Section I
METHODS FOR IBNR

Preamble

IBNR means "Incurred but not reported". The term refers to claims not yet known to the insurer, but for which a liability is believed to exist at the reserving date. That is simple enough in itself, but the four letters contain a wealth of meaning, and of ambiguity. The wealth arises from the fact that IBNR acts as the remainder term in General Insurance reserving. Many things which cannot be dealt with explicitly elsewhere can be left to fall into the IBNR bag — and as a result, one sometimes cannot be sure what it contains, or even is supposed to contain.

Apart from this, there is the slight mystery of how to go about reserving for claims which have not yet come in, and are still in some sense a figment of the future. The only certainty is that such claims *will* come in, and that there is a duty to make provision for them. Of course, there are ways of dealing with the problem, and fortunately many of these can be described through a single basic principle. It is largely a question of finding a surrogate measure which will stand in place of the IBNR, of substituting the known for the unknown, and justifying one's case for so doing.

An important concept is that of the IBNR run-off. Although IBNR claims are at first unknown, at some time in the future they must become manifest. At this stage, values can be recorded for them — payments, numbers, case reserves, and incurred amounts, just as for any other group of claims. Hence data can be built up on their development patterns, and the data used in projections. A few of the methods are described here, but in the end the reserver need only be limited by his or her own ingenuity. There are some pitfalls to be avoided, however, particularly when it comes to combining IBNR estimates with those for reported claims. It is very possible to double count certain of the elements, or to leave out others completely.

Contents

[I1]
IBNR — DEFINITION & AMBIGUITIES

IBNR is an acronym standing for "Incurred but not reported". IBNR claims are thus that group which are incurred before the reserving date, but not reported until after it. They are the claims for which, in symbols:

$$a] < v] < r]$$

where $a]$ = accident date, $v]$ = reserving date, $r]$ = reporting date.

The picture is simple enough, but as always in General Insurance some complications enter. To begin with, the process of claims reporting is not instantaneous. Claims will normally come in through branch offices, brokers or agents for the insurer or reinsurer concerned. There will be an interval between such actual first report and the later time when the claim is notified to head office and/or formally recorded in the insurer's main data base. For reserving purposes the latter event will be the more convenient one to take. Hence a "reported" claim is normally one which has already been processed to the extent that a central record on it is held. The corollary is that, at any time, there are likely to be claims in the pipeline, already reported at branch level, but still counting for reserving purposes as IBNR.

A second complication is that the cut-off date for IBNR purposes may well be distinct from the reserving date itself. This arises because the extraction of information from the claims data base takes time, and because month and year ends do not fall regularly with respect to the working week.

Hence in examining IBNR data, the reserver must take precautions. Possible variations in the time taken between first report of a claim and its formal recording should be watched for carefully, as should the gap between the IBNR cut-off and reserving dates. Either of these factors can act as a source of bias in the IBNR projections.

Ambiguities

IBNR is a term much used in General Insurance reserving, but which has a good deal of ambiguity about it. This ambiguity has its uses, but can lead to imprecision in the discussion. Hence it is important to be aware of the variations, and to know what is being referred to at any given time. In the first place, IBNR can refer either to the claims themselves in this category, or to the reserve which needs to be established for them. This ambiguity is seldom troublesome, and it is usually clear from the context which sense is intended.

Of far greater import is the following distinction. It can best be seen by approaching IBNR from two different points of view, that of: a) the direct insurer, and b) the reinsurer. The direct insurer should have fairly full information on the pattern of claims, including numbers of claims in each category, and individual case reserves on claims still open at the reserving date. Particular claims can be identified at will, and it is natural to consider the liability as divided into that for the group of known claims on the one hand, and for IBNR claims on the other. The known claims can be estimated by taking the total of case reserves, plus some adjustment for future development, while the IBNR can be estimated by a statistical method (e.g. as described later in this section of the Manual). The point is that the two groups of claims are logically separate, and can be estimated as such.

Coming now to the reinsurer, the amount of information on the claims pattern is likely to be much less. The ceding company or agent will not generally provide information on numbers of claims or individual case reserves — the simple facts of the paid and incurred claims to date will have to suffice. Hence the reinsurer will consider the liability as divided into that which has been reported to it to date (i.e. the incurred claims less his or her payments on account), and that which remains to be reported in the future. The latter will naturally be termed by the reinsurer the "IBNR". It is thus the liability rather than the claim groups which generates the logical distinction. The difference from the direct insurer's view is that the two parts of the liability no longer correspond to fully identifiable groups of claims. (The position is summarised with a diagram, on the next page.)

To this point, we have stated the logical distinction. In practical terms, the main difference is that for the reinsurer the IBNR liability will include the subsequent development in case reserves for the known claims, i.e. in addition to the liability for genuine IBNR claims. There are other elements which can complicate the position further (e.g. reserves for settled claims later re-opened), but this is the central feature.

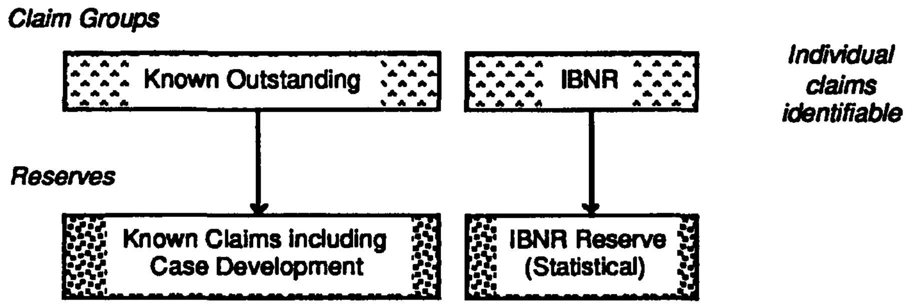
The discussion has been presented in terms of direct vs. reinsurance, but the definitions given above should not be thought of as in exclusive use in the two respective areas. In particular, direct insurers may also think of IBNR as including the subsequent development in case reserves. To help the distinction, the term "true IBNR" is sometimes used for the first version described above, with "IBNER" for the second. IBNER stands for "Incurred but not enough reserved". The algebraic relationship, in its simplest terms, is:

True IBNR + Development on Open Claims = IBNER

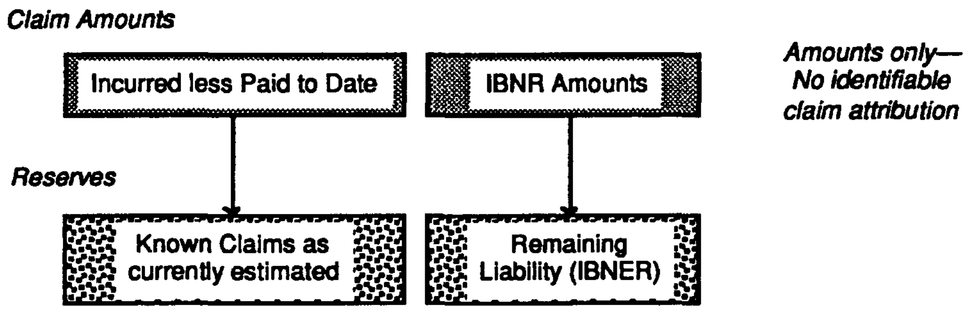
Finally the term IBNYR may also be encountered. This stands for "Incurred but not yet reserved", and has the same meaning as true IBNR.

Diagram — Clarification of the Ambiguity in IBNR

1) Direct Insurer



2) Reinsurer



[I2]
IBNR AS THE REMAINDER TERM

The majority of reserving methods described in the Manual thus far approach the estimation problem by a particular route. The main estimate that is made is of the full ultimate loss — by projecting some convenient quantity such as the paid or incurred claims. This done, subtraction of the paid claims to date then gives the required reserve. The general formula is:

$$CV = \hat{}L\text{-}ult \ - pC^*$$

As a matter of course, the reserve so determined includes provision for both the reported and the IBNR claims. Why then is there any need to find the IBNR reserve separately from the whole? A number of good reasons exist:

a) A breakdown of the overall claims reserve gives more information to decision takers, and helps with the control of business.

b) A separate statement of the IBNR liability is in any case needed for purposes of the returns to the supervisory authority.

c) It may be that a report year classification of data is being used in the main projections. These will then cover reported claims only, and a separate estimate of the IBNR must be made.

d) The chosen route for determining the overall reserve may be to build it up as the sum of parts, e.g. as case reserves plus IBNER element.

An example of d) is the Bornhuetter-Ferguson method on incurred claims, described in §G4 . But whatever the method, the first major analysis of the claims reserve will be into the reported and IBNR elements. In symbols:

$$CV = rV + ibV$$

Here, rV denotes the reserve for reported claims, while ibV means the complementary IBNR reserve.

The equation shows a simple algebraic relationship between three quantities. Any two of the quantities, if known, will determine the third. This leads directly to the first main method by which the IBNR reserve is determined. It is just the remainder, when the reported claims value is taken from the overall reserve, i.e:

$$\hat{}ibV = CV \ - rV$$

As was seen at the beginning of this section, many methods do develop an overall value for the claims reserve. Then, say, case reserves can be used for the estimate of rV, and the IBNR value is left as the residue. In this event, if case reserves are unadjusted for subsequent development, the IBNR value is correctly described as IBNER rather than true IBNR. If the case reserves *are* adjusted, then the value will be closer to true IBNR.

In the reinsurance world, it is very common to estimate IBNR by this route. As a result, what are often described as "IBNR methods" in this sphere are in fact general claims reserving methods. Thus a reinsurance paper on IBNR will often just describe the projection of paid or incurred claims, or of loss ratio, by the chain ladder and other familiar techniques. In the Manual, however, "IBNR methods" will be used to refer to techniques specifically aimed at estimating the IBNR component of the overall reserve.

<>

[I3]
IBNR ESTIMATES — THE UNDERLYING PRINCIPLE

Where IBNR is not being determined as a remainder, there is an underlying principle which applies quite generally. Almost by definition, direct statistical data on the current IBNR cannot be available. Hence it is necessary to find some other measure or base to which the IBNR value can be correlated. If this can be done, then the IBNR estimate will follow as some proportion of the alternative measure. In symbols, we are interested in the quotient:

$$ibV_y / M_y$$

where ibV denotes the IBNR value, M is the chosen measure, and y is a suffix to identify the current year (or other period).

Many different choices are possible for the alternative measure, and these are elaborated below. One simple example is quoted in Skurnick's 1973 paper, where a US Treasury formula at the time required an IBNR reserve for fidelity insurance of at least 10% of the premiums in force. For surety insurance the figure was 5% of the same quantity.

Once such a figure has been set, the IBNR estimation can follow automatically, e.g. by taking the reserve at the minimum required, or by adding a percentage margin for safety. But it is not necessary to assume that the ibV/M quotient is a constant. Indeed, there is bound to be variation in its true value over time. The simplest way of taking this variation into account is to assume that, for the current period, the value will be as for the immediately preceding period. Symbolically, this is:

$$ibV_y / M_y = ibV_{y-1} / M_{y-1}$$

Multiplying by M_y then gives:

$$\textit{Current IBNR Estimate} = \textit{Previous IBNR Value} \times \textit{Ratio}\ (M_y / M_{y-1})$$

The stated relationship is quite general. The ratio M_y / M_{y-1} can be thought of as a kind of growth factor, linking successive IBNR values in the sequence of years. The means of application of the formula will depend, however, on the nature of the line of business, and how the data are structured. The simplest cases arise for the short-tail lines of business, particularly those where all or most of the IBNR claims are run off during the year following the year of exposure. The formula can then be applied directly to the data in hand. But where there is a medium or long tail to the business, tables of IBNR run-off for earlier accident years may need to be constructed, and the formula applied in a more roundabout way. (Examples of possible procedures are given in §§I7,I8.) <>

[I4]
ALTERNATIVE BASES FOR IBNR PROJECTION

In the previous section, it was seen that it is generally necessary to find some alternative measure or base against which to correlate the IBNR value. One of the skills in IBNR estimation is the choice of this alternative base. Many different examples have been used in practice, and many are quoted in the literature. Such measures are usually either related to premiums or to claims, and may be in either money or unit terms. The general classification is thus a 2-way one:

	Premium Related	**Claim Related**
Money Terms	Earned Premium Written Premium Premium in Force	Paid Claims Incurred Claims Case Reserves Late Reported Claims
Unit Terms	Earned Exposure Units Written Exposure Units No. of Policies in Force	No. of Reported Claims No. of Open Claims

In deciding upon an adequate and appropriate measure, there are two main elements to consider. In brief, these are a) risk exposure, and b) inflation of claims. Both will play their part in the IBNR value which ultimately results. For this reason the money related measures tend to be easier to use, since they contain elements relating both to risk exposure and to inflation. The unit related measures only include the risk exposure part, and hence must be supplemented by some allowance for claims inflation. (The alternatives are for the reserver either to gather further evidence on the position, or make an inflation assumption explicitly.)

On the choice between a premium related and a claim related measure, the latter at first would seem to have the advantage, since it is well known that premium rates do not remain at a constant level of adequacy. However, the relationship between IBNR and reported claims is not constant either, and such events as a change in the claim reporting pattern can disturb the picture. A great deal could be said about each of the possible measures, but for now some brief notes on each of the four main categories in the table above may suffice.

Premium Related/Money Terms

The main problem here is that as the underwriting cycle takes its course, premium levels vary in relation to claims in general and to IBNR in particular. In addition, premium rates may react to influences which are not claim related at all, e.g. the general level of office expenses. One solution here is to use only the pure risk premium in the calculations.

On the choice of earned, written or in-force premium, it is largely a question of relating to the main data definition being used in the reserving projections. Thus, for accident year data the earned premium will be most appropriate, while for underwriting year the written premium is best.

Premium Related/Unit Terms

Exposure units are mainly of use in dealing with personal lines, such as motor and household insurance. In these lines, exposure units can be a good measure or base for IBNR claim numbers. This is in effect one half only of the determination, the other being the average amount of IBNR claim. No information is given about this latter element, or its progress with time, so that a separate estimate will be required.

As to the choice of earned, written or in-force units, similar considerations apply as given for premium in the paragraph above.

Claim Related/Money Terms

Paid claims can be useful for the short tail lines, but as the tail gets longer their relevance rapidly diminishes. For the medium and long tail lines, incurred claims will be more appropriate, and will give more information about the likely claims emergence. A problem that may arise here is the consistency over time of the case reserves which form part of the incurred value. Also, the extent to which such reserves include an allowance for inflation must be known, and adjusted for if necessary. Another problem is that variations in the IBNR cut-off date can produce distortions — if the date is later one year than another, say, then incurred claims will be increased by an amount equal to the reduction in IBNR. But the higher incurred value will lead to a *higher* projection for IBNR emergence.

Late reported claims can be a particularly useful measure, mainly for short tail lines. These claims are actual IBNR claims, emerging shortly after the valuation date, in a period in which information can still be brought into the accounting and financial statements. Since the period is likely to be short, it may frequently be the case that only claim numbers rather than the incurred values are available. If so, the measure is in unit terms only, and should strictly be re-classified in the paragraph below.

Claim Related/Unit Terms

Numbers of claims, either of the open or reported variety, can be used as a base for IBNR claim numbers. As with the use of exposure units, a separate estimate of the progress of IBNR claim amounts will have to be made. Of the two measures suggested here, number reported is probably the better choice, since variations in the claim payment pattern will have no disturbing influence.

◇

[I5]
TARBELL'S METHOD & IBNR RUN-OFF

Tarbell's method is included at this point, since it is a good practical illustration of the general principle set down in the previous section. It also happens to be a classic of the reserving literature, having first appeared in the *Proceedings of the Casualty Actuarial Society* as early as 1933. Finally, it introduces the useful concept of the IBNR run-off.

To explain IBNR run-off, consider the position for some given year of exposure, y. At the end of the year, the reported claims can be evaluated to their incurred value, iC_y, by adding paid claims to case reserves. Of the IBNR claims, denoted as ibC_y, nothing will yet be known. But during the following year, y+1, these claims will begin to emerge, moving effectively from IBNR to the reported group. However, their status of being IBNR at the end of year y will remain so that IBNR claims for the year y are those where the event took place in year y but which were not reported until after year y. As the emergence takes place, all the usual quantities of paid claims, incurred claims, case reserves, and so on can be applied to this IBNR group in its own right. Development tables can be constructed, just as has earlier been done for the overall claims run-off of given accident years.

It is useful to be able to represent the IBNR run-off in symbols. We have already used symbols such as *pC, iC, kV* for the main claims run-off. Coming to the IBNR, we will adapt these to *ibpC, ibiC, ibkV* and so on. The full symbol for incurred claims in the run-off will be:

$$ibiC_y(d)$$

where y is the year of exposure, and d is the development time following the end of year y. For example, d=1 will denote the position at the end of year $y+l$.

Tarbell assumes that the main part of the IBNR run-off will occur during the 12 months following the end of any given year. He is therefore dealing essentially with short tail business. However, he allows that the 12 months may not be sufficient, in which case a tail factor must be introduced, which we will denote as $f(1)$. In general:

$$f(1) = ibL\text{-}ult \,/\, ibiC(1)$$

i.e. $f(1)$ is the ratio of the final IBNR loss to the incurred amount after 1 year. Based on experience of earlier IBNR run-offs, it should be possible to estimate this factor. There is the assumption, however, that f will show reasonable stability from year to year.

We come now to the main question. What is the equivalent measure or base which Tarbell uses in his IBNR estimations? Effectively, it is the claims incurred during the last three months of the year of exposure. However, the value is expressed not directly, but as the product of number of claims reported and average size of claim. Tarbell's reason for doing this is to allow more flexibility in the treatment of the various claim distributions experienced in different lines of business. In particular, he is concerned to exclude abnormally large claims which may distort the picture. The resulting formula for the IBNR estimate is as follows:

$$\hat{}ibV_y = ibiC_{y\text{-}1}(1\ ) \times (nR_{y[10,12]} \times A_{y[10,12]}) \,/\, (nR_{y\text{-}1[10,12]} \times A_{y\text{-}1[10,12]})$$

The formula has a lot of detail in it, so it may be useful to re-define each term separately.

$\hat{}ibV_y$	–	Estimated reserve for IBNR claims for the year y as required at the end of year y.
$ibiC_{y\text{-}1}(1)$	–	Claims incurred by the end of year y in the run-off of the IBNR claims for year y-1.
$nR_{y[10,12]}$	–	Number of claims reported in the last three months of year y.
$A_{y[10,12]}$	–	Average size of claim for claims reported in the last three months of year y.
$nR_{y\text{-}1[10,12]}$	–	As above, for year $y-1$.
$A_{y\text{-}1[10,12]}$	–	As above, for the year $y-1$.

The expression of the formula here is essentially that given by Tarbell, but the notation has been adapted to the general form used throughout the Manual. Tarbell also restates the formula by combining the *nR* and *A* terms into their product, which is effectively the incurred claim value for the last three months of the respective year of exposure. The result is as follows (again in the Manual's notation):

$$\hat{}ibV_y = ibiC_{y\text{-}1}(1) \times iC_{y[10,12]} \,/\, iC_{\,y\text{-}1[10,12]}$$

Strictly speaking, both versions of the formula should have the factor $f(1)$ inserted, in order to provide for any further development in the $ibiC_{y\text{-}1}$ term beyond the first twelve months.

An important point, admitted by Tarbell, is that there is nothing sacrosanct about his choice of the last three months of the year over which to take the incurred claims measure. The point is to choose a period which is: a) recent enough to reflect current trends in claims numbers and claim sizes, and b) long enough to give dependable statistical data. Different length periods might therefore be selected for different lines of business, according to the balance between these two main requirements.

In conclusion, Tarbell cites his formula as being applicable to such lines as personal accident, motor and other miscellaneous property damage. These are essentially short tail, relatively high frequency lines. Medium and long tail lines are likely to need the more elaborate methods developed below in §§I7. I8.

[I6]
AVERAGE COST PER IBNR CLAIM

An obvious way to analyse the IBNR position is to look separately at the numbers of such claims and at their average cost. If both can be projected, then their product will give the estimated liability. The main question will be to find suitable data from which to make the projections. The technique suggested here, in fact, is to relate the IBNR claims to the reported claims, both by numbers and average cost.

Number of IBNR Claims

Provided that data of previous years' IBNR run-offs are available, projecting the number of claims should be quite straightforward. The simplest method will be to assume a stable ratio as between the number of claims which are IBNR at the end of a given year (symbol ibn_y), and the number reported during the year (nR_y). Suppose first of all that the run-off for any given year of exposure is completed during the following 12 months. Then the previous year's data can be used to estimate the ratio, i.e. as:

$$ibn_{y-1}/nR_{y-1}$$

leading to the projected IBNR number of:

$$\hat{}ibn_y = nR_y \times (ibn_{y-1}/\ nR_{y-1})$$

It will be better, however, to gather in data from years earlier than $y-1$, so that variations in the *ibn/nR* ratio can be assessed. Thus, an average or a weighted average might be taken over the last 3–5 years say, or a trend might be observed and extrapolated.

What has been described so far leads to fairly rough and ready estimates. A more refined method is to use data on a monthly or quarterly basis, rather than just the annual figures. By this means, a development table for claim numbers reported can be built up, in the familiar triangular or parallelogram form. The axis on the left hand side of the triangle gives the accident month or quarter, while the axis along the top gives the equivalent development period. The table so constructed is no different in principle from the annual-period triangles shown in previous sections of the Manual. It can therefore be evaluated by the usual methods, and sets of grossing up factors or link ratios derived in the usual way. Then, applying the appropriate factor to the current year reported claims, the estimated final number of claims is given, and the IBNR claims can be derived by subtraction.

To this point, we have been assuming that the IBNR run-off for any given year of exposure is completed during the following 12 months. For many lines, this will not be the case. But once the concept of the development table is introduced, there is no problem in obtaining the projection for number of claims by the standard methods. Indeed, number of reported claims is one of the more stable quantities in the field of claims reserving, so that this part of the IBNR estimation can often be done with confidence.

Average Cost per IBNR Claim

The second part of the estimation is likely to be the more difficult, at least in terms of reliability. The approach which most readily suggests itself is to assume a stable relationship between the average cost of IBNR claims and reported claims for each period of exposure. Then observed trends in the average reported claims can be used to project the required average IBNR values. If only the previous year's data are being used, the formula will be:

$$\hat{}ibiA_y = iA_y \times (ibiA_{y-1} / iA_{y-1})$$

Here, $ibiA_y$ is used to denote the average cost per IBNR claim for the year y, while iA_y is the incurred average for the same year.

The same remarks as previously made for numbers of IBNR claims apply, i.e. it will be better to examine the data from several past years. In this way, the stability of the *ibiA/ iA* relationship can be checked upon, and averages or trends calculated as appropriate. But it is likely that irregularities will be found in the ratio, particularly in times of varying inflation. The point is that IBNR claims will on average be settled at a significantly later date than will reported claims. Hence changes in inflation will soon disturb any otherwise steady relationship. Indeed, inflation is a factor which should specifically be taken into account in deriving the average cost for the IBNR claims.

An alternative to the above, where the *ibiA/iA* ratio is not stable, is to look purely at the trends in *ibiA* values themselves for recent years. Then taking account of current inflation rates and other known influences on the business, the trend can be carried forward to estimate the *ibiA* value for the current year.

Where the IBNR run-off takes longer than a year or two, the *ibiA/iA* ratio becomes more difficult to assess. This is because IBNR development for the most recent years will not be complete, and must itself be estimated on the way to making the current year's estimate. Of course, full development tables of the incurred claims or incurred average can be constructed and evaluated. But these will generate their own values for the ultimate loss, and hence for the IBNR by subtraction of the incurred claims to date. If that is done, the present method of evaluation is effectively made redundant, although the *ibiA/iA* ratios can be found as a by-product of the work.

The IBNR Estimate Itself

The final estimate of the IBNR liability follows simply as the product of the estimated number of claims and the average cost per claim. In symbols:

$$^{\wedge}ibV = {}^{\wedge}ibn \times {}^{\wedge}ibiA$$

◇

IBNR BY ACCIDENT YEAR PROJECTION

To this point in §I6, we have been looking at methods suited to the case where the IBNR run-off is mainly completed during the year or two following the year of exposure. But for many lines nowadays, particularly in the liability class, the tail is very much longer. The aim then should be to construct full development tables showing the IBNR emergence by accident or underwriting year, (or possibly by calendar year).

The example given here uses accident year data, and sets the IBNR emergence against a base measure of earned premium. The same procedure could be followed using underwriting year data and written premium. To begin with, the data must be put into the appropriate triangular form, as shown immediately below. These particular data are an extension from the main example used in the text of §§E–H.

				d		
		1	2	3	4	5
	1	604	806	948	1018	1044
	2	718	952	1080	1160	
a	3	776	1004	1196		
	4	868	1114			
	5	962			[*ibiC*]	
	6					

It can be seen that the table follows the usual pattern already established. The accident years appear down the left hand side, while the development periods run across the top. The period d=1 is the year immediately following the accident year itself, and so on. d=0 would coincide with the accident year, but by definition there can be no IBNR emergence at that stage. Year a=6 appears at the foot of the table, but without any entries in its row. This is to show the point in time at which the table is constructed, i.e. the end of the year a=6.

A word is also necessary about the nature of the IBNR run-off data. Generally speaking, such data will be in *incurred* form. That is, they will include both actual payments made on the IBNR claims, plus the reserves needed for the open IBNR claims at the development date. The latter may well be derived as the sum of case reserves held on such claims at the time in question. Finally, the symbol used to denote IBNR run-off data is *ibiC*. Individual elements in the table can then be identified, if necessary, by expanding the symbol to $ibiC_a(d)$.

Once the data are in development table form, the first step is to construct the year-by-year figures. This is done overleaf.

		d				
		1	2	3	4	5
	1	604	202	142	70	26
	2	718	234	128	80	
a	3	776	228	192		
	4	868	246			
	5	962			[$\Delta ibiC$]	
	6					

In the row a=1, 604 is just the value repeated from the first table above. Then 202 is found by subtracting 604 from 806, 142 is the value (948–806), and so on. (As usual, the symbol Δ is used to denote a step-by-step quantity, found by taking differences on the cumulative function.)

The second step in the work is to introduce the earned premium, aP, for each accident year. Then the step-by-step IBNR run-off figures are divided by the premium to which they relate. This is done in the table below, by working along each row in turn, using the earned premium figure from the left hand column. The results are expressed as percentages, appearing as the lower of the two figures in each cell of the table.

aP	a			d		
		1	2	3	4	5
4486	1	604 13.46	202 4.50	142 3.17	70 1.56	26 .58%
5024	2	718 14.29	234 4.66	128 2.55	80 1.59%	
5680	3	776 13.66	228 4.01	192 3.38%		
6590	4	868 13.17	246 3.73%			
7482	5	962 12.86%			[$ibiC$] [$ibic/aP$]	
8502	6					

Looking down the columns of the table, the percentage figures show a good degree of stability from year to year. That is, the IBNR emergence is closely correlated to the earned premium for each accident year. It is therefore reasonable to project the columns downwards, in this case by taking the average of the observed values. The projection is shown in the table below, which also gives the addition of the percentage figures along each row.

aP	*a*			*d*			*row*
		1	2	3	4	5	*sum*
4486	1	.	.	.	.	.	
5024	2	.	.	.	.	.58	.58%
5680	3	.	.	.	1.58	.58	2.16%
6590	4	.	.	3.03	1.58	.58	5.19%
7482	5	.	4.23	3.03	1.58	.58	9.42%
8502	6	13.49	4.23	3.03	1.58	.58	22.91%

The row sums show the amount of IBNR emergence still expected to occur for each accident year. Thus to make the full IBNR projection, it is only necessary now to multiply the added percentages by the respective earned premium figures. The calculations follow:

a	*row sum*	*aP*	*^ibV*	*kV**
1		4486		234
2	.58	5024	29	475
3	2.16	5680	123	969
4	5.19	6590	342	1796
5	9.42	7482	705	2881
6	22.91	8502	1948	3929

The table shows the current values of case reserves for each accident year, for comparison figures the IBNR figures. The overall value for the IBNR estimate is just the sum of the *^ibV* column, i.e. 3,108. This can, apparently, be added to the current case reserves to give the overall reserve required.

Overall Values:	∑*^ibV*	3,147
	∑*kV**	10,284
	Reserve	13,431

[I8]
ADJUSTED PROJECTION USING ACCIDENT & REPORT YEAR

This section presents a modified version of the calculations of §I7 above.

The method depends on having data which are classified both by accident and report period. Thus, for accident year a=1, data are analysed according to the claims reported in year a+1, a+2, and so on. For each group, the incurred claim development is tracked, and a table drawn up to show the current position. As usual, accident years are shown down the LHS, while across the top is plotted the interval between the respective accident and report years. Thus, using r for year of report, (r -a) takes values 1, 2, 3, ... and the familiar triangular form results:

		($r-a$) 1	2	3	4	5
	1	669	187	127	45	16
	2	788	214	108	50	
a	3	846	218	132		
	4	928	186			
	5	962			[iC*:a,r -a]	
	6					

The data in each cell can be characterised as $iC^*_{a,r-a}$, that is the incurred value to date for the given group of claims. E.g. 218 in cell a=3, r -a=2 is the incurred liability to date on those claims originating in year 3 and with report date in year 5, i.e. 2 years later. The actual time of evaluation of all the cells is the end of year 6.

The IBNR emergence can now be evaluated by comparing the values in the table with some suitable base measure. Using earned premium, as was previously done in §I7, the following percentages arise:

aP	a	(r -a) 1	2	3	4	5
4486	1	14.91	4.17	2.83	1.00	0.36%
5024	2	15.68	4.26	2.15	1.00%	
5680	3	14.89	3.84	2.32%		
6590	4	14.08	2.82%			
7482	5	12.86%		[iC*:a,r -a/aP]		
8502	6					

The percentage values in each column are very stable, and can easily be projected downwards. This is done in the table below, taking the average of the most recent three values in each case. The resulting figures are then summed along the rows:

aP	*a*			*(r -a)*			*row*
		1	2	3	4	5	*sum*
4486	1						
5024	2					.36	0.36%
5680	3				1.0	.36	1.36%
6590	4			2.43	1.0	.36	3.79%
7482	5		3.64	2.43	1.0	.36	7.43%
8502	6	13.94	3.64	2.43	1.0	.36	21.37%

It remains to apply the row sums to the baseline figures for earned premium to produce the IBNR estimates:

a	*row sum*	*aP*	*^ibV*	*^VR*
1		4486		244
2	0.36	5024	18	507
3	1.36	5680	77	1023
4	3.79	6590	250	1855
5	7.43	7482	556	3002
6	21.37	8502	1817	4150

The overall IBNR value is now 2,718, by addition over the accident years. Data on reserves for reported claims, *^VR*, are also shown for comparison. The full position is now:

Overall Values:	$\sum$*^ibV*	2,718
	^VR	10,781
	Reserve	13,499

One point remains to be made. The IBNR reserve, as constructed here, does not include any allowance for future development of the case reserves (i.e. the case reserves held at present on claims formerly in the IBNR category). This is because each cell in the development table is, as it were, an independent IBNR unit, and does not directly link with other cells in its row. Consequently, on projecting the figures down the columns, there is no "knock on" effect from previous columns — but which effect is present in the equivalent tables from §I7 above. Compared with §I7, the IBNR value provided here is an estimate of true IBNR, rather than a hybrid containing IBNER elements.

The result is the beneficial one that, when combining the IBNR estimate with case reserves to give the full liability, the case reserves can be treated consistently. In fact, the reserves *will* need to be adjusted for future development in this case. As an alternative, a projection of the main data on a report year basis can be used. This gives a direct estimate of the liability for reported claims, to which the IBNR reserve can be added.

◇