GIRO Conference and Exhibition 2012
Juggling uncertainty the actuary’s part to play

Date of presentation
Optimal Pricing in Solvency II
- A new era of Risk Adjusted Pricing?
    Ji Yao
## Agenda

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Any pricing decision essentially depends on risk model and demand curve.

调整价格以最大化预期利润。
Any models come with uncertainty

Risk model
- Model: GLM is standard practice with standard deviation around the estimation
- Trend: new claim (BI)

Demand curve
- Model: is GLM the right solution?
- Market is changing

Pricing could become a dangerous exercise to exploit errors of the model.

- Should these uncertainty be considered in pricing?
- How could it be considered?

YES
Practical solutions to uncertainty

- **Judgment**
  - Leave experienced underwriter/actuary to make a judgmental call

- **Constraints**
  - Volume of sales doesn’t drop
  - Modeled LR/Conversion/Retention within a range

- **Avoidance**
  - Underwriting rule
    - Young driver
    - High value car
Evidence from the market

At what loss ratio could this segment of 17-20 years old become appealing?
Capital Asset Pricing Model (CAPM)

Asset/investment management

Pricing in Insurance
Move into Risk world

Profitability

Demand curve

Capital requirement

Risk Adjusted Pricing (RAP)
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Standard formula and internal model

• For regulatory purpose
• For whole portfolio
• Objective:
  – Ensuring a high, effective and consistent level of prudential regulation and supervision;
  – Protect policyholder and customer
  – Protect the integrity, efficiency and orderly functioning of financial markets
  – Encourage risk management within insurance company
Right idea, right tool?

- Modify existing capital model
- Build new capital model for pricing
Pricing needs capital model at appropriate granularity

- It is important to have a model that works at the granular level that decision will be made.

- **Two** levels of granularity in pricing practice
  - Overall level for planning/budget
    - LoB/channel/Tenure (New Business vs. Renewal)
    - Similar level as capital model
    - Capital model results might already be reflected in good practice to certain extent
  - Detailed level pricing decision
    - Adjust relativity to achieve volume target/respond to market move
    - Much more granular than capital model
    - One simple solution is to apply the same capital requirement ratio for all every risks in the LoB
Decomposition of underwriting risk

• Granular capital model is all about understand individual risk better

• Insurance risk relates to the uncertainty about the results of the insurer
  – Premium risk
  – Reserve risk
  – Catastrophe risk

• For each risk, it can be split further
  – Model/Systematic risk
  – Parameter risk
  – Process risk (Randomness)
Different type of risk dominates capital models at different granularity

- Model/Systematic risk
  - The mathematical model of the process is inappropriate
  - Unexpected inflation/new trend in claim
  - Normally cannot be captured in historical data
- Parameter risk
  - Parameters are mis-estimated
  - Cannot be diversified by taking more same type of risk
  - But can be diversified by a balanced book
- Process risk (Randomness):
  - Actual results will vary from actuarial best estimates based on random chance
  - Will be diversified when there are large enough volume
  - But in practice, for a medium size book, there is still 1-5% left

<table>
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<tr>
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<th>LoB</th>
<th>Segment</th>
<th>Individual policy</th>
</tr>
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<tbody>
<tr>
<td>Model risk</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Parameter risk</td>
<td>1-10%</td>
<td>5-20%</td>
<td>5-20%</td>
</tr>
<tr>
<td>Process risk</td>
<td>1-5%</td>
<td>50%-200%</td>
<td>1000%+</td>
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Parameter risk in premium risk

- It is the uncertainty over rating pricing structures, both rate factors and relativities.

- In the context of GLM, it is about:
  - Is correct interaction picked?
  - How wide is standard deviation of estimated parameter?

- Do pricing actuary really appreciate how volatile GLM result is?
  - Use SD as a rating factor?
Process risk in premium risk

• For a enough large book, volatility over attritional and large claim will be diversified.
• The key is to model **very large claim** (>£100k), both frequency and severity
• In GLM, randomness are modelled by a **pre-chosen** distribution
  – A fix form of variance is assumed related to the mean of distribution – a key assumption of GLM
  – If it is believed that the randomness (variance) is
    – Same → Normal distribution
    – Linear with mean → Poisson distribution
    – Power of 2 of mean → Gamma distribution
    – Power of 3 of mean → Inverse Gaussian distribution
  – So GLM never answers the question whether young male is more volatile than old female?
    – Because we have already chosen the answer when set up the model
• If sticks to GLM,
  – Detailed model of perils
    – Number of claimants
    – Multi-layer of BI claim

Forget GLM
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Model set-up

- **Assumptions**
  - $C_0$ are capital requirement related all risk other than premium risk
  - $m$ is the expected claim
  - $P$ is the premium, and premium risk is a $k\%$ charge to premium
  - $\begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$ is the correlation matrix between premium risk and other risks
  - Future profit could offset capital requirement

- **Capital required is**
  \[
  C = \sqrt{(kP)^2 + 2\alpha (kP) C_0 + C_0^2} - (P - m)
  \]

- In addition to this, there are normal price optimisation set-ups.
  - Logistics demand curve
Features of profit vs. risk chart

- A shifted, rotated hyperbola shape curve
- Maximum profit (MP) point
- Run-off (RO) point: no future business is written
Risk is not always rewarded with profit

- For point beyond the maximum profit (MP) point, the risk is higher but expected profit is lower
- This happens when premium drop below the MP point, so
  - Volume is increased, which increase the base for capital calculation
  - Reduce future profitability, which deplete capital pool
- Contrast to CAPM
Possible to have a minimum capital point, but not always

- Increase premium, the volume will decrease, which has impact to
  - Reduce the base for capital calculation.
  - But might also increase/decrease future profitability.
- Minimum capital (MC) point is achieved where
  \[
  \text{Marginal profit} = \text{Marginal capital charge}
  \]
- Possible for LoB that is profitable and capital requirement is low.
Efficient frontier is between MP and MC point

- It always makes sense to price between MP and MC point
  - If not, it indicates that
    - Either too much risk is taken but not rewarded for profit
    - Or, volume is too low that there is opportunity to gain profit and minimise capital

- The size of this area depends on risk/profitable model, demand curve and capital requirement model.
Factors that affect shape of the efficient frontier

- Profitability
  - Loss ratio
  - Expense
- Demand curve
  - Conversion / Retention rate
  - Elasticity
- Capital requirement
  - Premium risk capital calculation
  - Other risk capital calculation
  - Correlation / dependency structure

Higher LR

Higher Elasticity

Higher Premium Risk Charge
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Portfolio management

- Each LoB can be plotted as well as the total portfolio
  - Portfolio profit = sum of each LoB profit
  - Portfolio required capital < sum of each LoB required capital, due to diversification.
Efficient frontier of portfolio

- Different combination of LoB’s can be simulated and termed ‘opportunity set’.
- Efficient frontier exists
Example

- Suppose we find a risky segment that not much business is written now.
  - Deliberately price higher
- Compared to whole book, this segment has
  - Premium is 10% of whole book
  - Conversion is 60% lower than whole book
  - Elasticity is 50% higher
  - LR is 15% lower
  - Capital charge is 100% higher
- Portfolio optimisation
  - Profit vs. Volume chart
    - Grow risky segment
    - Shrink other part of book
    - What extra risk is taken?
    - Any other options?
Example

• Profit vs Risk chart

But the volume might change significantly!
Example

- Efficient frontier with constraint on volume

- Reduce risky segment 4.1%, increase other segment 1% will slightly reduce the required capital, keep the volume same and increase expected profit by 1%!

### Table: Rate change

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<th>Rate change</th>
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<tr>
<td>Required Capital</td>
<td>Expected Profit</td>
</tr>
<tr>
<td>11.3%</td>
<td>12.5%</td>
</tr>
<tr>
<td>11.4%</td>
<td>13.0%</td>
</tr>
<tr>
<td>12.1%</td>
<td>14.0%</td>
</tr>
<tr>
<td>16.2%</td>
<td>14.4%</td>
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Conclusions

• Explicitly consider risk/uncertainty in pricing.
  – Risk Adjusted Pricing (RAP)
• This will happen step by step:
  – For Line of business optimisation: capital model is a good starting point
  – For individual price optimisation: a new capital model might be needed
    – Parameter risk and process risk are good starting point
    – Use model uncertainty as a rating factor
    – Review appropriateness of risk model structure
• RAP provides good potential to gain advantage in this highly competitive market.
  – Maximum profit
  – Minimum capital requirement
  – Optimal return on capital
  – Efficient frontier on return and required capital
Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

Ji Yao: jyao@uk.ey.com