Cost of Capital Method of Determining Risk Margins – Dealing with Neglected Issues: Observations from Three Empirical Analyses

GIRO, October 2007

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COST OF CAPITAL ANALYSIS FOR RISK MARGINS
ANALYSIS OF PRACTICAL ISSUES

1 Introduction

The 'cost of capital' method for calculating risk margins for supervisory and general purpose financial reporting has been suggested by a number of industry groups in Europe[1] and has been recognized as potentially appropriate by the International Association of Insurance Supervisors (IAIS)[2] and the International Accounting Standards Board[3].

In the "cost of capital approach" to setting risk margins for unpaid claim liabilities, the risk margin is determined by measuring the return on the capital required to protect against adverse development of those unpaid claim liabilities. It is quickly evident that application of the cost of capital method requires an estimate of the initial capital to support the unpaid claim liabilities and the required return on that capital (the cost of capital). A number of organizations have volunteered and/or are already doing research on those issues[4]. There are, however, are other important technical issues that are less often recognized.

In this paper we present three studies that use actual data to derive insights regarding those 'less often recognised' technical issues. The work also provides some information about required capital levels, although that is not its primary purpose. This work does not address the issue of required cost of capital. The explanations are aimed a reasonably knowledgeable audience. We assume the audience is familiar with standard actuarial risk-analysis terminology.

This reflects the work of the authors and not of any firms they are associated with it.

1.1 Issues Covered by our Work

We analyse motor insurance portfolios from Australia, the UK and the US, using a different model in each case. The portfolios are from entities of varying sizes. In addition to the indicated initial capital levels, the issues of interest to us are the following:

1. Release of capital – It is convenient to assume that required capital is proportional to the best estimate liability, but the International Actuarial Association (IAA) Risk Margin Working Group (RMWG) described good reasons to believe that assumption may not be correct for most general insurance lines of business.[5]

2. Latent claims – One special area in which required capital is largely or totally unrelated to the size of the liabilities is the exposure to latent injury claims.

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[1]For example, the CFO Forum, CRO Forum, identified in GIRO Risk Margin Working Party, *Interim report to GIRO September 2006, Vienna, General insurance reserves for accounting and solvency: incorporating provision for risk*, page 64, http://www.actuaries.org.uk/files/pdf/proceedings/giro2006/White.pdf

[2]IAIS, *Issues arising as a result of the IASB's Insurance Contracts Project – Phase II, Second Set of IAIS Observations*, 1 June 2006, page 12, http://www.iaisweb.org/__temp/IAIS_provides_second_comment_paper_to_IASB_on_Phase_II_project.pdf

[3] International Accounting Standards Board, *Preliminary Views on Insurance Contracts Part 2: Appendices*, 2007, Appendix F, item F9, page 36-37, http://www.iasb.co.uk/

[4] Committee of European Insurance and Occupational Pension Supervisors (CEIOPS), IAA, and Chief Risk Officer (CRO) Forum.

[5] IAA Risk Margin Working Group, *Measurement of Liabilities for Insurance Contracts: Current Estimates and Risk Margins*, Exposure Draft, February 23, 2007, page 63-64, http://www.actuaries.org/index.cfm?lang=EN&DSP=CTTEES_RISKMARGIN&ACT=DOCUMENTS

3. Diversification – Actuarial practice suggests that the effect of diversification across lines of business should reduce the risk and therefore the risk margin,[6] but some accounting treatments suggest that the effect of diversification should not necessarily be considered. We are interested in measuring how much diversification is likely to reduce risk margins, if the reductions were allowed.

4. Net, gross and outwards reinsurance (RI) liabilities/assets – While the formula Gross - RI = Net should apply from consistent assumptions regarding the three expected values, the same is not necessarily the case for risk margins and hence for provisions.[7] We are interested in exploring how this might be handled in practice.

5. Reference Portfolio Concept – Risk margins, whether measured by the cost of capital method, confidence levels or many other methods, have the property that a company writing fewer policies will have a larger risk margin (as a percentage of the expected value) than company writing more policies, even if the two companies write the same type of business. This can be awkward as the IAIS would like similar types of portfolios, regardless of size, to have the same percentage risk margin. To address this, the IAIS and IAA have discussed the concept of measuring risk margins based on the experience of a hypothetical portfolio of an assuming company, rather than based on the experience of the reporting company. The concept of a reference portfolio is also inherent in the IASB's concept of an exit value.

6. Marginal capital requirements – As the portfolio might be valued as if it were part of the reference portfolio, we consider the marginal additional capital that is required when the risk of the target company reserves are combined with the risks in a larger entity.

1.2 Structure of this paper

Section 2 of the paper summarises the data, methodology, assumptions and results from our analyses.

Section 3 describes the cost of capital methodology.

Sections 4, 5 and 6 describe the Australian, UK and US examples in more detail.

Sections 7-11 describe the results in the areas of latent injury, diversification, reinsurance, the implications of the reference company concept, and alternative cost of capital formulations.

1.3 What this paper does not do

This paper is an exploration of technical issues in the implementation of the cost of capital methodology. The paper does not take any position on the following issues:

1. Whether cost of capital is an appropriate method, or whether it is better or worse than other methods.

2. Appropriate rules for determination of required capital or cost of capital.

[6] IAA Risk Margin Working Group, page 78-79

[7] This issue is not unique to the cost of capital method of determining risk margins. Any method that determines risk margins from individual company expected variability is subject to this difficulty.

3. Whether cost of capital approach can properly be described as a 'market price' 'exit price' or 'fair value' for insurance liabilities.

4. The interaction of capital requirements and other regulatory rules.

In our examples, unless otherwise noted, we use a 6% cost of capital, a 99.5% Value at Risk (VaR) capital level and a 4% risk rate with a flat yield curve. Those are the levels that have typically been used to illustrate the concepts and therefore are the values shown in the IAA RMWG Exposure Draft. We are not recommending that those levels are necessarily appropriate. We also discuss the use of Tail Values at Risk (TVaR) and expected policyholder deficit (EPD) as alternative risk measures.

We show results from our models to illustrate various issues. None of the results are intended for actual use by companies in practice. Further research might change some of our conclusions.

1.4 Our Worksheets

Further details of our work are available in spreadsheets that can be provided on "as-is" and "at your own risk" basis by contacting any of the authors.

2 Summary of Data, Methods, Assumptions and Results

In this section we first show the data and methods of risk analysis that we used for our three analyses. We then summarise some of the main numerical and qualitative results.

2.1 The data, methods and assumptions

2.1.1 Data

Table 2-1 below compares the data used in each of our three analyses.

Table 2-1—Type of Business and Data

Item/Country	Australia	UK	US
Business Type	Australia New South Wales personal motor bodily injury liability	UK personal motor bodily injury	US commercial auto (motor) bodily injury and property damage liability
Data Portfolio Size-Annual Expected Claims in $USD	$500 million up to 1989, $50 million thereafter	Over $500 million	$50 million and $25 million (But paid loss development and claim size data from larger companies other sources also used.)
Data Format	Accident quarter by development quarter	Accident year by development quarter;	Accident year by development year
Level of detail available	Individual claim amounts with accident date, report date and closing (finalization) date	Aggregate paid data triangles	Aggregate data triangles "(Schedule P format)"
Type of data triangles	Unit record data, for claims open at any time in the period 1993 to 2003	Paid claim amounts	Paid claim amounts Target company data supplemented by comparable data from about 40 other companies and claim size information from other sources
Exposure Years	Claims open 1993-2003	AY 1994-2006 developed to 2006 Q4	1986-1995
Policy Limits and Reinsurance Treatment	Gross data Policy provides unlimited protection Claim size distribution used to model reinsurance effects Reinsurance with indexed retention	Gross data Policy provides unlimited protection	Net data. There is little reinsurance, but all data is affected by policy limits. Claim size distribution used to model reinsurance effects beyond those reflected in the data.

2.1.2 Models

For each of the examples we assume there is a best estimate of the unpaid claim liability. The details of the models used, while critical in practice, are not part of the scope of this paper. The main risk measurement features of the models are described in Table 2-2.

Table 2-2—Risk Models

Item/Country	Australia	UK	US
Model Type	Collective Risk[8]	Bootstrapping	Collective Risk plus Bayesian approach with aspects that serve purposes similar to bootstrapping.
Model description	The run-off of several portfolios of claims is simulated using duration dependent rates of settlement and settlement amount distributions. Two types of starting portfolio are considered: • cohorts of claims, each at a single duration since accident, with the same expected value in each cohort; • a mature portfolio of claims, at varying durations into run-off. These are considered in isolation and in the context of a much larger ongoing mature portfolio of claims, to assess the marginal impact on the capital requirements of a hypothetical purchaser.	A paid loss development factor model (DFM) is created. This involves adjusting for misleading ratios, averaging and also fitting a curve for the later points and the tail. The differences between the actual and estimated development factors are used to create standardised residuals from which pseudo data triangles are simulated. DFMs are then applied to these pseudo data triangles to produce a range of reserve estimates. This is described in England and Verrall (2006)[9].	The expected loss is determined by: • Accident Year Earned Premium • Expected loss ratio (ELR) • Incremental paid loss development factors (Dev_{Lag}) Lag=1…10 ELR and each Dev_{Lag} are selected from a random set of parameters derived from an observed 10-year loss development triangle using Bayesian estimation. The prior distribution for the Bayesian estimate comes from information of other companies. Errors are randomly distributed around the expected loss according to the collective risk model

[8] An individual claim frequency severity model.
[9] *Predictive Distributions Of Outstanding Liabilities In General Insurance,* Annals of Actuarial Science, Volume 1, Number 2, 2006 , pp. 221-270(50) England, P. D.; Verrall, R. J.

2.1.3 Sources of Risk Considered

The sources of reserve variability and the manner in which that variability in considered in the models is summarized in Table 2-3 below.

Table 2-3 – Risk Considerations

Item/Country	Australia	UK	US
Inflation	Explicitly included with several models Tables 2.5-2.11 use inflation “model 4” (defined in Table 4-2), unless otherwise indicated.	No explicit consideration. Past inflation patterns are implicitly assumed to continue into the future.	Implicitly reflected in a 5% annual premium growth.
Parameter risk	Embodied largely in the inflation models Inflation rate has initial value and then (a) random fluctuation around known value and/or (b) random walk from starting point.	Process and parameter risk measured together	Reflected in the random selection of the ELR and Dev_{Lag} parameters.
Process risk	Variability in claim size from log normal distribution.	See parameter risk	Reflected in the collective risk model for the random errors.

Summarising Tables, 2-2 and 2-3 above,

- For the Australian data, we applied a collective risk model. In that model variations in outcomes are determined by variations in inflation, variations in claim closing times and variations in claims sizes around an expected mean value.

- For the UK data we applied a bootstrapping model. In that model, variations in outcomes are determined by the observed variations between the assumed loss development model and the actual data and assumptions about the underlying error structure.
- For the US data we used a composite of Bayesian and collective risk approaches. The Bayesian model component was used to determine the variation in parameters in a loss development model given a collective risk model that measured the process risk around the expected values.

Section 7 on shock and latent injury claims, discusses the implication on risk margins of risks not included in the modelling.

2.1.4 Required Capital

Required capital is measured as described in Table 2-4

Table 2-4 – Measuring Capital requirements

Item/Country	Australia	UK	US
Measuring Capital	Simulation results (expected present value) are sorted by size. Selected VaR, TVaR and EPD values are determined.	Bootstrapping gives range of results around the mean from which VaR, TVaR or EPD can be calculated.	First select a {ELR, Dev_{Lag}} parameter set at random to calculate the expected loss and then use the collective risk model to get random outstanding losses over a period consisting of ten accident years.
The Base capital is 99.5% VaR. Other capital levels measured are:	VaR and TVaR at 95%, 99%, 99.5%, 99.9% and 99.95%, based on 10,000 simulations	99.5% VaR	99% TVaR and 99.5% VaR

The VaR and TVaR levels represent confidence levels for discounted claims. In the Australia analysis the confidence levels is calculated from discounted claims. In the UK and US analysis the cash flows are determined and then confidence levels are calculated from undiscounted cash flows. The payouts associated with the mean value and various confidence levels are discounted to produce indicated capital on a discounted basis.

2.2 Results

Some of our main quantitative results are summarised tables 2-5 to 2-14 below.

The issues we consider are the following:

- Basic cost of capital results
- The rate at which capital is released ("shape")
- Effects of reinsurance
- Size of reporting company
- "Marginal" capital – Implications for reference company
- Parameter risk
- Value at Risk vs. Tail Value at Risk
- Shock claims and latent injury claims
- Alternative cost of capital formulas

Unless otherwise indicated our starting points are as follows:

- Australian experience with unlimited policy coverage net of unlimited excess of loss (XoL) reinsurance attaching at 2.5m AUD (indexed))
- US experience with $1 m USD policy limits
- UK experience with unlimited policy coverage gross of reinsurance
- Capital means capital at 99.5% VaR levels of discounted ultimate claims above the mean discounted reserves with respect to run-off risk for the sample line of business with no diversification. No other risks are considered, e.g., no capital for asset risk, operational risk, etc.
- The cost of capital is assumed to equal 6% per year (in addition to a risk free rate of 4% per year) and the cost of capital methodology is the one illustrated in Table 3-2.
- Risk margin is expressed as a percentage of discounted mean reserves.

For each country the models are as described in the appropriate sections of the report and summarised in Tables 2-1 to 2-4 above.

2.2.1 Cost of Capital by Model

Table 2-5 below summarises the base case cost of capital risk margins for each of the three models and some comparative risk measures.

Table 2-5--Capital, Cost of Capital, Percentile and Amount of Interest Discount

1	2	3	4	5
Country	Capital	Risk Margin Cost of Capital	Percentile Equivalent	Interest discount (4% p.a.)
Aus	28.8%	4.6%	68%	10.8%[10]
UK	15.1%	2.2%	67%	6.2%
US	23.2%	4.3%	65%	7.0%
(2) At 99.5% VaR				
(5) Undiscounted Reserve/Discounted Reserve -100%;				

For our three models the cost of capital risk margins range from 2% to 5%. These are less than the interest discounts of 7% to 11% which are sometimes viewed as the risk margins in jurisdictions that present liabilities on an undiscounted basis. These are also less than the 75%-ile used as the confidence level in Australia.[11]

The UK results were based on a very large data set and the risk margin is relatively low because the development patterns were very stable. We cannot be certain whether or not the lower UK levels are "real' because of the large data set, due the choice of bootstrapping vs. collective modelling or due to details in the DFM method used in bootstrapping. Supporting the theory that the UK risk margin is due in part, at least, to the size of the portfolio, we observe (Table 2-9) that the risk margin for the US model is 3.1% for a company of $500m. Table 2-5 results apply to a single line of business. If diversification were considered in the calculation of cost of capital risk margins, as seems theoretically proper, then the cost of capital risk margins would be lower. The percentile level corresponding to the cost of capital risk margin would also be lower. The discount would not change.

On the other hand, the above does not include provision for regulatory minimum capital requirements or for capital related to shock or latent injury claims. Any such consideration would increase the cost of capital and percentile risk margins.

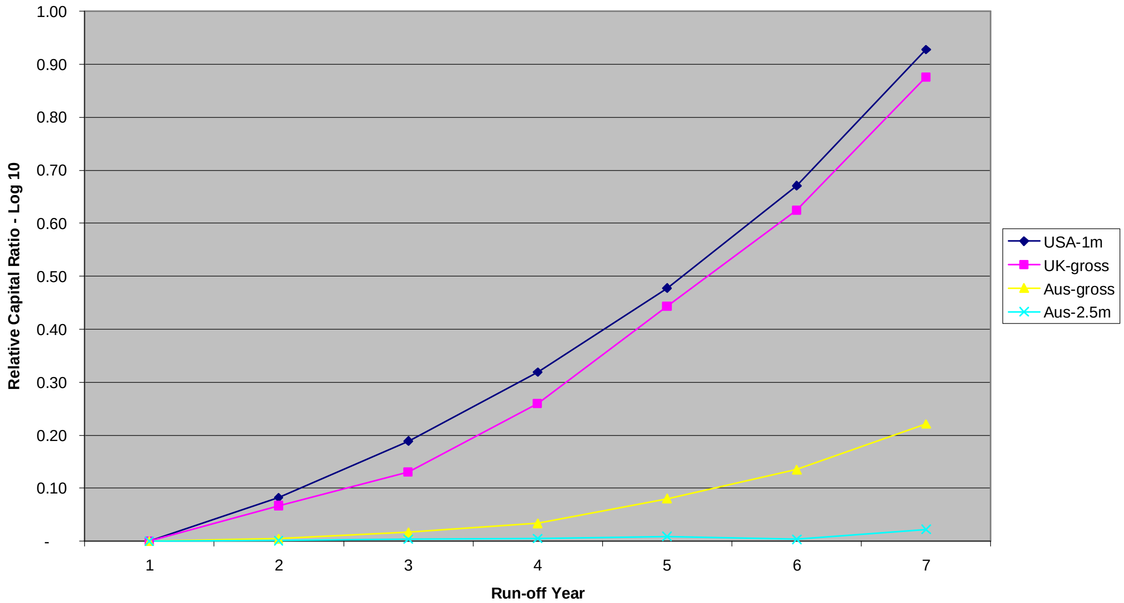
2.2.2 Variations in Capital Ratio by Year of Run-off

One issue this paper proposed to examine was the how the ratio of capital to liability changes by age of claim. With each of our models we measured this in two ways.

Firstly, as shown in Chart 2-6 we calculated the ratio of required capital to discounted liability by age of runoff. In most cases, as we had expected, the ratio increases significantly with age.

[10] This value is not quite comparable to the other values as the interest discount was determined with gross policy limit payments.
[11] Australia confidence level is applied on an entity basis rather than line of business basis.

Chart 2-6--Capital÷Discounted Liability by Run-off Year



The effect is very strong in the US and UK models. Because the effect is so strong we used a log-scale for the change in capital to liability.

The effect is smaller in the Australian model. The effect was so small on claims limited to 2.5mk AUD that we also looked at unlimited gross Australian claims. In that case the effect was more significant, although still smaller than in the US and UK models. We discuss this further in section 2.2.7 where we consider the effect of different inflation models on the Australian data.

We also measure the effect of the 'shape' of the capital release by considering how the cost of capital risk margin varies depending on whether the actual shape is used or whether we assume that the capital to liability ratio is constant. Table 2-7 shows the results of that calculation. The effect of the change in shape is apparent in the risk margin even for the Australian net experience where the effect was not particularly visible in Chart 2-6.

Table 2-7—Affect of variation of capital by age of claim

Country	Cost of Capital Risk Margins	
	Capital Ratio Constant	Capital Ratio Varies by Age
Aus-2.5m	4.0%	4.6%
Aus-unlimited	5.2%	7.2%
UK	1.5%	2.2%
US	3.1%	4.3%

2.2.3 Reinsurance

The Australian model provided the most data regarding the effect of reinsurance on risk margins. Table 2-8 below shows the net, ceded and gross risk margins by layer and also the risk margin for liabilities on an unlimited basis.

Table 2-8—Net, Ceded and Direct Risk Margins
Australian Model

	Risk Margins				% Liability-Net and Ceded		
Attachment (000 AUD)	Net	Ceded	Average	Unlimited	%Net	%Ceded	%Total
10,000	5.5%	89.4%	8.7%	7.2%	96%	4%	100%
5,000	5.0%	49.1%	8.5%	7.2%	92%	8%	100%
2,500	4.6%	29.7%	8.1%	7.2%	86%	14%	100%
1,000	3.9%	18.2%	7.7%	7.2%	73%	27%	100%
500	3.3%	14.1%	7.5%	7.2%	61%	39%	100%
250	2.7%	11.6%	7.4%	7.2%	48%	52%	100%
100	2.0%	9.6%	7.3%	7.2%	31%	69%	100%
50	1.5%	8.7%	7.2%	7.2%	21%	79%	100%

As expected, for net liabilities the risk margin is lower for lower attachment points. For ceded liabilities, the risk margin is higher for higher attachment points. Also, as expected, the weighted average of the net and ceded risk margins is greater than or equal to risk margin for the direct or "unlimited" business. Lastly, as expected, the difference between the average risk margin and the risk margin for the unlimited business increases as the attachment rises.

Thus if risk margins were determined from the reporting company perspective, then gross risk margins will not equal the sum of net plus ceded risk margins. This is a problem for financial reporting purposes.

Evaluating ceded risk margins from the reporting company perspective may not be appropriate as the assuming reinsurer has pooled these with other business, and has a lower risk margin than the reporting company. The weighted average of the ceded risk margin from the reinsurer's perspective and the net risk margin ratio from the reporting company perspective will lower than the average from the reporting company perspective. In fact, this weighted average risk margin could be lower than the gross risk margin from the reporting company perspective. This might also be a difficulty for financial reporting purposes.

To be sure that gross = net + ceded for risk margins we need to measure the risk margins using a carefully thought through, albeit somewhat arbitrary, reference portfolio or portfolios. Consistency between reporting companies would be achieved only through some regulatory or other mandated reference point.

The adverse financial reporting effects could be ameliorated by calculating the gross and net risk margins and establishing the ceded risk margin as the difference. While the ceded risk margin measured in that way would not reflect the reporting company risk for ceded business, it would not distort the reporting company's risk margins on gross and net business.

2.2.4 Size of Company and More on Reinsurance

Table 2-9 shows how the cost of capital risk margin varies by size of company and reinsurance level for the US example.

Table 2-9 Risk Margins by Company Size and Reinsurance
US Model

Layer	0-500	500-500	Average	1m Average
Size (USD m)				
25	4.7%	11.8%	5.8%	5.3%
50	3.9%	8.6%	4.6%	4.3%
100	3.5%	6.5%	3.9%	3.7%
250	3.1%	4.9%	3.4%	3.3%
500	3.0%	4.3%	3.2%	3.1%

As expected the risk margin decreases as the size company increases. This applies for a basic ground up 500k layer, for 500x500 layer and for total limits 1m layer.

As with the Australian example in Table 2-8, the risk margin for the direct 1m layer is less than the average of the risk margins for the 0-500 layer and the 500x500 layer.

2.2.5 Marginal Capital

In section 10, we note that a significant unresolved issue in the use of the cost of capital approach is whether the risk is to be measured from the perspective of the reporting company or from the perspective a reference company.

A simplified treatment of the reference company would be to measure the marginal risk associated with merging the reporting company experience into that of a company identical to the reporting company but large enough that the variability in runoff results is as small as practical. Therefore in the Australia and US examples, we have measured marginal capital and related risk margins as well as stand-alone amounts.

Table 2-10 shows the risk margins for various size reporting companies for gross 1m policy limits and for the excess 500x500 layer alone. The table also shows the risk margins based on the marginal additional capital required for a larger reference company (500m USD premium) to assume the risk from the reporting company.

Table 2-10 Risk Margins
Marginal Capital vs. Stand-Alone Capital
US Model

Size	1m Layer		500x500 Layer	
	Stand-Alone	Marginal	Stand-Alone	Marginal
25	5.3%	1.5%	11.8%	1.9%
50	4.3%	1.6%	8.6%	1.8%
100	3.7%	1.6%	6.5%	1.8%
250	3.3%	1.8%	4.9%	1.8%
500	3.1%	2.0%	4.3%	1.9%

Marginal risk margins by size of company are relatively constant by size of company, even for the excess layer where stand-alone risk margins vary substantially by size of company.
The marginal risk margins increase as the size of company, which might not be expected at first thought. However, this follows because we assume that the reporting company and reference company results are independent. In that case, the addition of small amounts of reporting company business requires very little additional capital. As the reporting company size increases, the marginal capital requirements begin to equal to average capital requirements for the reference company.

If the reporting company and reference company business were assumed to be 100% correlated, the marginal capital would equal the reference company capital and the reporting company risk margin would equal the reference company risk margin, regardless of the size of the reporting company.

We also observe that a margin based on the marginal capital requirement capital might be unrealistically low. It might be more appropriate to use the reference company total risk margin.

Table 2-11 below shows the standalone and margin risk margins for a 100m AUD reporting company and a 1000m AUD reference company, at various retentions. As expected, the variations in risk margin by retention are less for marginal risk margins than for stand-alone risk margins.

Table 2-11 Risk Margins
Marginal Capital vs. Stand-Alone Capital
Australian Model

Retention	Stand Alone	Marginal
25,000	1.2%	1.0%
50,000	1.7%	1.4%
100,000	2.2%	1.9%
250,000	3.1%	2.6%
500,000	3.8%	3.1%
1,000,000	4.6%	3.7%
2,500,000	5.9%	4.4%
5,000,000	7.2%	4.6%
10,000,000	8.6%	5.1%
Unlimited	12.7%	5.7%

2.2.6 Parameter Risk

In the Australia model, parameter risk is contained largely in the inflation component of the model. The extent to which this inflation component affects risk margins can be seen in the Table 2-12 below.

Table 2-12—Factors driving parameter risk (Australia)

Inflation Risk-Random around base-line as follows	Model # (Per Table 4-2)	Cost of Capital Risk Margin - 250k retention[12]	Cost of Capital Risk Margin – unlimited retention
Constant trend	0	Na	5.8%
Mean reverting trend-1	1	2.2%	6.2%
Mean reverting trend-2	2	2.4%	6.7%
Random walk-1	3	2.5%	7.1%
Random walk-2	4	2.7%	7.2%
Random walk-3	5	3.6%	9.0%
Random walk-4	6	7.0%	32.6%

We tested whether the release of capital would be slower as the risk in the trend increased. Interestingly, the rate which capital can be released did not change with the inflation rate. The change in risk with inflation appears to have similar affects at all ages.

2.2.7 Value at Risk (VaR) vs. Tail Value at Risk (TVaR)

While discussions of capital requirements are usually based on confidence levels, often referred to as Value at Risk levels (VaR). Tail Value at Risk (TVaR) levels, are statistically sound alternatives with certain desirable properties.

For each of our models we calculated risk margins based one or more combinations of VaR and TVaR. Table 2-13 shows the risk margins that result.

Table 2-13 – Affect of VaR and TVaR at varying levels

	Capital At:	Aus	UK	US
VaR	99.0%	4.0%		
VaR	99.50	4.6%	2.2%	4.3%
VaR	99.90	5.0%		
VaR	99.95	5.5%		
TVaR	99.00%	4.7%		4.5%
TVaR	99.50%	5.1%	2.4%	

From the table we can see that, as usually suggested, the risk margins from a VaR of 99.5% are similar to the risk margin from a TVaR of 99.0%. Equivalence between other VaR and TVaR levels could be constructed.

While not visible in this table, the VaR values for later periods in the runoff can fall below the mean value, creating negative capital requirements. This can occur because the runoff risk distributions become increasing skew as the runoff proceeds. In our calculations we retained the negative risk margins, as the relative amount of liabilities remaining at those points was so small that the resulting risk margin was not distorted. This would be avoided by using a TVaR throughout or, with less theoretical nicety, establishing a minimum capital level (e.g., something more than two standard deviations) above the mean.

[12] 250k retention shown in Table 2-13 because we did not analyze this issue for 2.5m retention.

2.2.8 Shock and Latent Claims

In section 7 we show sensitivity tests that lead us to observe that extreme events are a material consideration in the determination of risk margins. A relatively small shock (measured against premium) can produce a noticeable change in risk margin (measured against risk margins). We show how a shock potential equal to 0.25% of premium per year can produce an increase in risk margin in excess of 100 basis points.

As shock events are not reflected in the data, the parameters will likely need to be judgmental.

2.2.9 Comparison of Cost of Capital and Percentile Approaches

While the scope of our project did not include an assessment of the different approaches, as a result of our work we have the following observations:

1. The cost of capital method is 'practical' in the sense that the same types of analysis that would be needed to produce percentile risk margins can be used to produce cost of capital results.
2. Cost of Capital method is conceptually closer to exit value in theory than percentiles.
3. However, the Cost of Capital method is, on balance, more difficult to apply than percentiles because:
    a. It requires measurement of capital at each runoff age, rather than just the capital currently required.
    b. It is more sensitive to tail events than the percentile approach.
4. There is no agreed 'correct' percentile level for the risk margin. We observe that the results of the cost of capital methods, in these examples are in the 60-65th percentile reserve levels which are low compared to the 75th percentile minimum standard used in Australia.

    Increasing the cost of capital from 6% to higher amounts would increase the Cost of Capital risk margin, perhaps to the 75th percentile level or higher.

2.2.10 Minimum Capital Requirements and Cost of Capital Formulas

A test for asset adequacy could be formulated the several ways. Two of these are as follows:
Test 1 -consistent with the way we calculate capital in this paper

- Determine an asset level such that there is a sufficient probability (e.g., 99.5%) that the claim payouts will not exceed assets.
- Divide those assets into three parts:
    a. discounted mean,
    b. a risk margin calculated from the Capital Cash Flow (CCF) formula described in Section 11, and

c. capital equal to the total assets minus ((a)+(b))

Test 2 – consistent with the way the Swiss Solvency Test works

- Capital is determined so that at any time during the runoff there is a sufficient probability (e.g., 99.5%) that assets are sufficient to cover best estimate reserves and risk margins
- The risk margin is separately determined from the SST formula used in Section 3.
- Determining the get risk margin may require iteration as capital depends on risk margin and risk margin depends on capital.

Test 2 requires more total assets than Test 1 for several reasons. Firstly, in Test 2 capital needs to be sufficient to assure that assets cover risk margins as well as discounted liabilities during the course of the runoff. Secondly, in Test 2 the capital needs to be sufficient to cover the risk that reserves over-state the ultimate payout and create a 'false' projection of failure.

Test 1, however, is the way that UK ICAS calculations tend to work.

Neither Test 1 nor Test 2 fully considers the real world. Most jurisdictions have other minimum capital requirements and restrictions on the release of capital to shareholders. Also, rather than simply satisfy, say, 99.5% VaR based on ultimate claim payments, in practice, an insurer must maintain a reasonable degree of assurance that it will meet statutory requirements and will be allowed to continue operating. That is, it needs to hold

- Reserves (i.e. an estimate of the expected value remembering that reserves might be higher or lower than the actual ultimate); plus
- a risk margin; plus
- regulatory minimum capital or other rules that might go beyond Test 1 or Test 2;
- additional capital to ensure a good probability of statutory solvency and an appropriate financial rating

Finally, as a general matter the risks to be considered in assessing solvency include asset, credit and operational risks that are not reflected in the claim runoff. The degree to which those risks need to be considered in the cost of capital risk margin is not fully resolved.

It is beyond our scope to determine how to address those additional issues.

As a practical matter we did our analyses on the assumption that required capital is based on Test 1, but we used the SST cost of capital formula, as if we had determined capital based on Test 2.

Section 11 shows that if we had applied Test 1 logic consistently we have obtained slightly lower risk margins, e.g., 4.6 vs. 4.0% for the Australian model. The hybrid approach, though, may have produced results closer to those that would have been obtained from considering Test 2 and the other capital adequacy issues.

3 Cost of Capital Method

The cost of capital methodology as illustrated in the Swiss Solvency Test[13] and the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) QIS-3[14] material is as shown in Table 3-1 below.

Table 3-1- Cost of Capital Method --Fixed % Capital Required

1	2	3	4	5	6	7	8
Year	Liability	Req'd Capital %	Req'd Capital	Cost of Capital	Risk Margin	Risk Margin	Cash Flow - Confirm Margin
0	100	70%	70	4.2	20.6	20.6%	-70.0
1	89.0	70%	62	3.7	17.2	19.4%	14.7
2	77.0	70%	54	3.2	14.2	18.4%	14.6
3	66.0	70%	46	2.8	11.5	17.5%	13.1
4	54.0	70%	38	2.3	9.2	17.1%	13.0
5	43.0	70%	30	1.8	7.3	17.0%	11.5
6	37.0	70%	26	1.6	5.8	15.7%	7.2
7	31.0	70%	22	1.3	4.5	14.4%	6.8
8	26.0	70%	18	1.1	3.3	12.9%	5.7
9	20.0	70%	14	0.8	2.4	12.0%	6.0
10	14.0	70%	10	0.6	1.6	11.8%	5.6
11	11.0	70%	8	0.5	1.1	10.2%	3.1
12	9.0	70%	6	0.4	0.7	7.9%	2.2
13	6.0	70%	4	0.3	0.4	6.0%	2.7
14	3.0	70%	2	0.1	0.1	4.0%	2.5
15	-	70%					2.3
Tot	100.0	70%					IRR=10.0%

Notes

2	Selected	
3	Assumed	
4	=(2)*(3)	
5	=(A)*(4)	
6	NPV of upward sum of column (5); NPV at rate=B	
7	=(6)/(2)	
8	(4)-(year n minus year n+1) + (6)-(year n minus year n+1) + (B) * ((4)+(6))	
A	Cost of Capital	6.0%
B	Risk Free Rate	4.0%

[13] Swiss Federal Office of Private Insurance (FOPI), *The Swiss Experience with Market Consistent Technical Provisions - the Cost of Capital Approach*, 28 March 2006, http://www.cea.assur.org/cea/v2.0/uk/solvency/solvdocs/SST_SwissCostofCapital_20060328%5B1%5D.pdf

[14] Committee of European Insurance and Occupational Pension Supervisors, *QIS3*, *Technical Specifications, Part II: BACKGROUND INFORMATION*, April 2007, page 5-7, http://www.ceiops.org/media/files/consultations/QIS/QIS3/QIS3TechnicalSpecificationsPart2.PDF

This calculation assumes the following:

1. Initial capital (e.g., 70%) and cost of capital (6%) have been determined by some means; in the case of CEIOPS and SST, by the insurance supervisors.
2. Capital decreases in proportion to the discounted best estimate of the liability. This is discussed further below.
3. The method could be applied equally to liabilities gross or net of reinsurance.
4. Each line of business is analyzed separately.
5. The capital required in column (4) is the amount required in addition to the risk margin in column (6), and the cash flows in column (8) reflect the release of both capital and risk margin. This is discussed further below.

With those assumptions the indicated risk margin at the valuation date is 20.6% of the best estimate discounted unpaid claims at time zero (the valuation date).

Assumption 2-Discussion

Assumption 2, the assumption that future capital needs is a constant percentage of the current estimate may not be appropriate. In particular, for many property & casualty claim liabilities two factors make it likely that the probability distribution function for "late claims" would have a higher coefficient of variation and skewness than would the probability distribution function of the entire set of claims. First, the late settled claims are different than early settled claims, e.g., larger, subject to more disputes and more variability. Second, late settled claims will be more subject to uncertain economic effects, e.g., inflation, social inflation, and judicial activity, which increase the uncertainty in the estimate.

Consider, for sake of illustration, that the required capital as a percentage of liability (present value of the best estimate of future claim payments) increases smoothly at 10% a year. The analysis from Table 2-1 can be repeated, but changing column (3) from a constant 70% of liabilities to an increasing percentage of liabilities as shown below in Table 3-2.

Table 3-2— Cost of Capital Method--Varying % Capital Required

1	2	3	4	5	6	7	8
Year	Liability	Req'd Capital %	Req'd Capital	Cost of Capital	Risk Margin	Risk Margin	Confirm Margin
0	100	70%	70.0	4.2	29.7	29.7%	-70.0
1	89	77%	68.5	4.1	26.7	30.0%	8.5
2	77	85%	65.2	3.9	23.7	30.7%	10.2
3	66	93%	61.5	3.7	20.7	31.4%	10.2
4	54	102%	55.3	3.3	17.8	33.0%	12.3
5	43	113%	48.5	2.9	15.2	35.4%	12.4
6	37	124%	45.9	2.8	12.9	35.0%	7.4
7	31	136%	42.3	2.5	10.7	34.5%	8.2
8	26	150%	39.0	2.3	8.6	33.0%	7.5
9	20	165%	33.0	2.0	6.6	33.0%	9.9
10	14	182%	25.4	1.5	4.9	34.8%	10.9
11	11	200%	22.0	1.3	3.5	32.2%	6.0
12	9	220%	19.8	1.2	2.4	26.3%	4.4
13	6	242%	14.5	0.9	1.3	21.3%	7.2
14	3	266%	8.0	0.5	0.5	15.3%	8.0
15	0	292%	0.0	0.0	0.0		8.8
16							
IRR							10.0%

For this illustration, corresponding to a line of business with a long duration of payments, the change in risk margin is significant, from 20.6% to 29.7%.

Assumption 5-Discussion

Assumption 5 states that the capital required in column (4) is the amount required in addition to the risk margin in column (6), and the cash flows in column (8) reflect the release of both capital and risk margin.

This assumption follows from a view that capital is the amount required to provide sufficient confidence that assets will be sufficient to cover both the expected value of unpaid claims and the risk margin. In section 2.2.10 we called this Test 2.

The models we develop in this paper, typical of models used for general insurance purposes, measure the amount of capital required to assure that assets are sufficient to cover only unpaid claim obligations over the period required to settle the claims. We called this Test 1 in section 2.2.10. We have used the Test 1 capital ratio as it were the capital ratio that could be applied to unpaid claims plus risk margin measured over any time period.

We believe our approach is sufficient for purposes of exploring a number of important issues. A more complete analysis would need to consider not only the runoff risk (as we have), but other issues described in Section 2.2.10.

4 Australia – Third Party Bodily Injury Liability – Collective Risk Model

4.1 Background

Compulsory Third Party (CTP) insurance in Australia is a stand-alone product that provides cover against bodily injury (including psychological trauma) in motor vehicle accidents. In most states the at-fault driver is excluded. CTP insurance is a condition of registration. Unregistered or untraceable vehicles are insured under a Nominal Defendant scheme, funded out of insured premiums.

In New South Wales (NSW), CTP insurance became compulsory in 1942 and has operated under five regimes, four of which are still relevant to open claims.

- For accidents up to 30 June 1987, claims are subject to the tort principles of common law, with only limited legislative constraints. Initially there were competing insurers but most were squeezed out by premium controls. For all intents and purposes, the Government Insurance Office of NSW (an offshoot of the NSW Treasury) was the monopoly insurer well before 1980. When GIO was privatized and floated, this business was not included.
- From 1 July 1987 to 30 June 1989, statutory benefits were provided under the TransCover scheme for accidents from 1 July 1987. This provided limited lump sums for pain and suffering, with most benefits paid as and when needed.
- Upon a change in government, TransCover was repealed, “ab initio”, with effect from 1 July 1989. Claimants had the option of continuing on TransCover benefits or reverting to common law. Almost all switched and all remaining TransCover claims were finalised within a couple of years. While the remaining claims are nominally indistinguishable from older claims, their subsequent experience is clearly different.
- At the same time, new accidents came under the Motor Accidents Act, which provided basically common law benefits, subject to a range of restrictions and caps. Initially, claims were randomly allocated between insurers and premiums were set by the government.
- From 1 July 1991, the market was opened to competition among insurers licensed under the Act on a file and write basis for premiums, with limited scope for variation around a fixed set of relativities, applied to the base premium set by each insurer. Different marketing, proving and underwriting strategies quickly led to dramatic differences between different insurers’ experience remain. There has also been some fine (and not so fine) tuning of the provisions of the Act affecting benefits.

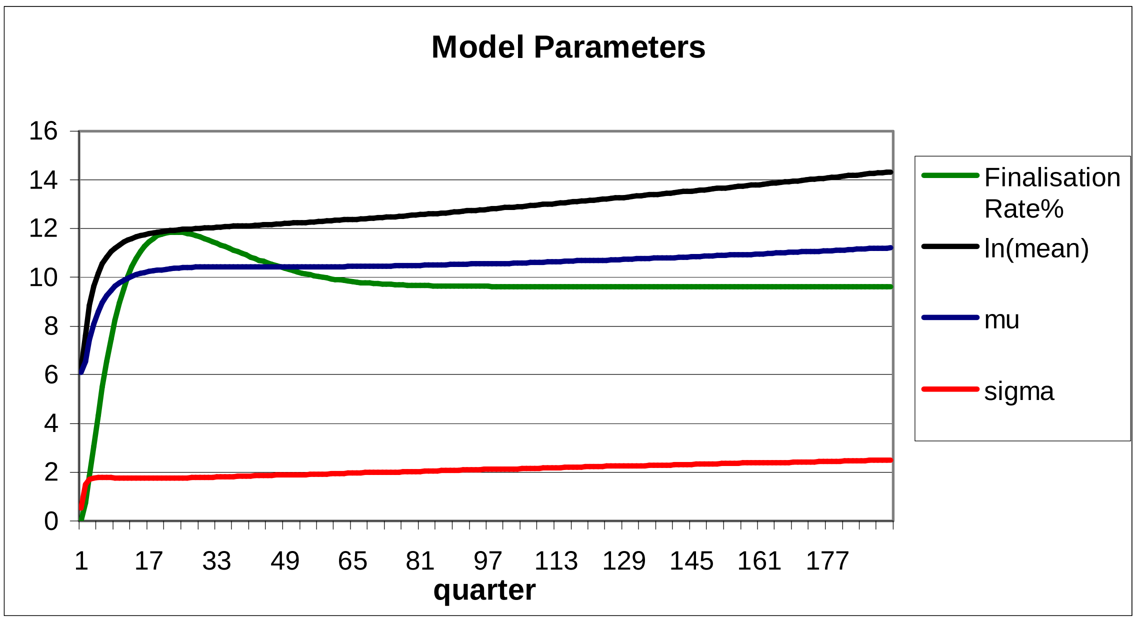
With the exception of the TransCover period, a small number of structured settlements and some claims shared across jurisdiction boundaries, claims are mostly settled by payment of a lump sum. While there are some advance payments and some delays, while legal costs on court settlements are negotiated, the bulk of the cost is typically paid close to the time of finalisation.

4.2 Base Model

We use *NSW* CTP data from about 1993 to about 2003, with claims from as far back as the early 1950s to create a plausible generic model for a lump sum liability class, rather than a definitive model of a real portfolio. Because of the scheme changes, it is not possible to simply fit model parameters to the available data. Rather, the model adopted is a composite, generated by rescaling the four subsets of data, so that they more or less coincide in the areas where they overlap.

The fitted parameters are:

- rates of finalisation, by development quarter;
- percentage of trivial claims;
- average amount of trivial claims;
- amounts of non-trivial claims, by development quarter;
- claim cost inflation, incorporating both wage and superimposed inflation.



In addition, the model incorporates a discount rate which is also used as the economic inflation rate for reinsurance indexation, where applicable.
The non-trivial claim amount parameters incorporate two fitted stochastic features.

- For each claim within each simulation, the claim amount is a log-normal variable. The log-normal mu and sigma parameters are functions of development quarter.
- For each finalisation quarter within each simulation, the log-normal mu parameters for all claims finalized in that quarter are subject to a common random adjustment.

The fitted rates of finalisation and trivial claims are also applied stochastically, as percentages. This adds a limited degree of systemic variation.

4.3 Projections

We prepared a number of groups of projections, exploring different aspects and incorporating different stochastic features. Each projection comprises 10,000 simulations of a starting portfolio comprising a number of claims at each development age. The present values of the claim costs for each starting age are summed within each simulation and sorted across simulations to allow the calculation of the following statistics for each starting age:

- expected value or mean;
- standard deviation;
- VaRx – the value for which x% of outcomes are smaller and (100-x)% are larger (Value at Risk);
- TVaRx – the average of the (100-x)% of largest outcomes (Tail Value at Risk, also called CTE, Conditional Tail Expectation);
- EPDx – the average, across all outcomes, of the (non-negative) excess over VaRx (Expected Policyholder Deficit).

VaR75 is the basis for regulatory provisions in Australia. VaRx for suitably high values of x (typically 99.5%) is proposed as a regulatory minimum capital requirement in a number of jurisdictions.

TVaR is a little harder to assess, as it requires knowledge of the shape of the distribution past VaR, but has certain superior theoretical properties. In particular, it cannot, as can happen for VaR with extremely skew distributions, lie below the expected value.

EPD is the portion of the liability that would not be covered by assets related to reserves or capital when capital is set at the VaRx level.[15]

Cash flows are discounted at 1.5% per quarter, reflecting Australian interest rates, and VaR and TVaR confidence levels calculated from the discounted cash flows. The cost of capital calculations are prepared with 4% risk free rate to be consistent with the US and UK results and to reflect global risk free interest rates.

4.3.1 Portfolios

The different projections can be grouped on the basis of the starting portfolio, additional stochastic features and reinsurance retentions.

The projections use either a (i) base portfolio alone, i.e., a stand-alone portfolio or (ii) a base portfolio and a supplementary portfolio. The base and supplementary portfolios may comprise a mature (ongoing) portfolio of claims, the run-off of such a portfolio, or cohorts of claims of equal value at each quarterly duration.

The mature portfolio is generated by deterministic application of the finalisation rates to a constant flow of new claims. Because this process generates fractional numbers, these are rounded and a calibration factor applied in the projection, after the simulation stage. Calibration factors are also applied in the extreme tail, where a minimum number of claims at each development age is applied.

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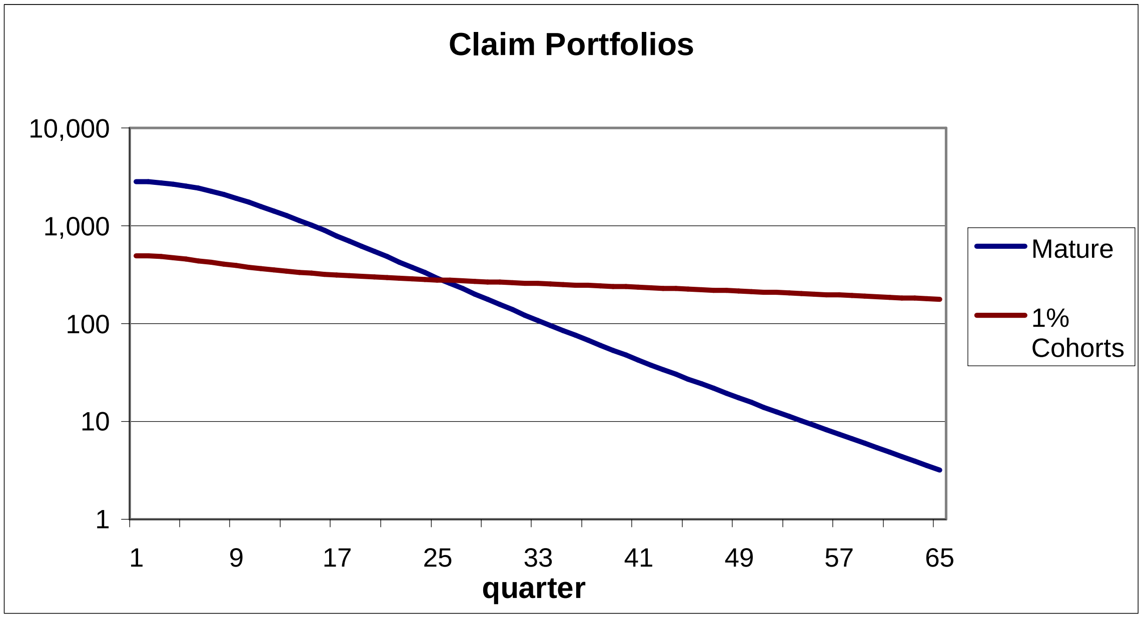
[15] The EPD can also be viewed as the expected cost of defaults to an insolvency fund or as the reduction in reserves that would be applied if liabilities were to reflect the credit standing of the reporting company. This paper does not address the “own credit standing” issue.

Where a minimum number of claims is applied, this understates the independent variation in this part of the projection. These cells are required to complete the expected value of the base portfolio in Portfolios A and F (defined below) but are not presented in the graphs.

The equal value portfolio comprises the number of claims required at each development age, to give a present value, for that age, of 1% of the total present value of the mature portfolio.



When a supplementary portfolio is used, the statistics (VaR, TVaR and EPD) are determined for the base portfolio on its own and for the base portfolio plus each element of the supplementary portfolio. Subtracting the base portfolio values gives the marginal additional values, when a cohort of claims is added to an ongoing portfolio.

We considered a variety of cases, set out in Table 4.1. For historical reasons, these are not labelled in the natural order.

Table 4.1- Definition of Portfolios

Case	Base	Supplementary	Reinsurance
D	Run-off of mature portfolio ($5 billion starting value)	None	without
E	Run-off of mature portfolio ($5 billion starting value)	None	With
F	Ongoing mature portfolio ($5 billion)	Run-off of mature portfolio ($500 million starting value)	with and without
B	Cohorts of equal value ($100 million at each development quarter)	None	with and without
A	Ongoing mature portfolio ($5 billion)	Cohorts of equal value ($100 million at each development quarter)	with and without

4.3.2 Systemic Variation

Additional systemic variation is introduced by adding stochastic features to the claim inflation rate. These are summarised in Table 4.2.

Table 4.2- Inflation Models

Case	
0	The inflation factors in each quarter vary (sigma = 41%) around a constant trend of a little under 4.5% per annum (the base level).
1	The inflation factors in each quarter vary (sigma = 41%) around a constant trend that varies (sigma = 0.25%), between simulations, around the base level.
2	The inflation factors in each quarter vary (sigma = 41%) around a constant trend that varies (sigma = 0.5%), between simulations, around the base level.
3	The inflation factors in each quarter vary (sigma = 41%) around a variable trend that follows a random walk (sigma = 1%) within each simulation around a trend that has a variable (sigma = 0.5%) starting level, around the base level.
4	The inflation factors in each quarter vary (sigma = 41%) around a variable trend that follows a random walk (sigma = 2%) within each simulation around a trend that has a variable (sigma = 0.5%) starting level, around the base level.
5	The inflation factors in each quarter vary (sigma = 41%) around a variable trend that follows a random walk (sigma = 5%) within each simulation around the base.
6	The inflation factors in each quarter vary (sigma = 41%) around a variable trend that follows a random walk (sigma = 2.5%) within each simulation around a trend that has a variable (sigma = 2.5%) starting level, around the base.

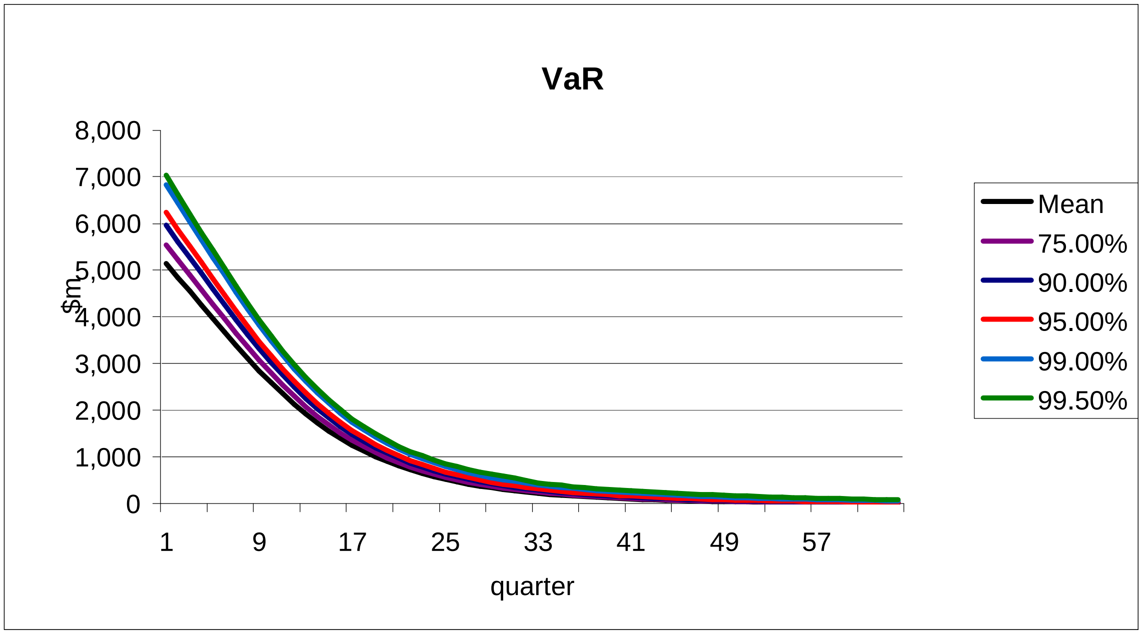
As claim inflation for NSW CTP has been highly erratic and is partially subject to political influences, it is difficult to assess what is a reasonable level of variability. Case 0 reflects actual quarter to quarter variation in average claim sizes around the short-term trends. Cases 1 and 2 probably understate the variability in the rate. Cases 3 and 4 are possibly plausible as to the level of variability. Case 4 is used as the default in the following figures. Cases 5 to 6 are intended to incorporate a high level of systemic variation, to better show how systemic variation can dominate independent variation.

4.4 Results

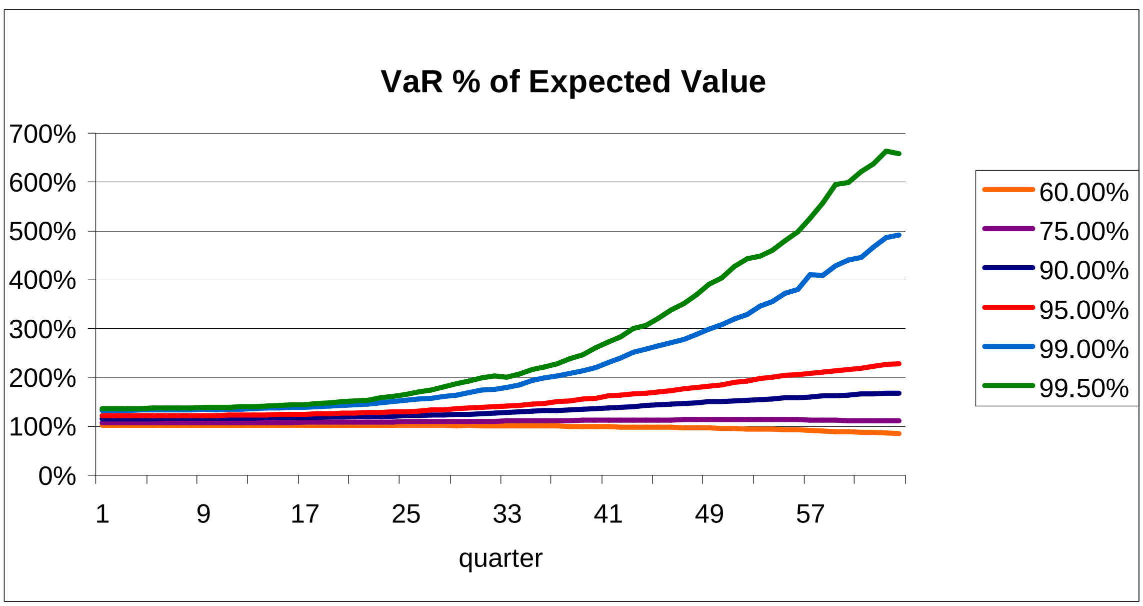
The projection results are presented in detail in a number of spreadsheets which are available from the authors. The following figures are selected to illustrate particular points.

4.4.1 Stand-alone Run-off of a Mature Portfolio

The following graph shows how the expected value, and VaR for selected probabilities of adequacy, run off when a $5 billion mature portfolio is run off.

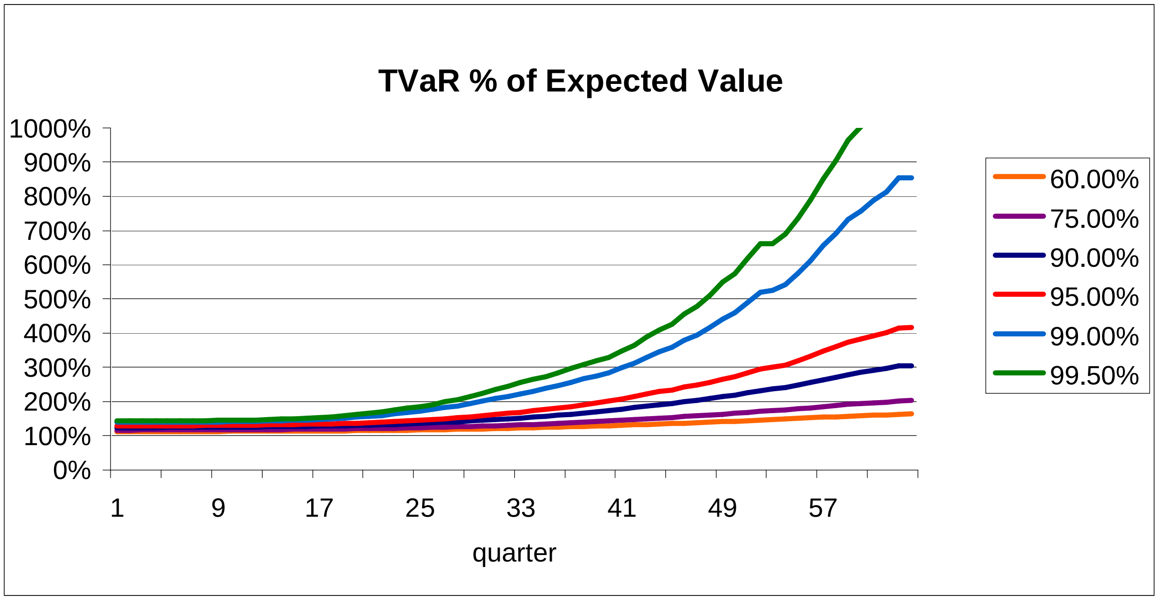


Because this graph is dominated by the reducing expected value, as the portfolio runs off, it is difficult to discern patterns in the relationship of VaR to the expected value. To show these, it is necessary to show VaR as a percentage of the expected liability, as in the following graph.



This shows that, at higher adequacy levels, VaR as a percentage of expected value rises strongly as a stand-alone portfolio runs off. In contrast, VaR60 falls at higher durations and can fall well below 100% of expected value. This is a consequence of increasing skewness, as the numbers of surviving claims drops. This drop is also observable for VaR75 and it can also be seen that the rate of increase in VaR90 is slowing and, if the numbers of claims fall sufficiently, ever high probability VaRs can fall below the expected value. For this reason, VaR is an unsatisfactory reserving or capitalisation basis for small portfolios.

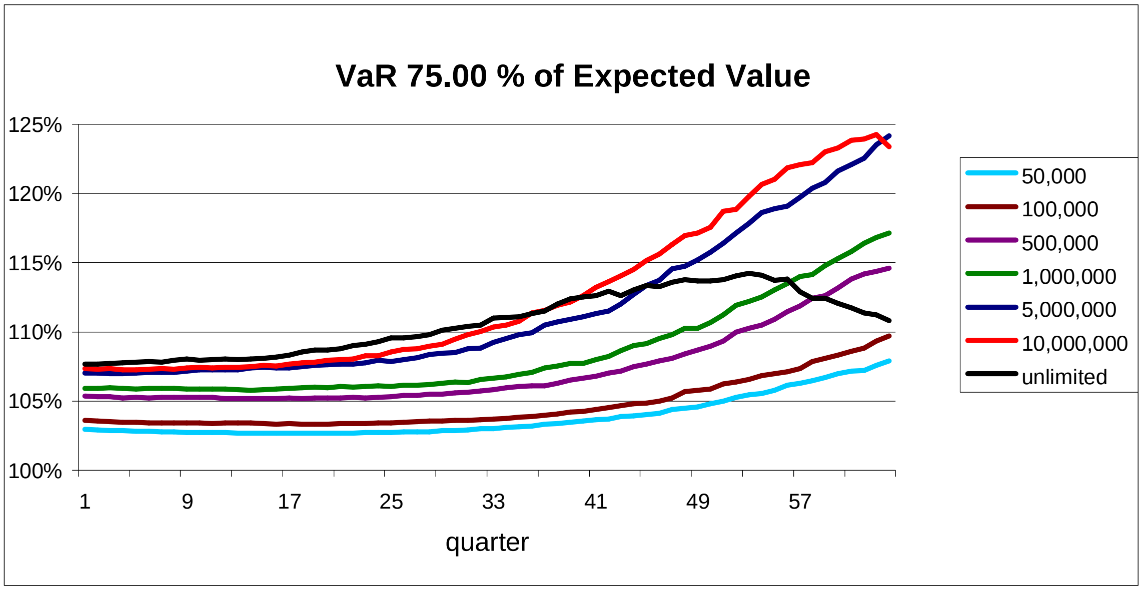
The following graph shows the equivalent TVaR values.



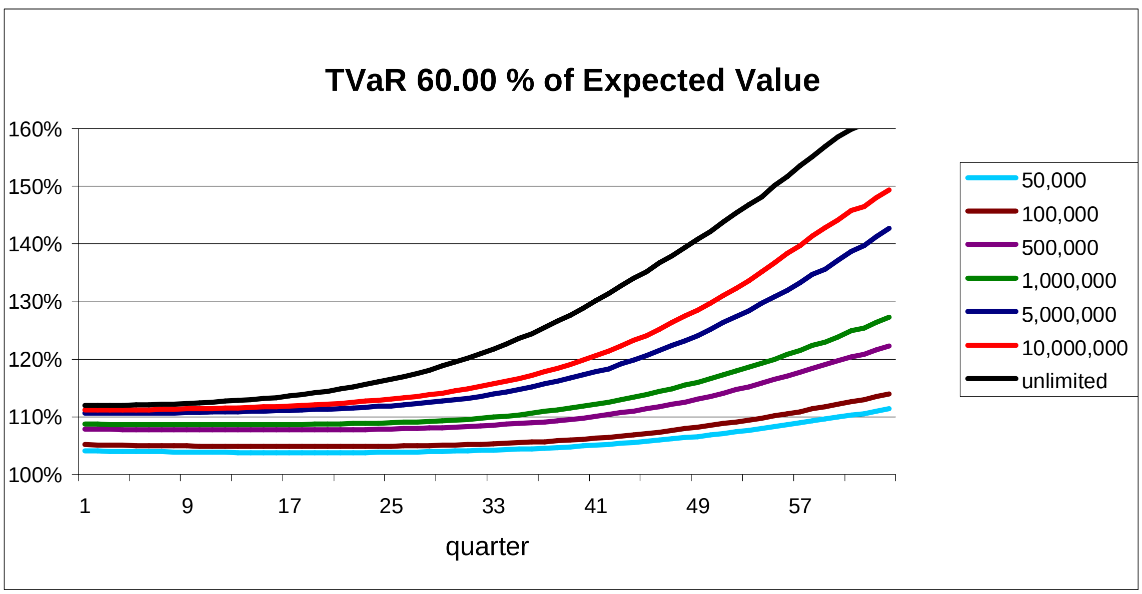
Again, there is a strong upwards trend but the droop for the lower probabilities of adequacy is no longer apparent.

4.4.2 Stand-alone Run-off of a Mature Portfolio with Reinsurance

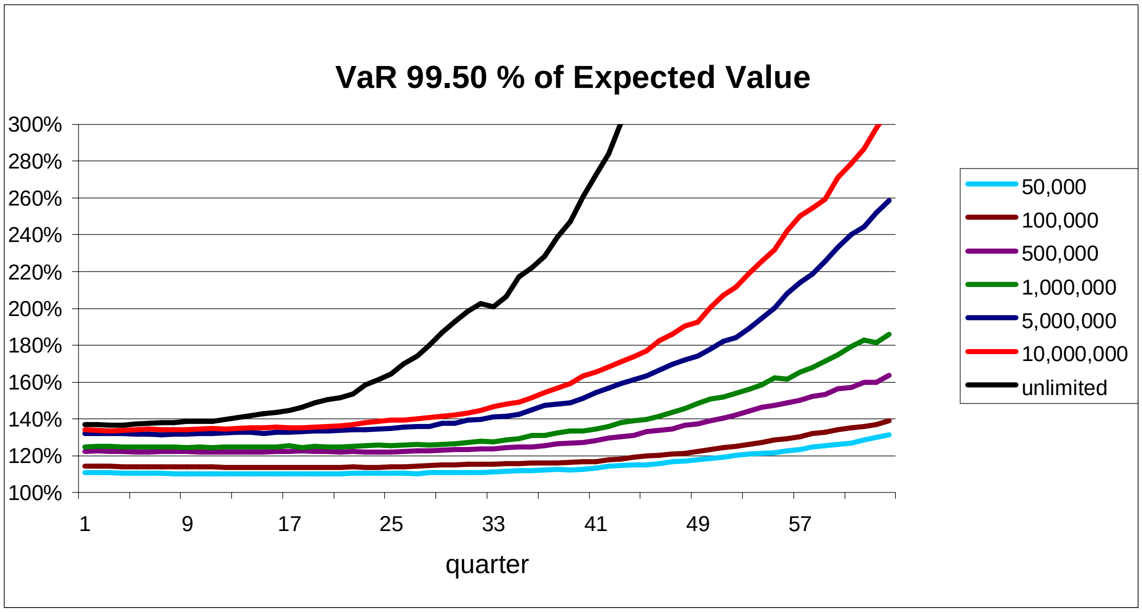
The following graphs show VaR and TVaR for selected probabilities of adequacy, for a mature stand-alone portfolio, in run-off, for various levels of excess-of-loss reinsurance.



VaR75 (albeit on a whole insurer basis) is the level specified for provisions under Australian general insurance regulation GPS 310. For moderate levels of reinsurance, VaR75 is essentially flat for nearly ten years. The dips in the unreinsured line and at the top of the $1million retention are a consequence of how the mean rises faster than moderate percentiles, as numbers drop and skewness increases. For the model chosen, and for moderate levels of retention, TVaR60 gives similar levels of margin to VaR75, but does not give anomalous results for high levels of skewness.

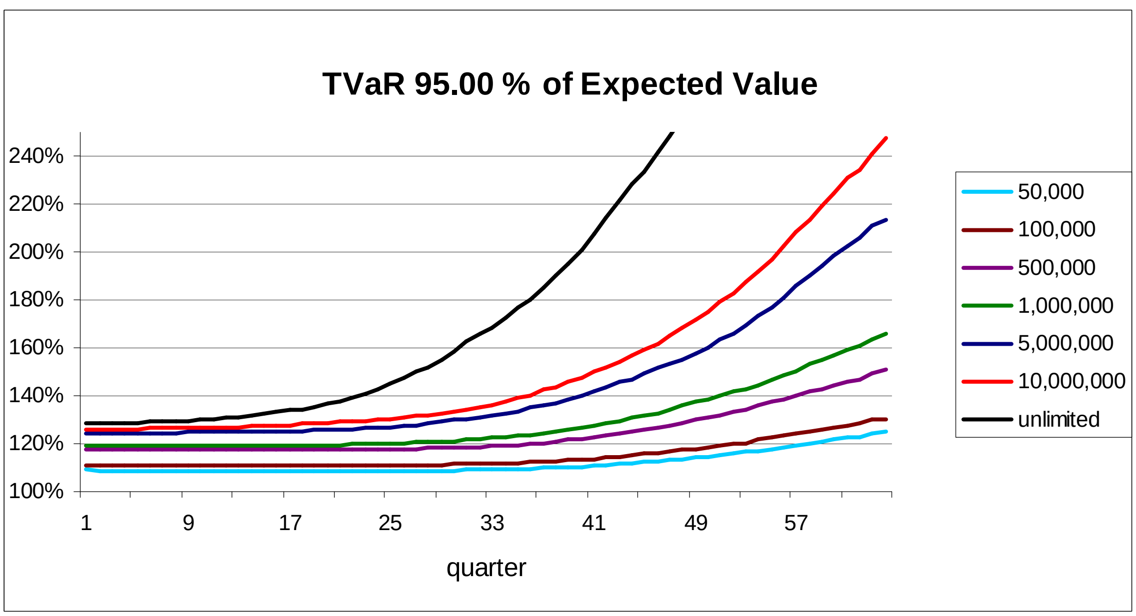


As a basis for required capital, higher adequacy levels are clearly required. VaR99.5 is the (whole insurer) level specified under the Australian internal model basis Minimum Capital Requirement.

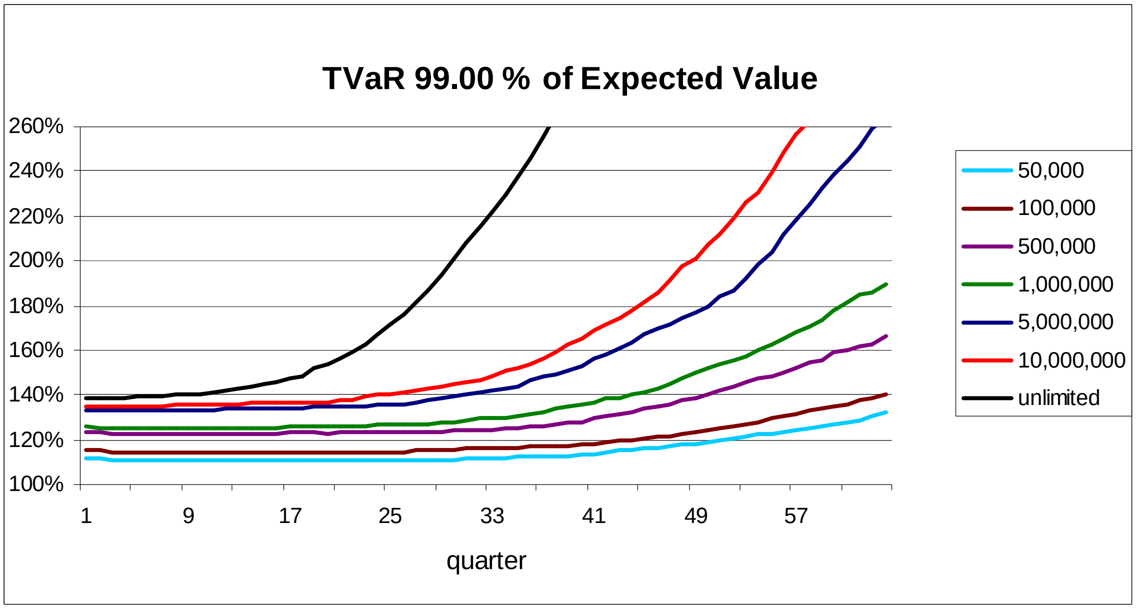


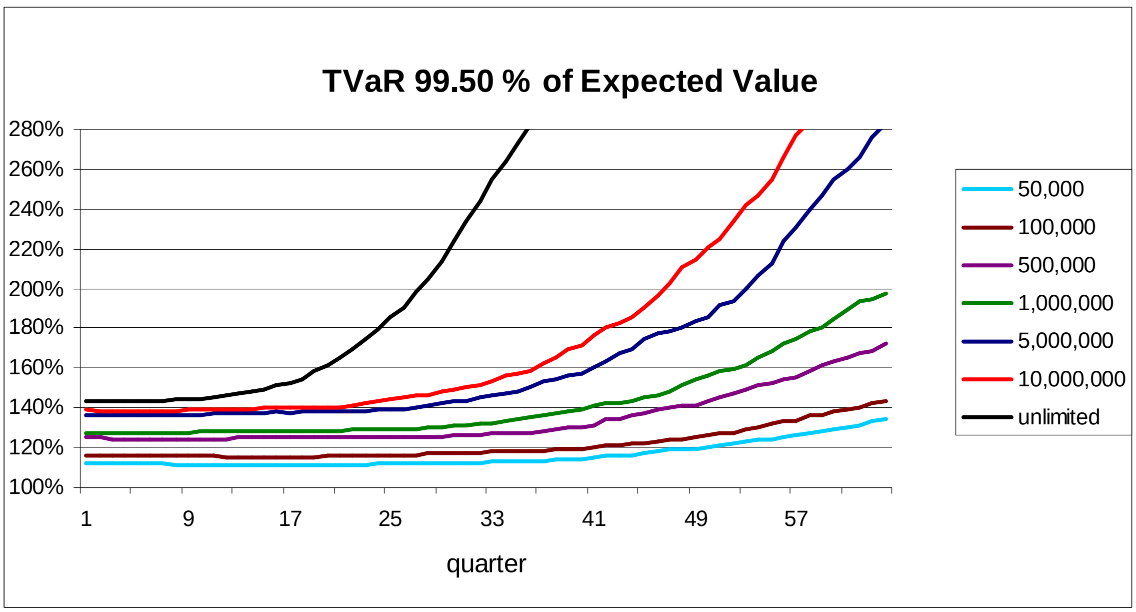
Again, for moderate levels of XoL reinsurance[16], there is a flat area for perhaps 10 years. Part of the explanation for this is presumably that these levels of reinsurance drastically reduce skewness.

Comparison with the following suggests that, for the model used here, depending on retention, VaR99.5 gives results somewhere in the range TVaR95 to TVaR99.



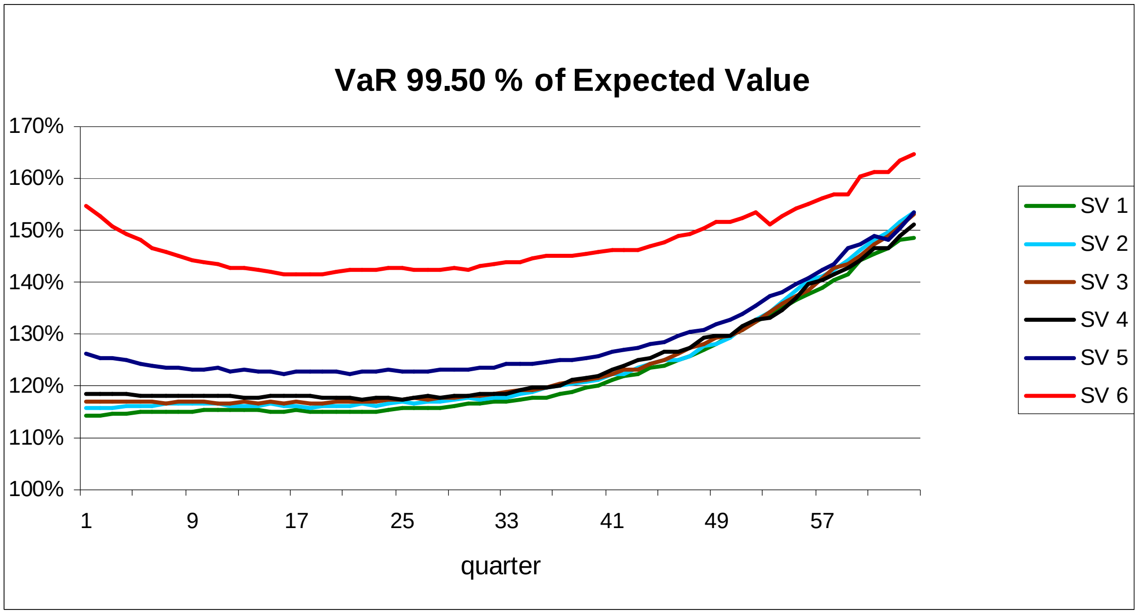
[16] The analysis assumes XoL reinsurance covers the claim amount above the attachment point without limit. If XoL cover had upper limits, the curve would not be as flat.


TVaR 99.00 % of Expected Value
260%
240%
220%
200%
180%
160%
140%
120%
100%
1
9
17
25
33
41
49
57
quarter
50,000
100,000
500,000
1,000,000
5,000,000
10,000,000
unlimited

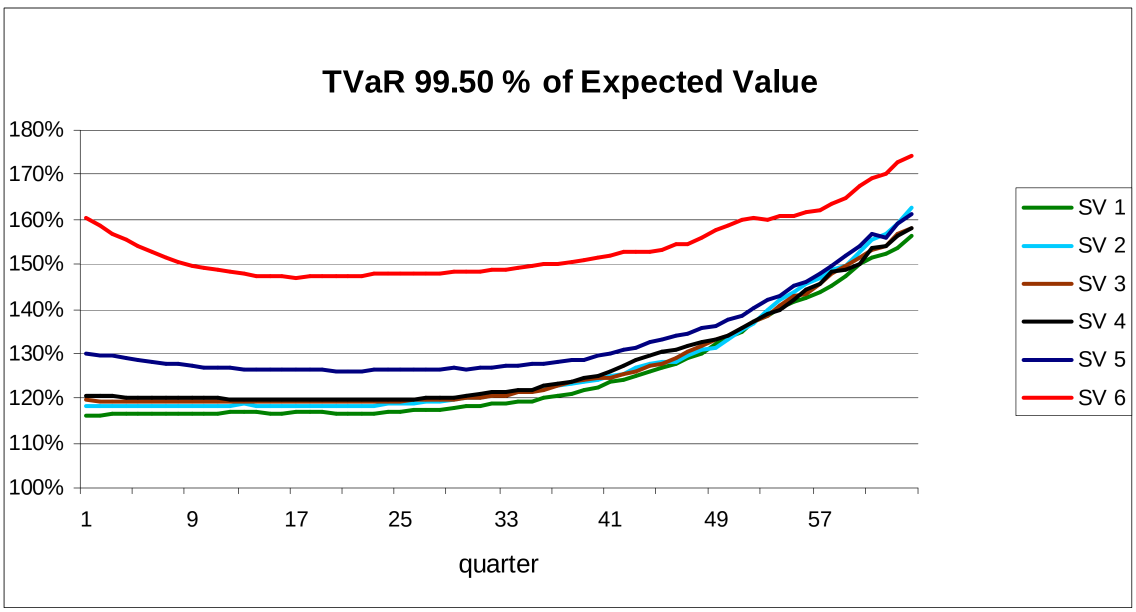

TVaR 99.50 % of Expected Value
280%
260%
240%
220%
200%
180%
160%
140%
120%
100%
1
9
17
25
33
41
49
57
quarter
50,000
100,000
500,000
1,000,000
5,000,000
10,000,000
unlimited

4.4.3 Stand-alone Run-off of a Mature Portfolio with Reinsurance – Variability Effects

The following graphs show how VaR99.5 and TVaR99.5 are affected by adding parameter risk, in the form of different levels of variability in the inflation assumption. See Table 4.2 for a description of these levels of variability.



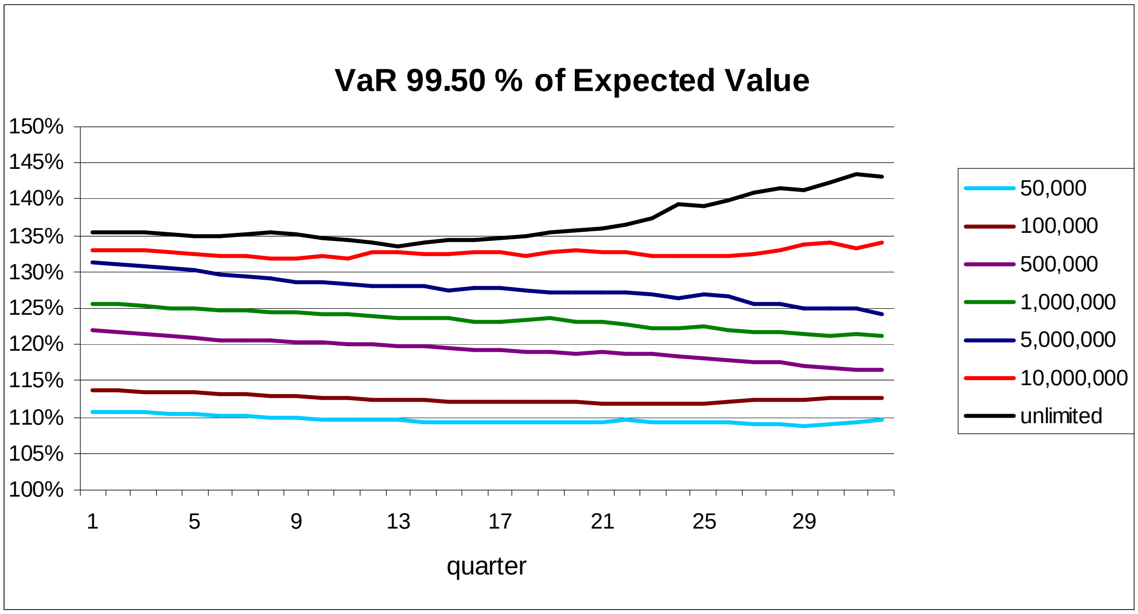
The black line (case 4) corresponds to the results in the previous section. Extreme parameter uncertainty (case 6) reduces the relative range of VaR, as well as increasing its general level.



The pattern is essentially the same for TVaR99.5.

4.4.4 Run-off of a Mature Portfolio in the Context of a Larger On-Going Portfolio

The following graphs show VaR99.5 on a marginal basis, when a closed portfolio with a starting liability of $500 million is added to a $5 billion ongoing portfolio. This graph is restricted to 8 years, because the variability is such that, even after smoothing, differencing simulated values of VaR and TVaR gives increasingly erratic results as the numbers of claims in the supplementary portfolio drops and the supplementary portfolio becomes a smaller and smaller fraction of the ongoing portfolio, which remains constant in real terms.

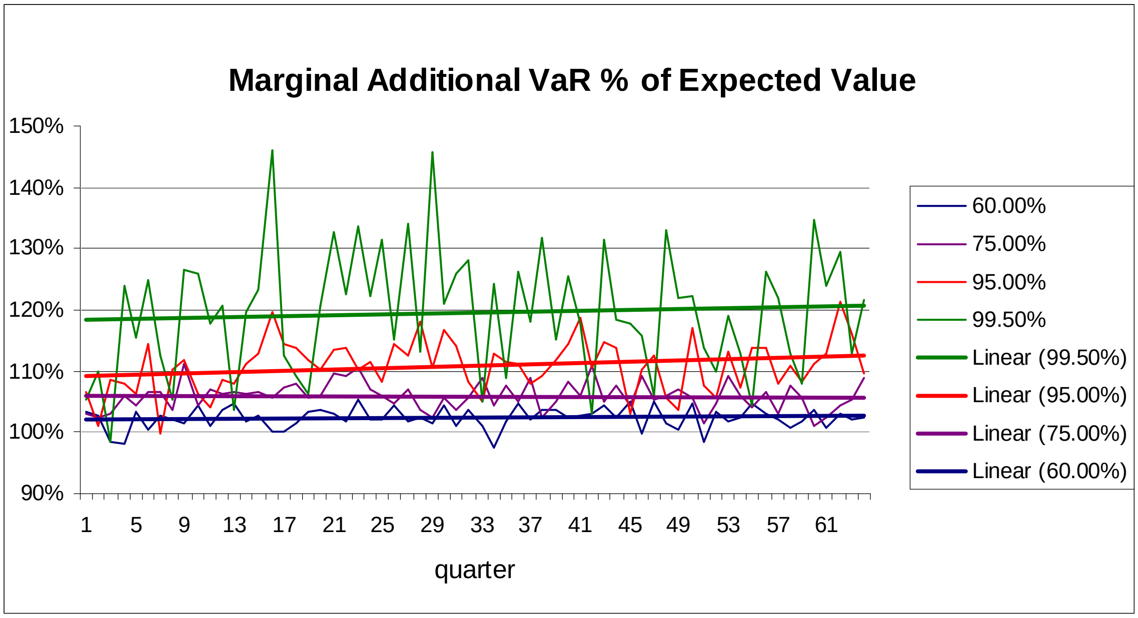


The apparent downward trend for retentions below $10 million may be a result of the increasingly dominant ongoing portfolio. It may also be, in part, an artifact of the smoothing process. The values for higher levels of adequacy are clearly affected by instability near the extremes of the smoothed distribution function.

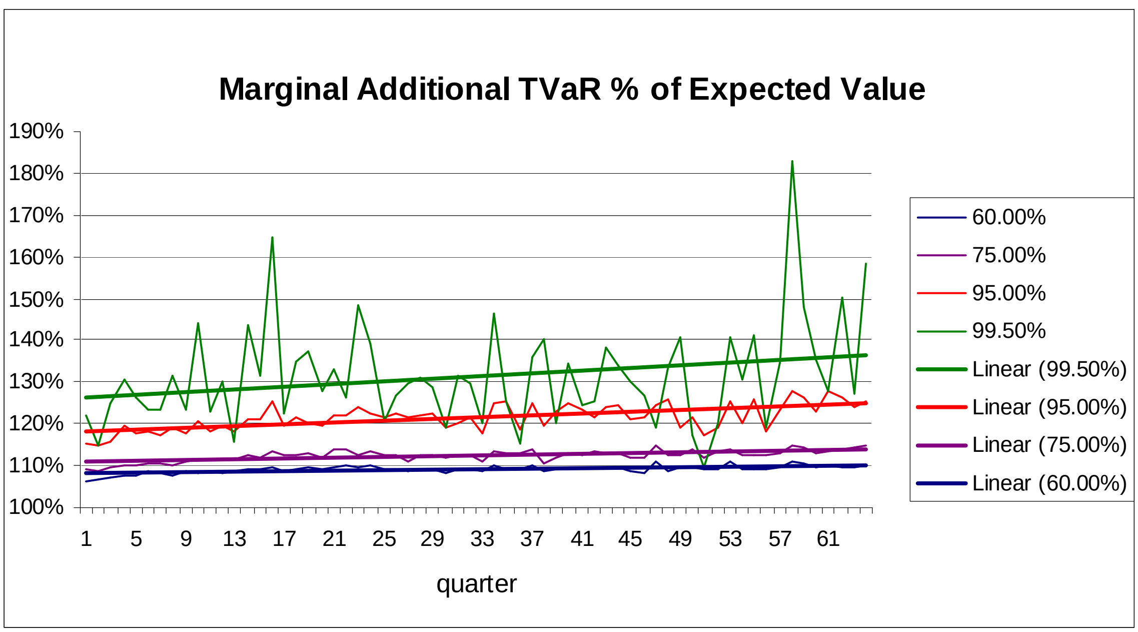
The pattern is similar for TVaR and for other levels of adequacy.

4.4.5 Run-off of a Cohort of Claims in the Context of an On-Going Portfolio

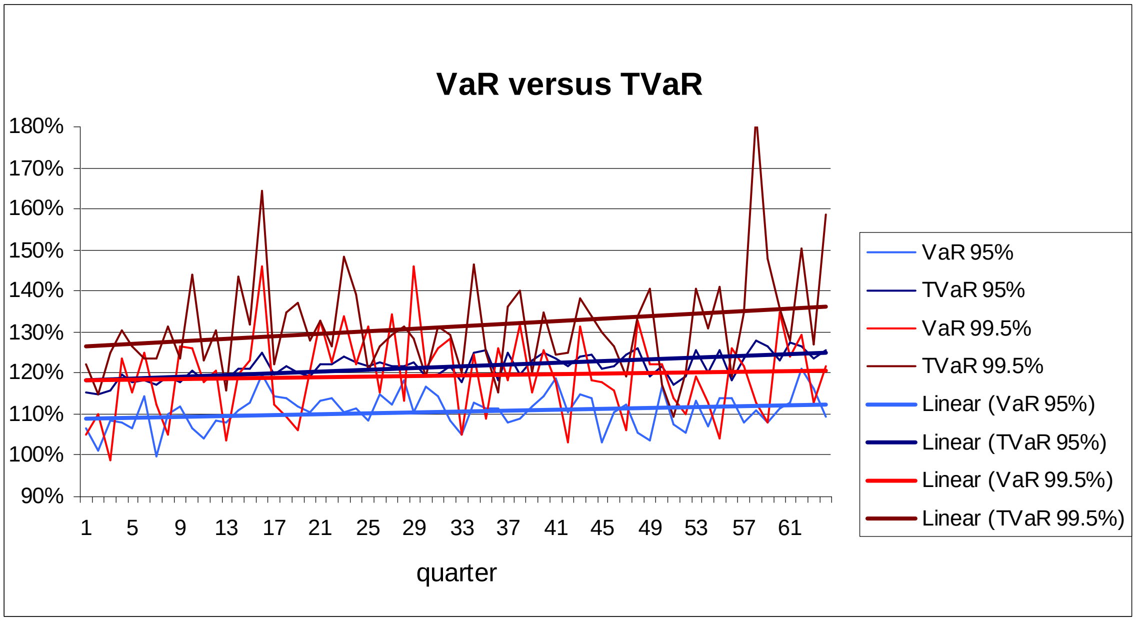
The following graphs show the marginal impact on VaR and TVaR for selected probabilities of adequacy, when a $50 million cohort of liabilities is added to a $5 billion stable portfolio, together with a comparison of VaR and TVaR.



Although trend lines are shown, it is clear that there is no particular pattern for a given level of adequacy.



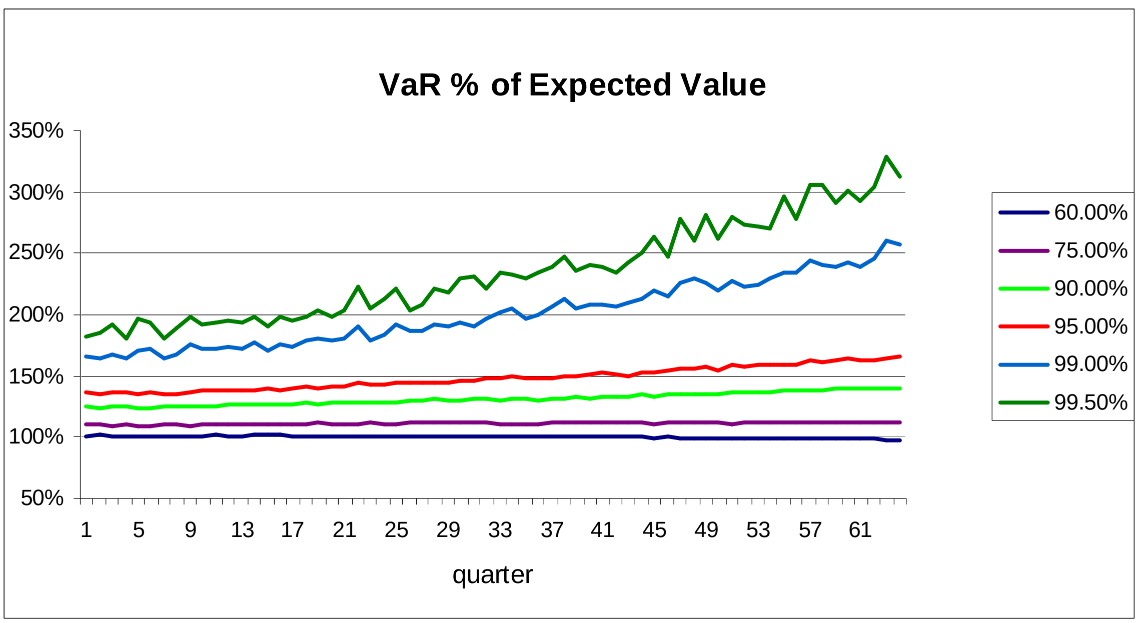
In this case, there does appear to be an upwards trend, though it is not clear that this is significant if any one level of adequacy is taken in isolation.



As expected, this shows that TVaR is clearly greater than VaR for a given level of adequacy, although there is a lot of variation.

4.4.6 Stand-Alone Run-off of a Cohort of Claims

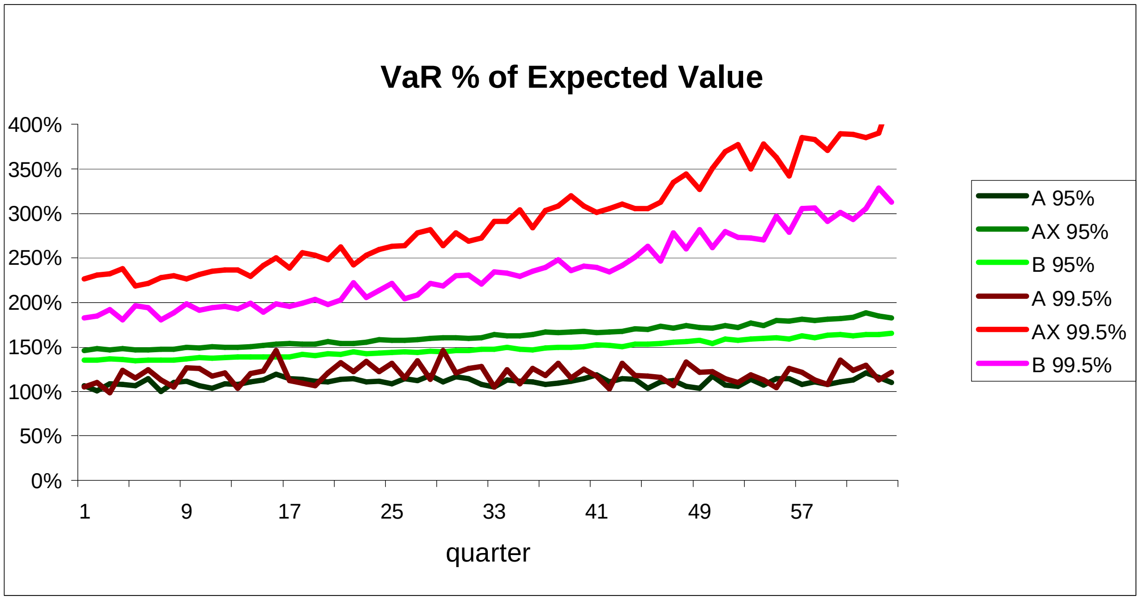
The following graphs show VaR and TVaR for selected probabilities of adequacy, for a stand-alone $100 million cohort of liabilities.

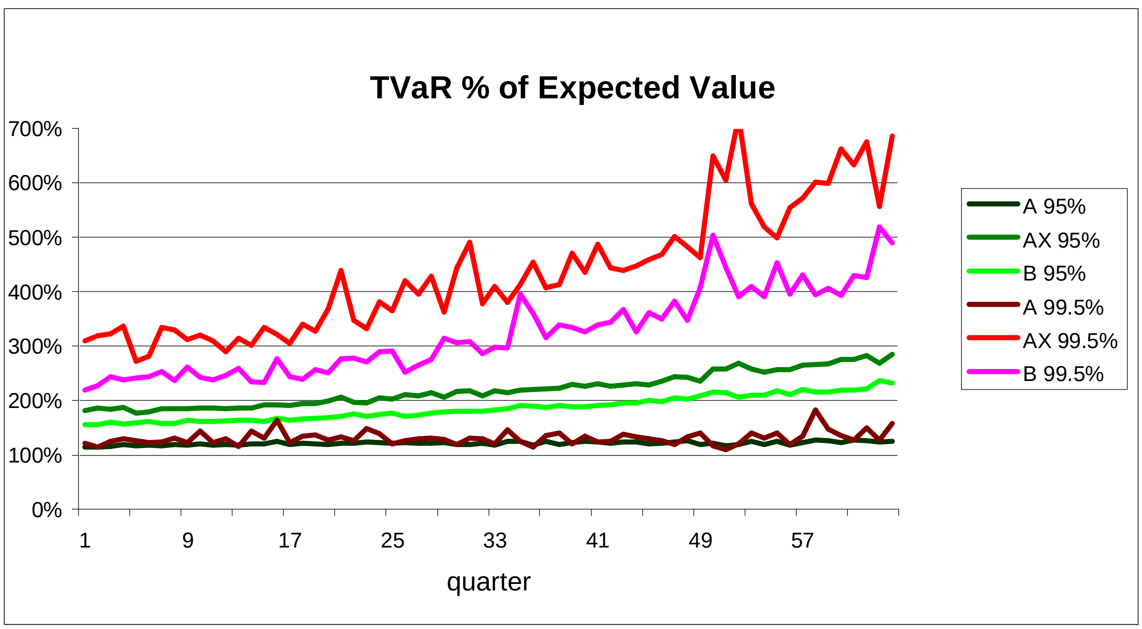


On a stand-alone basis, there is clearly an upward trend in VaR with duration. There also appears to be some shape to this trend. Both trend and shape are also visible for TVaR.

4.4.7 Comparison of Stand-Alone and Supported Run-off of a Cohort of Claims

The top four curves in the following graphs show VaR and TVaR for the stand-alone cohorts AX and B (twice the size of AX). The bottom two curves show the marginal VaR and TVaR when AX is combined with a large portfolio A (100 times the size of AX). These show the substantial impact that a host portfolio has on the capital needed to support a given block of business.





These clearly show the dramatic difference between marginal capital (A) and stand-alone capital (AX) for the same size portfolio, and also the difference that doubling the portfolio size makes (AX vs. B). Clearly, an insurer that set its capital requirements for individual portfolios on the basis of their marginal needs would be under-provided in aggregate.

5 UK– Third Party Bodily Injury Liability - Bootstrapping

5.1 Introduction and Overview

The initial aim of this section is to investigate how the capital ratio (capital / reserves) changes over time as a portfolio runs off. The capital required at each future time needs to be estimated if one is going to estimate a risk premium for a portfolio of liabilities.

In this section, all the analysis is undertaken gross of reinsurance and considers only the stand-alone capital requirements of the portfolio.

A very large claims dataset was assembled based on mixing large data sources, each scaled for confidentiality. The business analysed was UK private motor, focussing only on bodily injury claims arising from comprehensive policies. We estimate that in the most recent accident year, approximately 5 million vehicles were exposed. This gives far more stability than one would normally see in a typical triangle, leading to an unusually low capital ratio and hence risk margin.

Each of the source data sets was made up of quarterly accident origin periods from 1994 Q1 to 2006 Q2 inclusive with quarterly developments up to 2006 Q4.

A bootstrapping method was used in order to generate simulations of future cash flows from this data set. The next sub-section gives a brief description of bootstrapping and a reference for further reading.

The initial results clearly show that the capital ratio increases materially as the portfolio runs off. This section discusses some of the alternative ways that the future capital requirement could be calculated.

No risk margin calculations are shown in this section. Here we explore the capital requirement over the time that the portfolio runs off. Section 2 showed the corresponding risk margin results for comparison with the US and Australian models.

5.2 Bootstrapping description

The bootstrapping process can be used to determine the distribution of the incremental amounts for each origin period and each future development period, allowing the simulation of future cash flows.

It is important to realise that bootstrapping is simply a statistical procedure that can be applied to any well defined model. One commonly used is a special case of the so called "over-dispersed Poisson" model. It is also possible to bootstrap an alternative model, known as "Mack's model", which has been used in this analysis. Both models can be written as generalised linear models (GLMs).

The methodology for bootstrapping both models (in the context of the GLM framework) is described in *Predictive Distributions Of Outstanding Liabilities In General Insurance,* Annals of Actuarial Science, Volume 1, Number 2, 2006 , pp. 221-270(50) England, P. D.; Verrall, R. J.

The basic data required for the bootstrapping process is a triangle of claim amounts. A deterministic methodology is applied to the claims triangle. A basic chain ladder method is used and a curve fitted to the resulting development factors. The model includes a number of parameters which affect the curve fitting process; the choice of curve to fit, restrictions on which development factors are used for fitting, restrictions on which development ratios are smoothed, and the length of the tail to estimate.

The bootstrapping methodology generates pseudo triangles. The original chain ladder with tail method is then reapplied to these triangles. As the chain ladder model used for each pseudo triangle is the same as for the original triangle, the chain-ladder model should be general enough to cope with similar, but potentially different, data sets.

Future claim amounts are then simulated for each origin period and each future development period. The model calculates a variance factor for each development period. These are based on the residuals from the original DFM and are used to calculate the variance of the simulated results. These variance factors may be unduly affected by exceptional claims or unusual experience, although the data set used for this analysis was exceptionally large with no obvious odd issues. It is necessary to extend these variance factors into the tail where a tail is required.

Within the bootstrapping model we have excluded certain development factors. These factors have been excluded to smooth the residuals and hence reduce variability where it is being driven by erroneous data points. This can happen in the first four development quarters, which are quite erratic and not necessarily representative of the variability within the rest of the triangle. The development factors in the last four development quarters (in the oldest year) can also distort the variability since they are based on just one accident year's data.

These adjustments do not have an adverse effect on the soundness of the bootstrapping theory. However they are essential to ensure that the DFM is an appropriate model.

5.3 Methodology and results

The following methodology was applied:

- Take data for origin years 1994 to 2001
- Take quarterly paid development data as at the end of 2001 Q4 ("cut-off point")
- Fit a DFM, based on accident year origin periods and development quarters
- Bootstrap it and simulate 10,000 sets of reserves
- For each simulation, extract cash flows for 2002Q1 and subsequent quarters

From this data set it is fairly straightforward to create the distribution of undiscounted and discounted reserves at each future quarter, allowing the means and any other statistics such as value at risk or tail value at risk to be calculated.

The following simple table demonstrates how the mean undiscounted reserves are calculated at each future point:

Table 5.1: Simple example to demonstrate approach

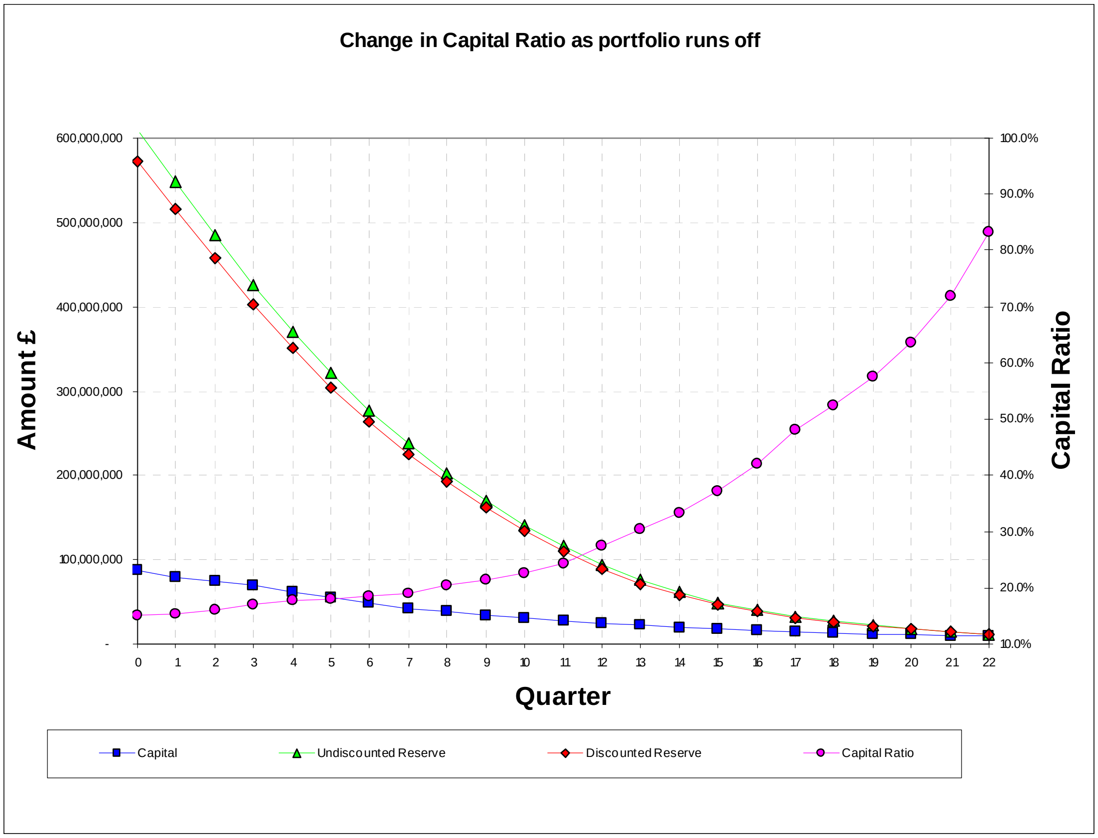
	Simulated cash flows (incremental)				
Period	**1**	**2**	**3**	**4**	**5**
Sim 1	10	8	6	4	2
Sim 2	11	6	7	3	1
Sim 3	7	4	3	1	0
Sim 4	13	8	5	3	1
Sim 5	9	5	5	2	0
Total	50	31	26	13	4
Total future	124	74	43	17	4
Mean reserve	24.8	14.8	8.6	3.4	0.8

Similarly, for example, the 99.5th percentile of the reserve and the discounted equivalents can be calculated at each point, allowing the capital ratio to be determined.

Discounting was undertaken using a flat rate of 4% per annum. The capital was defined as the discounted reserves at the 99.5th percentile less the mean discounted reserves. The capital ratio was defined as the capital divided by the mean discounted reserves.

The following chart shows the results of this analysis:

Graph 5.2: How capital ratio changes as portfolio ages



This chart clearly demonstrates that the capital ratio increases materially as the portfolio runs off. Alternative capital definitions such as 97.5th value at risk and tail value at risk were explored and similarly increasing capital ratio curves were produced.

5.4 Using hindsight

The analysis above used data from origin years 1994 to 2001 only, although data was available up to origin quarter 2006 Q2. This is to allow us to explore how the capital would be calculated at future points in time based on actual experience. By setting "now" equal to the end of 2001 Q4, giving us the benefit of hindsight this was possible.

The methodology described in section 5.3 was repeated (i.e. new DFMs created and new bootstraps run) but having added an extra four development quarters of actual paid data to the triangle – in other words at cut-off period 2002 Q4. This was then further repeated at cut off points 2003Q4, 2004 Q4 and 2005 Q4.

In all of these extra analyses, the origin years remained as 1994 to 2001, so it is the same portfolio being analysed. The following table summarises the results:

Table 5.3: Discounted reserves and capital ratios for same portfolio using different cut-off dates

As at:	Cut off point				
	2001 Q4	**2002 Q4**	**2003 Q4**	**2004 Q4**	**2005 Q4**
	Discounted reserves (£m)				
2001 q4	572.0				
2002 q4	350.1	353.0			
2003 q4	192.0	187.5	197.6		
2004 q4	89.4	80.1	88.4	71.3	
2005 q4	38.1	27.9	34.0	22.4	22.9
	Capital ratio				
2001q4	**15.1%**				
2002q4	17.7%	**9.1%**			
2003q4	20.4%	10.7%	**8.2%**		
2004q4	27.5%	17.7%	13.9%	**10.4%**	
2005q4	42.0%	39.2%	29.8%	14.8%	**14.6%**

The 2001 Q4 column is from the analysis explained in section 5.3 above. The additional columns reflect the results from the four additional DFMs and associated bootstrapping analyses, each of which had four extra actual paid development quarters added.

A number of observations jump out from the capital ratio table above. These are listed below with some suggested explanations:

A. Going down each column, the capital ratio increases. This is consistent with the findings in section 5.3 above. We believe that there are two reasons for this. Firstly a reserve estimate made as at a future point in time will have relatively more uncertainty around it than a current reserve estimate. Equivalently, it is more difficult to estimate a cash flow in 5 years time than it is to estimate next year's cash flow. The second reason we suggest is that as a book gets older and smaller, it becomes more volatile.

B. Going across each row, the capital ratio decreases. With extra information, the reserves can be estimated with more certainty.

C. Going down and right along the first capital ratio for each analysis, the capital ratio decreases (15.1% to 9.1% to 8.2%) and then increases (to 10.4% and 14.6%). We suggest that this is a combination of two of the effects described in A and B:

   - extra data giving more certainty and bringing the capital ratio down
   - increasing inherent volatility as the portfolio shrinks, pushing the capital ratio up.

One point of debate amongst this group was whether, when estimating the capital required at a future time, some allowance should be given for the reduced uncertainty that will most likely exist at that future time due to the extra information available (extra diagonals). We concluded that this was not appropriate. The extra uncertainty is a function of not knowing where the mean reserve will be at that future time.

[As an aside, we note that the discounted reserve as at 2005 Q4 based on the actual data to 2005 Q4 (£22.9m) is just below the 0.5th percentile of the distribution from the 2001 Q4 analysis (which has a mean of £38.1m and a 0.5th percentile of £23.6m). We believe that this is a function of model risk – the simulations produced by the bootstrapping approach include allowance for both parameter and process uncertainty, but assume that the underlying model is correct. This implies that the capital ratios above are all understated.

This raises interesting issues for practitioners who use bootstrapping to determine reserve variability. A DFM model (or other reserving approach) will never be perfect, so additional variability should be added to allow for this model risk – this is not explored further in this paper.]

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6 US - Third Party Bodily Injury and Property Damage Liability - Collective Risk Model

The stochastic loss reserving model discussed in this section is taken from Meyers[17] [2006]. In this paper, Meyers studies the US Commercial Auto line of business. The underlying data was taken from ten-year triangles of incremental paid losses from the accident years 1986-1995. Meyers fit a model to the data of over 100 insurers and validated the model on subsequent losses paid through 2001.

We use a similar model derived from the same data in this paper.

6.1 The Underlying Loss Model

For a given accident year, *a*, and settlement lag, *l*, the expected loss, is given by:

$$\text{Expected Loss}_{a,l} = \text{Premium}_a \cdot ELR \cdot Dev_l$$

Premium_a is specified by the insurer. The parameter set $\{ELR, Dev_l\}$ is estimated from the insurer's data.

The distribution of actual losses, given the expected loss for each cell, (a,l), is given by the collective risk model, described by the following simulation algorithm.

1. Select the expected claim count, $\lambda_{a,l}$ as the Expected $\text{Loss}_{a,l}$ divided by the expected claim severity for the settlement lag *l*.

2. Select a random number of claims, *N*, from a negative binomial distribution with mean λ and variance $\lambda + c \cdot \lambda^2$.

3. Select *N* random claims, $\{Z_i\}$, from the claim severity distribution for settlement lag *l*.

4. The loss for the cell (a,l) is given by $X_{a,l} = \sum_{i=1}^{N} Z_i$.

Here are some additional details about the fitting of this model.

- The parameter, c, was set equal to 0.01. Justification for the selection of this parameter can be found in study done by Meyers[18] [2007] on Commercial Auto data.

- The claim severity distributions were derived from Commercial Auto data reported to ISO, which serves as a statistical agent collecting data from the large number of insurers

[17] Meyers, Glenn G., "Estimating Predictive Distributions for Loss Reserve Models," *CAS Forum*, Fall 2006, http://www.casact.org/pubs/forum/06fforum/163.pdf

[18] Meyers, Glenn G., "The Common Shock Model for Correlated Insurance Losses," *Variance*, Volume 1, Issue 1, 2007. http://www.variancejournal.org/issues/?fa=article&abstrID=6373

in the US. These distributions vary by settlement lag. Figure 6-1 provides a graphical representation of these distributions.

- The fitting method uses a Fast Fourier Transform (FFT) to calculate the likelihood of a triangle of data. This paper uses Bayesian estimation[19] to produce a set of 100 $\{ELR, Dev_{Lag}\}$ parameters that represent the parameter risk for the fitted model. Figures 6-2 and 6-3 represent this parameter risk graphically.

- For a given parameter set ELR, Dev_{Lag}, the collective risk model represents the process risk associated with the parameter set.

- The final predictive distribution will be an equally weighted mixture of the collective risk models for each parameter set in $\{ELR, Dev_{Lag}\}$.

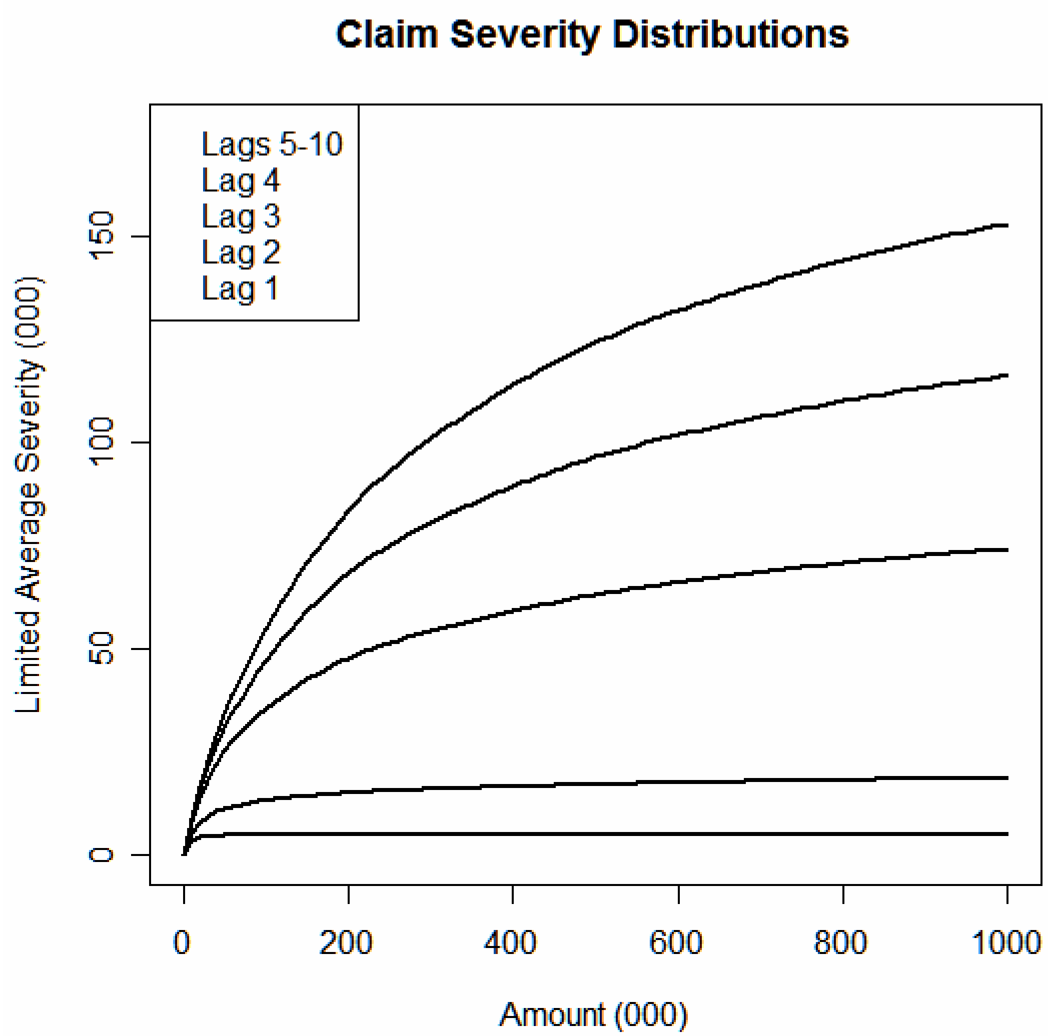
- The estimation was done on a simulated triangle with an annual premium of $50,000,000 using the model described above. Given the validation results obtained by Meyers [2006] we feel that it is a fair representation of a similarly sized Commercial Auto portfolio.

- For a given $\{ELR, Dev_{Lag}\}$ parameter set, it is possible to derive a predictive distribution for a portfolio of any given size. This assumes that the parameter risk is the same regardless of the portfolio that provided the data used in the estimation.

[19] In contrast to the Bayesian methodology described in Meyers [2006] that calculated the posterior probabilities on a fixed multidimensional grid of points, the methodology described in this paper used the Gibbs sampler, to calculate a random set of $\{ELR, Dev_{Lag}\}$ points. This methodology will be described in detail in Meyers, "Fitting the Compound Negative Binomial Model to Loss Reserve Triangles." (In preparation)

Figure 6-1

Claim Severity Distributions in the Collective Risk Model



- The Limited Average Severity statistic is described in Klugman, Panjer and Willmot[20] [2004]. In general, greater values of the limited average severity indicate a greater likelihood of large claims.

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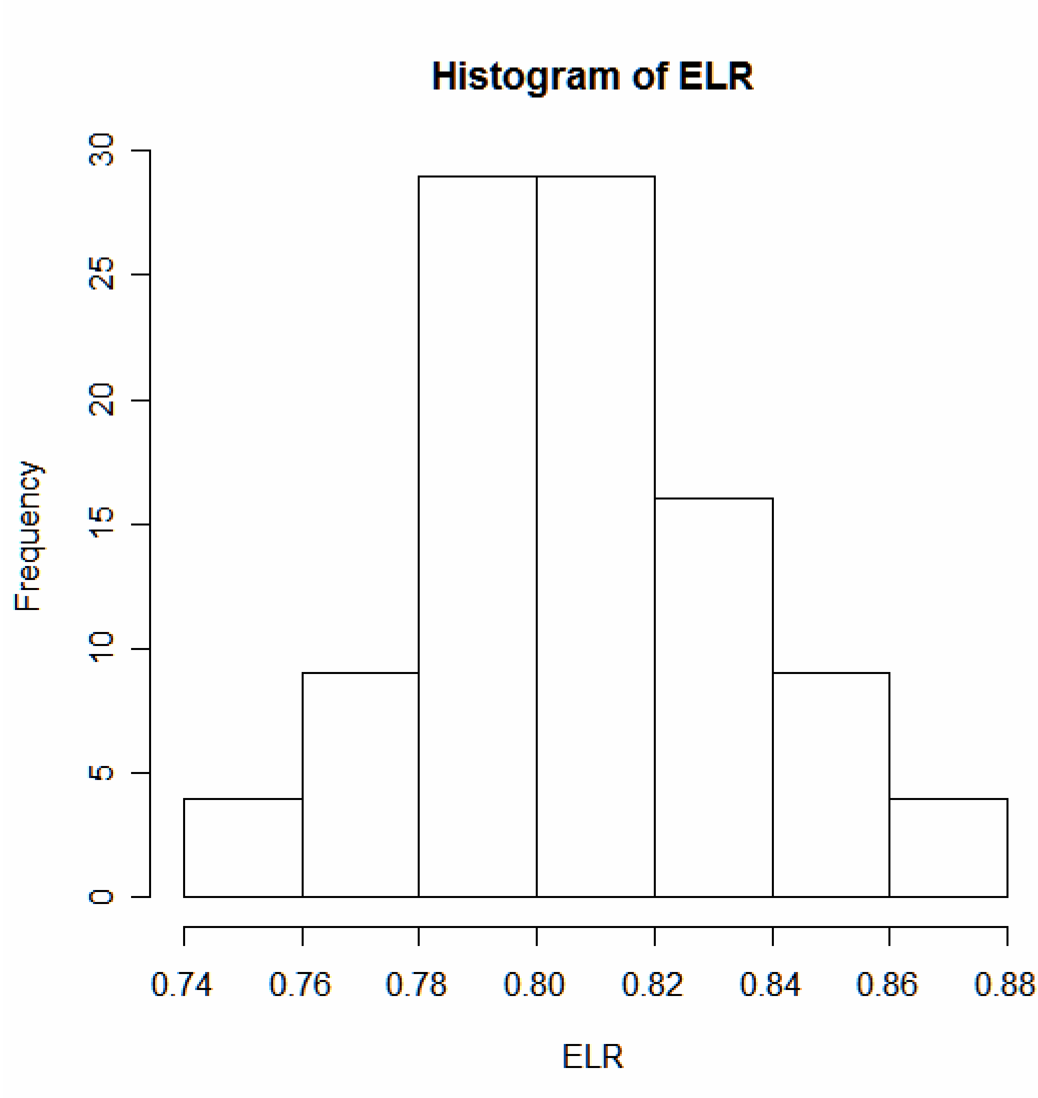
[20] Klugman, Stuart A,, Panjer, Harry H., and Willmot, Gordon E,, Loss Models, Wiley 2004, p. 30.

Figure 6-2

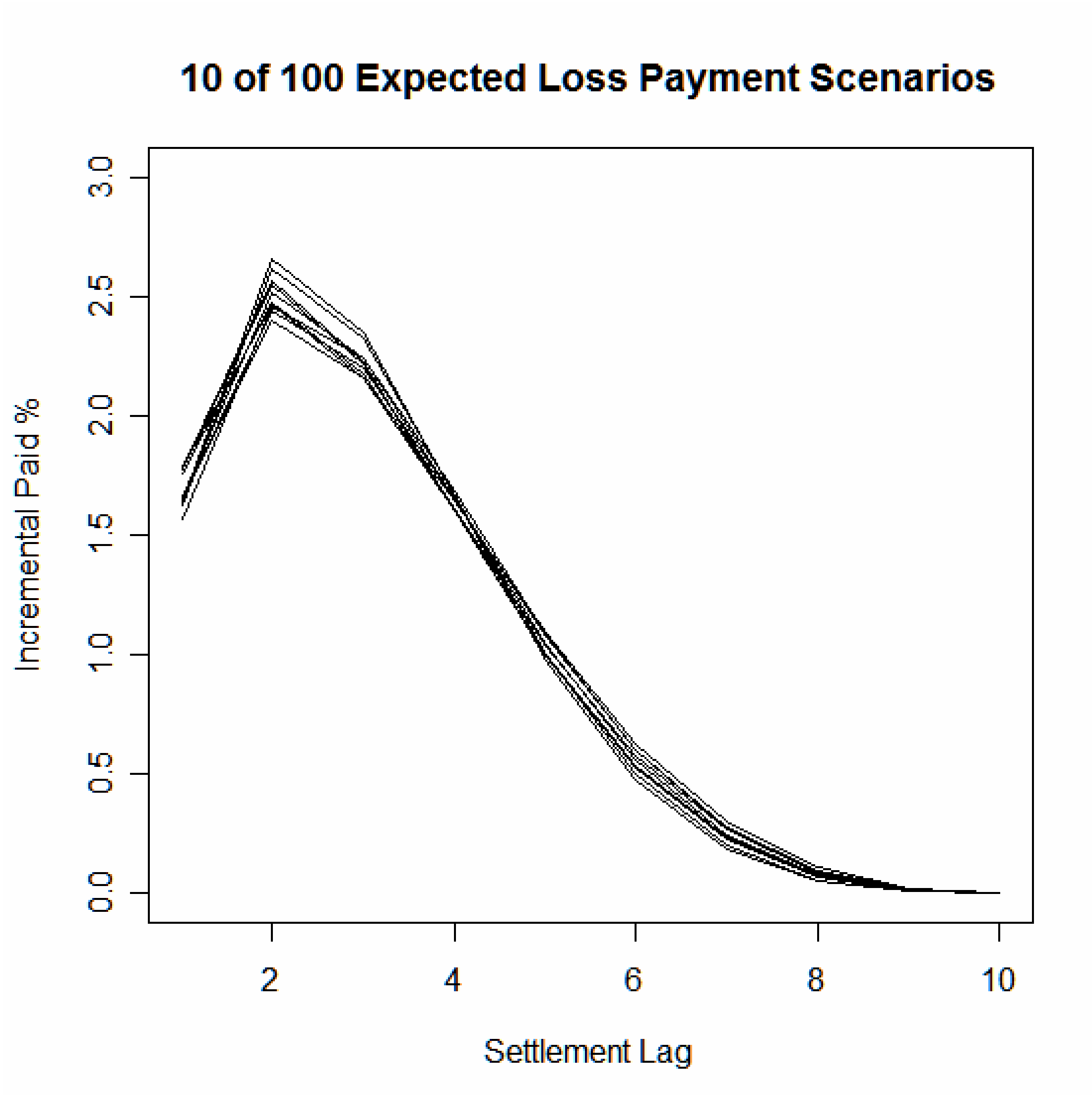
The Marginal Distribution of *ELR*



- This histogram represents 100 *ELR*s derived from the Gibbs sampler of the posterior distribution of the fitted model.

Figure 6-3

A Graphical Representation of {Dev_{Lag}}



- Plotting all 100 Dev_{Lag}'s produces a solid graph with roughly the same width. Taking a sample allows one to view the variability of the individual payment patterns.

6.2 Calculating the Capital for a Runoff Insurer

In this paper we use the cost of capital method to measure risk margins as shown below. This section considers two criteria for required capital.

1 Value at Risk at the 99.5% level – VaR@99.5%.

2 The Tail Value at Risk at the 99% level – TVaR@99%.

The VaR criterion has the advantage that more people are familiar with it. The TVaR criterion has the advantage of a stronger basis in actuarial theory. As we shall see, the theoretical difficulties of the VaR can lead to counterintuitive results for smaller insurers with skewed loss distributions.

The portfolios considered in the examples that follow start with a stated initial annual premium, P, and then grow at a rate of 5% per year. At the end of the tenth calendar we consider the run-off portfolios. The initial liabilities are the sum of random outcomes in the numbered cells of Table 6-1 below.

Table 6-1

AY	Premium\Lag	1	2	3	4	5	6	7	8	9	10
1	P	pd	pd	pd	pd	pd	pd	pd	pd	pd	1
2	$P\cdot 1.05$	pd	pd	pd	pd	pd	pd	pd	pd	1	2
3	$P\cdot 1.05^2$	pd	pd	pd	pd	pd	pd	pd	1	2	3
4	$P\cdot 1.05^3$	pd	pd	pd	pd	pd	pd	1	2	3	4
5	$P\cdot 1.05^4$	pd	pd	pd	pd	pd	1	2	3	4	5
6	$P\cdot 1.05^5$	pd	pd	pd	pd	1	2	3	4	5	6
7	$P\cdot 1.05^6$	pd	pd	pd	1	2	3	4	5	6	7
7	$P\cdot 1.05^7$	pd	pd	1	2	3	4	5	6	7	8
8	$P\cdot 1.05^8$	pd	1	2	3	4	5	6	7	8	9
10	$P\cdot 1.05^9/2$	1	2	3	4	5	6	7	8	9	10

- Payments in the n^{th} subsequent years are the random sums of the cells number *n* and above. In all, the insurer will be making random payments for 10 years in the future.

Using the Fourier methods described in Meyers [2006] we calculated the distribution of the sum of the random losses paid in each of the *n* subsequent years. This gives information of the form that would be obtained by simulation, but does that work much faster. We assumed that the losses were correlated within an accident year, but were independent between accident years.

Given the distribution of the losses to be paid in the next ten years, the capital required for each of the subsequent years was as follows.

VaR@99.5% – Expected Loss

TVaR@99% – Expected Loss

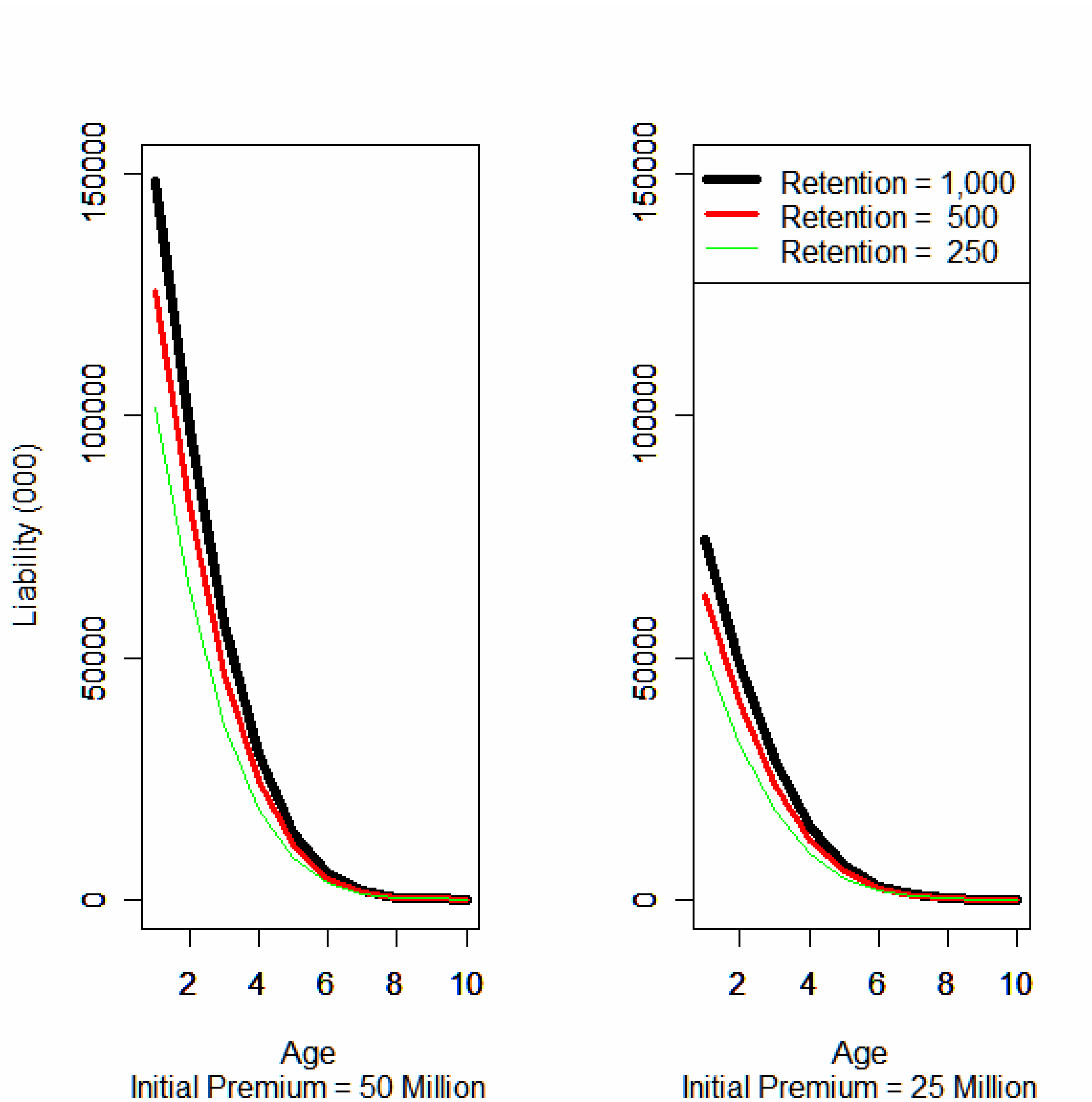
The required capital was calculated for insurers with initial premiums of $50 million and $25 million. To test the effects of reinsurance on that portfolio, we limited individual claims to $1 million, $500 thousand and $250 thousand.

We also considered the case where the runoff portfolio was ceded to a similar existing insurer with an ongoing portfolio with an initial premium of $500 million. In this case we expect the cost of capital to be proportional to the additional (or marginal) capital the insurer must raise to take on the risk.

The results and commentary of this analysis are in the following tables.

Figure 6-4

Standalone Liabilities for the Portfolio
Initial Premium = $50 & $25 Million

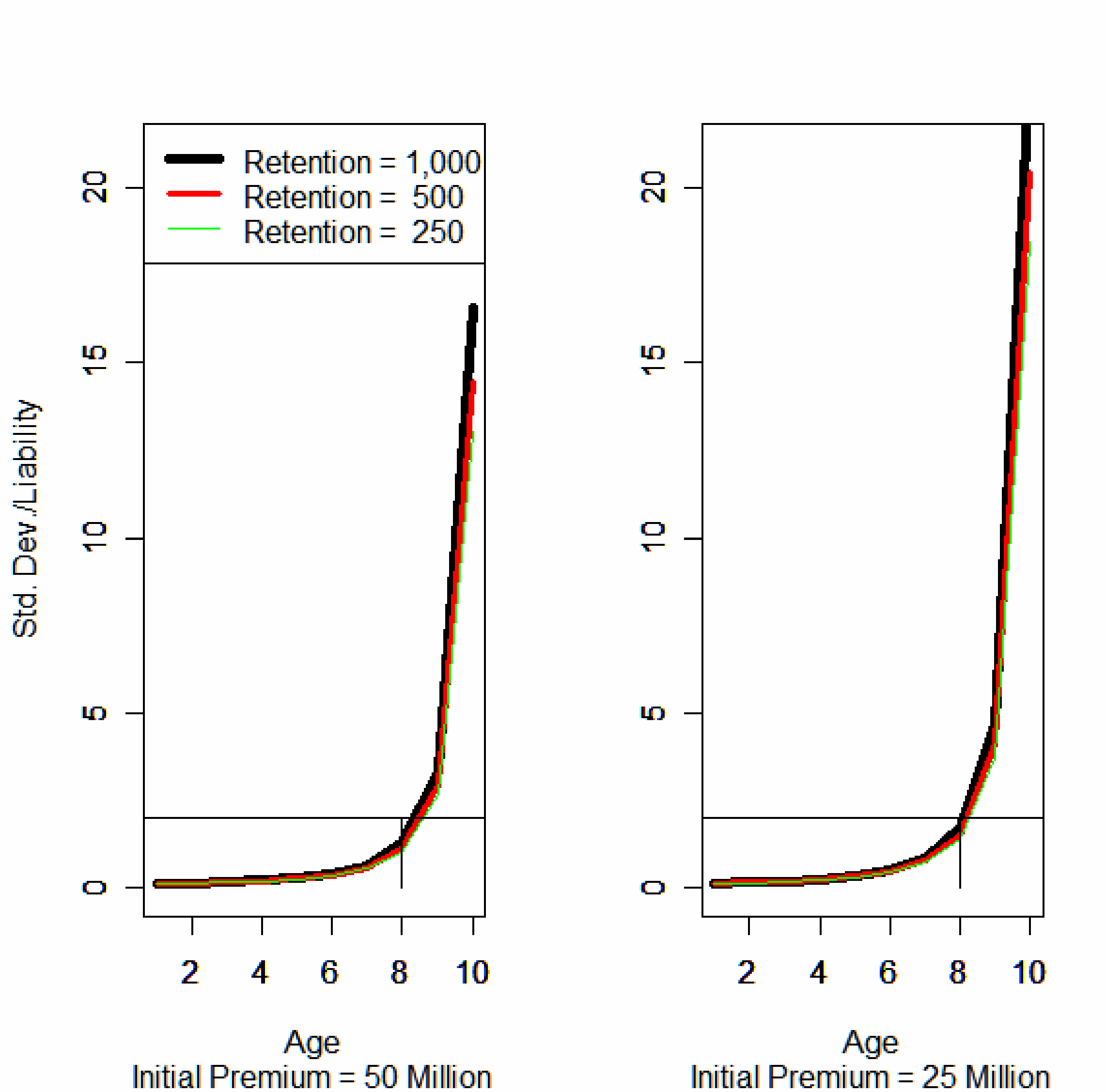


As expected:

- The liabilities decrease as the retention decreases.
- The liability for the $50 million portfolio is twice that of the $25 million portfolio.

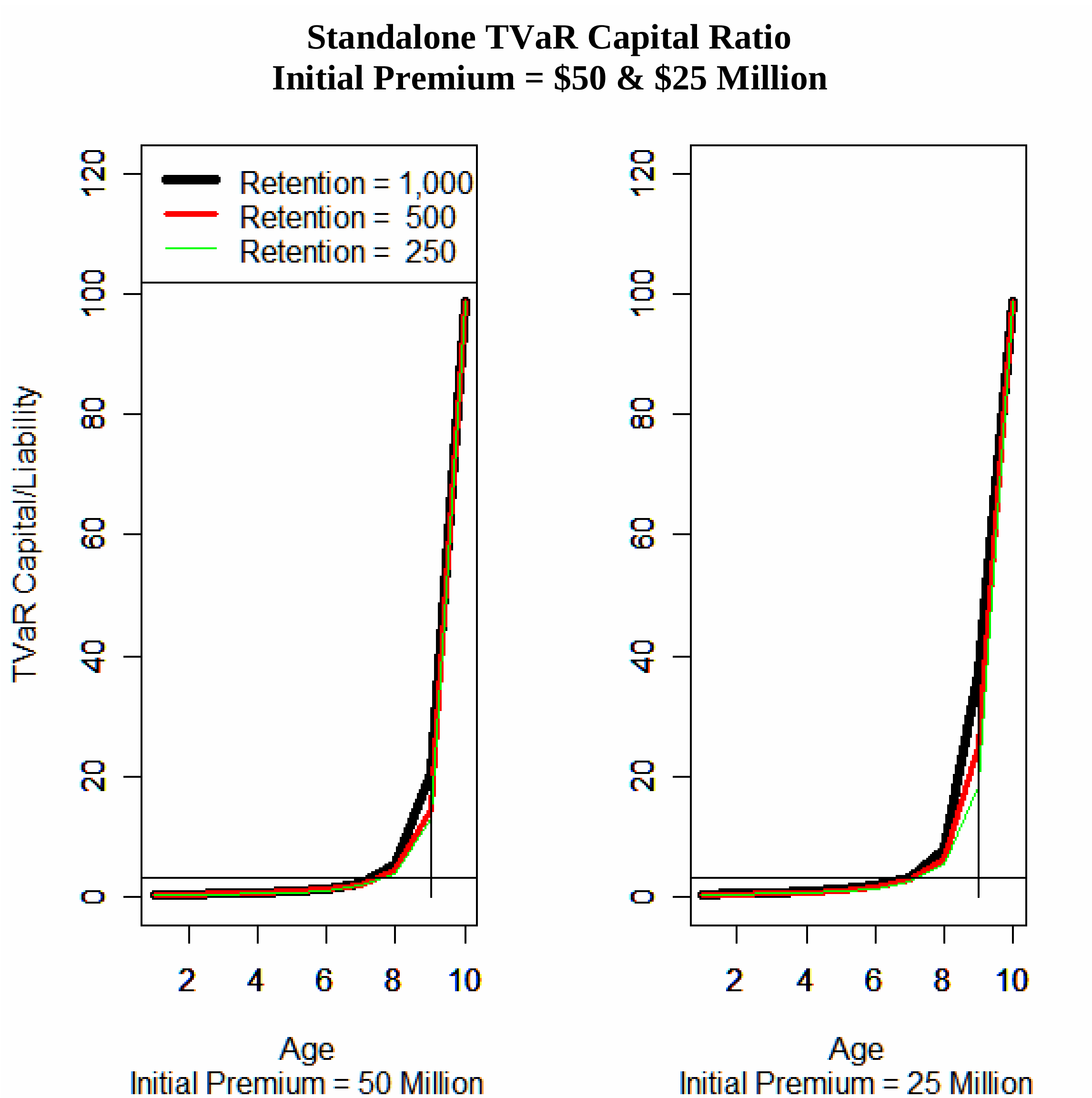
Figure 6-5

Coefficient of Variation of the Standalone Liabilities for the Portfolio
Initial Premium = $50 & $25 Million



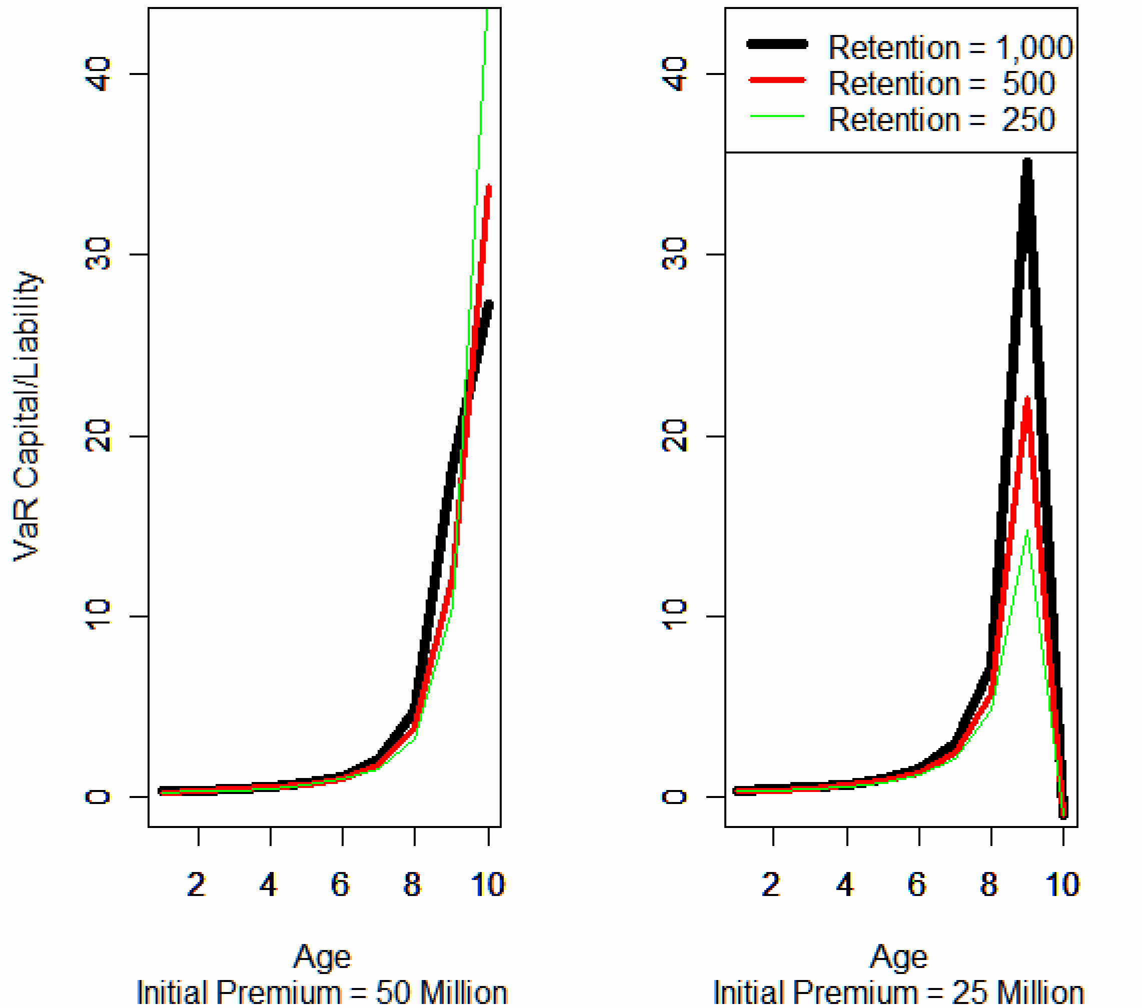
- The common reference lines on each graph help to identify the differences between them.
- The coefficient of variation grows rapidly with settlement lag because of the small number of claims that are potentially very large.
- Reinsurance can have a noticeable effect on the coefficient of variation.

Figure 6-6



- The common reference lines on each graph help to identify the differences between them.
- The liability to premium ratio grows very fast with the settlement lag because of the small number of claims that are potentially very large leads to a skewed distribution.
- Excess of loss reinsurance can have a noticeable effect on the TVaR capital ratio.

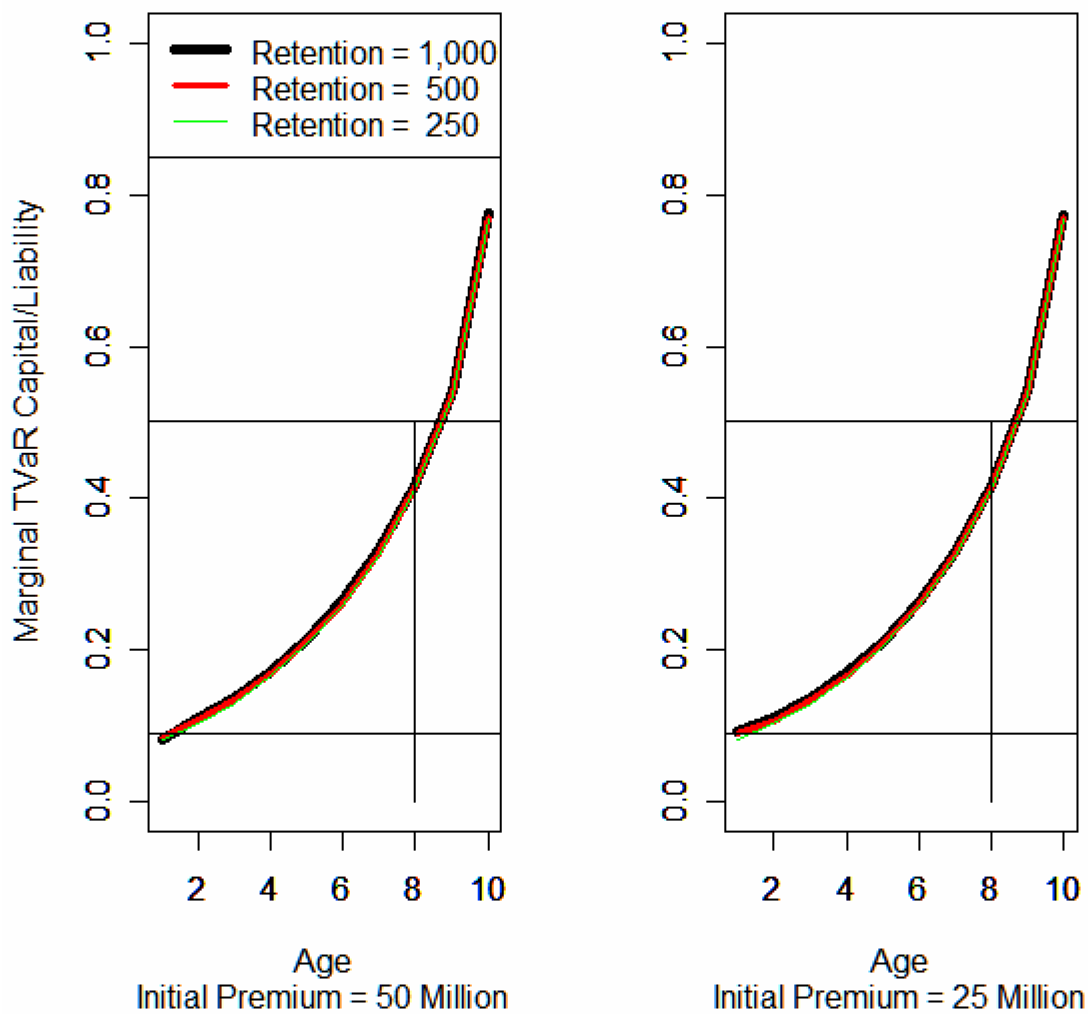
Figure 6-7
Standalone VaR Capital Ratio
Initial Premium = $50 and $25 Million



- These graphs illustrate a theoretical feature of the VaR measure of risk. For very skewed distributions, the mean can be less than the 99.5th percentile.
- For the smaller portfolio, the 99.5th percentile is equal to zero, which implies that the VaR Capital Ratio is negative one.
- The distortions of the VaR can be subtle for larger portfolios. Note that counterintuitive effect of excess of loss reinsurance in the $50 million portfolio.

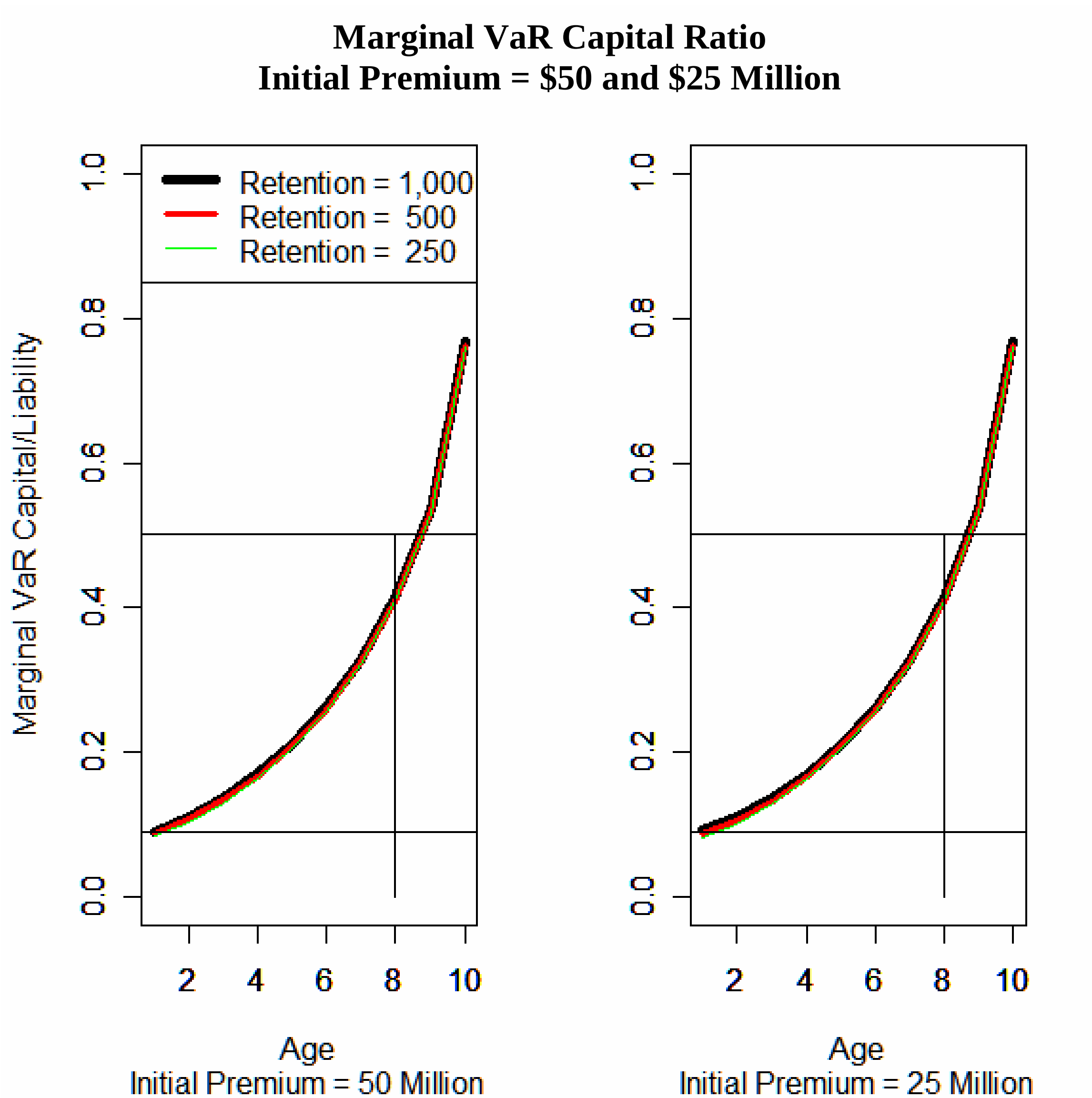
Figure 6-8

Marginal TVaR Capital Ratio
Initial Premium = $50 and $25 Million



- Compare this with Figure 6-9. In these examples, the capital to liability ratios are nearly equal for the TVaR@99% levels and VaR@99.5% levels. When a small portfolio is added to a large portfolio, the liability distribution is not very skewed.

Figure 6-9



- Compare this with Figure 6-8. In these examples, the capital to liability ratios are nearly equal for the TVaR@99% levels and VaR@99.5% levels. When a small portfolio is added to a large portfolio, the liability distribution is not very skewed.

6.3 Conclusions

These examples suggest the following conclusions.

- The assumption of a constant capital to liability ratio is not appropriate. This is caused by: (1) late-settling claims tend to be larger; and (2) the number of late-settling claim is generally smaller.

- Using the VaR to calculate capital has theoretical difficulties, which can lead to counterintuitive results for capital calculations for smaller insurers.

- Calculating the risk margin with a large, but similarly constituted, reference insurer can lead to tractable and intuitively appealing results in which the TVaR and VaR risk measures are in substantial agreement.

- One potential difficulty with the marginal approach is that the sum of the marginal capitals for sub portfolios will generally (always in the case of the TVaR) be less than the total capitals. We recommend that the marginal capital be multiplied by a constant factor to force additivity. This is not as arbitrary as it might appear. Meyers[21] [2003] provides an economic justification for this adjustment. The gist of the argument is that an efficient insurance market will provide a constant return on marginal capital inputs.

[21] Meyers, Glenn G., "The Economics of Capital Allocation," *CAS Forum*, Fall 2003. http://www.casact.org/pubs/forum/03fforum/03ff391.pdf

7 Shock and Latent Claims

For purposes of this paper we define shock claims as claims arising from major events that are not represented in the database (because they have been excluded or because they have not occurred) and therefore not reflected in the modelling techniques used to develop the capital requirements. We define latent claims as claims arising from events with latency periods during which no events emerge and then a period when claims from many past exposure years are reported and paid. Asbestos and US pollution liability are the most notable latent shock claims. Other examples include mould claims on US Homeowners, construction defect claims, and a variety of pharmaceutical/medical device claims.

It is more difficult to point to shock or latent claims for automobile exposures. However, to examine how sensitive the capital requirements, and therefore risk margins, are to such events we constructed the examples of shock claims (Table 7-1) and the Shock+Latent (Table 7-2) claims described below.

Table 7-1—Risk Margin from Shock (Non-latent) Claims

Row	Item	Amount	Notes
1	Expected annual cost	0.25%	Selected % annual premium
2	Reserves/Premium ratio	3.0	Selected
3	Frequency	0.5%	A 1/200 year event
4	Expected Claim size	16.7%	Percent initial reserves. (1)/(2)/(3)
5	Capital required:		
5a	Normal	26.5%	From US Example
5b	Stand alone shock	16.7%	Size of 1/200 event (Row 4)
5c	Shock + Normal	31.3%	5a and 5b combined as if shock event is independent of variability in normal claims
6	Risk Margin-Cost of Capital		
6a	Normal	4.9%	From US Example
6b	Stand alone shock	3.1%	(6a)*((5b)/(5a))
6c	Shock + Normal	5.8%	(6a)*((5c)/(5a))

For the shock claim in Table 7-1 we assume that normal and shock claims have the same payment pattern and variability during runoff.

For the latent shock example in Table 7-2 we assume that the latent claim has developed over a period of 10 years and would be paid over a further period of 10 years. We assume that the chance of a shock loss over the prior 10 years is constant at 1/200, but the expected size of the event decreases as the exposure runs off. Thus, after 1 year the company is exposed to only 90% of the initial shock amount, 80% of the initial event size after 2 years, and 0% after ten years. If shock and normal claims are combined as independent events, then risk margins from ‘shock-only’ and ‘latent shock’ are as follows in Table 7-2:

Table 7-2—Risk Margins from Shock and Latent Shock Events

(1)	(2)	(3)
Item	**Shock Event**	**Latent Shock Event**
Stand – Alone Shock Event	3.1%	4.7%
Shock + Normal	5.8%	7.0%

Column (2) from Table 7-1; Column (3) from detailed cost of capital calculation not shown.

In the analysis shown above, we use a 99.5% VaR perspective on required capital. The calculations would be different, but the concepts would be similar for other VaR and various TVaR levels. Also, we note that if shock event has probability of 1/200, then the average value of the shock event limited to 1/200 probability is less than the full shock event size. We have not considered the details of that issue in this conceptual example.

Our conclusion from these sensitivity tests is that extreme events that are not reflected in the data, and which may necessarily be judgmental, are a material consideration in the selected of the appropriate risk margin.

8 Effect of diversification

The cost of capital approach to setting risk margins for reserving has generally been illustrated by considering lines of business separately. The IASB has expressed views on the issue of when experience from different portfolios can be combined.

However, regardless of the accounting treatment, our experience shows that material diversification benefits on capital are appropriate. Properly measured capital for a multi-line company will be less, often significantly less, than the sum of the capital required line-by-line.

We have not investigated that subject empirically in this paper.

9 Reinsurance

In the Australian and US examples, we have shown that, as expected, reinsurance reduces the cost of capital risk margins. Our results are shown in the summary section and are not repeated here.

10 Reference Company or Own Company Analysis

The IASB has expressed a preference for an Exit Value view of measuring liabilities. In many respects, the IAIS has accepted that preference. Therefore, a significant unresolved issue in the use of the cost of capital approach is whether the risk is to be measured from the perspective of the reporting company or from the perspective a reference company which could be viewed as assuming the business from the reporting company.

In a related issue, one of the IAIS principles states “Similar obligations with similar risk profiles should result in similar liabilities”.[22] As size of portfolio and diversification among lines of business can affect the measurement of risk margins for an individual company, the IAIS principle would lead to looking at liabilities in a context of something beyond just the individual company, e.g. a reference company, regardless of exit value considerations.

The IAA introduces the reference company as follows:[23]

Before a cost of capital method can be applied, the fundamental question of the entity to which the capital should relate to needs to be answered. To be consistent with an exit value approach, it is reasonable to construct a reference entity to which the portfolio would be transferred. As a result, the reference company is preferred to the reporting entity. In addition, the use of a reference would promote increased comparability between preparers' financial statements.

If there were a deep liquid market, there would be no need to understand the nature of buyer of an asset or an obligation. The price would be determined by the market; end of story. However, as we are modelling the price that would be required by the entity to which the net obligations will be transferred, the relevant characteristics of the entity need to be defined.

While the choices to be made are discussed below, a focus will be on a reference company that is a large, multi-line, diversified, and highly rated insurer with business similar in nature to the portfolios subject to the valuation.

1. *“Large” means large enough that “process risk”[24], fluctuation about the expected value, is as small as practical. For many types of insurance, given that the reference company is large, process risk will be negligible compared to parameter and model risk. Process risk may be significant for some coverages, e.g., property-catastrophe*

[22] IAIS, *Issues arising as a result of the IASB’s Insurance Contracts Project – Phase II, Second Set of IAIS Observations*, 1 June 2006, page 20, http://www.iaisweb.org/__temp/IAIS_provides_second_comment_paper_to_IASB_on_Phase_II_project.pdf

[23] IAA Risk Margin Working Group, *Measurement of Liabilities for Insurance Contracts: Current Estimates and Risk Margins*, Exposure Draft, February 23, 2007, page 57-58

[24] IAA, A *Global Framework for Insurer Solvency Assessment*, 204. The concepts of “risk” and “uncertainty” are defined. Risk means the variability in outcomes in a process that is fully understood, e.g., the result of rolling a pair of fair dice. “Uncertainty” means the additional variability in outcomes that occurs because the process is not fully understand, the model used might be incorrect to some degree and/or the actual model parameters will vary from the estimated parameters. As the term “risk” is used in other ways, e.g., risk margin” in this report, the term “process risk” is used to refer to risk as defined in the *Global Framework for Insurer Solvency Assessment* “parameter and model risk” refers to uncertainty as it was defined in the *Global Framework for Insurer Solvency Assessment*.

and high-layer excess property or liability coverages. Parameter and model risk for the reference company is not expected to be small.

2. *"Multi-line, diversified" means the realistic benefits of risk diversification across portfolios and territories (including within and between countries to the extent that such diversification is observed in the market) are recognized in determining capital, cost of capital, variability in liabilities, margin setting, and other parameters.*
3. *'"Highly rated" means having resources that equal or exceed that of a typical insurance company, for example, an AA rating, that might depend on the types of business involved. It means a company with "minimum statutory surplus" would not be consistent with this example*[25]*.*
4. *"Business similar in nature" means that determination of the characteristics of the reference company is based on a review of an appropriate set of companies that are in the same business.*

One simplified treatment of the reference company would be to measure the marginal risk associated with merging the reporting company experience into a company identical to the reporting company but much larger. Therefore in the Australia and US examples, we have measured marginal capital and related risk margins as well as risk margins from a stand-alone company.

This paper does not address diversification issues, but those should be considered in a realistic reference company framework.

[25] The IAA indicated that the choice of financial strength level may not be significant if cost of capital is selected on a consistent basis, as cost of capital will increase as financial rating deteriorates.

11 Minimum Capital Requirements and Cost of Capital Formulas

In our examples we have used the cost of capital formulation described in section 3.

This can be described by the following formula

$$M_{SST} = (r - i)\sum_{t=0}^{\infty}\frac{C_t}{(1+i)^{t+1}} \qquad (1)$$

Where M_{SST} is the risk margin from the Swiss Solvency Test, and

- i = Risk-free rate of return on investments (e.g., 4% in our examples)
- r = Total rate of return demanded by investors for taking insurance risk. (This is the risk free rate plus an additional cost of capital provision, 4% plus 6%=10% in the examples)
- C_t = Amount of capital required to (or allocated to) support an insurance portfolio at time t.
- t=0 is the reporting date; t=1 is the end of the first year, etc.

Another formulation, which we call the capital cash flow (CCF) risk margin formula of the transaction, would be as described below.

Assume that Insurer #2 takes on the liability of Insurer #1. In return, Insurer #2 receives assets equal to the discounted liability plus a risk margin M_{CCF}.

First look at the cash flow of the insurance transaction.

- At the beginning of the first year, at time t = 0, investors contributes a sum of C_0 to Insurer #2 and earns a risk-free rate of return, i, over the next year.
- At time t = 0, Insurer #2 collects M_{CCF} from Insurer #1 and immediately transfers it to its investors. Equivalently, one could say that the investor contributes $C_0 - M_{CCF}$ to Insurer #2.
- At time t = 1, the investors are obligated to keep C_1 invested in the Insurer #2, and they expect to receive a cash flow $C_0(1+i) - C_1$ at the end of year 1. Since the losses the Insurer #2 is required to pay and C_1 are uncertain, they discount the value of the amount returned at the risky rate of return $r > i$.
- Continuing on to time t, the investors are obligated to keep C_t invested in Insurer #2, and they expect a cash flow of $C_{t-1}(1+i) - C_t$ at the end of year t.

Since the cash flows are uncertain, it is appropriate to discount the cash flow at the risky rate of return, r. This leads to the following expression.

$$C_0 = M_{CCF} + \sum_{t=1}^{\infty}\frac{C_{t-1}(1+i) - C_t}{(1+r)^t} \qquad (2)$$

This equation implies.

$$M_{CCF} = C_0 - \sum_{t=1}^{\infty} \frac{C_{t-1}(1+i) - C_t}{(1+r)^t}$$

$$= \frac{C_0(1+r-1-i)}{1+r} + \frac{C_1(1+r-1-i)}{(1+r)^2} + \frac{C_2(1+r-1-i)}{(1+r)^3} + \ldots \qquad (3)$$

$$= (r-i)\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}$$

Another way to consider this is given in Hart Buchanan & Howe, *Actuarial Practice of General Insurance (IAAust, 7th ed., 2007)*, Chapter 11 (HB&H), in the equivalent context of profit margins.

If we define R_t as the release of capital (referred to as the *transfer* in HB&H) at time t and note that

$$R_t = C_{t-1}(1+i) - C_t \qquad (5)$$

Then (2) can be re-written as

$$C_0 = M_{CCF} + \sum_{t=1}^{\infty} \frac{R_t}{(1+r)^t} \qquad (6)$$

If we now note that C_0 can also be expressed it terms of transfers (releases of capital) as

$$C_0 = \sum_{t=1}^{\infty} \frac{R_t}{(1+i)^t} \qquad (7)$$

We can derive the required margin as

$$M_{CCF} = \sum_{t=1}^{\infty} \frac{R_t}{(1+i)^t} - \sum_{t=1}^{\infty} \frac{R_t}{(1+r)^t} \qquad (8)$$

The first term is the value of the releases of capital, at the risk-free rate and is the amount of capital require at time 0. The second term is the value that shareholders, with their higher required earning rate, place on those same releases of capital. The required margin is the difference between these two and is the implied value of uncertainty.

Note the similarity between Equations 1 and 3. The only difference is the discount rate used for the C_t's.

The theoretical difference between the two formulas was described in Section 2.2.10. The difference between the two approaches for our examples is as follows:

Table 11-1--Risk margins—SST and CCF formulations

Model	SST	CCF
Australia	4.6%	4.0%
UK	2.2%	1.9%
US	4.3%	3.7%