Institute and Faculty of Actuaries

GIRO40

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Institute and Faculty of Actuaries

Back of the Envelope Price Optimisation

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Agenda

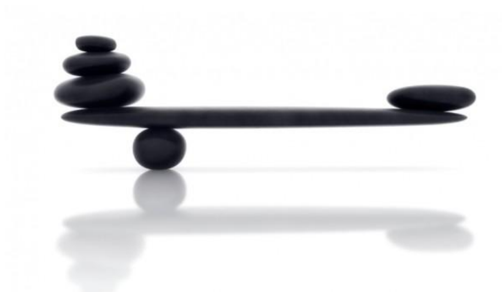
1. Challenges of price optimisation
2. How to predict the impact of a rate change with a pen and paper
3. How to sense check results of profit vs. volume optimisation
4. How to quickly derive simple profit vs. volume optimal rates
5. Discussion

Challenges of price optimisation

- It can take months
- Software can be complex
- ... so experts are usually required to perform the work
- The rates may be difficult to sense check
- And it is even more difficult to communicate why the specific rate changes are “optimal”

Solution

With decent accuracy, many optimisation questions can be answered with a pen and paper on the back of an envelope



There is a balance between accuracy of analysis and its speed.

Notation

- c_i – cost for policy i, usually claim cost, but other allocated fix costs can be included
- d – rate change, for example d=0.05 for a 5% rate increase
- e_i – elasticity for policy i defined as percentage change in volume per 1% rate change
- n – number of policies in the portfolio
- p_i – premium for policy i, net of all variable expenses

Typical problem no 1: Impact of a rate change

A form of this question arises for every rate change

What will happen to average premium and average cost after rates go up, say, by 10%?

Naïve answer: average premiums will go up 10%, average cost will not change

Labour & model intensive answer: let's run a scenario using a sample business mix, conversion models and elasticity models in a software package

Back of the Envelope answer: use the Covariance Rule from the next slide

The Covariance Rule

Claim. In a portfolio of n policies let $P = \{p_i\}$ be premiums, $C = \{c_i\}$ predicted burning cost and $E = \{e_i\}$ elasticities. After a rate change of d, if $d \cdot e_i$ are small, then

- the new burning cost is

$$\sum \frac{c_i}{n} - d\frac{Cov(C,E)}{1-d\sum\frac{e_i}{n}} \approx \sum\frac{c_i}{n} - d\,Cov(C,E)$$

Naïve burning cost

- the new average premium is

$$\sum\frac{p_i}{n}(1+d) - d\frac{Cov(P,E)}{1-d\sum\frac{e_i}{n}} \approx \sum\frac{p_i}{n}(1+d) - d\,Cov(P,E)$$

Naïve average premium

The Covariance Rule – intuitions

Think about average premium

High premium business is price sensitive → Low premium business is less elastic → Cov (P, E) is positive

→ Covariance Rule correction is negative → $\sum \frac{p_i}{n}(1+d) - d\,Cov(P,E)$ → Average premium goes up less than intended

Example – impact of a rate increase

Real life situation.

For a motor book

average premium	***£453***
average cost	***£335***
loss ratio	***74.0%***
average elasticity	***2.72***
rate change	***+ 10%***

Question. What will happen to average premium and average cost?

Example – impact of a rate increase

Solution. First, determine ***Cov*** between elasticities, premiums and burning cost.

We assume that modelled burning costs and elasticities are scored onto a dataset.

Then ***Cov***'s is easy to determine. SAS example:

```
proc corr data=portfolio noprob cov;
var model_elasticity premium model_cost;
/* by age_group; [if rate change varies by age_group] */
run;

/*  Results:
Cov ( model_elasticity <-> premium)    = 164;
Cov ( model_elasticity <-> model_cost) =  59;
*/
```

(The covariances can be tabularised periodically for future use.)

Example – impact of a rate increase

Average cost predicted by the Covariance Rule is:

$$\left(\sum \frac{c_i}{n} - d\,\frac{Cov(C,E)}{1-d\sum \frac{e_i}{n}}\right)$$

is: ***£335 – 10% * 59 / (1 – 10% * 2.72) = £327***.

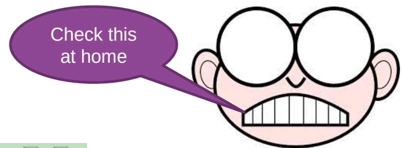
…and average premium

$$\left(\sum \frac{p_i}{n}(1+d) - d\,\frac{Cov(P,E)}{1-d\sum \frac{e_i}{n}}\right)$$

is: ***£453 * (1 + 10%) – 10% * 164 / (1 – 10% * 2.72) = £476***

Therefore loss ratio reduces from ***74.0%*** to ***68.7%*** and not by 10% ☹

The Covariance Rule – proof for costs



New average cost equals
$$\frac{\sum c_i(1-de_i)}{\sum(1-de_i)}$$

unwind and multiply by ***n***
$$=\frac{n\sum c_i - nd\sum c_ie_i}{n(n-d\sum e_i)}$$

plus and minus two terms
$$=\frac{n\sum c_i - nd\sum c_ie_i + d\sum c_i\sum e_i - d\sum c_i\sum e_i}{n(n-d\sum e_i)}$$

regroup terms
$$=\frac{n\sum c_i - d\sum c_i\sum e_i}{n(n-d\sum e_i)}+\frac{-nd\sum c_ie_i + d\sum c_i\sum e_i}{n(n-d\sum e_i)}$$

reduce first term
$$=\frac{\sum c_i(n-d\sum e_i)}{n(n-d\sum e_i)}+\frac{-nd\sum c_ie_i + d\sum c_i\sum e_i}{n(n-d\sum e_i)}$$

group
$$=\sum\frac{c_i}{n}-d\,\frac{n\sum c_ie_i - \sum c_i\sum e_i}{n(n-d\sum e_i)}$$

divide by ***n²***
$$=\sum\frac{c_i}{n}-d\,\frac{\sum\frac{c_ie_i}{n}-\sum\frac{c_i}{n}\sum\frac{e_i}{n}}{1-d\sum\frac{e_i}{n}}$$

since
Cov (X,E) = E(X·E) - E(X)·E(E):
$$=\sum\frac{c_i}{n}-d\,\frac{Cov(C,E)}{1-d\sum\frac{e_i}{n}}$$

The Covariance Rule – proof for premiums

New average premium equals
$$\frac{\sum p_i(1+d)(1-de_i)}{\sum(1-de_i)}$$

unwind and multiply by ***n***
$$=\frac{n\sum p_i + nd\sum p_i - nd\sum p_ie_i - nd^2\sum p_ie_i}{n(n-d\sum e_i)}$$

plus and minus two terms
$$=\frac{n\sum p_i + nd\sum p_i - nd\sum p_ie_i - nd^2\sum p_ie_i + d\sum p_i\sum e_i - d\sum p_i\sum e_i + d^2\sum p_i\sum e_i - d^2\sum p_i\sum e_i}{n(n-d\sum e_i)}$$

regroup terms
$$=\frac{n\sum p_i + nd\sum p_i - d\sum p_i\sum e_i - d^2\sum p_i\sum e_i}{n(n-d\sum e_i)}+\frac{-nd\sum p_ie_i - nd^2\sum p_ie_i + d\sum p_i\sum e_i + d^2\sum p_i\sum e_i}{n(n-d\sum e_i)}$$

reduce first term
$$=\frac{\sum p_i(1+d)(n-d\sum e_i)}{n(n-d\sum e_i)}+\frac{-nd\sum p_ie_i - nd^2\sum p_ie_i + d\sum p_i\sum e_i + d^2\sum p_i\sum e_i}{n(n-d\sum e_i)}$$

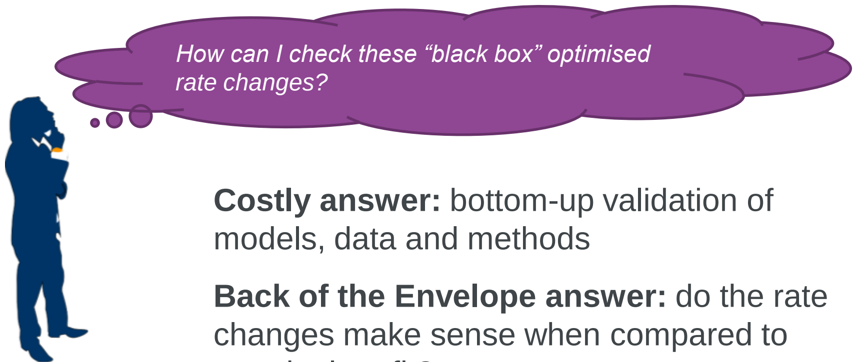
group
$$=\sum\frac{p_i}{n}(1+d)-d\,\frac{n\sum p_ie_i - \sum p_i\sum e_i}{n(n-d\sum e_i)}-d^2\,\frac{n\sum p_ie_i - \sum p_i\sum e_i}{n(n-d\sum e_i)}$$

divide by ***n²***
$$=\sum\frac{p_i}{n}(1+d)-d\,\frac{\sum\frac{p_ie_i}{n}-\sum\frac{p_i}{n}\sum\frac{e_i}{n}}{1-d\sum\frac{e_i}{n}}-d^2\,\frac{\sum\frac{p_ie_i}{n}-\sum\frac{p_i}{n}\sum\frac{e_i}{n}}{1-d\sum\frac{e_i}{n}}$$

and
Cov (X,E) = E(X·E) - E(X)·E(E):
$$=\sum\frac{p_i}{n}(1+d)-d\,\frac{Cov(P,E)}{1-d\sum\frac{e_i}{n}}-d^2\,\frac{\sum\frac{p_ie_i}{n}-\sum\frac{p_i}{n}\sum\frac{e_i}{n}}{1-d\sum\frac{e_i}{n}}$$

and ***d²*** is close to zero

Typical problem no 2: Checking optimised rates



Costly answer: bottom-up validation of models, data and methods

Back of the Envelope answer: do the rate changes make sense when compared to marginal profit?

Marginal profit

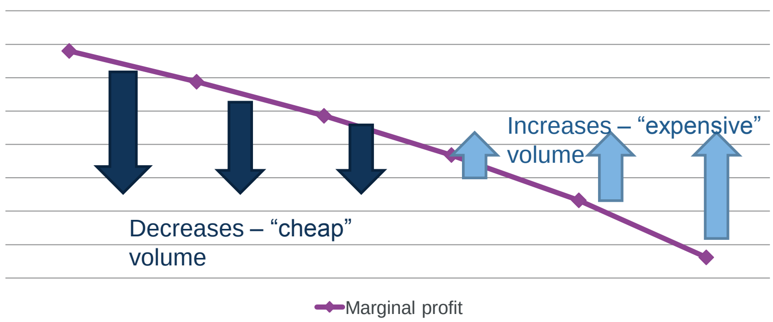
Marginal profit = change in profit from price cut sufficient to gain one policy

Let ***p*** be the average premium of a segment, ***e*** the elasticity and ***p – c*** the average profit. Marginal profit ***MP*** is:

$$MP = p - c - \frac{p}{e}$$

Marginal profit – intuition

- "Cost" of volume
  - Buy "cheap" volume
  - Sell "expensive" volume



Example – optimising # bedrooms

We have optimised number of bedrooms for home insurance.

Optimisation is volume vs. profit.

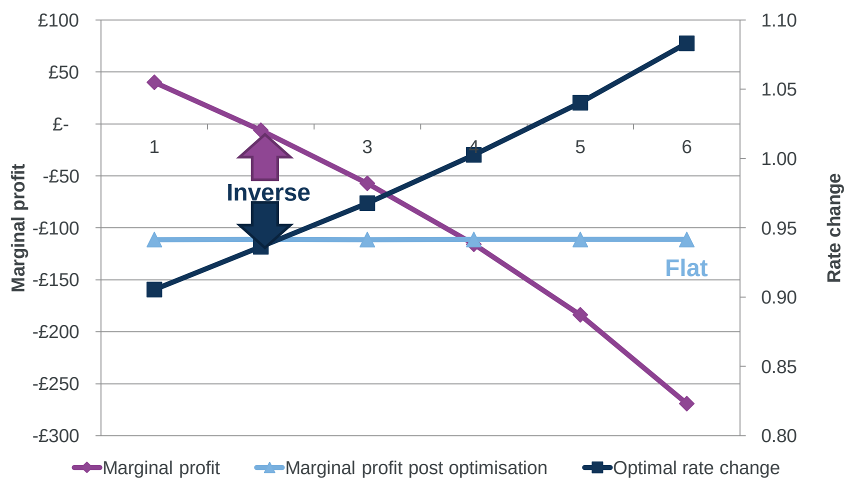
Each age group has its own average premium, cost and elasticity.

Rooms	Premium	Cost	Elasticity	Marginal profit
1	p1	c1	e1	p1-c1-p1/e1
2	p2	c2	e2	p2-c2-p2/e2
3	p3	c3	e3	p3-c3-p3/e3
4	p4	c4	e4	p4-c4-p4/e4

Calculate marginal profit before and after applying rate change.

Compared on next slide.

Example: Marginal profit vs. optimal rates



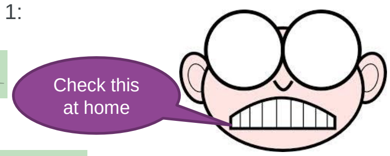
Marginal profit – applications

- Deep understanding of the value added by different segments both in terms of profit and volume
- Verifying if proposed rate changes are indeed optimal
- Explaining to others what the optimal rates do
- Choosing most valuable factors for optimisation

Marginal profit - derivation

Price decrease **d** s.t number of policies **n** increases by 1:

$$n(d) = n + 1$$



From definition of elasticity:

$$e = -\frac{\%\Delta n}{d} = \frac{\frac{n - n(d)}{n}}{d}$$

Solving for d:

$$d = \frac{1}{en}$$

Marginal profit - derivation

Definition of marginal profit:

$$MP = \pi(d) - \pi = n(d)\left[p(1-d) - c\right] - n(p-c)$$

Substituting n(d) and d:

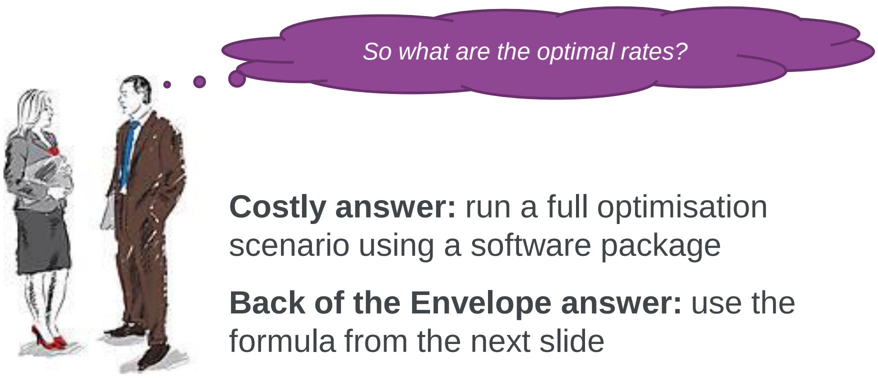
$$MP = (n+1)\left[p\left(1 - \frac{1}{ev}\right) - c\right] - n(p-c)$$

Multiplying out:

$$MP = p - c - \frac{p}{e} - \underbrace{\frac{p}{ve}}_{\sim 0} + \underbrace{np - nc - n(p-c)}_{=0}$$

Typical problem no 3: Optimal rates

The ultimate question is:



Costly answer: run a full optimisation scenario using a software package

Back of the Envelope answer: use the formula from the next slide

Formula for optimal rates

Claim. Let ***p*** denote average premium net of variable cost, ***c*** be the average claim cost and ***e*** be the elasticity of a portfolio. Let $\{p_i\}$, $\{c_i\}$ and $\{e_i\}$ represent average premium, cost and elasticity for each pricing cell ***i***.

In addition let's assume that the overall rate level is already optimal.

Then the optimal rate in a volume vs. profit optimisation for segment ***i*** is

$$p_i + \frac{1}{2}\left(p - c - \frac{p}{e} - p_i + c_i + \frac{p_i}{e_i}\right)$$

Formula for optimal rates – limitations

The formula does not handle:

- Multiple constraints, but it is not recommended to use too many constraints anyway
- Constraints in subsequent years, but policy ageing is hard to model correctly anyway
- Big changes in business mix, but maybe such changes are unlikely to happen most of the time
- Non-linear forms of elasticity, but maybe one can do without them

Example – optimising age groups

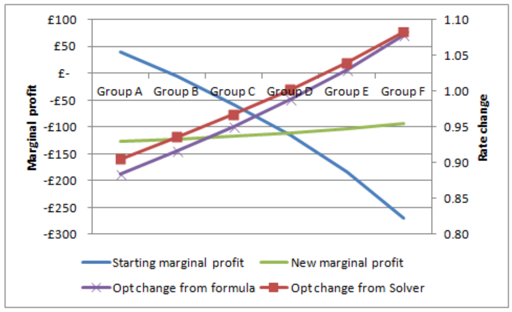
- We are optimising age group for motor insurance
- Optimisation is volume vs. profit
- Each age group has its own average premium, cost and elasticity

Age group	Premium p	Cost c	Elasticity e
Group A	£ 800	£ 600	2.95
Group B	£ 825	£ 652	3.23
Group C	£ 840	£ 697	3.52
Group D	£ 870	£ 757	3.84
Group E	£ 900	£ 819	4.18
Group F	£ 950	£ 903	4.54
TOTAL	**£ 881**	**£ 751**	**3.77**

Optimal rate change
0.8842
0.9171
0.9512
0.9890
1.0304
1.0779

Example – optimising age groups

New rates are derived using the proposed formula & using Excel Solver



- The two are very close
- With the new rates the marginal profit is ~ flat

Formula for optimal rates – proof

To attract k_i extra policies from segment i, we need to reduce rates in this segment by a multiplier of

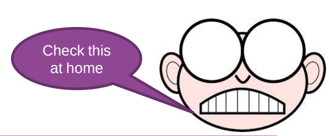
$$1-\frac{k_i}{e_i v_i}$$

Profit will change by

$$(v_i+k_i)\left(p_i(1-\frac{k_i}{e_i v_i})-c_i\right)-v_i(p_i-c_i)=k_i\left(p_i-c_i-\frac{p_i}{e_i}(1-\frac{k_i}{v_i})\right)$$

Optimal rates mean that trading volume between segments does not increase profit. Hence derivatives over k_i must equal across all segments. For simplicity, let's assume that it equals the current average across all segments.

$$\forall i \quad p_i-c_i-\frac{p_i}{e_i}(1-2\frac{k_i}{v_i})=p-c-\frac{p}{e}$$



Formula for optimal rates – proof

After transformations and dividing both sides by e_i we have

$$\frac{k_i}{v_i e_i} = \frac{1}{2e_i} + \frac{1}{2p_i}\left(p - c - \frac{p}{e} - p_i + c_i\right) = \frac{1}{2p_i}\left(p - c - \frac{p}{e} - p_i + c_i + \frac{p_i}{e_i}\right)$$

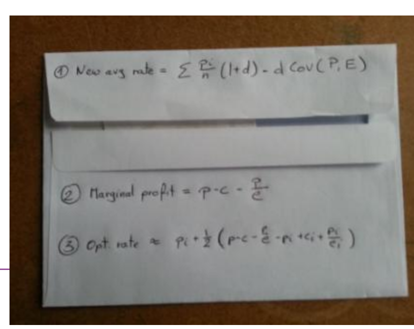
The left hand side equals a percentage rate change. Therefore to get back-of-the-envelope optimal rates for segment i, we need to correct the rates by a multiplier of

$$1 + \frac{1}{2p_i}\left(p - c - \frac{p}{e} - p_i + c_i + \frac{p_i}{e_i}\right)$$

Conclusion

- ✓ Accurate predictions of rate change impact
- ✓ Explaining rate optimisation
- ✓ Optimising rates for volume vs. profit

…can be done with a pen and paper (and 3 lines of SAS code)



Questions Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.