How to Net Down an Aggregate Gross Distribution

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Agenda

• What is the Challenge?

• Conditional Ceded Approach
  ➢ Round 1 - Simple Solution
  ➢ Round 2 - Concept of Ceded Curve
    • Graphical representation
    • Mathematical derivation
  ➢ Round 3 – Simulation
    • Insight to the dynamics between Excess of Loss structure and Ceded Curve

• Ceded Ratio Approach
  ➢ Beta distribution and dependency assumption
  ➢ Ceded Ratio graphs
  ➢ Ceded Ratio vs Gross Aggregate Copula

• Comparison of Results

• Questions / Comments
What is the Challenge

- **How to derive a distribution of reserves on a net basis (net of reinsurance)**
  - Direct use of net data
  - Use gross data then “net it down”

- **Issues with using net data directly**
  - Net data contains reinsurance impact
  - What if a line hasn’t had any reinsurance recoveries?
  - Does the history have an above average or below average reinsurance recoveries?
  - Is a low residual in a particular cell due to low noise OR large noise but dampened by RI?

- **Gross analysis then “Netting it down”**
  - Unless individual claim analysis is performed, the result will be an aggregate claim distribution
  - Even if Large Losses were treated separately, Attritional claims might developed to Large!
  - The challenge is to incorporate RI program to convert the distribution from a Gross to a Net basis
What is the Challenge

- Given a Gross empirical aggregate reserve distribution (10,000 points), derive a Net empirical aggregate reserve distribution (10,000 points)
- Reserving actuaries provided Booked Reserves
- Objective: Produce Net reserve distribution on a trial consistent basis by incorporating reinsurance program (XoL) by line by year

<table>
<thead>
<tr>
<th>AY 2012 Reserve</th>
<th>GROSS $</th>
<th>NET $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim#</td>
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<td>1</td>
<td>2,001</td>
<td>?</td>
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<tr>
<td>7</td>
<td>8,692</td>
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<td>10000</td>
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</table>

Mean = 49,820 = Gross Booked Ultimate

Mean = 44,838 = Net Booked Ultimate
Conditional Ceded Approach
Conditional Ceded Approach - Round 1a

- Look at Ultimate, not Reserve
- Look at Ceded, not Net
- Ceded Aggregate $ using constant percentage (like Quota Share)

<table>
<thead>
<tr>
<th>AY 2012 Ultimate</th>
<th>GROSS $000</th>
<th>CEDED $000</th>
<th>NET $000</th>
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</thead>
<tbody>
<tr>
<td>#</td>
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<tr>
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<td>25,715</td>
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<td>31,082</td>
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<td>3,146</td>
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<td>32,406</td>
<td>3,241</td>
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<tr>
<td>10000</td>
<td>87,540</td>
<td>8,754</td>
<td>78,786</td>
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Mean 49,820 = Gross Booked Ultimate
Mean 4,982 = Ceded Booked Ultimate
Conditional Ceded Approach - Round 1b

- Increasing Ceded% but start after a threshold as a Starting Point
- Linear interpolate
  - 0% if Gross Aggregation Ultimate < StartingPoint
  - MaxCeded is a free parameter
  - Constrain: Average Ceded $ must match Ceded Booked Ultimate

<table>
<thead>
<tr>
<th>AY 2012 Ultimate</th>
<th>GROSS $000</th>
<th>CEDED $000</th>
<th>NET $000</th>
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<tr>
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<td>21,416</td>
<td>57,609</td>
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<tr>
<td>10000</td>
<td>87,540</td>
<td>23,811</td>
<td>63,729</td>
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Mean 49,820 4,982

=Gross Booked Ultimate = Ceded Booked Ultimate
Conditional Ceded Approach - Round 2

- Concept of Ceded Curve
  - Visual representation allows us to generalize the approach
  - Need a mathematical curve to formulate and implement the approach

![Diagram of Conditional Ceded Approach]

- Conditional Ceded Approach
- Visual representation
- Mathematical curve
- Gross
- Booked
- Ceded
- Ultimate
- XOL
- CEDED

Institute and Faculty of Actuaries
Conditional Ceded Approach - Round 2a

- Derivate the Ceded Curve mathematically using Frequency & Severity

\[ X_i \sim Severity \ e.g. \ LogNorm(\mu, \sigma) \]

\[ N \sim Frequency \ e.g. \ Poisson(\lambda) \]

\[ G = \sum_{i=1}^{N} X_i \quad C = \sum_{i=1}^{N} \min(\max(X_i - Attachment, 0), Limit) \]

- Ceded Curve is a conditional expectation on Gross

\[ E[C \mid G = g] = \int_c c \cdot f(c \mid G = g), \text{where} \ f(c \mid G = g) = \frac{f(c, g)}{\int_c f(c, g)} \]

- Unlikely the integral has a close-form solution but it must be a transformation of Exponential
Conditional Ceded Approach - Round 2b

- **StartingPoint**: At what Gross aggregate $ likely to start benefiting from RI?
- **EndingPoint**: At the very extreme worst Gross, what is the max benefit from RI?
- **Convexity**: Freedom parameter to control the steepness
- Even included **Quote Share!** \((\text{StartingPoint} = 0, \text{Convexity} = 0)\)
Conditional Ceded Approach - Round 3
No Reinstatements

Ceded Curve - 10m xs 10m X1 - $0m

- G Agg
- C Agg
- C Agg Mean
- C Agg StdDev
Conditional Ceded Approach - Round 3
1 Reinstatement

Ceded Curve - 10m xs 10m X2 - $0m

- G Agg
- C Agg
- C Agg Mean
- C Agg StdDev

Gross Aggregate $

Ceded Aggregate $

Standard Deviation $

Gross Aggregate $
Conditional Ceded Approach - Round 3
Unlimited (99) Reinstatements

Ceded Curve - 10m xs 10m X100 - $0m

- G Agg
- C Agg
- C Agg Mean
- C Agg StdDev

Gross Aggregate $ vs Ceded Aggregate $ vs Standard Deviation $
Conditional Ceded Approach - Round 3
Unlimited Reinstatements; Decrease Limit to $5m

Ceded Curve - 5m xs 10m X100 - $0m

- Gross Aggregate $
- Ceded Aggregate $
- C Agg
- C Agg Mean
- C Agg StdDev

Gross Aggregate $
0 20,000 40,000 60,000 80,000 100,000

10,000 20,000 30,000 40,000 50,000 60,000 70,000 80,000 90,000 100,000

10,000 20,000 30,000 40,000 50,000 60,000 70,000 80,000 90,000 100,000

12,000 14,000

0 2,000 4,000 6,000 8,000 10,000 12,000 14,000

X Standard Deviation $

Gross Aggregate $

Gross Aggregate $
Conditional Ceded Approach - Round 3
Unlimited Reinstatements; Limit $5m; Increase Attach to $20m

Ceded Curve - 5m xs 20m X100 - $0m

- G Agg
- C Agg
- C Agg Mean
- C Agg StdDev

Gross Aggregate $
Ceded Aggregate $

Standard Deviation $

Institute and Faculty of Actuaries
Conditional Ceded Approach - Round 3

• Every line of business must have a Ceded Curve

• Characteristics of Ceded Curves
  ▪ Starts at zero when Gross is zero: \( C(0) = 0 \)
  ▪ Monotonic increasing: \( C(g_1) \leq C(g_2) \) if \( g_1 < g_2 \)
  ▪ Slope at any point can’t be greater than slope of Gross curve: \( C'(g) \leq 1 \) for \( g>0 \)

• Quota Share
  ▪ Slope is a constant: \( C'(g) = k \) for \( g>0 \)

• Excess of Loss
  ▪ Initially flat
  ▪ Increasing slope then flattened
  ▪ Reflect RI exhaustion
Conditional Ceded Approach - Round 3

- Approximations
  - Flat in the beginning stays until a take-off point = $StartingPoint$ (constrained by Booked Ceded Ultimate)
  - Then flattens off after $EndingPoint$ (provided by reserving actuary)
Ceded Ratio Approach
Ceded Ratio Approach

- Generate the Ceded % directly with a distribution
- Require a dependency assumption with the Gross Aggregate $
- Need a parameter on the ceiling $

<table>
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<th>Ceded%</th>
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<td>25,715</td>
<td>?</td>
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<table>
<thead>
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</table>

Mean = 49,820

Gross Booked Ultimate

4,982

Ceded Booked Ultimate

Institute and Faculty of Actuaries
Ceded Ratio Approach

Ceded Ratio - 10m xs 10m X1 - $0m

- Ceded Ratio
- Ceded Ratio Mean
- Paid
- Incurred
- C Ratio StDev

Gross Aggregate $
Ceded Ratio Approach

Ceded Ratio - 10m xs 10m X2 - $0m

- Ceded Ratio
- Ceded Ratio Mean
- Paid
- Incurred
- C Ratio StDev

Gross Aggregate $
Ceded Ratio Approach

Copula (Gross Agg v Ceded Ratio) - 10m xs 10m X1 - $0m
Ceded Ratio Approach

Copula (Gross Agg v Ceded Ratio) - 10m xs 10m X2 - $0m

Gross Aggregate %ile

Ceded Ratio %ile
Ceded Ratio Approach

• True Distribution is modelled by a Beta Distribution

4 Parameters Needed:
- Mean
- SD/CV
- Max Recovery Cap
- Correlation Factor
Comparison of Methods
Comparison of Methods – Ceded vs Gross Aggregate $
Comparison of Methods – Ceded Ratio vs Gross Aggregate $
Comparison of Methods – Copulas

Conditional Ceded Approach

Ceded Ratio Approach

True Result
### Comparison of Methods

<table>
<thead>
<tr>
<th></th>
<th>True Aggregate Net</th>
<th>Conditional Ceded Approach</th>
<th>Ceded Ratio Approach</th>
<th>Fixed Ceded Ratio Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>26,060</td>
<td>26,385</td>
<td>26,244</td>
<td>26,087</td>
</tr>
<tr>
<td><strong>StDev</strong></td>
<td>15,267</td>
<td>16,007</td>
<td>15,477</td>
<td>16,775</td>
</tr>
<tr>
<td><strong>CoV</strong></td>
<td>58.6%</td>
<td>60.7%</td>
<td>59.0%</td>
<td>64.3%</td>
</tr>
<tr>
<td><strong>10.0%</strong></td>
<td>8,937</td>
<td>8,889</td>
<td>8,680</td>
<td>7,861</td>
</tr>
<tr>
<td><strong>25.0%</strong></td>
<td>15,154</td>
<td>15,155</td>
<td>15,064</td>
<td>14,009</td>
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<tr>
<td><strong>50.0%</strong></td>
<td>23,937</td>
<td>23,562</td>
<td>24,232</td>
<td>23,289</td>
</tr>
<tr>
<td><strong>75.0%</strong></td>
<td>34,021</td>
<td>34,794</td>
<td>34,808</td>
<td>34,667</td>
</tr>
<tr>
<td><strong>90.0%</strong></td>
<td>45,086</td>
<td>46,896</td>
<td>46,107</td>
<td>47,508</td>
</tr>
<tr>
<td><strong>95.0%</strong></td>
<td>53,704</td>
<td>56,052</td>
<td>54,236</td>
<td>57,637</td>
</tr>
<tr>
<td><strong>99.5%</strong></td>
<td>86,564</td>
<td>84,192</td>
<td>83,516</td>
<td>89,197</td>
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</table>
## Comparison of Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
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</thead>
<tbody>
<tr>
<td>Conditional Ceded Approach</td>
<td>• Capture the true dynamics of reinsurance&lt;br&gt;• Produce Gross and Net trial consistent results&lt;br&gt;• Useful on Deterministic basis in rationalising the ceded reserves&lt;br&gt;• Force actuaries to have a view of potential recoveries&lt;br&gt;• Features such as Aggregate Deductibles can be explicitly captured&lt;br&gt;• Can be used to help set the mean for the Ceded Ratio Approach</td>
<td>• Not trivial to parameterise the Ceded Curve&lt;br&gt;• Tricky to modelling the noise around the Ceded Curve</td>
</tr>
<tr>
<td>Ceded Ratio Approach</td>
<td>• Simple to Implement&lt;br&gt;• Produce Gross and Net Trial Consistent results&lt;br&gt;• Assumptions can be validated</td>
<td>• Not trivial to parameterise the Ceded Ratio distribution&lt;br&gt;• Tricky to set dependency structure between Ceded Ratio and Gross Aggregate $&lt;br&gt;• Can only cap maximum recovery</td>
</tr>
<tr>
<td>Fixed Ceded Ratio Approach</td>
<td>• Simple to implement&lt;br&gt;• Simple to explain</td>
<td>• Incorrect distribution for Excess of Loss reinsurance&lt;br&gt;• Can be very wrong at the tail (but prudent)</td>
</tr>
<tr>
<td>Model Net Directly</td>
<td>• Simple to implement&lt;br&gt;• Simple to explain</td>
<td>• Cannot produce Gross and Net trial consistent results without additional assumptions and modelling Gross as well.&lt;br&gt;• <em>All issues in Slide 3</em></td>
</tr>
<tr>
<td>Full Individual Claim modelling</td>
<td>• Potentially the most accurate&lt;br&gt;• Netting down strategy simple</td>
<td>• Large Computational power needed&lt;br&gt;• Many more assumptions needed</td>
</tr>
</tbody>
</table>
Colour palette for PowerPoint presentations

- Dark blue: R176, G52, B88
- Gold: R217, G171, B22
- Mid blue: R64, G150, B184
- Light grey: R63, G69, B72
- Pea green: R121, G163, B42
- Forest green: R0, G132, B82
- Bottle green: R17, G179, B162
- Cyan: R0, G156, B200
- Light blue: R124, G179, B225
- Violet: R128, G118, B207
- Purple: R143, G70, B147
- Fuscia: R233, G69, B140
- Red: R200, G30, B69
- Orange: R238, G116, B29
- Dark grey: R63, G69, B72

Secondary colour palette

- Thank you for your attention!