How to Calibrate Risk Distributions
Andrew D Smith (andrewdsmith8@deloitte.co.uk)

Agenda
Modelling Extreme Events

Standard Formula, Internal Models and Scope of Validation

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SCR Calculation: Standard Formula vs Internal Model
How firms and the regulator calculate the Solvency Capital Requirement

Standard Formula (SF)

Largest EU (re)insurers / groups OR
Complexity not captured by SF e.g. VAs

Internal Model

*IMAP

QIS5

IMAP

IMAP

Standard formula understates required capital in regulators' view

SF + Add-ons

*IMAP = internal model application process

Internal Model validation
Regulatory requirements & purpose

Insurers seeking approval to use an internal model to calculate their capital requirements will have to demonstrate, as part of their IMAP, that they have had their internal model “independently validated” (Article 112 and Article 124).

The specific “validation standards” requirements are outlined in Article 124 of the Directive and in the detailed text that supports the Level 2 and emerging Level 3 texts. The requirements include:

- having a regular cycle of model validation;
- monitoring the performance of the internal model;
- reviewing the on-going appropriateness of its specification;
- testing its results against experience;
- analysing the stability of the internal model;
- reviewing the sensitivity of the results to changes in key underlying assumptions;
- demonstrating the model's use – “use test”; and
- assessing the accuracy, completeness and appropriateness of the data used.
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**10-Year Spot Rates by Term, 1970-2011**

Source: Federal Reserve, Bank of England, Bundesbank

![Graph showing 10-Year Spot Rates by Term, 1970-2011](chart.png)

- US
- UK
- Germany
Predictive Distribution for \( \Delta \) Bund Spot Curve
Comparison of Lognormal Model with QIS 5

Historic Equity Returns
Total return MSCI in Euro (reconstructed basket prior to 1999)

Source: Datastream
Fitted Return Distributions
Based on EGB2 distribution family

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UK
Europe
World
Extreme Histories: The Living Memory Test

Largest Falls in MSCI Europe € Index since 1970

Return period = 1/probability

-40.34% to October 2008
-40.47% to March 2003
-41.78% to November 2008
-42.89% to December 2008
-43.39% to February 2009
-43.67% fitted 1-in-200 event

Fitting to Overlapping Intervals

Monthly time series analysis may fail the “Living Memory” test

Feb 2008 – Feb 2009
Aug 2010 – Aug 2011
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**P-P Plot (10 year interest rate)**

Visualising Goodness of Fit

Fitted CDF

\[ y = 0.832 = \text{lognormal cdf}(1.088) \]

\[ \mu = -0.061; \sigma = 0.152 \]

Outcome / Forward = 1.088
Rank 370 of 456 observations
Plot x = 370/457
Kolmogorov-Smirnov Goodness of Fit Test

Look at largest deviation in P-P Plot

Skew & Kurtosis to fit Equity Returns

All distributions have mean = 0; standard deviation = 1
References
Some textbooks to read in your spare time!

- Continuous Univariate Distributions by Johnson, Kotz and Balakrishnan (JKB), published by Wiley in two volumes (Norman Johnson was an actuary).

- EGB2 distributions. Covered in JKB, vol 2 p141
- Generalised hyperbolic distributions. Treated by McNeil et al p78. See also http://cran.r-project.org/web/packages/ghyp/vignettes/Generalized_Hyperbolic_Distribution.pdf
- MULE distributions. You won’t find these in the literature because they are my invention. Unlike the other classes, the MULE (mixed exponential uniform logistic) permits distributions with negative kurtosis, with uniform logistic and exponential distributions as special cases. The inverse CDF is a linear combination of \{1, x, ln(x), ln(1-x)\}.

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**Pro and Contra-Cyclical Tests**

Arguments to avoid pro-cyclicality

- Unconditional estimates = “Through the cycle”
  - Average over states of the world
  - Estimate through historic distributions, as in this presentation
  - Can satisfy “1-in-200” test
  - Capital requirements can increase following large market moves

- Conditional estimate = “Point in Time”
  - Given current state of the world
  - Empirical validation by comparing to historic periods with the same starting point
  - Can satisfy “1-in-200” test
  - Sensitive to time series model formulation
  - Capital requirements may rise suddenly from small market moves, making this approach commercially unattractive
  - Arguments against this approach based on fear of “pro-cyclicality”

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**Fitting Methods: In Search of Stability**

<table>
<thead>
<tr>
<th>Method</th>
<th>Feature Replication</th>
<th>Fit Optimisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>Method of moments</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td></td>
<td>Modal fit</td>
<td>Minimax cdf difference (minimise Kolmogorov-Smirnov)</td>
</tr>
<tr>
<td>Pros</td>
<td>Can prove it has worked</td>
<td>Most powerful for large n</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Parameter standard error known for large n</td>
</tr>
<tr>
<td>Cons</td>
<td>Need Plan B outside feasible set</td>
<td>Solution may not exist</td>
</tr>
<tr>
<td></td>
<td></td>
<td>May not converge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Difficult to demonstrate method has worked</td>
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Rolling Estimates
Effect of 2008 Crisis on Estimated Stress Tests (fitted to data since 1970)

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Integrated Modelling Framework

**Preparation**
- Risk Definition: Identify and define the risk being modelled
- Model Selection: Design model approach
- Data Selection & Preparation: Identify and prepare the data which supports the model design
- Model Fitting Mechanism: Determine approach for model fitting and criteria for evaluation of the fit

**Calibration**
- Calibration Procedure: Design procedure for model fitting
- Testing Procedure: Outline results of goodness of fit testing
- Expert Judgement: Review and assess model output, include expert judgement

**Reporting**
- Onwards Reporting: Outline how model output will be communicated to relevant stakeholders
- Governance: Identify and define the risk being modelled

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- Fit to Past Data: Histogram, P-P plot, moments, KS Test
- Stability / Contra-Cyclical: Rolling estimates, Through-Cycle Methodology
- Consistency: Preparation » Calibration » Reporting Process
- Ownership / Use Test: Self-sufficiency
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- Practical challenges, Conclusions and Questions
Effect of Skewness and Kurtosis on EGB2
Contours of 99.5%-ile expressed as number of standard deviations
To consider a 0.5%-ile (eg equity fall), reflect this diagram left to right.

Example Equity Stress Test Calculation
Fitted Gamma Power Distribution to MSCI Europe Returns

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fitted Parameters</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>1.346608</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.999722</td>
</tr>
<tr>
<td>power</td>
<td>0.185368</td>
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<table>
<thead>
<tr>
<th>Quantile q</th>
<th>Gamma Inverse $G = \text{gammainv}(q, \alpha, \beta, \text{true})$</th>
<th>Return $R = G^{\text{power}-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5%</td>
<td>0.045219</td>
<td>-43.67%</td>
</tr>
<tr>
<td>10.0%</td>
<td>0.455908</td>
<td>-13.55%</td>
</tr>
<tr>
<td>50.0%</td>
<td>2.063342</td>
<td>14.37%</td>
</tr>
<tr>
<td>90.0%</td>
<td>5.761522</td>
<td>38.35%</td>
</tr>
<tr>
<td>99.5%</td>
<td>12.17842</td>
<td>58.94%</td>
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| Practical challenges, Conclusions and Questions |

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### Validation under Basel - Banks

- Banks have different rules: 10 day VaR at 99% Confidence
  - Look back over last year (250 trading days, overlapping periods each looking 10 days back) in which both VaR and profit are updated

- What does this process test?
  - The “back test” includes implicit tests of model and parameter error as well as outcomes
  - Although it won’t test risks that didn’t materialise in the last year

![Graph showing green, amber, and red zones with unbiased counts and number of exceptions in a year.](image-url)
Monte Carlo Calibration Test

- **Model #1**
  - Years 1-25
  - Sim #1
  - Sim #2
  - Sim #200

- **Model #2**
  - 10,000 runs
  - Prepared by validator

- **Model #50**
  - Years 26
  - 8 fitted percentiles (e.g., 0.5%, 1%, 5%, 10%, 90%, 95%, 99%, 99.5%)
  - 10,000 times
  - Model user submits

Test Yr26 outcome against percentiles

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**Probability {next observation ≤ estimated percentile}**

Coverage probability aka back-test probability

Coverage Probabilities for Estimated 99.5%-ile

- **Lognormal Parameter**
  - \( \lambda = -0.5 \)
  - \( \lambda = -0.25 \)
  - \( \lambda = 0 \)
  - \( \lambda = 0.25 \)
  - \( \lambda = 0.5 \)

Threshold Rank
(EVT estimation methodology)
Using Simulations for Model and Parameter Risk
Adjusting estimated 99.5%-ile for 99.5% coverage

The theoretical coverage bias correction depends on the underlying distribution which is (sadly) unknown. We can quantify model risk using a robust estimate based on a bias correction for the “worst case” distribution. This then overstates required capital (coverage probability > 99.5%) for other distributions.

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### Internal Model validation

#### Practical challenges

#### Timing

- The Solvency II Directive requires an insurer's internal model to be independently validated at regular intervals once Solvency II is fully implemented. However, validation is also important before Solvency II implementation:
  - Integrating validation modules into the process of developing, building and testing the model provides greater confidence in the model and reduces the risk that late stage validation identifies major re-working of the model.
  - A complete independent validation must be provided to the Board as part of the evidence to support their approval of the model before it is submitted for review by the CBI.

#### Board involvement

- The validation policy and report will be used by the Board when reporting to the regulator. The validation report will need to be accessible to all members of the Board, taking into account their varying experience and familiarity with Solvency II.
- The validation report should address the scope of the validation, the strengths and weaknesses of the model and the data and tools used in the validation process.

#### Documentation

- Detailed and complete validation documentation will help facilitate internal model approval.
- Validation documentation should address model theory, model implementation and model governance.

#### Risk assessment

- "Expert judgement" and "data" are likely to be high risk areas given the subjectivity and regulatory scrutiny respectively around these inputs.

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### Internal Model validation

#### Practical challenges

#### Independence

- An effective internal model validation process requires independent and objective challenge. The use test should provide evidence that the model has been challenged.

- Independence is a strict requirement but can be achieved in a number of ways, or through a combination of:
  - Existing resources
  - An internal audit team with specialist skills
  - External resources / auditors

- Care should be taken to ensure that model validators are independent from those who have been involved in designing and building the model. Reporting lines should also be independent.
Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged. The views expressed in this presentation are those of the presenter.