170	177.1	96.0	-0.53	0.28
52 weeks	66	64.4	102.5	+0.20	0.04
Total	689	689.0			3.32

6.8 Tables 2(a) and 2(b) give values of  $v$  for specimen ages and durations for (a) DP1 and (b) DP4-DP52. The values in Table 2(a) are simply equal to those in Table 2(b) multiplied by the value of  $q$  (0.744739).

The graduated intensities show little variation by age at falling sick, except at older ages, for sickness durations over the short to medium term (up to 2 or 3 years). In Figure 10 mortality intensities for DP4-DP52 are plotted as a function of duration for specimen ages 20 and 60 at falling sick for durations up to 10 years and 5 years respectively.

For durations over 5 years the graduated intensities are functions of attained age  $x$  only. Figure 11 gives graphs for DP1 and DP4-DP52 respectively in comparison with AM92(ult). For the younger part of the age range the graduated intensities are roughly the same as AM92(ult) with a constant addition of about 0.0039 for DP1 or

0.0054 for other deferred periods, with the difference becoming wider at older ages. The size of these differences is linked to the decision, for practical purposes, to assume that rates change only by attained age after 5 years. The effect of this is that the Weibull element of the graduation formula contributes a level amount to the calculated graduation intensities at all ages and durations after 5 years, equal to its 5-year value, which is 0.004335 for DP1 and 0.005803 for other deferred periods.

It is assumed that for all deferred periods the graduated rates may be extrapolated by the formula down to zero sickness duration.

6.9 A comparison of actual deaths with those expected by the graduated rates for DP1, DP4-DP52 combined and all deferred periods combined and in sufficiently broad age and duration groupings to be meaningful is given in Tables 4(a) to 4(c). Continuity corrections have been applied in calculating standardised deviations.

A  $\chi^2$  test was carried out for goodness of fit of the results in Table 5. For this purpose data in adjacent cells were amalgamated where necessary to ensure a minimum of an expected 5 deaths in a cell. This resulted in a total of 48 cells and a  $\chi^2$  value, allowing for continuity corrections, of 43.40. Given that 6 parameters were estimated from the data, the result is satisfactory. There are 2 cells out of the 48 for which the standardised deviation between actual and expected deaths exceeds 2.0, which is again satisfactory.

For durations over 5 years there were 116 actual deaths compared with 101.8 expected. In light of this comparison, the method of determining graduated intensities after 5 years, referred to in Section 6.8, does not appear to be unreasonable.

Tables with a more detailed breakdown of results than in Tables 4(a) to 4(c) would not be worthwhile in view of the number of deaths. However Figure 12 gives a visual comparison of actual and expected mortality ratios by duration (for all deferred periods and ages combined) in more detail than the final column of Tables 4(a) to 4(c).

## 7. DISABILITY ANNUITIES – CONTINUATION TABLES

7.1 From the graduated recovery and mortality intensities, double decrement tables were calculated for three specimen ages for each deferred period, as set out in Tables 5(a) to 5(e). In the tables,  $l(y, z)$  represents the number of claims remaining in force at exact sickness duration  $z$  from 100,000 claims commencing at the end of the specified deferred period for lives of exact age  $y$  at the date of falling sick. Decrements by recovery and death in the succeeding interval of sickness duration are labelled  $r(y, z)$  and  $d(y, z)$  respectively. As the tables are given for illustrative purposes only, they show only quinquennial durations after 10 years.

7.2 The tables were constructed by numerical integration, based upon the computational procedures developed for the multi-state model in *C.M.I.R.* 12, Part D.

For a life that fell sick at exact age  $y$ , let the probability that the life remains sick continuously for a period  $z$  until attained age  $x$ , where  $x = y + z$ , be  $ps(x)$ .

Similarly,  $ps(x + h)$  is the probability that the life survives to age  $x + h$  having been continuously sick up from age  $y$  up to age  $x + h$ . Let  $pr(x + h)$ ,  $pd(x + h)$  be the respective probabilities that a life falling sick at age  $y$  will recover or die in the interval  $(x, x + h)$ .

Applying a trapezium rule,  $pr(x + h)$  and  $pd(x + h)$  may be approximately evaluated by the formulae:

$$pr(x+h) = \{\rho(x).ps(x) + \rho(x+h).ps(x+h)\}.h/2$$

$$pd(x+h) = \{v(x).ps(x) + v(x+h).ps(x+h)\}.h/2$$

from which:

$$ps(x+h) = ps(x). \{1 - [\rho(x) + v(x)].h/2\} / \{1 + [\rho(x+h) + v(x+h)].h/2\}$$

By choosing a sufficiently small value for  $h$ , any required degree of accuracy may be obtained. In practice it was found necessary to use extremely small values for  $h$ , especially in the early durations of sickness where recovery rates are changing rapidly. The results of computations made for such small successive intervals were accumulated to build up the double decrement tables.

7.3 Table 6 shows the probability of surviving as sick for a number of years of sickness once a claim arises at the end of the deferred period, based on the graduated rates of recovery and mortality. Figures are given for quinquennial ages from 20 to 60 at start of sickness to show how the survivorship probabilities vary by age.

For all deferred periods (leaving out DP52) the probability of being still sick at the end of one year declines from age 20, reaches a minimum around age 30 and then rises with increasing age at falling sick. The pattern continues into the 5-year claim survivorship rates, though with the minimum age shifting towards age 25 for the longer deferred periods. These patterns reflect the fact shown by Tables 1(a) to 1(e), that graduated recovery rates are lower at age 20 than at age 30 during the first year of sickness at least.

Ongoing investigations into cause of sickness in the offices' experience may help to explain this phenomenon.

Table 1(a). Graduated values of  $\rho(y+z, z)$ : deferred period 1 week.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
0	81.820836	92.377251	72.113672	43.900002	23.504376
1	52.817553	63.330990	52.505613	33.946073	19.302368
2	31.645483	40.223157	35.350214	24.227147	14.603258
3	17.728568	23.844663	22.174824	16.081388	10.257089
4	9.349279	13.283433	13.049503	9.997053	6.735770
5	8.124983	11.430643	11.119118	8.434598	5.627242
6	7.090005	9.879520	9.518677	7.151740	4.725895
7	6.211171	8.574792	8.185122	6.092857	3.988909
8	5.461739	7.472336	7.068598	5.214402	3.383086
9	4.820037	6.536736	6.129471	4.482078	2.882525
10	4.268421	5.739422	5.336059	3.868721	2.466899
11	3.792454	5.057229	4.662903	3.352698	2.120166
12	3.380274	4.471285	4.089445	2.916692	1.829592
13	3.022088	3.966147	3.599001	2.546769	1.585018
14	2.709781	3.529120	3.177973	2.231662	1.378300
15	2.436597	3.149724	2.815223	1.962219	1.202872
16	2.196893	2.819272	2.501592	1.730973	1.053419
17	1.985938	2.530535	2.229516	1.531803	0.925620
18	1.799748	2.277475	1.992724	1.359674	0.815943
19	1.634959	2.055029	1.785995	1.210422	0.721491
20	1.488721	1.858935	1.604968	1.080596	0.639878
21	1.358608	1.685596	1.445987	0.967319	0.569131
22	1.242554	1.531966	1.305975	0.868190	0.507612
23	1.138789	1.395455	1.182332	0.781192	0.453955
24	1.045796	1.273853	1.072860	0.704630	0.407021
25	0.962269	1.165274	0.975689	0.637072	0.365850
26	0.887080	1.068098	0.889224	0.577305	0.329638
27	0.855014	1.023749	0.847551	0.547182	0.310696
28	0.824777	0.982162	0.808688	0.519246	0.293226
29	0.796236	0.943118	0.772399	0.493301	0.277089
30	0.769268	0.906421	0.738471	0.469172	0.262160
40	0.565583	0.636014	0.494525	0.299851	0.159904
50	0.439201	0.475288	0.355634	0.207512	0.106493



Table 1(a). (continued)

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.418427	0.449489	0.333864	0.193381	0.098514
2	0.188902	0.179844	0.118388	0.060773	0.027438
3	0.124559	0.111321	0.068790	0.033149	0.014049
4	0.096404	0.082870	0.049255	0.022830	0.009307
5	0.081044	0.067854	0.039282	0.017733	0.007041
6	0.081228	0.065191	0.036614	0.016230	
7	0.081025	0.062409	0.034047	0.014837	
8	0.080447	0.059540	0.031589	0.013549	
9	0.079513	0.056614	0.029246	0.012362	
10	0.078243	0.053660	0.027023	0.011270	
15	0.067854	0.039282	0.017733	0.007041	
20	0.053660	0.027023	0.011270		
25	0.039282	0.017733	0.007041		
30	0.027023	0.011270			
35	0.017733	0.007041			
40	0.011270				
45	0.007041				

Table 1(b). Graduated values of  $\rho(y+z, z)$ : deferred period 4 weeks.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
4	2.003475	2.846533	2.796403	2.142288	1.443421
5	2.349019	3.304720	3.214655	2.438531	1.626895
6	2.765468	3.853524	3.712776	2.789548	1.843343
7	3.268540	4.512362	4.307303	3.206279	2.099107
8	3.877657	5.305116	5.018475	3.702056	2.401881
9	3.699004	5.016437	4.703893	3.439646	2.212114
10	3.540768	4.761003	4.426402	3.209206	2.046358
11	3.400529	4.534598	4.181022	3.006219	1.901060
12	3.276226	4.333656	3.963568	2.826914	1.773275
13	3.166103	4.155150	3.770508	2.668133	1.660551
14	3.068654	3.996503	3.598852	2.527214	1.560837
15	2.982588	3.855512	3.446057	2.401912	1.472411
16	2.906795	3.730289	3.309954	2.290318	1.393820
17	2.627672	3.348250	2.949959	2.026789	1.224724
18	2.381317	3.013417	2.636650	1.799038	1.079606
19	2.163279	2.719089	2.363120	1.601557	0.954633
20	1.969784	2.459629	2.123597	1.429778	0.846648
21	1.797627	2.230278	1.913242	1.279898	0.753039
22	1.644071	2.027005	1.727986	1.148736	0.671641
23	1.506776	1.846381	1.564390	1.033626	0.600646
24	1.383734	1.685485	1.419543	0.932324	0.538545
25	1.273216	1.541820	1.290972	0.842935	0.484071
26	1.173731	1.413243	1.176567	0.763855	0.436156
27	1.131302	1.354563	1.121427	0.723998	0.411094
28	1.091295	1.299537	1.070007	0.687035	0.387979
29	1.053531	1.247876	1.021991	0.652706	0.366627
30	1.017849	1.199321	0.977100	0.620779	0.346875
40	0.748345	0.841535	0.654326	0.396744	0.211575
50	0.581123	0.628872	0.470553	0.274567	0.140905

Table 1(b). (continued)

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.553638	0.594737	0.441749	0.255871	0.130347
2	0.249944	0.237959	0.156644	0.080411	0.036304
3	0.164809	0.147293	0.091019	0.043861	0.018589
4	0.127556	0.109649	0.065172	0.030207	0.012314
5	0.107232	0.089781	0.051975	0.023464	0.009316
6	0.107476	0.086257	0.048445	0.021475	
7	0.107207	0.082576	0.045049	0.019632	
8	0.106443	0.078780	0.041796	0.017928	
9	0.105206	0.074909	0.038696	0.016357	
10	0.103526	0.070999	0.035755	0.014912	
15	0.089781	0.051975	0.023464	0.009316	
20	0.070999	0.035755	0.014912		
25	0.051975	0.023464	0.009316		
30	0.035755	0.014912			
35	0.023464	0.009316			
40	0.014912				
45	0.009316				

Table 1(c). Graduated values of  $\rho(y+z, z)$ : deferred period 13 weeks.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
13	0.479442	0.629213	0.570967	0.404035	0.251457
14	0.679347	0.884757	0.796724	0.559482	0.345542
15	0.965316	1.247838	1.115318	0.777380	0.476546
16	1.375382	1.765026	1.566140	1.083689	0.659501
17	1.964755	2.503543	2.205734	1.515464	0.915747
18	1.780551	2.253182	1.971468	1.345171	0.807239
19	1.617520	2.033108	1.766944	1.197511	0.713795
20	1.472841	1.839106	1.587849	1.069069	0.633053
21	1.344116	1.667617	1.430563	0.957001	0.563060
22	1.229300	1.515625	1.292044	0.858929	0.502197
23	1.126642	1.380570	1.169721	0.772859	0.449113
24	1.034641	1.260265	1.061416	0.697114	0.402679
25	0.952005	1.152845	0.965281	0.630277	0.361948
26	0.877618	1.056705	0.879739	0.571147	0.326121
27	0.845893	1.012829	0.838510	0.541346	0.307382
28	0.815979	0.971685	0.800062	0.513707	0.290098
29	0.787743	0.933058	0.764160	0.488039	0.274133
30	0.761063	0.896753	0.730594	0.464167	0.259364
40	0.559550	0.629230	0.489250	0.296652	0.158198
50	0.434516	0.470218	0.351840	0.205298	0.105357

Table 1(c). (continued)

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.413964	0.444695	0.330303	0.191319	0.097463
2	0.186887	0.177926	0.117125	0.060125	0.027145
3	0.123231	0.110133	0.068056	0.032795	0.013899
4	0.095376	0.081986	0.048730	0.022586	0.009207
5	0.080179	0.067131	0.038863	0.017544	0.006966
6	0.080362	0.064496	0.036223	0.016057	
7	0.080161	0.061743	0.033684	0.014679	
8	0.079589	0.058905	0.031252	0.013405	
9	0.078665	0.056010	0.028934	0.012230	
10	0.077408	0.053087	0.026734	0.011150	
15	0.067131	0.038863	0.017544	0.006966	
20	0.053087	0.026734	0.011150		
25	0.038863	0.017544	0.006966		
30	0.026734	0.011150			
35	0.017544	0.006966			
40	0.011150				
45	0.006966				

Table 1(d). Graduated values of  $\rho(y+z, z)$ : deferred period 26 weeks.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
26	0.741013	0.892224	0.742803	0.482245	0.275359
27	0.714226	0.855177	0.707992	0.457083	0.259536
28	0.688968	0.820438	0.675528	0.433746	0.244943
29	0.665127	0.787823	0.645215	0.412073	0.231463
30	0.642600	0.757169	0.616874	0.391917	0.218993
40	0.472453	0.531287	0.413096	0.250477	0.133574
50	0.366881	0.397027	0.297074	0.173343	0.088957
<i>Years</i>					
1	0.349529	0.375476	0.278890	0.161539	0.082292
2	0.157797	0.150231	0.098894	0.050766	0.022920
3	0.104049	0.092990	0.057463	0.027691	0.011736
4	0.080530	0.069225	0.041145	0.019071	0.007774
5	0.067699	0.056681	0.032813	0.014813	0.005882
6	0.067853	0.054457	0.030585	0.013558	
7	0.067683	0.052133	0.028441	0.012394	
8	0.067201	0.049736	0.026387	0.011318	
9	0.066420	0.047292	0.024430	0.010327	
10	0.065359	0.044824	0.022573	0.009414	
15	0.056681	0.032813	0.014813	0.005882	
20	0.044824	0.022573	0.009414		
25	0.032813	0.014813	0.005882		
30	0.022573	0.009414			
35	0.014813	0.005882			
40	0.009414				
45	0.005882				

Table 1(e). Graduated values of  $\rho(y + z, z)$ : deferred period 52 weeks.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.247493	0.265866	0.197475	0.114382	0.058269
2	0.111733	0.106375	0.070025	0.035946	0.016229
3	0.073675	0.065844	0.040688	0.019607	0.008310
4	0.057022	0.049016	0.029134	0.013503	0.005505
5	0.047936	0.040135	0.023234	0.010489	0.004165
6	0.048045	0.038560	0.021657	0.009600	
7	0.047925	0.036914	0.020138	0.008776	
8	0.047583	0.035217	0.018684	0.008014	
9	0.047031	0.033487	0.017298	0.007312	
10	0.046280	0.031739	0.015983	0.006666	
15	0.040135	0.023234	0.010489	0.004165	
20	0.031739	0.015983	0.006666		
25	0.023234	0.010489	0.004165		
30	0.015983	0.006666			
35	0.010489	0.004165			
40	0.006666				
45	0.004165				

Table 2(a). Graduated values of  $\nu(y+z, z)$ : deferred period 1 week.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
0	0.002301	0.002376	0.002709	0.004196	0.010822
1	0.004906	0.004981	0.005315	0.006806	0.013452
2	0.008595	0.008670	0.009006	0.010501	0.017165
3	0.013152	0.013228	0.013564	0.015064	0.021748
4	0.018279	0.018355	0.018692	0.020196	0.026899
5	0.023673	0.023748	0.024087	0.025595	0.032317
6	0.029074	0.029150	0.029489	0.031002	0.037743
7	0.034285	0.034361	0.034701	0.036218	0.042979
8	0.039165	0.039242	0.039583	0.041104	0.047885
9	0.043629	0.043706	0.044048	0.045574	0.052374
10	0.047630	0.047707	0.048050	0.049580	0.056399
11	0.051152	0.051229	0.051573	0.053107	0.059946
12	0.054200	0.054277	0.054623	0.056161	0.063020
13	0.056794	0.056872	0.057218	0.058761	0.065639
14	0.058963	0.059041	0.059388	0.060936	0.067834
15	0.060740	0.060818	0.061166	0.062718	0.069636
16	0.062160	0.062239	0.062588	0.064144	0.071082
17	0.063260	0.063338	0.063688	0.065249	0.072207
18	0.064072	0.064151	0.064502	0.066067	0.073045
19	0.064631	0.064710	0.065062	0.066631	0.073629
20	0.064965	0.065044	0.065398	0.066972	0.073990
21	0.065104	0.065183	0.065537	0.067116	0.074154
22	0.065072	0.065151	0.065506	0.067090	0.074148
23	0.064891	0.064971	0.065327	0.066915	0.073994
24	0.064583	0.064663	0.065021	0.066613	0.073712
25	0.064166	0.064246	0.064604	0.066201	0.073320
26	0.063655	0.063735	0.064094	0.065696	0.072835
27	0.063064	0.063145	0.063505	0.065111	0.072271
28	0.062407	0.062488	0.062850	0.064460	0.071641
29	0.061694	0.061776	0.062138	0.063754	0.070955
30	0.060936	0.061017	0.061381	0.063001	0.070223
40	0.052257	0.052341	0.052715	0.054382	0.061814
50	0.043863	0.043950	0.044334	0.046050	0.053698



Table 2(a). (continued)

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.042230	0.042317	0.042705	0.044431	0.052126
2	0.018658	0.018759	0.019208	0.021213	0.030149
3	0.010185	0.010302	0.010824	0.013152	0.023528
4	0.006380	0.006516	0.007123	0.009825	0.021874
5	0.004371	0.004529	0.005233	0.008372	0.022363
6	0.004379	0.004562	0.005380	0.009024	
7	0.004387	0.004600	0.005550	0.009782	
8	0.004397	0.004645	0.005747	0.010661	
9	0.004409	0.004696	0.005976	0.011683	
10	0.004422	0.004756	0.006242	0.012869	
15	0.004529	0.005233	0.008372	0.022363	
20	0.004756	0.006242	0.012869		
25	0.005233	0.008372	0.022363		
30	0.006242	0.012869			
35	0.008372	0.022363			
40	0.012869				
45	0.022363				

Table 2(b). Graduated values of  $\nu(y+z, z)$ : deferred periods greater than 1 week.

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Weeks</i>					
0	0.003089	0.003190	0.003638	0.005634	0.014532
1	0.006588	0.006688	0.007137	0.009139	0.018063
2	0.011541	0.011642	0.012092	0.014100	0.023049
3	0.017661	0.017762	0.018213	0.020227	0.029202
4	0.024544	0.024646	0.025099	0.027118	0.036119
5	0.031786	0.031888	0.032343	0.034367	0.043394
6	0.039039	0.039141	0.039597	0.041628	0.050680
7	0.046036	0.046138	0.046595	0.048631	0.057710
8	0.052589	0.052692	0.053150	0.055193	0.064297
9	0.058583	0.058686	0.059146	0.061194	0.070325
10	0.063955	0.064059	0.064520	0.066574	0.075731
11	0.068684	0.068788	0.069250	0.071310	0.080493
12	0.072777	0.072881	0.073345	0.075411	0.084620
13	0.076261	0.076365	0.076830	0.078902	0.088137
14	0.079173	0.079278	0.079744	0.081821	0.091084
15	0.081559	0.081664	0.082131	0.084215	0.093504
16	0.083466	0.083571	0.084040	0.086130	0.095445
17	0.084942	0.085047	0.085517	0.087613	0.096956
18	0.086033	0.086139	0.086610	0.088712	0.098081
19	0.086783	0.086889	0.087362	0.089469	0.098866
20	0.087232	0.087338	0.087813	0.089927	0.099350
21	0.087418	0.087525	0.088000	0.090120	0.099570
22	0.087375	0.087482	0.087959	0.090085	0.099562
23	0.087133	0.087240	0.087718	0.089851	0.099355
24	0.086719	0.086827	0.087307	0.089445	0.098977
25	0.086159	0.086266	0.086747	0.088892	0.098451
26	0.085472	0.085580	0.086063	0.088213	0.097800
27	0.084680	0.084788	0.085272	0.087429	0.097043
28	0.083797	0.083906	0.084391	0.086554	0.096196
29	0.082840	0.082950	0.083436	0.085605	0.095275
30	0.081822	0.081931	0.082419	0.084595	0.094292
40	0.070168	0.070281	0.070783	0.073021	0.083001
50	0.058897	0.059013	0.059530	0.061834	0.072103

Table 2(b). (continued)

Exact duration of sickness	Exact age $y$ at falling sick				
	20	30	40	50	60
<i>Years</i>					
1	0.056705	0.056822	0.057342	0.059659	0.069992
2	0.025053	0.025188	0.025792	0.028484	0.040482
3	0.013676	0.013833	0.014534	0.017659	0.031592
4	0.008567	0.008750	0.009564	0.013193	0.029372
5	0.005870	0.006082	0.007027	0.011242	0.030029
6	0.005880	0.006126	0.007224	0.012118	
7	0.005891	0.006177	0.007452	0.013135	
8	0.005904	0.006237	0.007717	0.014316	
9	0.005920	0.006306	0.008025	0.015687	
10	0.005938	0.006386	0.008382	0.017280	
15	0.006082	0.007027	0.011242	0.030029	
20	0.006386	0.008382	0.017280		
25	0.007027	0.011242	0.030029		
30	0.008382	0.017280			
35	0.011242	0.030029			
40	0.017280				
45	0.030029				

Table 3(a). Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates: all deferred periods combined but excluding the first 4 claim weeks of DP4, DP13 and DP26.

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
Sickness period 1-2 weeks								
Exposed to risk (days)	1094	2868	5086	6894	8636	9211	8088	41877
Actual recoveries	128	406	681	783	828	620	402	3848
Expected recoveries	155.7	411.3	668.5	773.7	789.2	656.0	439.2	3893.7
Standardised deviation	-2.18	-0.24	0.46	0.31	1.36	-1.39	-1.75	-0.72
Sickness period 2-3 weeks								
Exposed to risk (days)	555	1222	2265	3387	4883	5882	5760	23954
Actual recoveries	46	111	200	260	267	337	252	1473
Expected recoveries	47.5	108.3	189.9	250.2	300.2	292.8	224.4	1413.4
Standardised deviation	-0.14	0.21	0.70	0.58	-1.89	2.55	1.81	1.57
Sickness period 3-4 weeks								
Exposed to risk (days)	369	716	1334	2256	3639	4185	4448	16947
Actual recoveries	15	36	83	95	129	150	115	623
Expected recoveries	17.7	36.8	66.8	102.0	141.0	135.1	115.7	615.0
Standardised deviation	-0.52	-0.04	1.92	-0.64	-0.97	1.24	-0.01	0.30
Sickness period 4-5 weeks								
Exposed to risk (days)	291	519	904	1702	2923	3362	3798	13499
Actual recoveries	9	20	35	59	72	82	69	346
Expected recoveries	9.3	17.9	30.7	53.2	79.5	77.2	71.3	339.1
Standardised deviation	0.00	0.37	0.68	0.73	-0.79	0.49	-0.22	0.35
Sickness period 5-6 weeks								
Exposed to risk (days)	226	416	729	1416	2497	2969	3434	11687
Actual recoveries	13	12	21	34	51	41	50	222
Expected recoveries	6.3	12.3	21.2	37.7	57.5	57.5	54.2	246.7
Standardised deviation	2.48	0.00	0.00	-0.52	-0.80	-2.11	-0.50	-1.54
Sickness period 6-7 weeks								
Exposed to risk (days)	160	336	637	1175	2123	2629	3072	10132
Actual recoveries	4	10	14	26	53	44	47	198
Expected recoveries	3.9	8.6	15.9	26.8	41.6	43.1	40.9	180.8
Standardised deviation	0.00	0.30	-0.36	-0.05	1.68	0.05	0.88	1.24

Table 3(a). (continued)

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
Sickness period 7-8 weeks								
Exposed to risk (days)	139	288	541	1003	1843	2425	2797	9036
Actual recoveries	4	13	14	19	38	21	26	135
Expected recoveries	2.9	6.4	11.7	19.6	30.9	33.9	31.5	137.0
Standardised deviation	+	2.41	0.53	-0.03	1.19	-2.13	-0.89	-0.12
Sickness period 8-9 weeks								
Exposed to risk (days)	1016	1127	1581	3083	4217	5125	5426	21575
Actual recoveries	20	23	32	43	46	45	59	268
Expected recoveries	14.1	17.2	24.1	42.4	51.0	52.4	44.9	246.0
Standardised deviation	1.45	1.28	1.52	0.02	-0.63	-0.95	2.03	1.37
Sickness period 9-10 weeks								
Exposed to risk (days)	914	1035	1392	2820	3901	4814	5125	20001
Actual recoveries	11	11	22	32	40	44	32	192
Expected recoveries	11.9	14.7	19.3	35.3	42.5	43.8	37.6	205.0
Standardised deviation	-0.11	-0.82	0.49	-0.47	-0.30	0.00	-0.83	-0.88
Sickness period 10-11 weeks								
Exposed to risk (days)	841	980	1266	2601	3631	4592	4930	18841
Actual recoveries	10	10	19	25	30	33	25	152
Expected recoveries	10.3	12.9	16.2	29.8	35.8	37.6	32.3	174.9
Standardised deviation	0.00	-0.68	0.57	-0.79	-0.88	-0.67	-1.20	-1.70
Sickness period 11-13 weeks								
Exposed to risk (days)	1488	1685	2292	4672	6646	8576	9339	34698
Actual recoveries	21	30	20	53	55	54	48	281
Expected recoveries	16.9	20.3	26.3	47.7	57.2	60.5	52.3	281.3
Standardised deviation	0.88	2.05	-1.14	0.70	-0.23	-0.78	-0.53	0.00
Sickness period 13-15 weeks								
Exposed to risk (days)	1285	1390	2056	4059	5917	7831	8545	31083
Actual recoveries	9	13	15	28	42	46	56	209
Expected recoveries	13.3	15.0	20.7	35.9	43.1	46.1	39.5	213.6
Standardised deviation	-1.04	-0.38	-1.15	-1.24	-0.10	0.00	2.55	-0.28

Table 3(a). (continued)

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
Sickness period 15-17 weeks								
Exposed to risk (days)	1143	1231	1873	3644	5451	7347	7890	28579
Actual recoveries	8	8	13	22	26	37	37	151
Expected recoveries	10.8	11.8	16.6	28.1	33.9	36.3	30.3	167.7
Standardised deviation	-0.69	-0.97	-0.76	-1.06	-1.27	0.04	1.14	-1.25
Sickness period 17-21 weeks								
Exposed to risk (days)	3104	4439	7065	12772	19862	27159	30315	104716
Actual recoveries	24	20	35	68	87	81	64	379
Expected recoveries	19.8	27.7	40.0	63.1	78.6	84.6	72.8	386.6
Standardised deviation	0.83	-1.36	-0.71	0.56	0.89	-0.34	-0.97	-0.36
Sickness period 21-26 weeks								
Exposed to risk (days)	2143	3121	5116	8954	14412	20199	22552	76497
Actual recoveries	7	14	10	23	32	24	36	146
Expected recoveries	8.9	12.4	18.3	27.3	34.8	37.9	32.3	172
Standardised deviation	-0.45	0.30	-1.82	-0.73	-0.39	-2.18	0.55	-1.94
Sickness period 26-30 weeks								
Exposed to risk (days)	1831	2786	4705	8236	13547	19166	21117	71388
Actual recoveries	10	11	15	17	29	42	34	158
Expected recoveries	5.9	8.4	12.6	18.6	23.9	26.2	21.7	117.3
Standardised deviation	1.49	0.73	0.52	-0.25	0.93	3.00	2.52	3.71
Sickness period 30-39 weeks								
Exposed to risk (days)	4269	7388	12800	24643	43174	61306	66970	220550
Actual recoveries	12	10	24	45	66	53	47	257
Expected recoveries	10.1	16.0	24.4	38.4	51.7	56.3	45.7	242.7
Standardised deviation	0.43	-1.38	0.00	0.98	1.93	-0.37	0.12	0.89
Sickness period 39-52 weeks								
Exposed to risk (days)	5803	8837	16738	32401	56841	82674	95139	298433
Actual recoveries	7	15	24	35	45	50	34	210
Expected recoveries	9.8	13.4	21.5	33.1	44.0	48.2	40.4	210.4
Standardised deviation	-0.73	0.29	0.43	0.24	0.07	0.19	-0.92	0.00

Table 3(a). (continued)

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
Sickness period 1-2 years								
Exposed to risk (days)	18755	31407	63847	125813	253093	343758	406443	1243116
Actual recoveries	15	23	34	61	81	90	64	368
Expected recoveries	16.2	23.4	37.4	55.6	80.3	79.6	67.6	360.2
Standardised deviation	-0.19	0.00	-0.47	0.65	0.02	1.11	-0.38	0.38
Sickness period 2-3 years								
Exposed to risk (days)	13774	23125	53987	104887	220939	297453	358830	1072995
Actual recoveries	6	8	17	17	32	27	29	136
Expected recoveries	6.1	8.2	15.0	20.4	29.9	27.6	23.1	130.3
Standardised deviation	0.00	0.00	0.40	-0.65	0.30	-0.02	1.11	0.46
Sickness period 3-5 years								
Exposed to risk (days)	17833	35033	80433	166228	335823	457253	591720	1684323
Actual recoveries	8	11	15	22	20	29	4	109
Expected recoveries	5.1	7.4	13.2	18.4	25.0	22.5	19.3	110.9
Standardised deviation	1.05	1.16	0.36	0.73	-0.89	1.26	-3.37	-0.13
Sickness period over 5 years								
Exposed to risk (days)	33484	91111	194601	365178	553581	695344	526863	2460162
Actual recoveries	4	7	12	24	24	35	3	109
Expected recoveries	5.2	12.5	19.2	22.7	23.8	20.3	11.5	115.2
Standardised deviation	-0.31	-1.40	-1.54	0.17	0.00	3.14	-2.36	-0.53

Table 3(b). Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates: first 4 claim weeks of DP4.

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
Sickness period 4-5 weeks								
Exposed to risk (days)	1227	1154	1609	2601	3188	3413	3500	16692
Actual recoveries	9	4	13	16	17	24	14	97
Expected recoveries	9.6	9.9	13.6	20.2	21.6	19.5	16.4	110.8
Standardised deviation	-0.03	-1.72	-0.03	-0.82	-0.88	0.91	-0.47	-1.26
Sickness period 5-6 weeks								
Exposed to risk (days)	1182	1175	1553	2587	3146	3265	3379	16287
Actual recoveries	15	14	17	16	24	30	25	141
Expected recoveries	10.8	11.7	15.2	23.1	24.4	21.2	17.9	124.3
Standardised deviation	1.13	0.52	0.34	-1.37	0.00	1.80	1.56	1.46
Sickness period 6-7 weeks								
Exposed to risk (days)	1094	1107	1444	2469	2970	3083	3240	15407
Actual recoveries	12	13	16	26	28	19	27	141
Expected recoveries	11.7	12.9	16.4	25.4	26.4	22.9	19.5	135.2
Standardised deviation	0.00	0.00	0.00	0.01	0.20	-0.71	1.57	0.45
Sickness period 7-8 weeks								
Exposed to risk (days)	1002	1034	1310	2319	2773	2996	3012	14446
Actual recoveries	13	12	25	18	32	20	22	142
Expected recoveries	12.6	14.1	17.3	27.7	28.5	25.6	20.8	146.5
Standardised deviation	0.00	-0.41	1.73	-1.75	0.57	-1.00	0.16	-0.33



Table 3(c). Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates: first 4 claim weeks of DP13.

Age group	Under 40	40-44	45-49	50-54	55-59	Totals
Sickness period 13-15 weeks						
Exposed to risk (days)	2655	2230	3482	4736	5219	18322
Actual recoveries	5	4	6	10	7	32
Expected recoveries	6.3	4.6	6.0	6.5	5.7	29.1
Standardised deviation	-0.32	-	0.00	1.16	0.33	0.44
Sickness period 15-17 weeks						
Exposed to risk (days)	2546	2175	3400	4698	5143	17962
Actual recoveries	13	6	8	9	8	44
Expected recoveries	12	8.7	11.4	12.5	10.8	55.3
Standardised deviation	0.16	-0.74	-0.85	-0.85	-0.70	-1.46

Table 3(d). Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates: first 4 claim weeks of DP26.

Age group	Under 40	40-44	45-49	50-54	55-59	Totals
Sickness period 26-30 weeks						
Exposed to risk (days)	2844	3613	6998	9342	10014	32811
Actual recoveries	8	3	7	9	6	33
Expected recoveries	5.9	6.1	9.3	9.8	7.9	39.0
Standardised deviation	0.66	-1.04	-0.60	-0.10	-0.50	-0.88

Table 3(e). Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates: totals of Tables 3(a)–3(d).

Age group	Under 30	30-34	35-39	40-44	45-49	50-54	55-59	Totals
All sickness periods								
Exposed to risk (days)	116254	228043	471464	905818	1593536	2104793	2226108	7646016
Actual recoveries	446	871	1440	1880	2215	2106	1642	10600
Expected recoveries	456.1	879.2	1404.5	1895.8	2223.0	2093.7	1647.6	10600.0
Standardised deviation	-0.45	-0.26	0.93	-0.35	-0.16	0.26	-0.13	0.00

Note: The standardised deviations in the above tables include continuity adjustments.

Table 4(a). Exposed to risk and comparison of actual deaths with those expected according to the graduated rates: deferred period 1 week.

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
<b>Sickness period 1-13 weeks</b>							
Exposed to risk (days)	23715	21562	33602	40766	43025	26501	189171
Actual deaths	2	1	0	6	7	5	21
Expected deaths	1.2	1.2	2.2	3.1	3.8	2.8	14.3
Standardised deviation	+	-	-	+	+	+	1.65
<b>Sickness period 13-26 weeks</b>							
Exposed to risk (days)	4818	5937	12620	19583	22731	11402	77091
Actual deaths	0	4	0	4	3	3	14
Expected deaths	0.8	1.0	2.2	3.6	4.3	2.4	14.4
Standardised deviation	-	+	-	+	-	+	0.00
<b>Sickness period 26-39 weeks</b>							
Exposed to risk (days)	3889	4910	10107	16884	19572	9236	64598
Actual deaths	1	2	3	1	5	4	16
Expected deaths	0.6	0.8	1.7	2.9	3.5	1.8	11.3
Standardised deviation	+	+	+	-	+	+	1.25
<b>Sickness period 39-52 weeks</b>							
Exposed to risk (days)	3481	4406	9025	16421	20158	8556	62047
Actual deaths	1	0	1	0	0	3	5
Expected deaths	0.5	0.6	1.2	2.3	3.0	1.4	9.0
Standardised deviation	+	-	-	-	-	+	-1.16
<b>Sickness period 1-2 years</b>							
Exposed to risk (days)	11012	14515	34583	57487	78405	29231	225233
Actual deaths	0	1	2	3	10	2	18
Expected deaths	0.9	1.2	2.9	5.1	7.7	3.4	21.1
Standardised deviation	-	-	-	-0.70	0.65	-	-0.56
<b>Sickness period 2-3 years</b>							
Exposed to risk (days)	9905	13914	31249	49650	71612	21994	198324
Actual deaths	1	0	1	0	5	2	9
Expected deaths	0.4	0.6	1.4	2.5	4.4	1.8	10.9
Standardised deviation	+	-	-	-	+	+	-0.43

Table 4(a). (continued)

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
Sickness period 3-4 years							
Exposed to risk (days)	10012	15137	29054	42791	62450	12568	172012
Actual deaths	1	0	0	0	3	0	4
Expected deaths	0.2	0.4	0.8	1.5	3.1	0.8	6.9
Standardised deviation	+	-	-	-	-	-	-0.92
Sickness period 4-5 years							
Exposed to risk (days)	10097	14219	25554	38338	59054	5204	152466
Actual deaths	0	0	1	0	2	0	3
Expected deaths	0.2	0.3	0.6	1.2	2.8	0.3	5.2
Standardised deviation	-	-	+	-	-	-	-0.76
Sickness period 5-11 years							
Exposed to risk (days)	53403	64855	88234	143160	125785	0	475437
Actual deaths	1	1	2	7	10	0	21
Expected deaths	0.7	1.1	2.1	5.4	6.5	0.0	15.9
Standardised deviation	+	-	-	0.48	1.17	null	1.16
Sickness period over 11 years							
Exposed to risk (days)	19793	22175	28348	16018	0	0	86334
Actual deaths	0	0	0	1	0	0	1
Expected deaths	0.3	0.7	1.2	0.9	0.0	0.0	3.1
Standardised deviation	-	-	-	+	null	null	-
All sickness periods							
Exposed to risk (days)	150125	181630	302376	441098	502792	124692	1702713
Actual deaths	7	9	10	22	45	19	112
Expected deaths	5.8	7.9	16.2	28.3	39.0	14.7	112.0
Standardised deviation	0.27	0.22	-1.42	-1.09	0.87	0.99	0.00

Table 4(b). Exposed to risk and comparison of actual deaths with those expected according to the graduated rates: deferred periods 4, 13, 26 and 52 weeks.

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
<b>Sickness period 1-13 weeks</b>							
Exposed to risk (days)	27488	19423	23414	25761	26323	15541	137950
Actual deaths	0	4	2	2	4	3	15
Expected deaths	3.8	2.8	3.4	3.9	4.3	2.9	21.3
Standardised deviation	-	+	-	-	-	+	-1.25
<b>Sickness period 13-26 weeks</b>							
Exposed to risk (days)	34349	27897	39904	52387	56933	29237	240707
Actual deaths	5	7	6	16	18	9	61
Expected deaths	8.0	6.5	9.5	12.8	14.5	8.2	59.5
Standardised deviation	-0.88	0	-0.97	0.77	0.78	0.12	0.13
<b>Sickness period 26-39 weeks</b>							
Exposed to risk (days)	32734	31582	53612	72930	78529	39509	308896
Actual deaths	4	5	7	15	19	12	62
Expected deaths	7.1	6.9	11.9	16.6	18.9	10.4	71.9
Standardised deviation	-0.98	-0.54	-1.28	-0.27	0	0.33	-1.1
<b>Sickness period 39-52 weeks</b>							
Exposed to risk (days)	27897	27995	47816	66253	74981	36457	281399
Actual deaths	3	9	12	16	9	5	54
Expected deaths	4.9	5.0	8.7	12.4	15.0	8.2	54.1
Standardised deviation	-	+	0.97	0.88	-1.41	-0.94	0
<b>Sickness period 1-2 years</b>							
Exposed to risk (days)	102997	111298	218510	286271	328038	138642	1185756
Actual deaths	8	22	29	45	35	29	168
Expected deaths	11.0	12.1	24.4	33.8	43.3	21.8	146.4
Standardised deviation	-0.75	2.71	0.84	1.84	-1.19	1.43	1.75
<b>Sickness period 2-3 years</b>							
Exposed to risk (days)	80981	90973	189690	247803	287218	94295	990960
Actual deaths	7	5	9	10	20	5	56
Expected deaths	4.3	5.0	11.1	16.5	23.7	10.1	70.6
Standardised deviation	+	+	-0.47	-1.48	-0.66	-1.45	-1.68

Table 4(b). (continued)

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
Sickness period 3-4 years							
Exposed to risk (days)	63696	74653	154158	205276	250765	52208	800756
Actual deaths	1	2	8	7	15	3	36
Expected deaths	2.0	2.5	5.9	9.8	16.7	4.7	41.7
Standardised deviation	–	–	0.65	–0.74	–0.3	–	–0.8
Sickness period 4-5 years							
Exposed to risk (days)	49494	62219	127057	170848	219451	20674	649743
Actual deaths	2	2	1	9	15	2	31
Expected deaths	1.0	1.5	3.8	6.9	13.9	1.8	28.9
Standardised deviation	+	+	–	0.6	0.17	+	0.3
Sickness period 5-11 years							
Exposed to risk (days)	174371	202329	359205	502506	401078	4	1639493
Actual deaths	4	5	17	22	30	0	78
Expected deaths	3.3	4.7	11.4	24.8	27.5	0.0	71.7
Standardised deviation	+	+	1.5	–0.46	0.39	–	0.69
Sickness period over 11 years							
Exposed to risk (days)	71629	75819	77794	33660	0	0	258902
Actual deaths	8	2	5	1	0	0	16
Expected deaths	1.6	2.9	4.2	2.5	0.0	0.0	11.1
Standardised deviation	+	–	+	–	null	null	1.32
All sickness periods							
Exposed to risk (days)	665636	724188	1291160	1663695	1723316	426567	6494562
Actual deaths	42	63	96	143	165	68	577
Expected deaths	47.0	49.9	94.2	140.0	177.9	68.1	577.0
Standardised deviation	–0.66	1.78	0.14	0.21	–0.93	0	0

Table 4(c). Exposed to risk and comparison of actual deaths with those expected according to the graduated rates: all deferred periods combined.

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
<b>Sickness period 1-13 weeks</b>							
Exposed to risk (days)	51203	40985	57016	66527	69348	42042	327121
Actual deaths	2	5	2	8	11	8	36
Expected deaths	5.0	4.0	5.6	7.0	8.1	5.7	35.5
Standardised deviation	-1.12	+	-1.33	0.19	0.83	0.75	0.00
<b>Sickness period 13-26 weeks</b>							
Exposed to risk (days)	39167	33834	52524	71970	79664	40639	317798
Actual deaths	5	11	6	20	21	12	75
Expected deaths	8.8	7.6	11.7	16.3	18.9	10.5	73.8
Standardised deviation	-1.12	1.06	-1.52	0.79	0.38	0.30	0.08
<b>Sickness period 26-39 weeks</b>							
Exposed to risk (days)	36623	36492	63719	89814	98101	48745	373494
Actual deaths	5	7	10	16	24	16	78
Expected deaths	7.8	7.7	13.6	19.5	22.4	12.3	83.2
Standardised deviation	-0.81	-0.08	-0.84	-0.67	0.24	0.92	-0.51
<b>Sickness period 39-52 weeks</b>							
Exposed to risk (days)	31378	32401	56841	82674	95139	45013	343446
Actual deaths	4	9	13	16	9	8	59
Expected deaths	5.4	5.6	9.9	14.7	18.0	9.6	63.1
Standardised deviation	-0.38	1.24	0.84	0.21	-2.00	-0.36	-0.45
<b>Sickness period 1-2 years</b>							
Exposed to risk (days)	114009	125813	253093	343758	406443	167873	1410989
Actual deaths	8	23	31	48	45	31	186
Expected deaths	11.9	13.3	27.2	38.9	51.0	25.2	167.5
Standardised deviation	-0.98	2.54	0.63	1.38	-0.77	1.05	1.39
<b>Sickness period 2-3 years</b>							
Exposed to risk (days)	90886	104887	220939	297453	358830	116289	1189284
Actual deaths	8	5	10	10	25	7	65
Expected deaths	4.6	5.5	12.4	19.0	28.1	11.9	81.5
Standardised deviation	+	0.00	-0.55	-1.95	-0.49	-1.27	-1.78

Table 4(c). (continued)

Age group	Under 40	40-44	45-49	50-54	55-59	60-64	All ages
Sickness period 3-4 years							
Exposed to risk (days)	73708	89790	183212	248067	313215	64776	972768
Actual deaths	2	2	8	7	18	3	40
Expected deaths	2.2	2.9	6.7	11.3	19.8	5.5	48.6
Standardised deviation	–	–	0.29	–1.14	–0.30	–0.86	–1.16
Sickness period 4-5 years							
Exposed to risk (days)	59591	76438	152611	209186	278505	25878	802209
Actual deaths	2	2	2	9	17	2	34
Expected deaths	1.2	1.8	4.3	8.1	16.6	2.1	34.1
Standardised deviation	+	+	–	0.14	0.00	–	0.00
Sickness period 5-11 years							
Exposed to risk (days)	227774	267184	447439	645666	526863	4	2114930
Actual deaths	5	6	19	29	40	0	99
Expected deaths	4.0	5.9	13.5	30.2	34.0	0.0	87.6
Standardised deviation	+	0.00	1.35	–0.12	0.95	–	1.17
Sickness period over 11 years							
Exposed to risk (days)	91422	97994	106142	49678	0	0	345236
Actual deaths	8	2	5	2	0	0	17
Expected deaths	1.9	3.6	5.3	3.3	0.0	0.0	14.2
Standardised deviation	+	–	0.00	–	null	null	0.62
All sickness periods							
Exposed to risk (days)	815761	905818	1593536	2104793	2226108	551259	8197275
Actual deaths	49	72	106	165	210	87	689
Expected deaths	52.8	57.8	110.4	168.3	216.9	82.8	689.0
Standardised deviation	–0.46	1.81	–0.37	–0.22	–0.43	0.41	0.00

Table 5(a). Graduated double decrement tables of claim terminations: deferred period 1 week.

Sickness duration	Exact age $y$ at falling sick								
	20			40			60		
	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$
<i>Weeks</i>									
1	1000000	549447	82	1000000	566878	86	1000000	277537	246
2	450471	166956	72	433036	181894	69	722218	152558	236
3	283443	63205	74	251073	70875	64	569423	84875	243
4	220164	33898	81	180134	37172	65	484306	53968	258
5	186185	25225	87	142897	25595	66	430080	40519	274
6	160873	19226	92	117235	18261	66	389287	31114	289
7	141555	14961	94	98908	13428	65	357884	24353	301
8	126499	11854	96	85415	10131	64	333230	19375	311
9	114550	9543	96	75220	7814	63	313544	15634	319
10	104911	7789	96	67343	6144	61	297592	12771	325
11	97026	6437	95	61138	4911	60	284496	10545	329
12	90494	5379	93	56167	3984	58	273622	8791	332
13	85022	4539	92	52126	3273	56	264498	7391	334
14	80391	3864	90	48796	2720	55	256774	6262	334
15	76436	3316	88	46022	2283	53	250177	5342	334
16	73032	2867	86	43686	1934	52	244502	4586	333
17	70079	2494	84	41700	1652	50	239584	3959	331
18	67501	2184	82	39998	1421	49	235294	3436	328
19	65236	1922	80	38529	1230	47	231530	2996	325
20	63234	1701	78	37251	1072	46	228208	2625	322
21	61455	1512	76	36133	939	45	225261	2309	318
22	59868	1350	74	35148	827	44	222633	2039	315
23	58444	1210	72	34277	732	42	220279	1808	311
24	57163	1089	70	33503	651	41	218161	1608	306
25	56004	983	68	32811	581	40	216247	1435	302
26	54953	910	66	32190	531	39	214510	1312	297
27	53977	862	64	31620	498	38	212901	1228	293
28	53051	818	63	31084	467	37	211380	1152	288
29	52171	777	61	30580	439	36	209941	1082	283
30	51333	739	59	30105	414	35	208576	1017	278
31	50535	704	57	29656	391	34	207280	958	274
32	49774	671	56	29231	369	33	206048	904	269
33	49047	640	54	28829	349	32	204874	854	264
34	48353	612	53	28448	331	31	203756	808	259
35	47688	586	51	28086	314	30	202689	765	255





Table 5(b). Graduated double decrement tables of claim terminations: deferred period 4 weeks.

Sickness duration	Exact age $y$ at falling sick								
	20			40			60		
	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$
<i>Weeks</i>									
4	1000000	40768	528	1000000	55886	535	1000000	28955	751
5	958703	45754	635	943579	60499	630	970295	31688	861
6	912314	51136	723	882451	65156	702	937746	34709	957
7	860454	56811	787	816592	69668	748	902080	38013	1033
8	802856	56207	825	746177	66343	766	863034	37294	1089
9	745825	49971	846	679067	56858	770	824651	32946	1131
10	695007	44709	855	621439	49174	765	790574	29321	1162
11	649444	40235	853	571500	42885	751	760091	26275	1182
12	608355	36407	843	527864	37687	732	732634	23696	1193
13	571106	33109	826	489445	33354	709	707745	21497	1197
14	537171	30251	804	455382	29712	683	685052	19610	1194
15	506116	27761	778	424987	26629	655	664247	17982	1186
16	477577	24639	750	397703	23122	627	645079	15955	1174
17	452187	21168	723	373954	19466	600	627949	13689	1161
18	430296	18317	697	353888	16533	576	613099	11815	1146
19	411282	15954	672	336779	14154	553	600139	10253	1130
20	394656	13979	649	322072	12206	532	588756	8942	1113
21	380028	12317	626	309334	10594	512	578701	7834	1097
22	367085	10907	605	298227	9251	494	569770	6894	1079
23	355573	9705	584	288482	8123	477	561797	6091	1062
24	345284	8673	565	279882	7168	461	554645	5401	1044
25	336047	7781	546	272253	6355	445	548200	4806	1026
26	327719	7157	528	265452	5779	431	542367	4383	1008
27	320034	6742	511	259242	5385	417	536976	4093	990
28	312781	6361	494	253440	5028	403	531892	3830	972
29	305925	6011	478	248008	4704	390	527090	3589	954
30	299437	5688	462	242914	4409	378	522547	3369	936
31	293287	5390	447	238127	4140	365	518242	3168	918
32	287450	5115	432	233622	3894	354	514156	2983	900
33	281903	4860	417	229375	3668	342	510273	2812	883
34	276626	4623	403	225364	3461	332	506578	2655	865
35	271600	4403	390	221572	3270	321	503058	2510	848
36	266807	4198	377	217980	3095	311	499700	2376	832
37	262231	4007	365	214575	2932	301	496492	2252	815
38	257859	3829	353	211341	2782	292	493425	2137	800



Table 5(c). Graduated double decrement tables of claim terminations: deferred period 13 weeks.

Sickness duration	Exact age $y$ at falling sick								
	20			40			60		
	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$
<i>Weeks</i>									
13	1000000	10929	1482	1000000	12897	1492	1000000	5654	1713
14	987588	15281	1511	985611	17722	1516	992633	7719	1750
15	970797	21305	1520	966373	24275	1521	983164	10547	1771
16	947972	29542	1508	940577	33058	1503	970846	14408	1777
17	916923	32288	1475	906015	35500	1465	954660	15604	1769
18	883159	28263	1439	869051	30540	1423	937287	13539	1756
19	853457	24877	1402	837087	26448	1383	921992	11803	1740
20	827178	22007	1366	809257	23043	1344	908450	10336	1721
21	803805	19560	1330	784871	20188	1306	896392	9089	1701
22	782915	17460	1294	763377	17777	1270	885601	8024	1680
23	764160	15649	1260	744331	15727	1235	875897	7110	1658
24	747251	14079	1226	727368	13975	1201	867129	6321	1634
25	731946	12710	1193	712192	12468	1169	859174	5639	1610
26	718043	11758	1161	698555	11403	1137	851925	5153	1585
27	705124	11137	1129	686015	10683	1107	845187	4822	1560
28	692858	10563	1098	674225	10027	1076	838804	4520	1535
29	681197	10033	1067	663122	9427	1046	832750	4244	1509
30	670098	9541	1036	652649	8878	1017	826997	3990	1482
31	659520	9084	1007	642754	8374	989	821524	3758	1456
32	649429	8660	978	633392	7910	961	816310	3543	1430
33	639792	8265	949	624521	7483	934	811337	3346	1404
34	630578	7896	922	616104	7088	908	806587	3163	1379
35	621760	7552	895	608108	6724	883	802045	2994	1353
36	613313	7230	869	600501	6386	858	797698	2838	1329
37	605214	6929	844	593257	6072	834	793531	2693	1304
38	597441	6646	819	586351	5781	812	789535	2558	1280
39	589975	6381	796	579758	5510	789	785697	2432	1256
40	582798	6132	773	573459	5257	768	782008	2315	1233
41	575893	5898	751	567434	5022	747	778459	2206	1211
42	569245	5677	730	561665	4801	727	775042	2105	1189
43	562839	5468	709	556137	4595	708	771749	2009	1167
44	556661	5272	689	550834	4402	689	768572	1920	1146
45	550700	5086	670	545743	4221	671	765506	1837	1126
46	544944	4910	652	540852	4050	654	762544	1758	1105
47	539382	4744	634	536148	3890	637	759680	1685	1086



Table 5(d). Graduated double decrement tables of claim terminations: deferred period 26 weeks.

Sickness duration	Exact age $y$ at falling sick								
	20			40			60		
	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$	$l(y, z)$	$r(y, z)$	$d(y, z)$
<i>Weeks</i>									
26	1000000	13844	1619	1000000	13800	1630	1000000	5109	1862
27	984537	13146	1579	984569	12962	1590	993029	4786	1834
28	969813	12499	1538	970018	12195	1550	986409	4490	1805
29	955775	11899	1499	956273	11491	1511	980113	4219	1776
30	942378	11342	1459	943271	10845	1472	974118	3970	1747
31	929577	10823	1420	930954	10251	1434	968401	3741	1717
32	917334	10339	1382	919270	9703	1396	962943	3530	1688
33	905612	9888	1345	908171	9196	1360	957725	3336	1658
34	894380	9466	1308	897615	8727	1324	952731	3156	1629
35	883606	9071	1273	887564	8293	1290	947946	2989	1600
36	873262	8700	1238	877981	7890	1256	943357	2834	1572
37	863323	8353	1205	868836	7515	1223	938951	2691	1543
38	853766	8027	1172	860098	7166	1191	934717	2557	1516
39	844567	7720	1140	851741	6840	1160	930644	2433	1489
40	835708	7430	1109	843741	6536	1131	926722	2317	1462
41	827168	7158	1079	836074	6252	1102	922943	2209	1436
42	818930	6901	1051	828721	5985	1074	919297	2108	1410
43	810979	6658	1022	821662	5736	1046	915779	2014	1385
44	803299	6428	995	814880	5502	1020	912380	1925	1361
45	795875	6211	969	808358	5282	995	909094	1842	1337
46	788696	6005	944	802082	5075	970	905916	1764	1314
47	781747	5809	919	796038	4880	946	902838	1691	1291
48	775019	5624	895	790212	4696	923	899857	1622	1269
49	768500	5448	872	784593	4523	901	896966	1557	1247
50	762180	5280	850	779170	4359	879	894163	1495	1226
51	756049	5840	945	773932	4792	979	891441	1639	1376













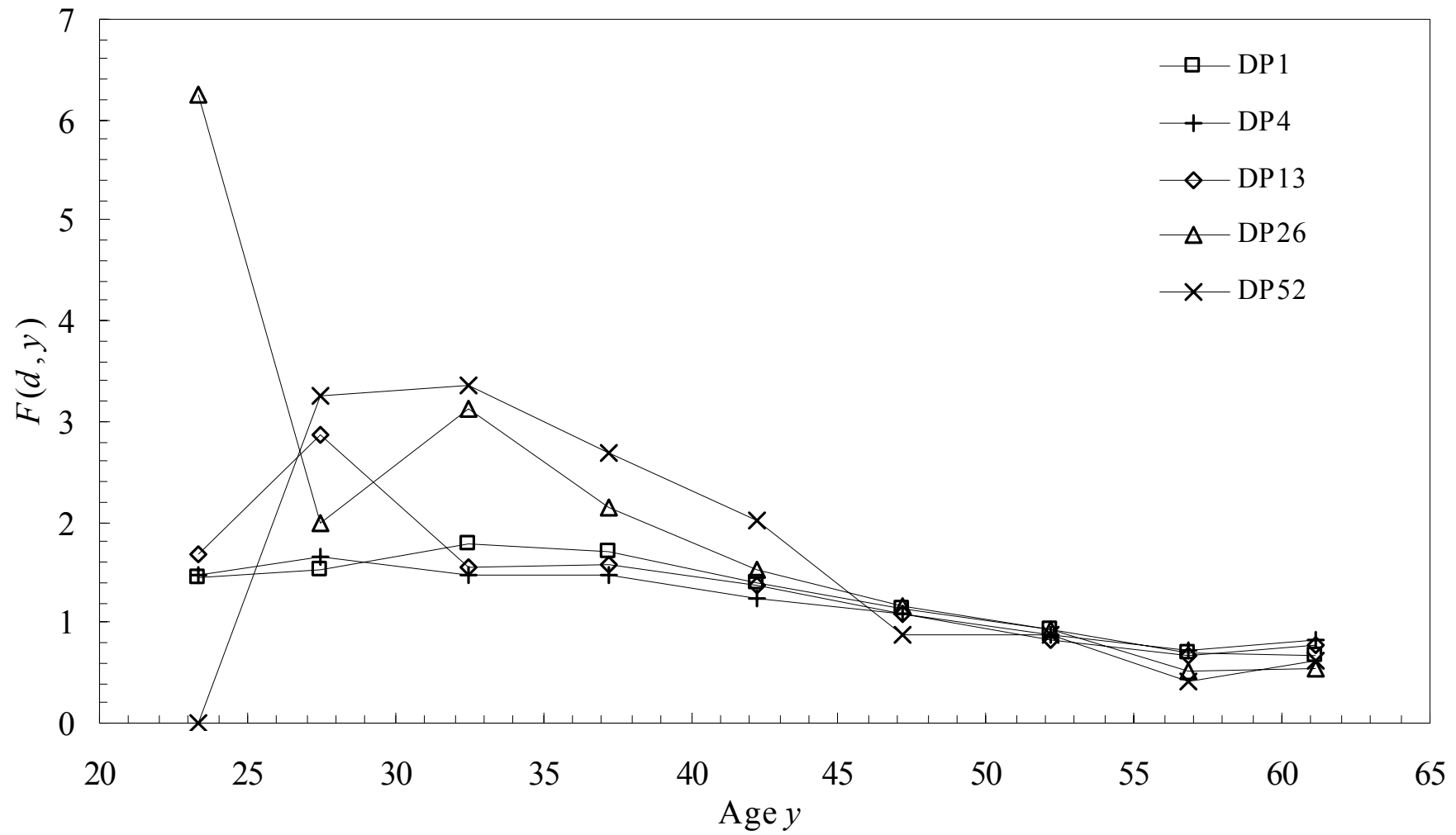


Figure 1. Recoveries –  $F(d, y)$  factors: computed for separate deferred periods.

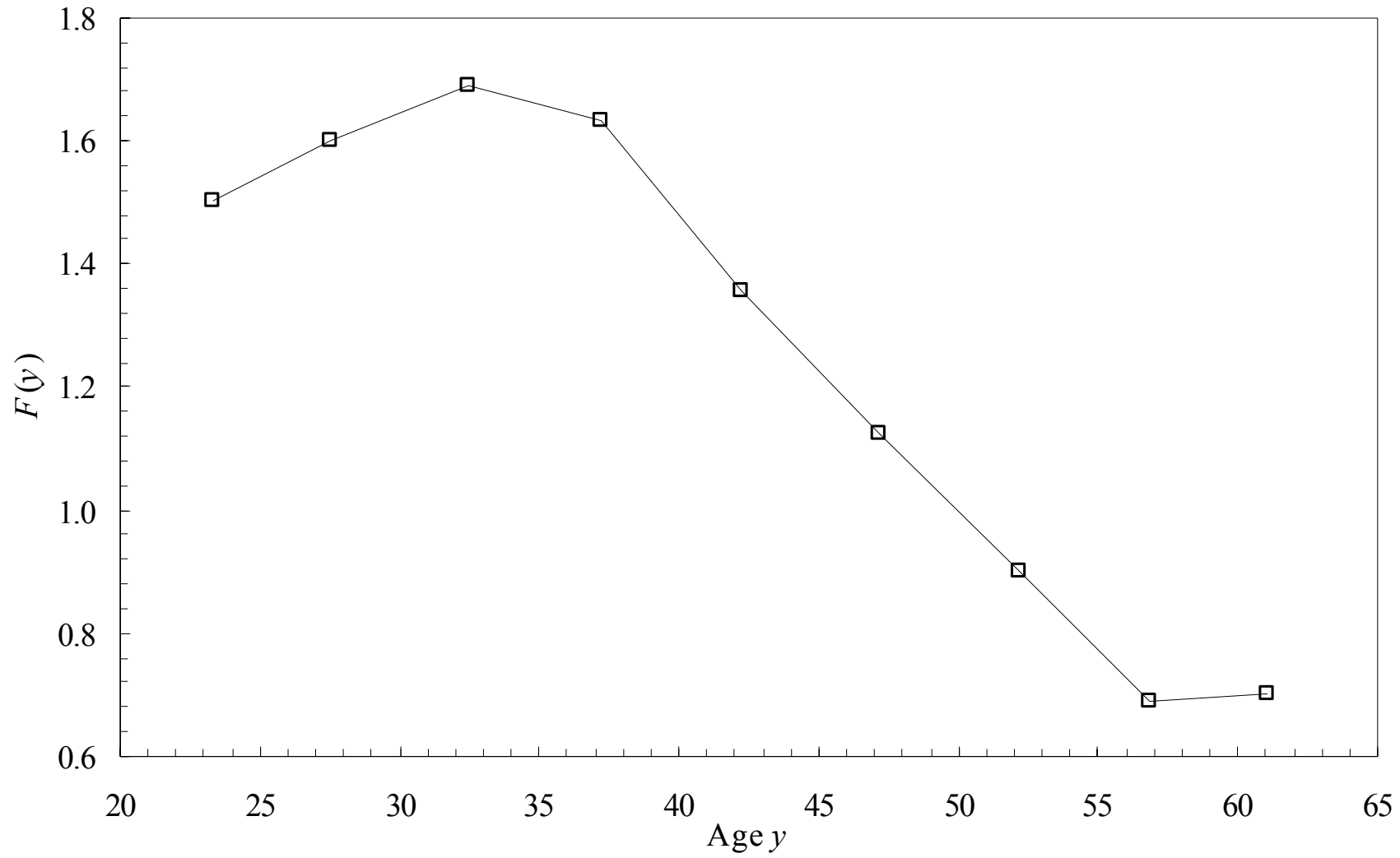


Figure 2. Recoveries –  $F(y)$  factors: all deferred periods combined.

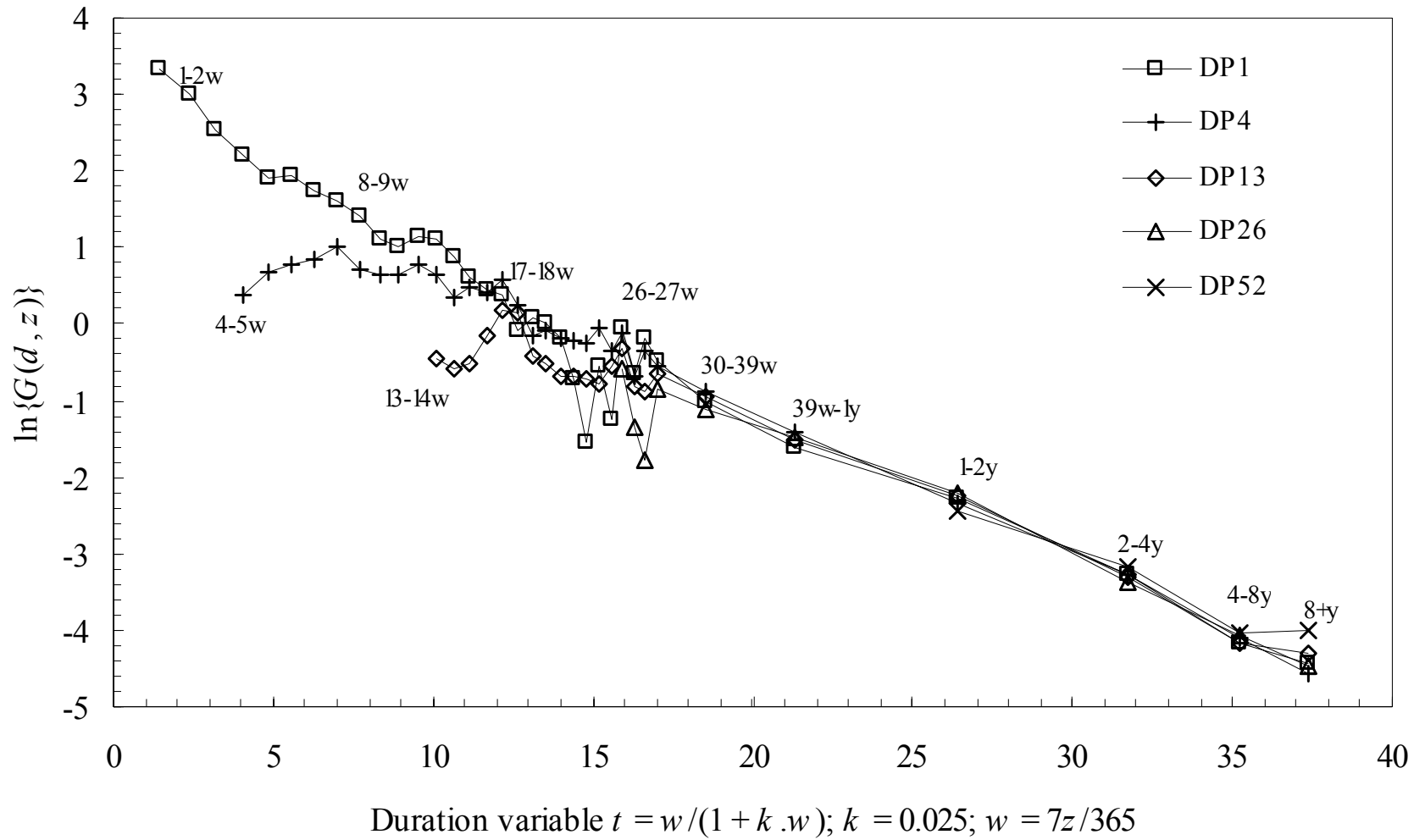


Figure 3. Recoveries –  $G(d, z)$  factors: computed for separate deferred periods.

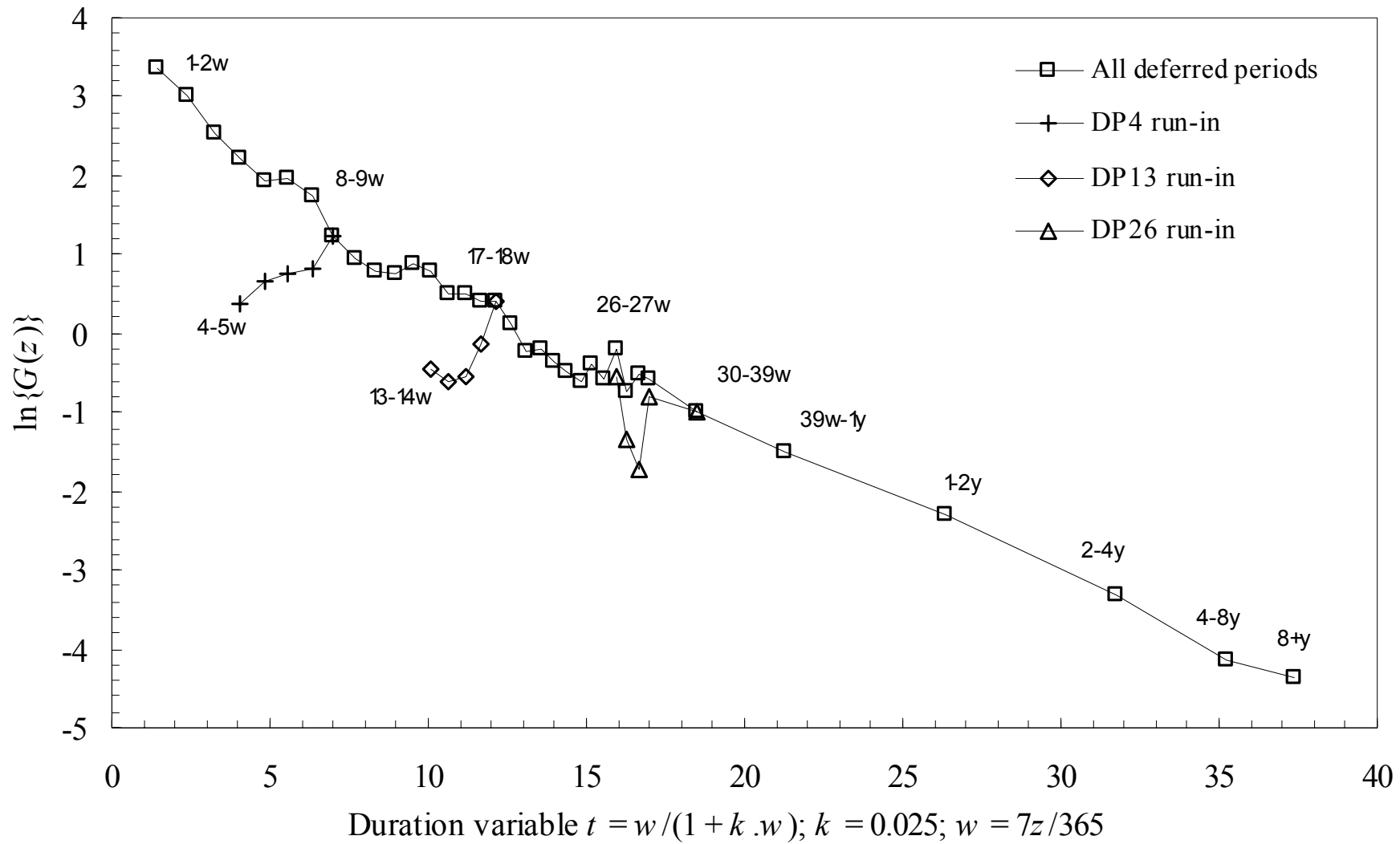


Figure 4. Recoveries –  $G(z)$  factors: all deferred periods combined.

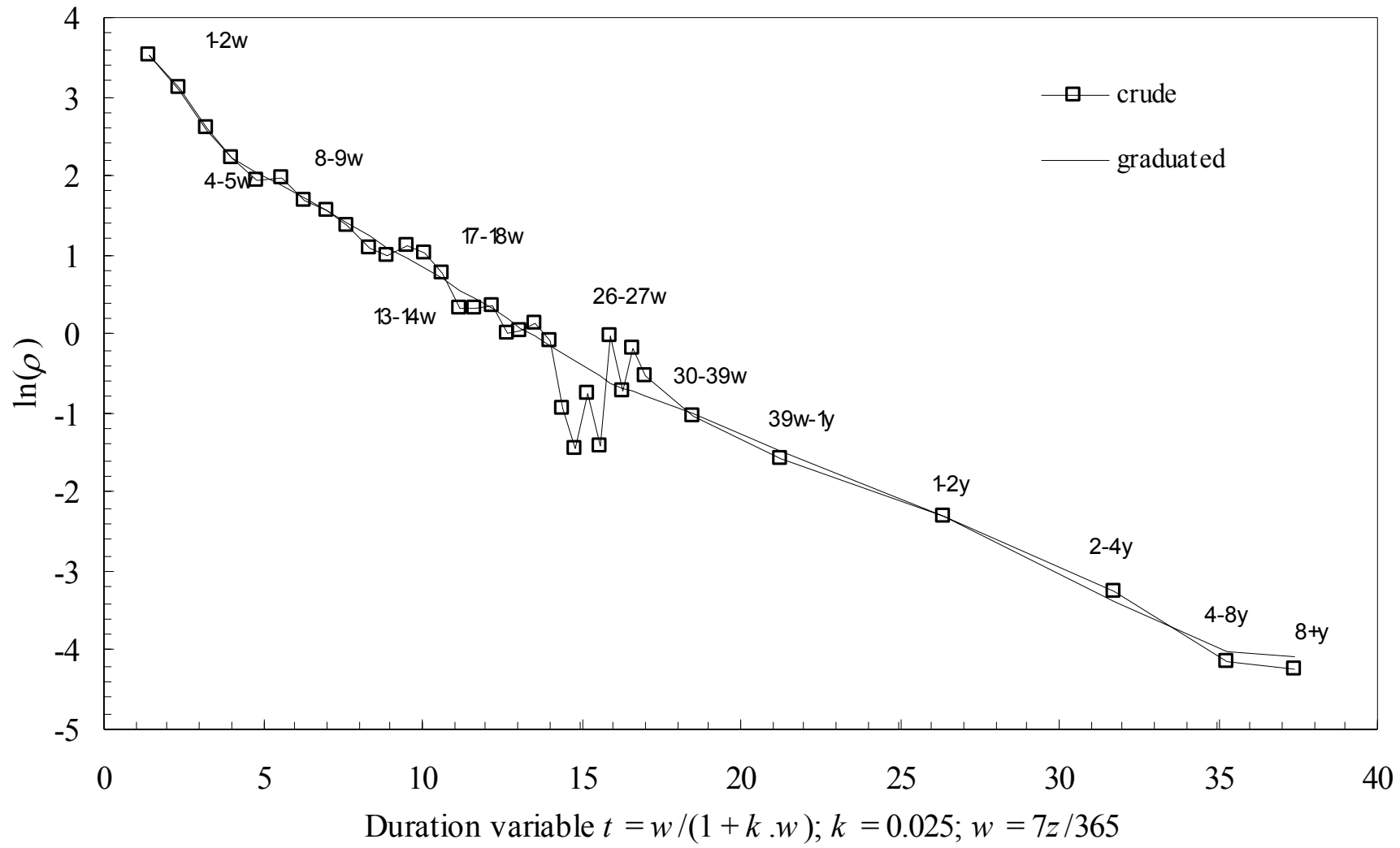


Figure 5A. Recovery rates by duration – DP1: all ages combined.



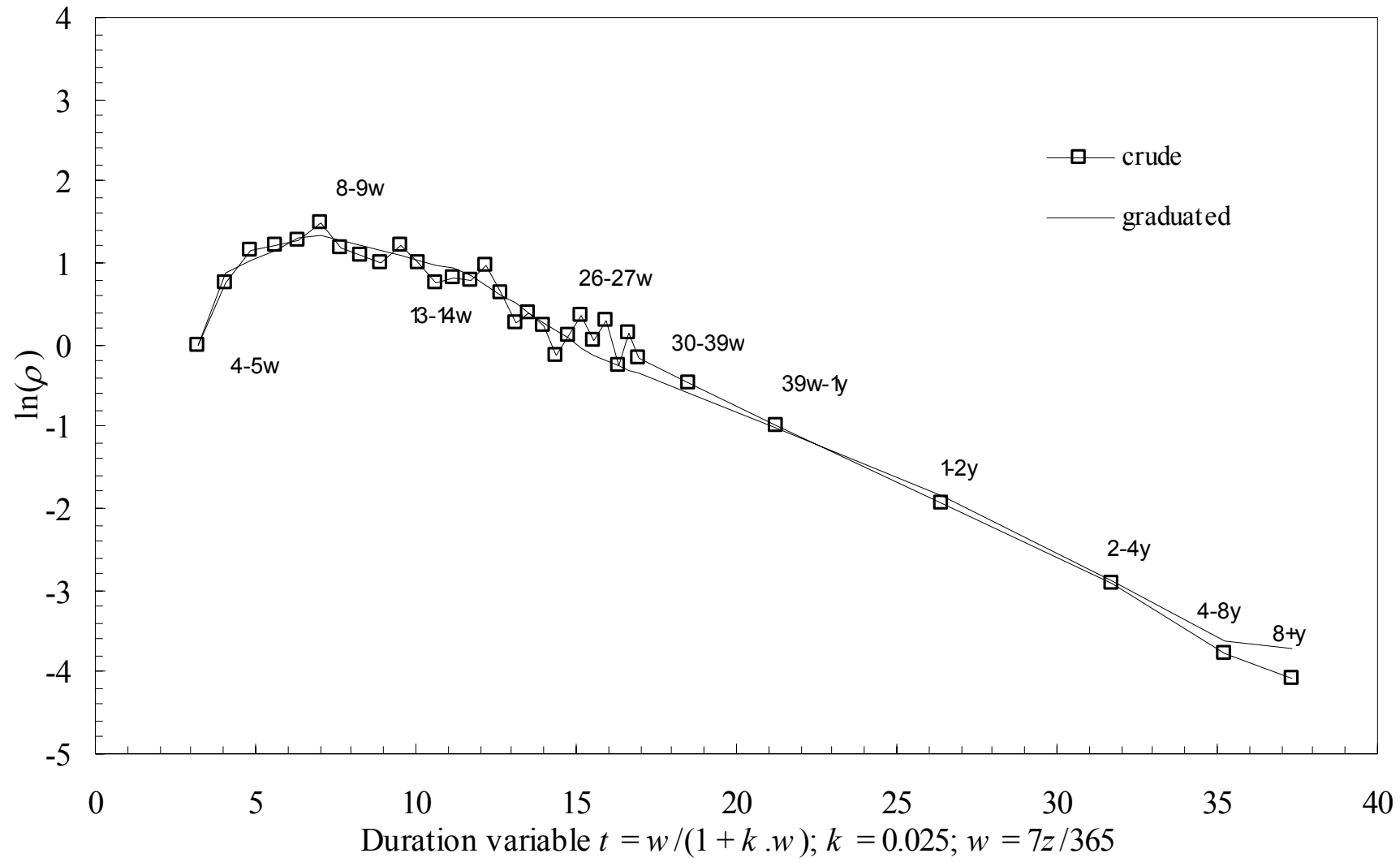


Figure 5B. Recovery rates by duration – DP4: all ages combined.

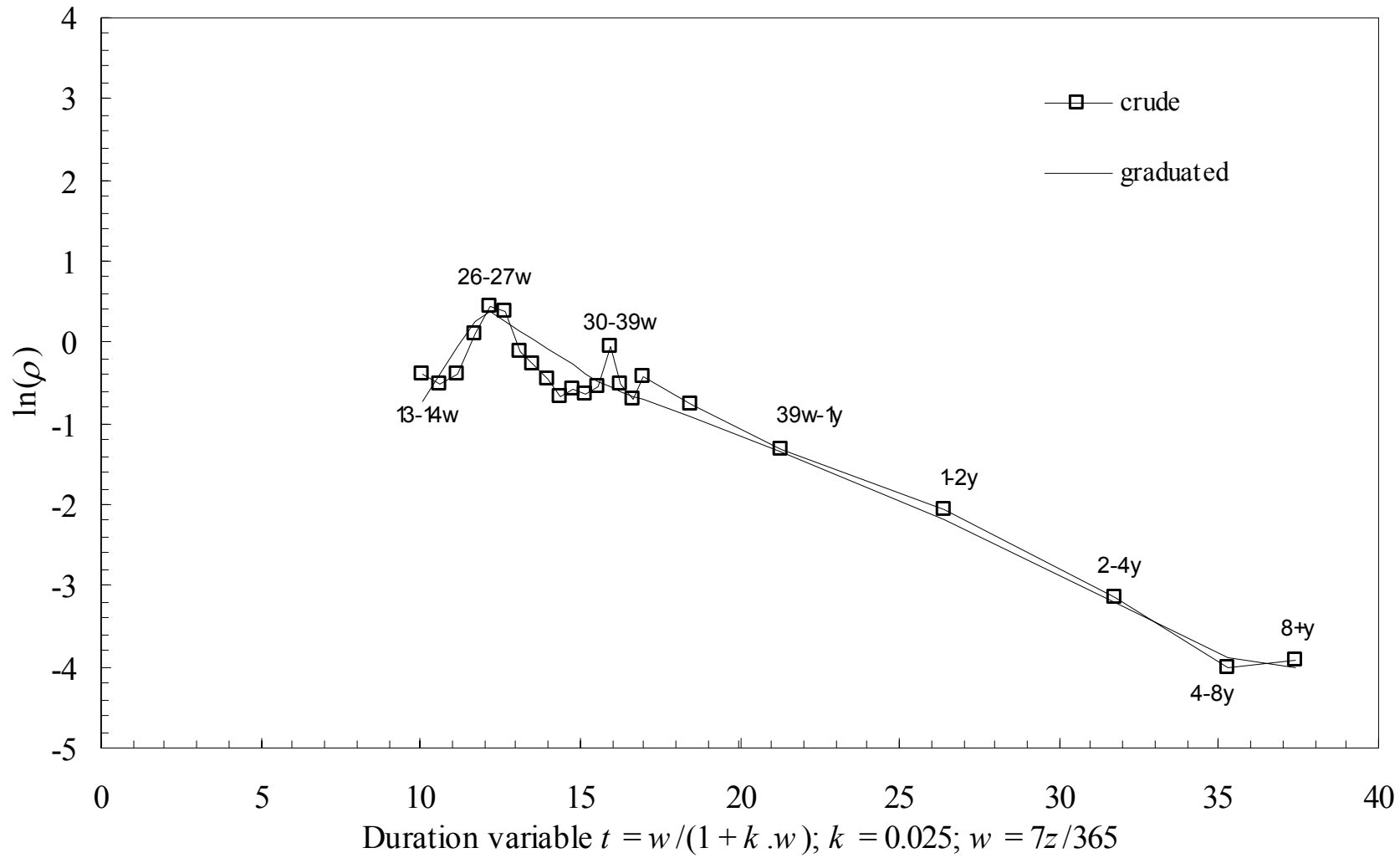


Figure 5C. Recovery rates by duration – DP13: all ages combined.

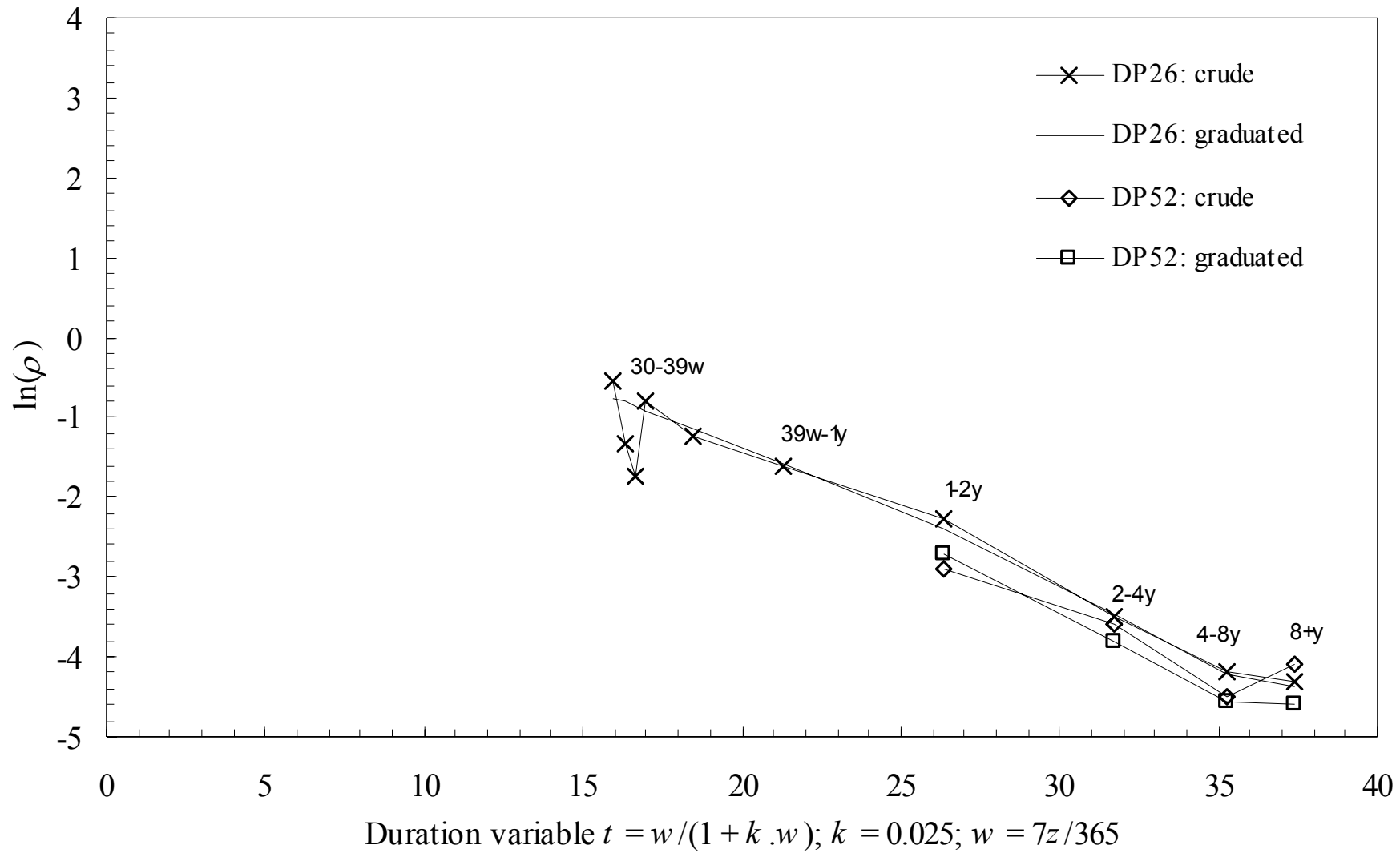


Figure 5D. Recovery rates by duration – DP26 & DP52: all ages combined.

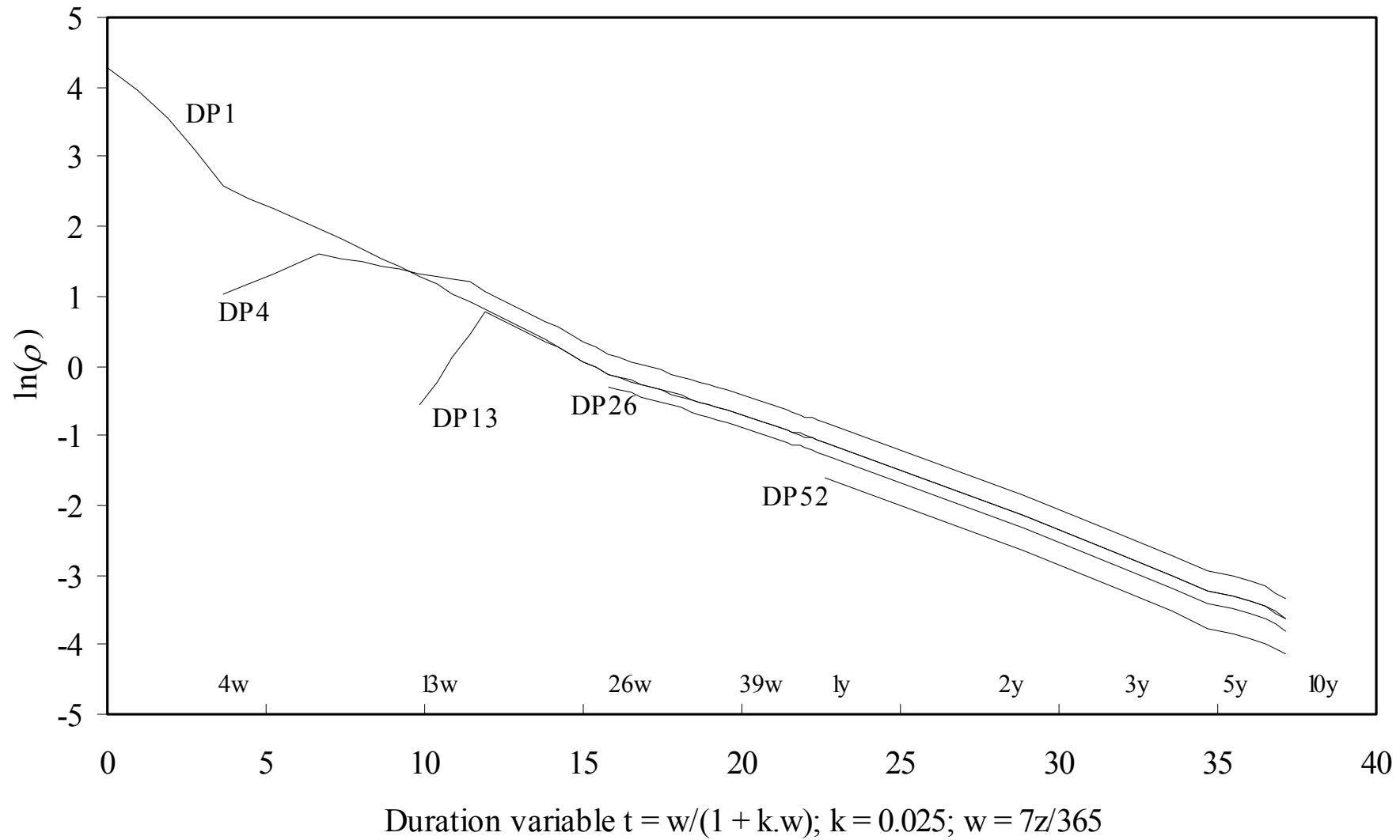


Figure 6. Graduated recovery rates, age 40: for separate deferred periods.

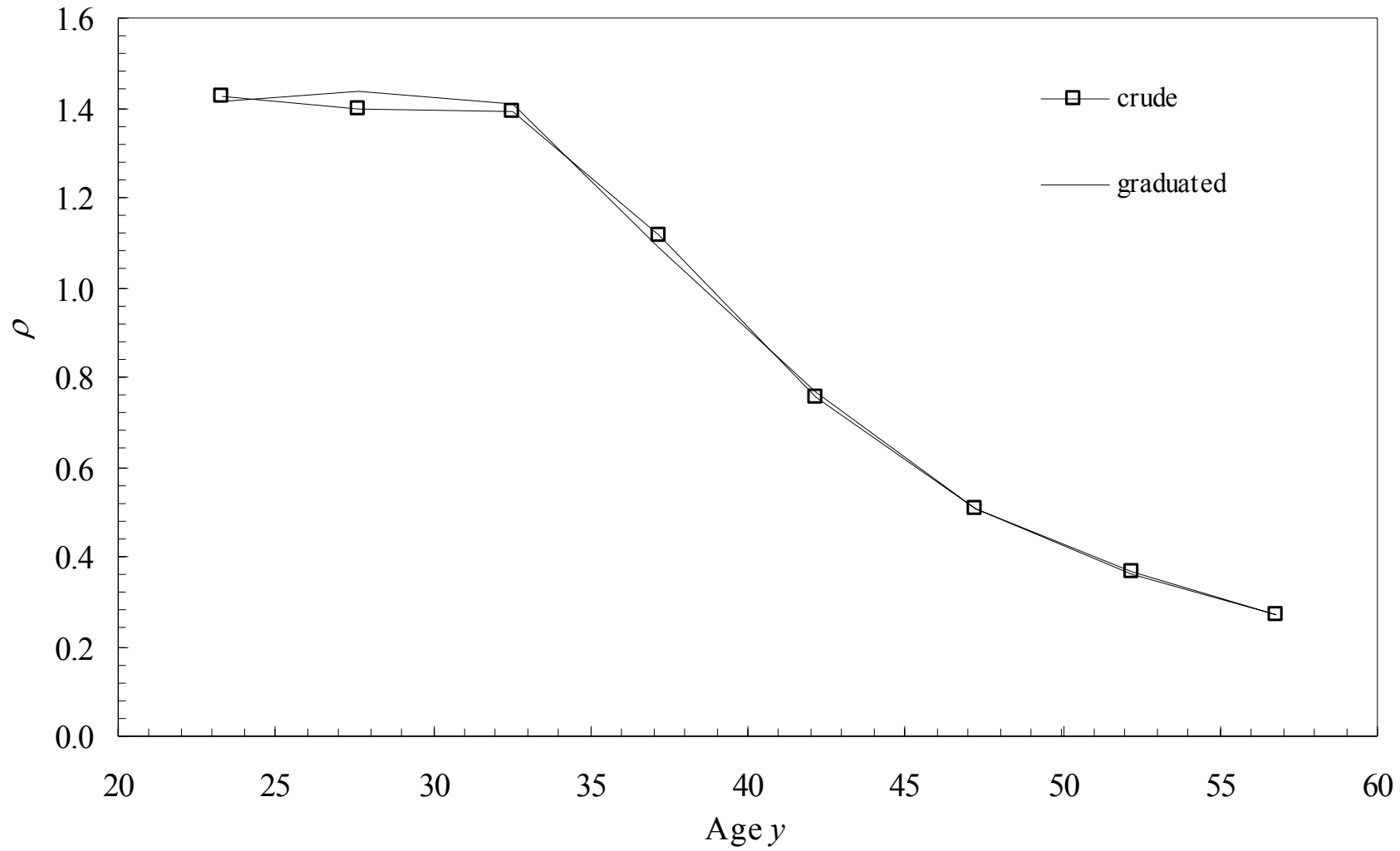


Figure 7. Recovery rates by age: all deferred periods/durations combined.

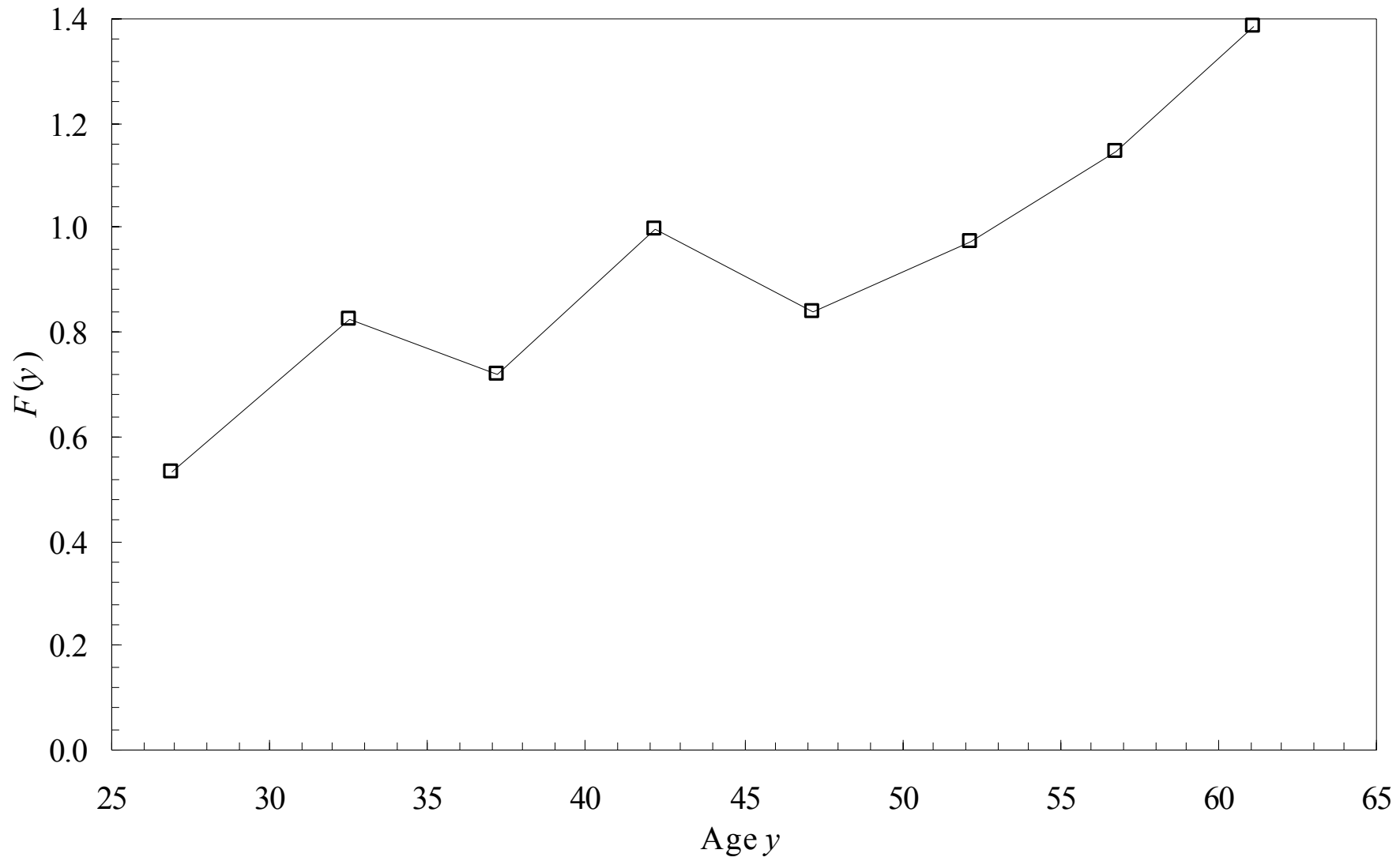


Figure 8. Deaths –  $F(y)$  factors: all deferred periods combined.

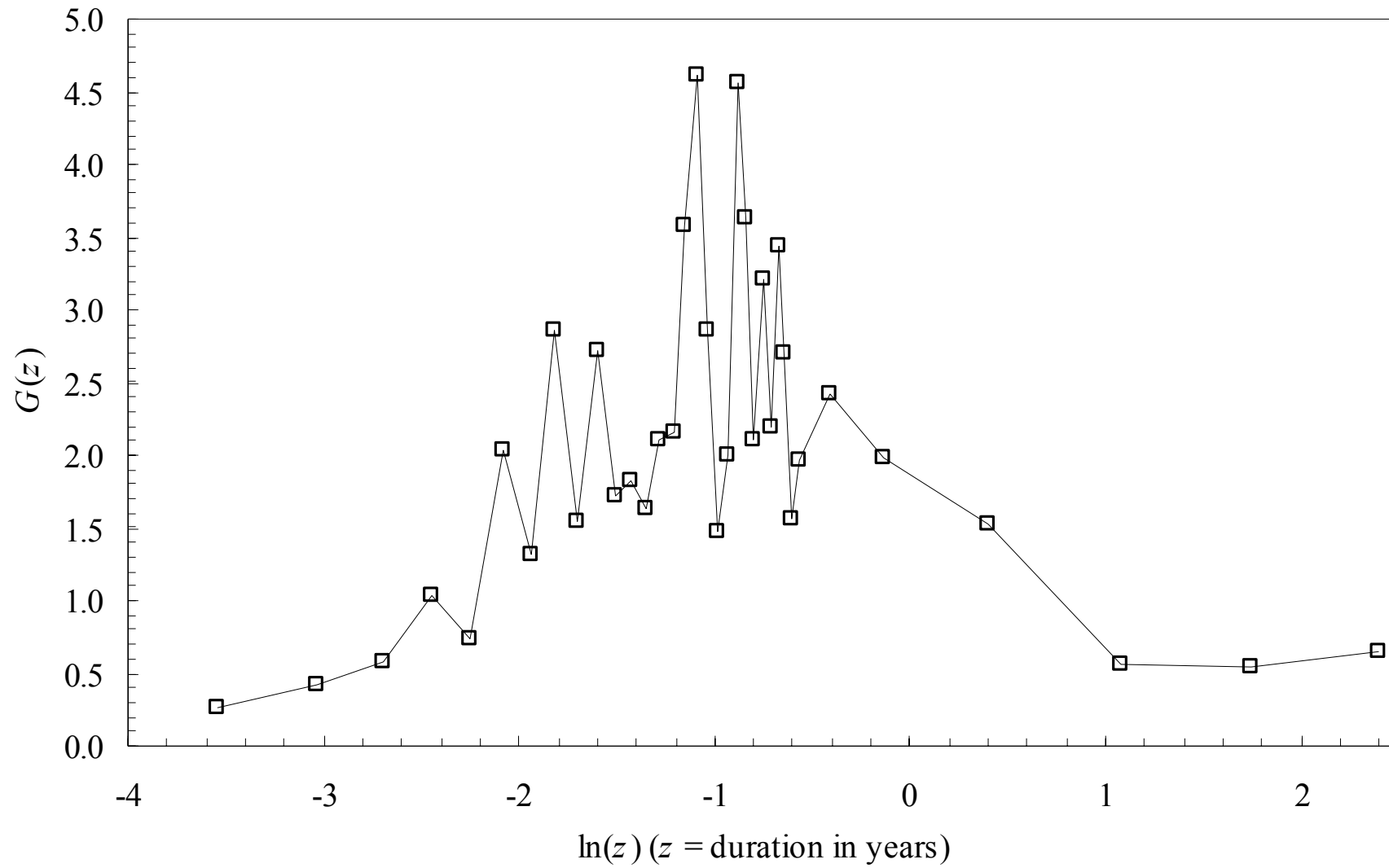


Figure 9. Deaths –  $G(z)$  factors: all deferred periods combined.

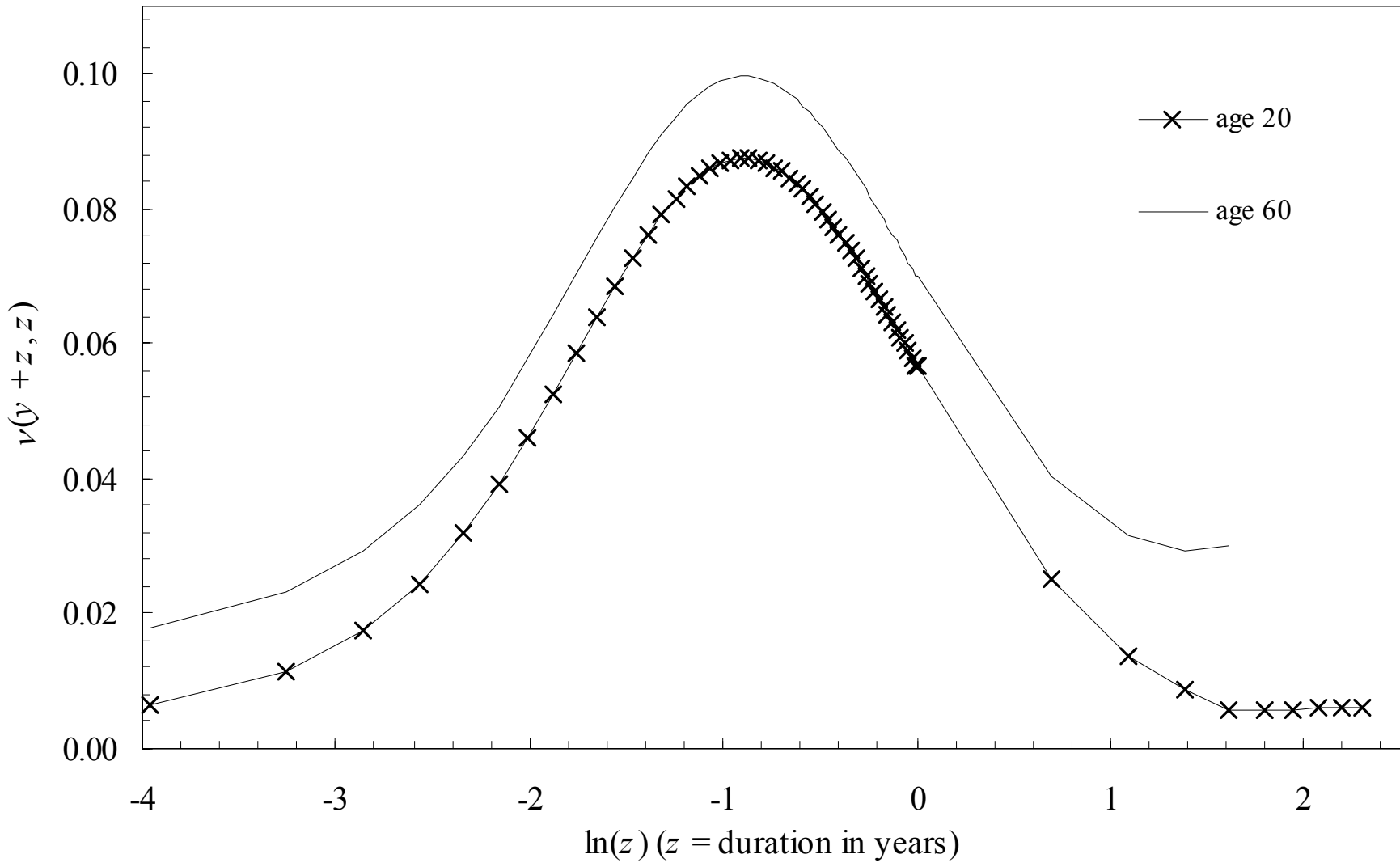


Figure 10. Graduated mortality intensities: all deferred periods except DP1.



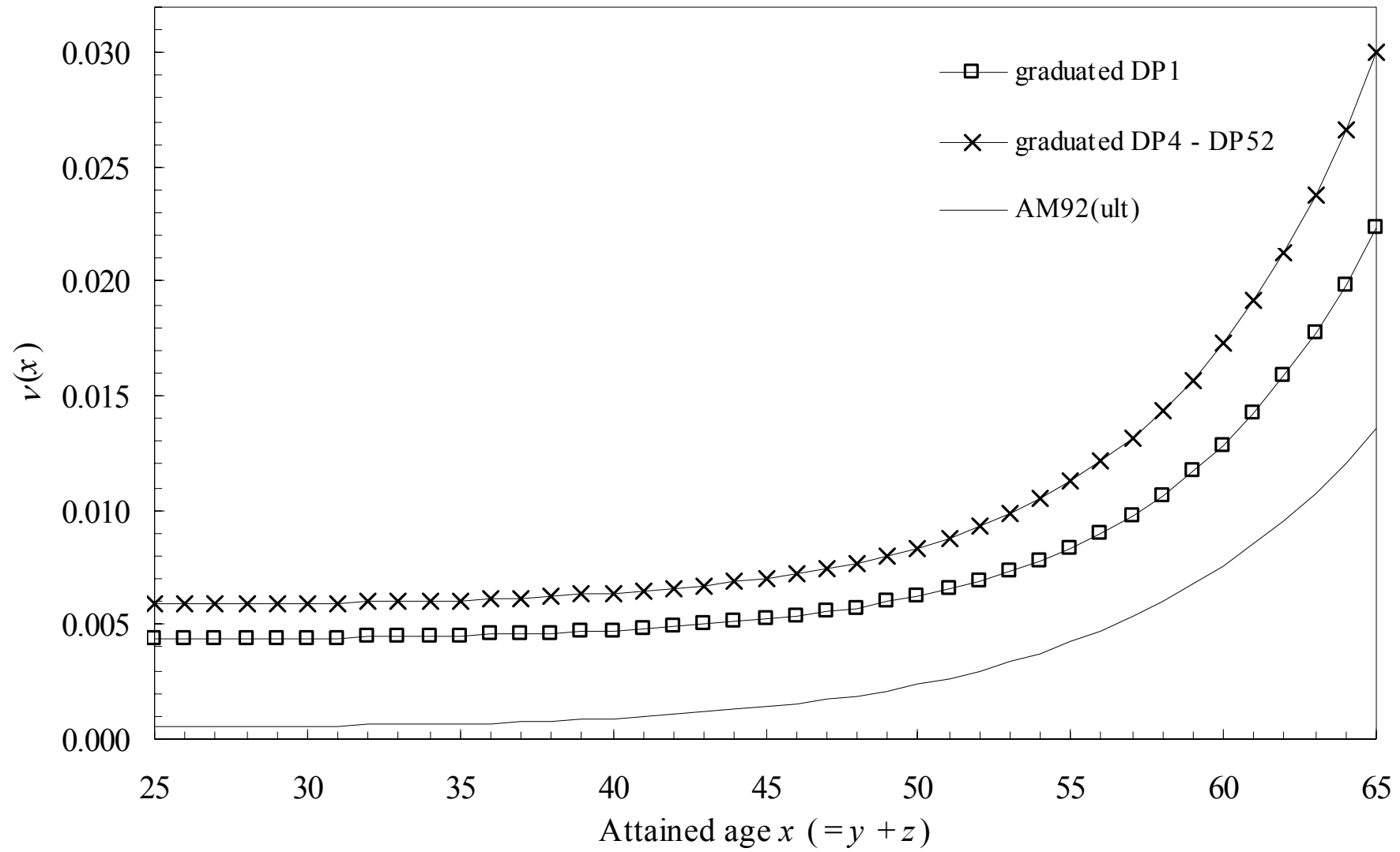


Figure 11. Mortality rates after 5 years sickness: intensities compared with AM92 (ult).

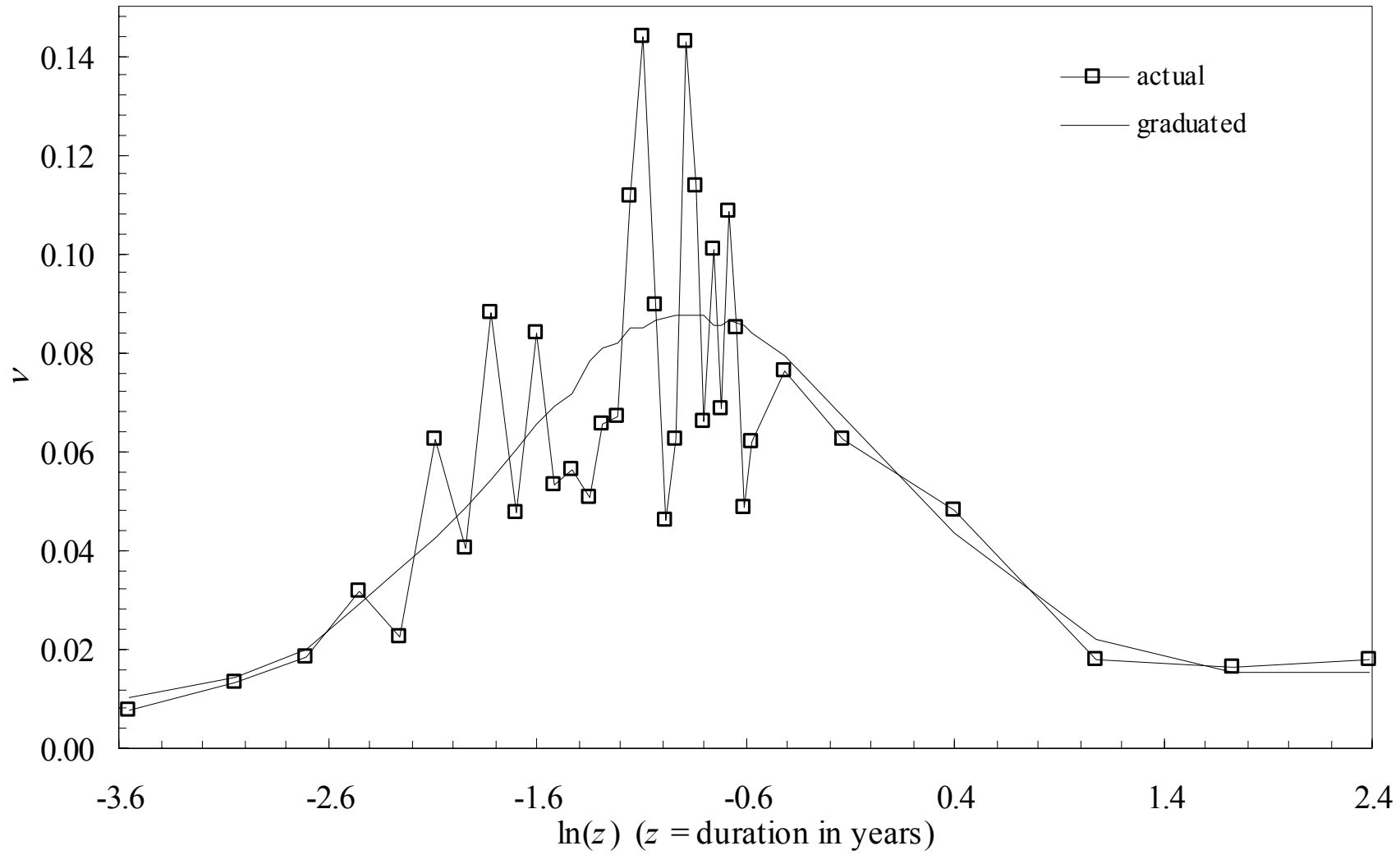


Figure 12. A & E mortality rates by duration: all deferred periods and ages combined.

## APPENDIX

### AGE DEFINITIONS

A1.1 In the course of these investigations it was discovered that the way in which ages had been calculated by the CMI Bureau, since analyses incorporating the disability annuity approach were introduced (including the 1975-78 experience analysed in *C.M.I.R.* 12), had been incorrect, by on average a little less than half a year. The results, however, had always been consistent with one another, so there is no need to correct past published figures, if the adjustment to be described is borne in mind. Nevertheless it was felt best to correct the position for the future. This deserves some explanation.

A1.2 On the data submitted for each policy, whether in-force or claim, there is space for an age definition code, and a month and year of birth (the last two digits). The current instructions to offices about the coding of ages allow three age definitions (coded 0, 1, and 2), and state:

- 0 = Age exact – month and year of birth must be coded in field 12.
- 1 = Nearest birthday at the end of the record year in field 3.
- 2 = Next birthday at the end of the record year in field 3.

Blank is equivalent to zero and if used the month and year of birth must be coded in field 12. If either 1 or 2 are coded then field 12 should contain 00YY, where YY is the office year of birth.

Field 3 gives the record year (i.e. the year ending 31/12/YY to which the in-force or claims relate); field 5 gives the age definition code; field 12 gives the month and year of birth; field 21 gives the date of falling sick in the format DDMMYY.

A1.3 For field 12, month and year of birth or office year of birth, the instructions state:

If age definition (field 5) = blank or zero, then code the month of birth in columns 23-24 and the year of birth in columns 25-26.

If age definition (field 5) = 1, then code '00' in columns 23-24 and the last two digits of the year of birth or the last two digits of the year of birth minus one in columns 25-26, depending on whether the month of birth fell prior to July or not.

If age definition (field 5) = 2, then code '00' in columns 23-24 and the year the policy was taken out – the age next birthday in that year, in columns 25-26.

If possible offices should adopt defining age exact in preference to the other options.

A1.4 It is assumed that offices interpret the instructions for age definition = 1 sensibly, that is, deducting one from the year of birth if the birthday falls prior to July, and not the reverse as the instructions might be interpreted.

A1.5 Age definition = 0 has always been the most popular, and in recent years has accounted for 95% of all claims records. Age definition = 2 has fallen out of use, with the last cases being submitted in 1990.

A1.6 Two definitions of age at commencement of sickness are required. One is an integer, and is used to classify results in age groups, e.g. 20-24, 25-29, etc. The other is a real number, i.e. including possible fractions, and is used to give the assumed exact age as at the date of sickness. The date of sickness is given in full, with day, month and year. We denote the integer age as  $iAge$ , and the exact age as  $dAge$ .

A1.7 The method in use by the CMI Bureau until now to calculate  $iAge$  has been, using an informal algorithmic formulation:

```

if age definition = 0 then
  if month of sickness  $\geq$  month of birth then
     $iAge = \text{year of sickness} - \text{year of birth}$ 
  else
     $iAge = \text{year of sickness} - \text{year of birth} - 1$ 
  endif
elseif age definition = 1 then
   $iAge = \text{year of sickness} - \text{year of birth}$ 
else
   $iAge = \text{year of sickness} - \text{year of birth} - 1$ 
  (so age definition = 2)
endif

```

A1.8 The method in use by the CMI Bureau until now to calculate  $dAge$  has been:

```

if age definition = 0 then
   $dAge = iAge$ 
elseif age definition = 1 then
   $dAge = iAge - 0.5$ 
else
   $dAge = iAge + 0.5$ 
  (so age definition = 2)
endif

```

A1.9 Careful analysis of these algorithms shows that, for age definition 0, where the month of birth and the month of sickness are not equal,  $iAge$  and  $dAge$  are both taken as age last birthday, so  $dAge$  is too small by one half of a year. Where the months are equal, however, on average half the cases give age last birthday and half age next birthday, so  $dAge$  is on average correct. The net effect is that  $dAge$  has been too small on average by  $11/24$  of a year, assuming even distributions of dates across months.

A1.10 A better algorithm for age definition 0 would be:

```

if age definition = 0 then
  if month of sickness  $>$  month of birth then
     $iAge = \text{year of sickness} - \text{year of birth}$ 
  elseif month of sickness  $<$  month of birth then
     $iAge = \text{year of sickness} - \text{year of birth} - 1$ 
  else
    (so months are equal)

```

```

    if day of sickness ≤ 15 then
        iAge = year of sickness – year of birth
    else
        iAge = year of sickness – year of birth – 1
    endif
endif
dAge = iAge + 0.5
else ...

```

A1.11 If the months are not equal this gives iAge as age last birthday in all cases. If the months are equal and the sickness commences in the first half of the month (assuming that all months have 30 days), then (assuming even distributions of all dates across the month) for 3/4 of these cases iAge is age last birthday, and for 1/4 it is age last birthday + 1 (or age next birthday). Where the sickness commences in the second half of the month, for 3/4 of these cases iAge is age last birthday, and for 1/4 it is age last birthday – 1. The quarters balance, so on average iAge is age last birthday. We then have  $dAge = iAge + 0.5$ , correctly the average exact age at commencement of sickness.

A1.12 Careful analysis of the algorithms for age definition 1 shows that iAge is on average age next birthday, and dAge is correctly calculated. However, the effect is that, for age definition 0, age group 20-24 means age last birthday 20 to 24; whereas for age definition 1 it means age next birthday 20 to 24, or age last birthday 19 to 23. This is inconsistent. A better algorithm for age definition 1 is therefore:

```

elseif age definition = 1 then
    iAge = year of sickness – year of birth – 1
    dAge = iAge + 0.5
else ...

```

A1.13 The description for age definition 2 is ambiguous since the date at which the age next birthday is to be taken is not defined. Is it the date of sickness, or the date at the end of the office year? We assume that it is taken as age next birthday at the date of sickness. We then see that iAge is age last birthday, and dAge is correctly calculated, so no alterations need to be made. As noted already, no such cases are currently being submitted. The algorithm can therefore be concluded:

```

else (so age definition = 2)
    iAge = year of sickness – year of birth – 1
    dAge = iAge + 0.5
endif

```

A1.14 In fact we now have for all cases that iAge is age last birthday so  $dAge = iAge + 0.5$  is correct on average.

A1.15 However, it is possible to calculate dAge more accurately, since for cases with age definition = 0 we know both month of birth and month of sickness, and for the other age definitions we know the month of sickness. A more exact algorithm for dAge would be:

```

if age definition = 0 then
  dAge = year of sickness – year of birth
        + (month of sickness – month of birth )/12
else
  dAge = year of sickness – year of birth – 1
        + (month of sickness/12) – 1/24
endif

```

A1.16 For age definition 0 this gives for all cases the age at commencement of sickness to the nearest month (assuming that all months have the same length) correctly, and for the other age definitions the age to the nearest month on average. In fact it was doing calculations on this more exact basis, and finding that they did not reconcile with the traditional method that exposed the error in the older method.

A1.17 All three methods can now be used and have their function. The older method may be necessary to replicate past results and to provide a comparison between methods. The newer method, but without going to exact months, is in fact necessary for the graduation process, in order to keep the volume of data down, and has been used for the graduations described in this report. The method with exact months for ages of sickness is convenient for routine analyses.