Section E
THE PROJECTION OF PAID CLAIMS

Preamble

As a starting point for the simpler statistical methods, the projection of paid claim amounts is ideal. The idea underlying the method is a simple one, but it is quite fundamental. Thus, we can watch the claims for a given accident or report year developing to the ultimate value, and see the pattern that is established over the intervening years. The pattern can be expressed in terms of the proportion of the final amount which is paid out as the years progress. If subsequent accident or report years can be shown or assumed to follow a similar pattern, then we have a simple and direct means for arriving at the claims estimate.

When projecting claims in this way, there are two main techniques which can be followed. These are respectively the Grossing Up and Link Ratio methods, and on each a number of variations can be used. In fact, the two methods are opposite sides of the same coin, and will normally give very similar results. The skill comes in the choice of variation, and in the assessment as to how far the data conform to the basic assumption of a stable claim payment pattern. The methods are easy to follow in principle, and are illustrated in the text by means of an extended numerical example.

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[E1]
THE GROSSING UP METHOD — INTRODUCTION

This method of treating the claims has been called the "Iceberg" technique (Salzmann 1984). The analogy is that the whole mass of the iceberg (or the ultimate value of the claims) is related by proportion to the visible part (the claims paid to date). Hence the unseen portion of the iceberg, which is the amount still to be paid, can be derived by subtraction. This is a good description, but the term "Grossing Up" is more common in Britain, and so will be used here.

Grossing up is best illustrated by means of a numerical example. Take the following table of paid claims data. (The figures are in £1,000s, but are not supposed to be based on any particular class or grouping of business.)

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

In the table as shown, the data are on a cumulative basis. In accordance with the format developed in §B3, the origin years are represented by the rows, and the development years by the columns. Origin is taken as accident year, and these years are listed down the left hand side from 1 to 6 (the current year). The development years from 0 to 5 are listed along the top of the table — year 0 being the accident year itself in each case.

The symbols *a* and *d* in the table are to denote "accident year" and "development year" respectively. In calendar time, we are standing at the end of year 6, and seeking a means for establishing the reserves at this date.

The first question to be asked is: How complete is the development of the paid claims after year $d = 5$? The data array itself gives no information, hence additional evidence is needed. If this shows that the development is effectively complete, all well and good. But it might be, for example, that more development years are needed for the completion of the run-off. In this case, some further information from earlier years will be needed, or some assumption about the remaining tail of claims must be made. Suppose, in the example, that the following information can be found from 3 earlier accident years:

		$pC_{(d=5)}$	*L-ult*	%
	1st	2969	3166	93.8
Earlier Yrs	2nd	3075	3257	94.4
	3rd	3200	3412	93.8
		9244	9835	94.0

where $pC_{(d=5)}$ represents the cumulative claims paid to the end of development year 5.

L-ult represents the ultimate liability.

The evidence points strongly to the pattern for paid claims at d=5 to stand at 94% of the ultimate liability. Thus, for accident year 1:

Estimated *L-ult* = 3483/.94 = 3705

Given this value for *L-ult*, we can now calculate the claim payment pattern for all development years of a=1:

d	0	1	2	3	4	5	*ult*
pC	1001	1855	2423	2988	3335	3483	3705
%	27.0	50.1	65.4	80.6	90.0	94.0	100

(% line indicates value of pC / *L-ult* e.g. 27.0% = 1001/3705 %)

Now, applying the basic assumption that this claim development pattern will hold in the subsequent years, we can gross up the claims to date from the later accident years a = 2, 3, . . 6. (These amounts are the figures appearing in the main diagonal of the original data array.)

a	6	5	4	3	2	1
pC	1889	3261	3880	3977	3844	3483
g	.270	.501	.654	.806	.900	.940
^*L-ult*	6996	6509	5933	4934	4271	3705

Here, g denotes the grossing factor obtained from the % line in the previous table. For each accident year we have used the formula:

^*L-ult* = pC / g, where ^ is the symbol for an estimate.

The estimated reserve V then follows by subtraction:

^V = ^*L-ult* − pC

The full figures are:

a	6	5	4	3	2	1
$\hat{}L\text{-}ult$	6996	6509	5933	4934	4271	3705
pC	1889	3261	3880	3977	3844	3483
$\hat{}V$	5107	3248	2053	957	427	222

It remains to accumulate the figures for all the accident years to arrive at the overall estimate:

Overall Values:	$\sum L\text{-}ult$	32,348
	$\sum pC$	20,334
	Reserve	12,014

It may also be useful to look at the *proportion* of the overall reserve which is attributable to each of the accident years:

a	6	5	4	3	2	1	Total
$\hat{}V$	5107	3248	2053	957	427	222	12,014
%	42.5	27.0	17.1	8.0	3.6	1.8	100.0

The estimated liability is heavily concentrated in the most recent two or three accident years. This is a common feature in much claims reserving. The implication is that the validity of the stable claim pattern assumption must be particularly scrutinised in relation to these latter years.

The reader may find it useful to review the calculations as a whole. They can be set out in an array of this form (pC^* denotes the amounts on the main diagonal):

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261					
	6	1889						
$pC_{(a=1)}$		1001	1855	2423	2988	3335	3483	3705
%		27.0	50.1	65.4	80.6	90.0	94.0	100
pC^*		1889	3261	3880	3977	3844	3483	
g		.270	.501	.654	.806	.900	.940	
$\hat{}L\text{-}ult$		6996	6509	5933	4934	4271	3705	
pC^*		1889	3261	3880	3977	3844	3483	
$\hat{}V$		5107	3248	2053	957	427	222	

Overall Values:	$\sum L\text{-}ult$	32,348
	$\sum pC^*$	20,334
	Reserve	12,014

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[E2]
GROSSING UP—VARIATIONS 1 & 2

The objection to the example given in §E1 is that the grossing up is all based on the pattern of a single accident year, a=l. If this year is exceptional in any way, then a bias will be introduced into the grossing up factors. Also, there are bound to be variations from year to year, but data from a single year only can give no idea of the extent of the possible fluctuation. For these reasons, it may be better to bring in data from a number of years, so that an average can be taken. Alternatively, a conservative or worst-case estimate can be made based upon the least favourable grossing up factors observed in the set of data.

Variation 1 — Averaging

Suppose that full data from 3 earlier accident years become available. The claim development pattern for each can be found, just as was previously done for year a=l itself. The following table of values of *pC/L-ult* % might emerge:

d	0	1	2	3	4	5
1st yr	27.5	52.0	66.8	81.1	90.3	93.8%
2nd yr	28.1	51.7	66.0	82.3	91.2	94.4%
3rd yr	26.1	49.6	64.9	79.5	90.4	93.8%
Yr a=1	27.0	50.1	65.4	80.6	90.0	94.0%

The years show a strongly consistent pattern, but there is some variation as well. Using the additional information, we can construct a set of factors based on the average of the 4 years. These are as follows:

Avge	27.2	50.8	65.8	80.9	90.5	94.0%

They are close, but not coincident with, the original factors for year a=l. Applied to the paid claim figures (as in the previous example) they produce a slightly lower overall reserve:

a	6	5	4	3	2	1
*pC**	1889	3261	3880	3977	3844	3483
g	.272	.508	.658	.809	.905	.940
^L-ult	6945	6419	5897	4916	4248	3705
*pC**	1889	3261	3880	3977	3844	3483
^V	5056	3158	2017	939	408	222

Overall Values:	$\sum L\text{-}ult$	32,130
	$\sum pC^*$	20,334
	Reserve	11,796

Variation 2 — Worst-Case Estimate

The lower the percentage figures are for the paid claims at any given point in the development, the greater will be the effect when grossing up takes place. Hence the worst-case estimate follows from selecting the *lowest* observed percentage in each column from the table of claim patterns. This gives:

Lowest	26.1	49.6	64.9	79.5	90.0	93.8%

Following which, the calculations for the estimated reserve take place as before:

a	6	5	4	3	2	1
pC^*	1889	3261	3880	3977	3844	3483
g	.261	.496	.649	.795	.900	.938
$\hat{}L\text{-}ult$	7238	6575	5978	5003	4271	3713
pC^*	1889	3261	3880	3977	3844	3483
$\hat{}V$	5349	3314	2098	1026	427	230

Overall Values:	$\sum L\text{-}ult$	32,778
	$\sum pC$	20,334
	Reserve	12,444

Summary

We now have 3 possible estimates for the reserve:

Basis	*Value*
Accident Year 1 Pattern	12,014
Average of 4 Years	11,796
Worst Case from 4 Years	12,444

There is variation between the latter two which lie above and below the original estimate: while the lowest result is about 2% less than our first estimate of 12,014, the highest result is about 4% greater. It is a common situation in claims reserving to find a band of possible values within which the answer is likely to lie, although it must be borne in mind that the actual result could of course be even higher than the "worst case" estimate. Selection of the final value will depend on:

a) further analysis of the reliability of the various estimates, and
b) the degree of conservatism it is appropriate to include in the figure.

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[E3]
GROSSING UP — VARIATIONS 3 & 4

In spite of producing a variety of estimates, the method as used so far is still open to objection. In fact, the work has been based on information from old accident years. There has been no attempt to use the data from later years, as they appear in the paid claims triangle, apart that is from accident year 1 itself. But claims payment patterns may be changing, and the reserver should be abreast of the *current* situation. Hence it is time to drop the earlier accident years, and concentrate on what may be discovered from the paid claims triangle itself.

To recap, the triangle is:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

To make use of the information, one convenient way is to work back through the triangle, starting from the top right hand corner. This will be done in Variations 3 & 4.

Variation 3 — Averaging

For Year 1 the ultimate value of the claims is estimated at 3705, and analysis of its payment pattern gives the following set of figures:

d	0	1	2	3	4	5	*ult*
pC%	27.0	50.1	65.4	80.6	90.0	94.0	100

This can be applied to the latest development value for Year 2, i.e. 3844, attained at d=4. The appropriate grossing factor is 90.0%, giving a final estimated loss for Year 2 of:

$$3844 \,/\, .900 = 4271$$

Using this value, the whole payment pattern can be derived for Year 2 as well:

d	0	1	2	3	4	*ult*
pC	1113	2103	2774	3422	3844	4271
%	26.1	49.2	64.9	80.1	90.0	100

(Note: it is not necessary to write down the *pC*% value for d=5, although if needed it would be taken as 94.0% directly from the Year 1 figure.)

Coming to Year 3, we now have 2 different payment patterns to choose from. The vital value is that for d=3, and the available figures are 80.6% from Year 1, and 80.1 % from Year 2. The obvious step is to take an average, which gives 80.4% as the grossing factor. (80.35% could be used, but one decimal place will be quite sufficient in the example.) Hence the estimated final loss for Year 3 is:

$$3977 \,/\, .804 = 4947$$

This leads immediately to the payment pattern for Year 3, this time taken only to $d = 3$:

d	0	1	2	3	*ult*
pC	1265	2433	3233	3977	4947
%	25.6	49.2	65.4	80.4	100

We now have 3 values for the $pC\%$ at d=2: 65.4, 64.9, 65.4%. The average is 65.2%, which can be applied to the latest claims figure for Year 4, i.e. 3880. The process continues automatically until the whole triangle has been covered, and all claims projected to their ultimate values. It is most convenient to set the procedure out in a single display, as follows:

		d						
		0	1	2	3	4	5	*ult*
	1	1001 27.0	1855 50.1	2423 65.4	2988 80.6	3335 90.0	3483 <u>94.0%</u>	3705
	2	1113 26.1	2103 49.2	2774 64.9	3422 80.1	3844 <u>90.0%</u>		4271
a	3	1265 25.6	2433 49.2	3233 65.4	3977 <u>80.4%</u>			4947
	4	1490 25.0	2873 48.3	3880 <u>65.2%</u>				5951
	5	1725 26.0	3261 <u>49.2%</u>					6628
	6	1889 <u>25.9%</u>						7293
								32,795

Overall Values:	$\sum L\text{-}ult$	32,795
	$\sum pC^*$	20,334
	Reserve	12,461

If this display looks somewhat elaborate, it may help to recall that the backbone of it is just the given triangle of paid claims values. The percentage values, and the final column to the right are the ones which have to be calculated.

As for the method of working through the display, this should be clear from the preceding development. Remember that we begin from the top right hand corner, and work down the leading diagonal. Each underlined % estimate on the diagonal is found as the average of the % values above it in the same column i.e. in the same year of development. Then it is applied as a grossing up factor to the current paid claims figure which is immediately beside it. This gives the estimate of the ultimate loss for the accident year, which is written in the final column of the display (e.g. for a = 6, 7293 = 1889/25.9%). With this figure, the other % values in the row can then be calculated, working in what may be called Arabic fashion from right to left. That done, we move to the next lower position on the leading diagonal, and repeat the process, until the work is finished on the lowest rank of the diagram.

Variation 4 — Worst-Case Estimate

As with Variations 1 & 2 above, a conservative or worst case estimate can be made using the data in the triangle. All that is necessary, when working down the columns of %s, is to choose the lowest value found rather than taking the average. E.g. in the above display in column d=2, use 64.9% instead of 65.2% as the choice. This new value will then appear in the 3880 cell, and be used for grossing up the Year 4 claims. The new procedure will of course affect the whole progress of the calculations, and the display must be redrawn. It appears as follows:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		4271
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			4965
a		25.5	49.0	65.1	80.1%			
	4	1490	2873	3880				5978
		24.9	48.1	64.9%				
	5	1725	3261					6780
		25.4	48.1%					
	6	1889						7586
		25.9%						
								33,285

Overall Values:	$\sum L$-*ult*	33,285
	$\sum pC^*$	20,334
	Reserve	12,951

The value obtained for the reserve is higher than that with simple averaging, by some 3.9%. A full comparison of the grossing up results appears in the next section.

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[E4]
GROSSING UP — COMPARISON OF RESULTS

The claims reserve for the given data has now been calculated by the grossing up technique in 5 different ways. The ways do not really have separate names in the literature, but it may help to list them here as follows:

1st Trial:	Grossing up by top row claims pattern.
Variation	1: Earlier year claims patterns — Averaged.
	2: Earlier year claims patterns — Worst case.
Variation	3: Arabic method, with averaging.
	4: Arabic method, worst case factors.

(The term "Arabic method" is used to denote the technique of working through the claims triangle from right to left.)

The estimates obtained by the different grossing up techniques are summarised here:

		1st Trial	Variation			
			1	2	3	4
	6	5107	5056	5349	5404	5697
Accident	5	3248	3158	3314	3367	3519
Year	4	2053	2017	2098	2071	2098
	3	957	939	1026	970	988
	2	427	404	427	427	427
	1	222	222	230	222	222
Total estimate		12,014	11,796	12,444	12,461	12,951

The variation between the highest and lowest figures is now almost 10%. Which estimate should be chosen, or are there yet more calculations to be done? The "Top Row" method is certainly weak, because it depends on the claims pattern of a single accident year. Also, the "Earlier Year" methods use elderly data which have probably been superseded by now. Ceteris paribus, one would be likely to prefer the estimates from Arabic variations (Nos 3 & 4), since these use the most recent information. They place the estimate in the top half of the range which has so far been exposed, and hence err on the side of caution if at all. They are useful in that they begin to define a credible range for the reserve, with say £12,461 being taken as the best estimate and £12,951 as the conservative value.

But it is possible that more can be learnt about the data, and this will be explored in the following sections through the use of link-ratio methods. ◇

[E5]
THE LINK RATIO METHOD — INTRODUCTION

The Link Ratio method is a close relation of the Grossing Up method just described. In a real sense, it is the reciprocal — the difference is effectively in the direction of working through the data triangle. In the Arabic version of grossing up, we worked from right to left, from projected final losses back to the vector of claims percentages. In the link ratio method, we work from left to right, using succeeding development ratios to build up towards the ultimate loss. The principle is most easily seen by working through the same example as in §E1–E4 above.

The basic triangle of paid claims is:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

The link ratios then operate along the rows, relating each value of paid claims to the value attained one development year later. Thus, working along the top row, the ratios are:

d	0	1	2	3	4
r	1.853	1.306	1.233	1.116	1.044

Hence,
$$1.853 = 1855 / 1001$$
$$1.306 = 2423 / 1855$$
$$\ldots\ldots$$
$$1.044 = 3483 / 3335$$

The general formula is: $r_a(d) = pC_a(d+1) / pC_a(d)$

where $pC_a\,(d)$ represents in respect of accident year a the cumulative claims paid to the end of development year d.

and $pC_a\,(d + 1)$ represents in respect of the same accident year a the cumulative claims paid to the end of development year $d + 1$.

The top row of the data triangle is not quite complete — as was seen in §E1, the data cover only the first 5 years of development following the year a=l. The ultimate loss was there estimated as 3705 for Year 1, using additional evidence from earlier accident years. With this same value, the link ratio needed from d=5 to ultimate is:

$$3705 / 3483 = 1.064$$

Moving on to year a=2 in the triangle, in a similar way we can develop the link ratios as:

d	0	1	2	3
r	1.889	1.319	1.234	1.123

where 1.889 = 2103/ 1113 etc.

Working through the whole triangle yields the following array of ratios:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

(No ratio exists yet for year a=6.)

The ratios show a good degree of regularity — each column has values confined to a relatively narrow spread. So the essential hypothesis that the claims development pattern is similar from year to year is supported in this case. It remains only to project the ratios down the columns, and apply them to the succeeding accident years' claims data.

How shall the projections be done? Following the ideas of §B7, either averaging or trending methods could be used. A little later on, we will apply both of these techniques to the data. For the time being, however, let us take a conservative view. A worst case scenario is easily generated by using the *highest* values to appear in each column, and projecting forward with these.

The values in question are:

d	0	1	2	3	4	5
r	1.928	1.351	1.234	1.123	1.044	1.064

In order to project forward, we need to multiply up these ratios, starting from the right hand side:

d	0	1	2	3	4	5
f	4.010	2.080	1.539	1.247	1.111	1.064

Here:

$$1.111 = 1.064 \times 1.044$$
$$1.247 = 1.064 \times 1.044 \times 1.123$$
$$\ldots\ldots\ldots$$
$$4.010 = 1.064 \times 1.044 \times 1.123 \times 1.234 \times 1.351 \times 1.928$$

In symbols, r is being used to denote the simple one-stage ratios, and f for the product of the r's. The mnemonic is:- "f is the ratio from the current claims to the final value $L\text{-}ult$." The related algebraic formulae are:

$$r_a(d) = pC_a(d+1) \,/\, pC_a(d) \text{ as before}$$

and

$$f_a(d) = r_a(d) \,.\, r_a(d+1) \,\ldots\ldots\, r_a(u-1)$$

from which it may be seen that:

$$f_a(d) = L_a\text{-}ult \,/\, pC_a(d)$$

i.e. $f_a(d)$ is the ratio from the current cumulative claims $pC_a(d)$ to the final ultimate value $L\text{-}ult$ as given above.

The further relationship:

$$f_a(d) = r_a(d) \,.\, f_a(d+1)$$

may also be useful. (A full understanding of the algebra is not necessary in order to follow the methods described here.)

The final stage in the link ratio method is to use the f-ratios for multiplying up the given paid claims to their projected final values. The particular claims figures which are needed are of course those on the leading diagonal of the claims triangle. The calculations are as follows:

d	0	1	2	3	4	5
f	4.010	2.080	1.539	1.247	1.111	1.064
pC^*	1889	3261	3880	3977	3844	3483
$\hat{}L\text{-}ult$	7575	6783	5971	4959	4271	3706

The required reserves then follow by deduction of the paid claims to date:

^L-ult	7575	6783	5971	4959	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^V	5686	3522	2091	982	427	223

Overall Values:	$\sum L\text{-}ult$	33,265
	$\sum pC^*$	20,334
	Reserve	12,931

For a conservative reserve, this compares with the £12,951 obtained by the relevant Arabic variation in §E3.

The whole link ratio process becomes clearer when set out as a full working array. The reader may check through the procedure, referring back if necessary to the details given above.

d

a	0	1	2	3	4	5	*ult*
	1.853	1.306	1.233	1.116	1.044	1.064	
1	1001	1855	2423	2988	3335	3483	3705
	1.889	1.319	1.234	1.123			
2	1113	2103	2774	3422	3844		
	1.923	1.329	1.230				
3	1265	2433	3233	3977			
	1.928	1.351					
4	1490	2873	3880				
	1.890						
5	1725	3261					
6	1889						
r	1.928	1.351	1.234	1.123	1.044	1.064	
f	4.010	2.080	1.539	1.247	1.111	1.064	
*pC**	1889	3261	3880	3977	3844	3483	
^L-ult	7575	6783	5971	4959	4271	3706	
*pC**	1889	3261	3880	3977	3844	3483	
^V	5686	3522	2091	982	427	223	

Overall Values:	$\sum L\text{-}ult$	33,265
	$\sum pC^*$	20,334
	Reserve	12,931

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[E6]
LINK RATIOS WITH SIMPLE AVERAGE

Having developed the example this far, it is a simple matter to work through the link ratios using averaged values. The idea will be to obtain what may be regarded as a best estimate in comparison with the conservative estimate derived above.

The changes which are necessary in the main array are minimal. It is only necessary to average the ratios in each column, instead of taking the highest of the values. Thus:

		d						
		0	1	2	3	4	5	*ult*
		1.853	1.306	1.233	1.116	1.044	1.064	
	1	1001	1855	2423	2988	3335	3483	3705
		1.889	1.319	1.234	1.123			
	2	1113	2103	2774	3422	3844		
		1.923	1.329	1.230				
	3	1265	2433	3233	3977			
a								
		1.928	1.351					
	4	1490	2873	3880				
		1.890	3261					
	5	1725						
	6	1889						
r		1.897	1.326	1.232	1.120	1.044	1.064	

The full calculations then follow with these values for *r* in place:

r	1.897	1.326	1.232	1.120	1.044	1.064
f	3.856	2.032	1.533	1.244	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^L-ult	7284	6626	5948	4947	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^V	5395	3365	2068	970	427	223

Overall Values:	$\sum L\text{-}ult$	33,782
	$\sum PL^*$	20,334
	Reserve	12,448

There seems to be quite good agreement with the Arabic best estimate variation of £12,461. (There was also agreement on the worst-case estimates.) Does this lead to a strengthening of confidence in the estimates? Unfortunately, the answer is no, since the algebra behind the Grossing Up and the Link Ratio methods will show them to be near equivalents of one another. However, they are not quite identical, and certainly appear to give a different emphasis in the working. This point will become a little more apparent later on. It may be remarked that when we use the expression "worst-case estimate" we do not exclude the possibility that the actual result could be higher.

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[E7]
LINK RATIOS WITH WEIGHTED AVERAGE

To recap the position so far, we have the basic triangle of paid claims data:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	†3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261					
	6	1889						

† estimated value

and from this we have calculated the one-stage link ratios:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

We have then made a conservative estimate of losses by using the highest ratio from each column, and a best estimate by taking the average value of each column. Does this exhaust the possibilities, or could other options be open to us? In fact, plenty more can be done, and this will be the subject of the next 3 sections.

To begin with, evidence of changes in the business may suggest that it will be apt to place a heavy emphasis on the most recent experience. In this case, we can for example decide to use the last three years' data only, and take a weighted average with descending weights of 3, 2, 1. The last three years in this case refers to accounting years rather than accident or development years, and clearly involves taking the last 3 elements in each column of the triangle:

		d					
		0	1	2	3	4	5
	1	.	.	1.233	1.116	1.044	1.064
	2	.	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

In each column, the latest element will be given weight 3, the middle weight 2, and the earliest element weight 1. (For the column *d*=3, weights 3 & 2 only will be used.) This yields the following set of ratios:

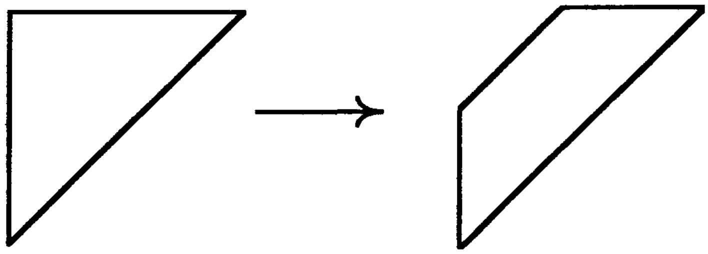
r	1.908	1.338	1.232	1.120	1.044	1.064

The full calculations then follow with these values for *r* in place:

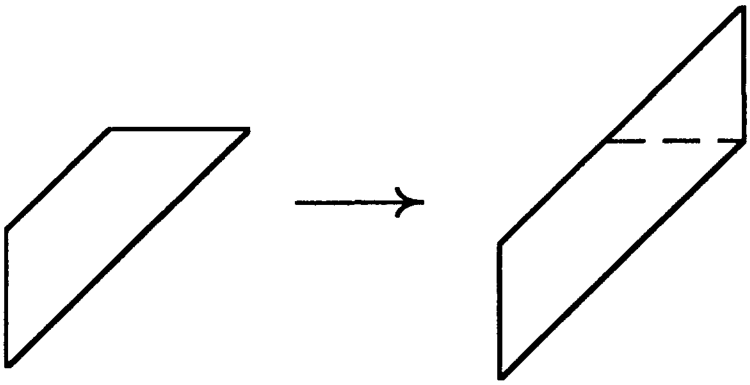
r	1.908	1.338	1.232	1.120	1.044	1.064
f	3.913	2.051	1.533	1.244	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7392	6688	5948	4947	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^*V*	5503	3427	2068	970	427	223

Overall Values:	$\sum$*L-ult*	32,952
	$\sum$*pC**	20,334
	Reserve	12,618

The liability here is about 1.5% higher than that from taking the simple average of the link ratios. The difference is not great because the data are relatively well behaved, and the triangle is of a modest size only. With a larger triangle, the exclusion of the data in the top left hand portion would be more likely to make a significant difference to the results. Indeed, in such cases it is right to question the continuing relevance of the figures in this part of the triangle. A trimming exercise on the data, leaving an array of the form shown below may well be in order. (Diagram overleaf.)



A corollary to this will be to seek additional data where possible in order to extend back the top right hand corner of the triangle. Such data will help improve the robustness of the later ratios in the link sequence:



We are here restating the point made in §B3.4, concerning the format of the data. The final "most desirable shape" for a claims reserving data triangle, in fact, may not be a triangle at all! If using recent data is the key, then the shape may be a parallelogram.

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[E8]
ORIGINAL WEIGHTINGS — THE CHAIN LADDER METHOD

Many other ways of weighting the ratios in the columns can be devised. But one of these stands out as being of particular importance from the mathematical point of view. This is the use of "original" weightings, by which is meant that each ratio in the column is weighted by the claims value from which it arises. Thus, in the first column of the example, the weights will be:

Ratio	Weight	Ratio × Weight
1.853	1001	1855
1.889	1113	2103
1.923	1265	2433
1.928	1490	2873
1.890	1725	3261
	6594	12525

Hence weighted average:

$$12525 / 6594 = 1.899$$

The figures in the Ratio × Weight column are familiar. Of course they must, from the definition of the link ratio itself, be just the paid claims figures from the second column of the data triangle. It follows that the weighted ratio in this case can be obtained simply by summing the second column, and dividing by the summed first column omitting its last element. A table will make this plain:

			d					
		0	1	2	3	4	5	*ult*
	1	† 1001	1855	2423	2988	3335	3483	3705
	2	. 1113	2103	2774	3422	3844		
a	3	. 1265	2433	3233	3977			
	4	. 1490	2873	3880				
	5	† 1725	3261					
	6	1889						
Sum († ... †)								
		6594	12525	Ratio = 1.899				

A similar procedure gives the link ratios for the later columns of the triangle. Thus the second pair of columns yields:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	† 1855	2423	2988	3335	3483	3705
	2	1113	. 2103	2774	3422	3844		
a	3	1265	. 2433	3233	3977			
	4	1490	† <u>2873</u>	<u>3880</u>				
	5	1725	3261					
	6	1889						
Sum († .. †)			9264	12310		Ratio = 1.329		

The full working for the chain ladder variation is:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	<u>3335</u>	3483	3705
	2	1113	2103	2774	<u>3422</u>	3844		
a	3	1265	2433	<u>3233</u>	3977			
	4	1490	<u>2873</u>	3880				
	5	<u>1725</u>	3261					
	6	1889						
$\sum -e$		6594	9264	8430	6410	3335		
$\sum$		8483	12525	12310	10387	7179	3483	3705

r		1.899	1.329	1.232	1.120	1.044	1.064
f		3.868	2.037	1.533	1.244	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483	
^L-ult	7307	6643	5948	4947	4271	3706	
*pC**	1889	3261	3880	3977	3844	3483	
^V	5418	3382	2068	970	427	223	

In the above scheme:

$\sum -e$ represents the sum of the items in the column excluding the last item
$\sum$ represents the sum of all the items in the column

The values of r are calculated from $\sum$-e and $\sum$ as follows:

r for column $d = 0$ is equal to $12525/6594 = 1.899$
for column $d =$ is equal to $12310/9264 = 1.329$

and so on.

Overall Values:	$\sum L\text{-}ult$	32,822
	$\sum pC^*$	20,334
	Reserve	12,488

In this example the result is very close indeed to that obtained by the straight averaging method, i.e. £12,448.

Comments on the Chain Ladder Method

The term "chain ladder" is sometimes used in a general sense, to describe any method which uses a set of link ratios to evaluate the claim development pattern. But it is also used in a very particular sense, to refer to the "original weightings" technique shown above. The variation is an important one — it is commonly encountered in the literature, and often used as the starting point in describing a sequence of methods. However, its logical appeal does not necessarily mean that it gives the best statistical answer.

Indeed, from the point of view of practical reserving, the chain ladder variation is far from being the be-all and end-all. The main criticism is that it can be operated blindly to produce the answers without any further thought. Of course, any method can be handled in this way, but it is particularly true of the chain ladder in this particular form. It tends to produce a single rigid estimate, without any indication of how to look for the possible variations.

To pursue the point, when using the chain ladder algorithm, one calculates the averaged link ratios directly from the column sums, without needing to look at the individual accident year ratios. Information can thereby be lost to the reserver, and important features in the data can easily be missed. It should be a rule, if the chain ladder is used, also to calculate the individual accident year ratios. These should then be inspected carefully for anomalies, evidence of trends and so on. It is always important to examine the data critically and to use informed judgment.

[E9]
LINK RATIOS WITH TRENDING

So far we have projected the link ratios by means of taking averages of various kinds. But suppose that a genuine trend is present in the data — in that case, a projection into the future will not be realistic unless it recognises and makes allowance for the trend. Let us look again at the example set of link ratios with this point in mind:

				d			
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

Down each column, with the exception of $d = 2$, the figures do show some evidence of a trend to increase. Even with the limited amount of information available, it would not be unreasonable to hypothesise that a real trend is present (i.e. as opposed merely to statistical variation about a fixed mean). Knowledge of the business will be of assistance here.

Given some positive evidence then, how is the trend to be projected in the link ratios? The most straightforward way is to fit a least squares trendline to each column in turn. Taking the general formula for the line as:

$$y = bx + c$$

and the points for fitting as (x_i, y_i), $i = 1, 2 \ldots n$, the required solution for b and c is:

$$c = \bar{y} - b\bar{x}$$

$$b = \{\sum x_i y_i - n\bar{x}\bar{y}\} \;/\; \{\sum x_i^2 - n\bar{x}\}^2$$

where $\bar{x}$, $\bar{y}$ are the mean values of the x_i and the y_i

The formulae are easiest to work with if the x-origin is chosen such that $\bar{x} = 0$. Then they reduce to:

$$c = \bar{y} \qquad b = \sum x_i y_i \,/\, \sum x_i^2$$

Calculations using these formulae now follow. Beginning with the first column of link ratios, we choose $x = 0$ for accident year 3 so that $\bar{x} = 0$.

a	x_i	y_i	x_i^2	$x_i y_i$
1	-2	1.853	4	-3.706
2	-1	1.889	1	-1.889
3	0	1.923	0	0
4	1	1.928	1	1.928
5	2	1.890	4	3.780
$\sum$	0	9.483	10	.113

Hence: $c = \bar{y} = 9.483 / 5 = 1.8966$

$b = \sum x_i y_i / \sum x_i^2 = .113 / 10 = .0113$

The projected ratio required for column 1 has $a = 6$ and $x = 3$, and is thus:

$$1.8966 + 3 \times .0113 = 1.931$$

For the second column of link ratios, we require $x = 0$ midway between accident years 2 and 3. So the calculations are:

a	x_i	y_i	x_i^2	$x_i y_i$
1	-1.5	1.306	2.25	-1.959
2	-.5	1.319	.25	-.660
3	.5	1.329	.25	.665
4	1.5	1.351	2.25	2.027
$\sum$	0	5.305	5	.073

Hence: $c = \bar{y} = 5.305 / 4 = 1.3263$

$b = \sum x_i y_i / \sum x_i^2 = .073 / 5 = .0146$

This time, two projected ratios are needed. They are for years $a = 5$ and $a = 6$, so that the respective x's are 2.5 and 3.5. The values are:

$$1.3263 + 2.5 \times .0146 = 1.363$$
$$1.3263 + 3.5 \times .0146 = 1.377$$

For the third column of link ratios, we need to put x = 0 for accident year 2.

So the calculations are:

a	x_i	y_i	x_i^2	$x_i y_i$
1	-1	1.233	1	-1.233
2	0	1.234	0	0
3	1	1.230	1	1.230
$\sum$	0	3.697	2	-.003

Hence: $c = 3.697/3 = 1.2323$
$b = -.003/2 = -.0015$

As before, the projected values follow. Three of them are needed, with $x = 2, 3, 4$:

$$1.2323 + 2 \times -.0015 = 1.229$$
$$1.2323 + 3 \times -.0015 = 1.228$$
$$1.2323 + 4 \times -.0015 = 1.226$$

Coming now to the fourth column of ratios, this can be projected immediately by inspection, using an increment of .007. The resulting values are:

1.130
1.137
1.144
1.151

We can now construct the full table of ratios:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123	***1.044***	***1.064***
a	3	1.923	1.329	1.230	***1.130***	***1.044***	***1.064***
	4	1.928	1.351	***1.229***	***1.137***	***1.044***	***1.064***
	5	1.890	***1.363***	***1.228***	***1.144***	***1.044***	***1.064***
	6	***1.931***	***1.377***	***1.226***	***1.151***	***1.044***	***1.064***

The values in the upper triangle are the link ratios as found directly from the paid claims data. The ratios in the lower triangle are the projected values just obtained by the least squares method.

Before proceeding, one comment should be made. That is, it is probably unreasonable to infer a trend given only 2 data points, as has been done in the column $d = 3$. It may be better to use an average here, or just rely on the latest ratio value of 1.123. We will take the latter course, so that the lower triangle now reduces to:

	d: 0	1	2	3	4	5
a = 2					1.044	1.064
3				1.123	1.044	1.064
4			1.229	1.123	1.044	1.064
5		1.363	1.228	1.123	1.044	1.064
6	1.931	1.377	1.226	1.123	1.044	1.064

From this, the cumulative ratios are found simply by multiplying along each row in turn. This gives the set of values:

f	4.067	2.088	1.533	1.247	1.111	1.064

where:

$$4.067 = 1.931 \times 1.377 \times 1.226 \times 1.123 \times 1.044 \times 1.064$$
$$2.088 = 1.363 \times 1.228 \times 1.123 \times 1.044 \times 1.064$$

and so on.

The final step is to apply these cumulative ratios to the figures for the paid claims to date. This yields the estimates of the final losses and hence the required reserves:

d	0	1	2	3	4	5
f	4.067	2.088	1.533	1.247	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7683	6809	5948	4959	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^*V*	5794	3548	2068	982	427	223

Overall Values:	∑*L-ult*	33,376
	∑*pC**	20,334
	Reserve	13,042

The value for the reserve is here almost 5% greater than that obtained with simple averaging, i.e. £12,448. This is quite a large difference compared with the differences we have seen so far, and if there is good evidence for the trending hypothesis, it will call into question the validity of the lower value.

It may be useful to set out the earlier part of the calculation again in summary form. This is done below, beginning from the basic triangle of paid claims data:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261					
	6	1889						

We calculate the link ratios and extend them downwards in each column by fitting the least squares trendline (except for column $d = 3$):

		d 0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123	***1.044***	***1.064***
a	3	1.923	1.329	1.230	***1.123***	***1.044***	***1.064***
	4	1.928	1.351	***1.229***	***1.123***	***1.044***	***1.064***
	5	1.890	***1.363***	***1.228***	***1.123***	***1.044***	***1.064***
	6	***1.931***	***1.377***	***1.226***	***1.123***	***1.044***	***1.064***

This then enables us to produce the key *f*-ratios by multiplying along each line:

f	4.067	2.088	1.533	1.247	1.111	1.064

The claims estimates are then obtained as on the previous page.

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[E10]
LINK RATIOS — COMPARISON OF RESULTS

The claims reserve for the given data has now been calculated by the link ratio technique in 5 different ways. To summarise what has been done, the following list may help:

Case
1) Worst case — selection of highest values
2) Link ratios with simple average
3) Weighted average of last 3 years only
4) Original weightings: the Chain Ladder method
5) Link ratios with trending

The values obtained by the different techniques are as follows:

		Worst Case	Simple Avge	W'td Avge	Chain Ladder	Trend
	6	5686	5395	5503	5418	5794
Accident	5	3522	3365	3427	3382	3548
Year	4	2091	2068	2068	2068	2068
	3	982	970	970	970	982
	2	427	427	427	427	427
	1	223	223	223	223	223
		12,931	12,448	12,618	12,488	13,042

The variation between the highest and lowest figures is just under 5%. In all this would be a comfortable range to be presented with in practice. The interesting features, *in this particular case,* are:

a) The chain ladder value is extremely close to that obtained from simple averaging of the ratios.

b) Using a weighted average based on the most recent data in the loss triangle produces only a small increase to the estimate.

c) The conservative estimate originally obtained, and that from trending the data, are very close in value.

As always, the final conclusion will depend on what further evidence is to hand — e.g. whether the apparent trend in the data is supported from other sources. But the range is well defined, and unless there is external evidence to support the trend it will be reasonable to set the best estimate at £12,488 (higher of chain ladder and simple averaging). Otherwise it would be prudent to set the estimate at a conservative value at £13,042.

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[E11]
LINK RATIOS v GROSSING UP

Essentially, link ratio and grossing up methods are just opposite sides of the one coin. That this is so is partly confirmed by the close agreement of the estimates obtained. Under both techniques, we have arrived at a best estimate of approximately £12,500 and a conservative one of £13,000. The point can be even more forcibly demonstrated by taking the cumulative ratios from, say, the simple averaging method, and inverting them. The ratios and their reciprocals are:

f	3.856	2.032	1.533	1.244	1.111	1.064
$1/f$	.259	.492	.652	.804	.900	.940

Now look at the paid loss percentages which arose in the Arabic variation with averaging. They will be recalled as:

$pC\%$	25.9	49.2	65.2	80.4	90.0	94.0%

The two sequences are identical. Mathematically, this is no surprise, from the very properties which the cumulative ratios f and the grossing up factors g have been given in the first place. These are such that the ultimate loss for any given origin year can be estimated as:

$$\hat{}L\text{-}ult = f \times pC$$

or

$$\hat{}L\text{-}ult = pC/g$$

Hence: f is an equivalent of $1/g$.

However, this equivalence for the single origin year does not mean that the two methods will give identical results in practice. The fact is that averaging the factors over a number of years will introduce at least small discrepancies. The average in one case is being taken of quantities which are the reciprocals of the quantities occurring in the other case. In general:

Reciprocal of {Average of A, B, C . . . }
$\neq$ Average of {Reciprocals of A, B, C . . . }

But where A, B, C ... are close to each other in value, the difference is small.

Whether to use the Grossing Up or the Link Ratio techniques, then, is a rather academic question. What is more important is that they do have a different feel when in use. Grossing Up has the advantage in that it requires slightly fewer calculations. Also, it deals in quantities (the g-factors) which speak directly of the

% of the losses incurred at each stage. The reserver who gains an insight into the way these percentages behave will certainly be gaining an understanding of the behaviour of the given class of business.

The Link Ratio methods, on the other hand, do bring out slightly more of the features of the data. This is because they examine the one-stage ratios r, as well as the final ratios f. The existence of the possible trend in the data, brought out by the last link ratio variation above, was not quite so apparent when using the grossing up technique alone. But there is an exception to this rule. Unfortunately, the chain ladder method, probably the most popular from the link ratio group, does more to conceal any clues or peculiarities in the data than any other variation here considered.

On balance, the reserver should choose between Link Ratio and Grossing Up techniques according to the one which gives them the best feel for it and what lies behind it.

[E12]
PAID CLAIM PROJECTIONS & THE CLAIM SETTLEMENT PATTERN

The paid claim projection methods have the advantage of simplicity — they are dependent only on the assumption of the stability of the payment pattern for succeeding accident (or report) years. For the given example, the pattern is expressed through the set of percentages of the ultimate loss paid at any given development time:

d	0	1	2	3	4	5
pC%	25.9	49.2	65.2	80.4	90.0	94.0%

The reserver, however, should always examine the basic assumptions for their validity. For the paid claim projection, the question is always: Is the pattern elicited from the figures a stable one? What influences could be operating to affect it, and how seriously can they bias the results?

In fact, there are at least 3 major ways in which the paid claim pattern can be disturbed:

a) A speeding up or slowing down of the whole rate at which the settlement of claims occurs.

b) A change in the relative severities of the losses paid out on the early-settled and the late-settled claims.

c) Fluctuations in the rate of inflation as it affects the average payments made on claims generally.

Inflation, particularly as it manifested in the 1970s, can be an unsettling factor indeed. Much more will be said about ways of tackling it in §J of the Manual. But for the time being, it may be useful to illustrate the kind of effects which can be produced in the data by influences a) and b).

Change in the Claims Settlement Rate

Consider the problem of the speeding up of the settlement rate. Taking the data from the main example, let us suppose that all claims are normally paid out by the end of $d = 8$, but not before. The whole development period, including the original accident year, is thus 9 years long. Now say that the settlement rate speeds up proportionately so that the whole period contracts from 9 years to 8. How will the recorded claims pattern be changed?

Some crude calculations follow, which should be fairly self-explanatory. The idea is that l/8th of the paid claims in the second year move into the first year, 2/8ths of the payments in the 3rd year move forward into the second year, and so on. The first step, however, is to move from the cumulative values for paid claims to the increments which occur in each succeeding year. These increments are given by the row labelled ΔpC, where Δ is the usual symbol for "change in" or "increase of".

d	0	1	2	3	4	5	6	7	8
$pC\%$	25.9	49.2	65.2	80.4	90.0	94.0	96.0	98.0	100
$\Delta pC\%$	25.9	23.3	16.0	15.2	9.6	4.0	2.0	2.0	2.0
–	.	(2.9)	(4.0)	(5.7)	(4.8)	(2.5)	(1.5)	(1.8)	(2.0)
+	2.9	4.0	5.7	4.8	2.5	1.5	1.8	2.0	
$\Delta pC\%$	28.8	24.4	17.7	14.3	7.3	3.0	2.3	2.2	
$pC\%$	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100	

In the table, the central line shows the losses which are being shifted forward. This enables the recalculation of the $\Delta pC\%$'s, which are then put back into cumulative form in the last line.

The direct comparison of the original and new claim patterns is:

d	0	1	2	3	4	5	6	*ult*
$pC\%$	25.9	49.2	65.2	80.4	90.0	94.0	96.0	100
$pC\%$	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100

This can also be put in terms of the link ratios which would be given by such claim patterns:

d	0	1	2	3	4	5	6
r	1.900	1.325	1.233	1.119	1.044	1.021	1.042
r	1.847	1.333	1.202	1.086	1.032	1.024	1.022

Note that the ratios given here are the *one-step* ratios, r, rather than the final ratios. They have changed appreciably — but not out of all recognition. In general, the speeding up of the settlement rate has *reduced* the values of the ratios, but with the exception of the value for $d = 1$ and, marginally, at $d = 5$. It is to be expected that a slowing down of the settlement rate would have the opposite effect, i.e. a general increase in the one-step link ratios. (But anomalies could still occur in particular cases.)

Change in Early/Late Claims Relativity

Again, the effect will be illustrated by a simple example. Take the case where early payments increase in severity relative to the later ones. Say the first 20% of the claims increase in value by 20%, and the last 20% decrease by the same margin. A set of calculations involving change in the incremental claims ΔpC can again be made. These are as follows:

d	0	1	2	3	4	5	6	7	8
$pC\%$	25.9	49.2	65.2	80.4	90.0	94.0	96.0	98.0	100
$\Delta pC\%$	25.9	23.3	16.0	15.2	9.6	4.0	2.0	2.0	2.0
–				(.1)	(1.9)	(.8)	(.4)	(.4)	(.4)
+	4.0	.	.						
$\Delta pC\%$	29.9	23.3	16.0	15.1	7.7	3.2	1.6	1.6	1.6
$pC\%$	29.9	53.2	69.2	84.3	92.0	95.2	96.8	98.4	100

The direct comparison of the original and new claim patterns is:

d	0	1	2	3	4	5	6	*ult*
$pC\%$	25.9	49.2	65.2	80.4	90.0	94.0	96.0	100
$pC\%$	29.9	53.2	69.2	84.3	92.0	95.2	96.8	100

Again, we can make the comparison of the link ratios which would be given by such claim patterns:

d	0	1	2	3	4	5	6
r	1.900	1.325	1.233	1.119	1.044	1.021	1.042
r	1.779	1.301	1.218	1.091	1.035	1.017	1.033

Once more, the link ratios have been *reduced* systematically, but retain an affinity with the original pattern. The most surprising feature, perhaps, is the similarity of this pattern with the one obtained for the speeding up of settlement rates. It is worth bringing the figures together:

d	0	1	2	3	4	5	6	*ult*
$pC\%$	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100
$pC\%$	29.9	53.2	69.2	84.3	92.0	95.2	96.8	100

d	0	1	2	3	4	5	6
r	1.874	1.333	1.202	1.086	1.032	1.024	1.022
r	1.779	1.301	1.218	1.091	1.035	1.017	1.033

The patterns are so similar that they could well be thought to have arisen merely from random variations in the data, rather than from quite different, systematically operating causes. The only means we have for knowing the difference is through the means of their construction.

The implication is that a study of claim development patterns and their change is not sufficient *on its own* to diagnose the causes of the changing. What, then, can be done if changes in the claim development pattern are suspected? A satisfactory reply is hard to give, but two possible tactics would be:

a) Weaker Option

Rely on the ability to observe correctly and follow any trends which may appear in the data. E.g. in the main example, there is a trend of increasing link ratios. This is the opposite of the effects demonstrated in the present section. Hence the observed trend could indicate either:

i) Slowing down of the claim settlement rate, or
ii) Later claims increasing in severity relative to earlier ones.

Since the effect of these factors on the link ratios can be very similar, it may not be fully necessary to distinguish the true cause of the change — at least, provided the trend is picked up sufficiently early on and properly monitored.

b) Stronger Option

Despite the above reasoning, a better alternative will be to seek further information so as to diagnose the cause of the variation. This could take the form of:

i) Evidence on possible trends from underwriters & claim managers, and/or
ii) Statistical information on claim numbers, claim severities, and frequency in relation to original exposure.

The aim would be particularly to monitor the settlement pattern through claim numbers and frequencies, and the early/late relativity plus inflation through claim severities. In this way, influences known to be operating, but not yet fully apparent in the paid claims figures, may be discovered. Then rational hypotheses about the quantity and direction of future change can be made, leading to the adjustment of the grossing up factors and link ratios. By such means, the paid claims projection method may yield an improved forecast of the ultimate losses, and a more reliable indication of the necessary reserves.

[E13]
FITTING TAILS BEYOND THE OBSERVED DATA

Until now, it has been assumed that the reserver has enough historical data to determine the duration and development pattern of claims from the origin year (which may typically be the accident year, depending on the choice of cohort used). In practice, this may not always be the case (for example, the company may be new to the class of business). In such cases, it will be necessary to extrapolate the projected claim cost beyond the last development period covered by the base data.

Presented below are three approaches commonly found in practice for doing this. The methods shown have been applied to paid claims data. They may also be applied to incurred claims data, though Sherman's method may lead to problems if the individual link ratios straddle 1.

Graduation

The reserver uses the pattern of published industry figures to apply a trend to their own figures. Sources of such information may include

- Supervisory Returns
- Claims Run-off Patterns, published by General Insurance Study Group (taken from the Supervisory Returns)
- LIRMA and ABI annual summary of business performance
- RAA annual summary of performance (covering US reinsurance company data)

Consider as an example the data used in section E, with the link ratios from the chain-ladder method. Assume we have no further historic information on which to extrapolate future run-off (in previous sections it was assumed some historic information was available to produce a run-off factor of 1.064).

The chain-ladder ratios are:

Dev Year	0	1	2	3	4
r	1.899	1.329	1.232	1.120	1.044

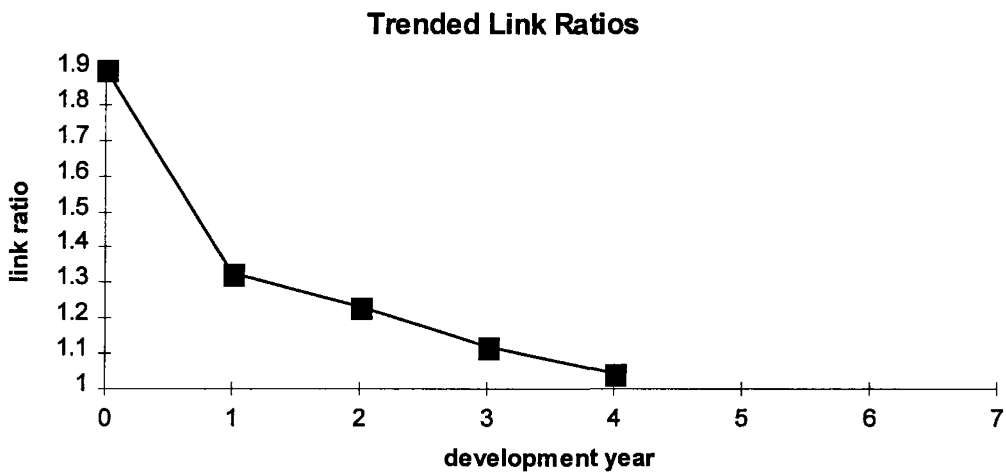
Suppose, for this particular class of business, the Claims Run-off Patterns publication shows a development factor from Development Year 5 to Ultimate of 10.1% for a similar class, then we could adopt a similar value for our own projection if we felt that it was likely to develop in the same way.

One problem with such data is that the classification used may not match that used by the insurer (both with respect to class and policy coverage), and the data quality may not be comparable.

Graphical

The actual development ratios for the above data are shown graphically below. It is tempting to extrapolate the graph by fitting a curve, either mathematically (see Sherman's method below for a particular application) or by eye, as done below.

It is important, however, to check that the extrapolation is consistent with the data available. The extrapolation may seem to fit well but, if the actual outstanding claims contain (say) structured settlements, the extrapolation period would need to extend to at least the full term of those payments. Alternatively, if claims are near to some aggregate limit, there may be little scope left for further development of the size suggested by the graph.



Sherman[1]

The paper by Sherman recommends fitting a curve of the following form to the development ratios:

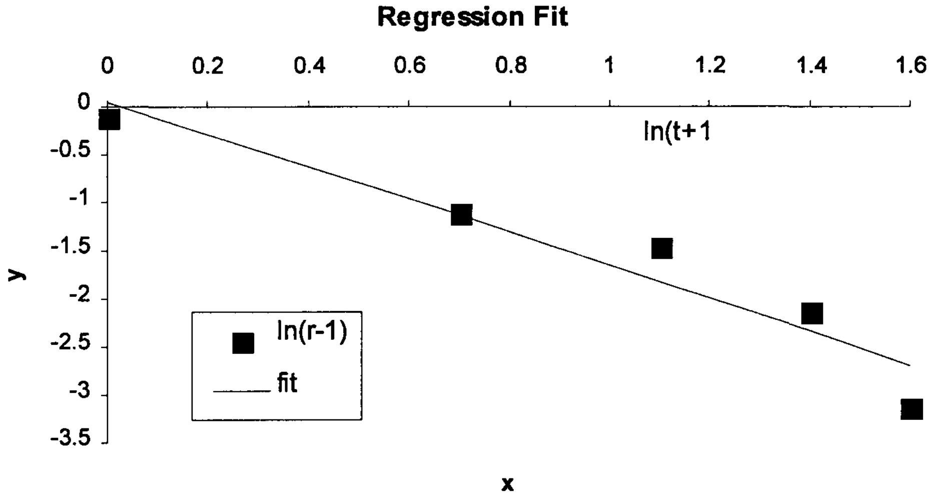
$$\hat{r}_i = (1 + at^b)$$

where
a, b are parameters fitted using regression and
t = development year

[1]see paper by R E Sherman

For example, with the data above (using linear regression fitted to the logs of $(r-1)$ against the logs of $(t+1)$):

$$a = 1.05$$
$$b = -1.7$$



The indicated tail factor here is 17.4% (using the product of factors fitted to development years 5 to 9).

Whilst this approach is conveniently simple to fit, it depends on

- all ratios being greater than one
- an arbitrary limit on the projection period, as it is unlikely that the product of the run-off factors will converge sensibly.

◇