Estimating and Communicating Reserving Uncertainty
4th Younger Members Convention

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Agenda
- Section 1: Working party findings
- Section 2: Models in Allianz
- Section 3: Beazley – useful models?

Section 1: What did we learn?
- “Reasonable” actuaries come up with variable results - wider than would be expected even allowing for blind reserving conditions
- Wide range of results from different methods/models
- Range still wide even when same method/model used
- No “correct” method/model apparent
Approach vs. needs

Sophisticated vs. Pragmatic

<table>
<thead>
<tr>
<th>London Market Companies</th>
<th>Personal &amp; commercial Better data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limited claim &amp; exposure data</td>
<td>Massive corporation</td>
</tr>
<tr>
<td>Longer tail</td>
<td></td>
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</tbody>
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Why measure reserve uncertainty?

Increasingly, we are being asked to quantify:

- a range of reasonable best estimates
- a range of reasonable outcomes around the actuarial best estimate
- what confidence level the held reserve is compared to the actuarial best estimate
- how likely future payments will be X% higher than the held reserve

Why measure reserve uncertainty?

In part, these questions are the result of new regulations and accounting rules, such as:

- ICA
- Solvency II
- IFRS
- Sarbanes Oxley, Morris, etc.
Reserving uncertainty

Reserving is about forecasting unpaid claims:

- The variability of a forecast includes the estimation variance and the process variance:
  \[ \text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2} \]
- However, what we are really interested in is a predictive distribution of outstanding claims (ultimate claims and the associated cash flows)

A simple example

- Data sample Y = {3, 8, 5, 9, 5, 8, 4, 8, 7, 3}
- Expected value = 6
- What is the best estimate of a new forecast value?
- What is the prediction error of a new forecast value?
- What is the predictive distribution of a new forecast value?

Analytic Solution

- Random variable is Poisson distributed
- \[ \text{standard error} = \frac{\sigma}{\sqrt{n}} \]
- \[ \text{process variance} = \mu \]
- \[ \text{prediction error of forecast} = (\mu + \frac{\sigma^2}{n})^{1/2} \]
### Parameter Uncertainty (Bootstrapping)

- Simple method to obtain a distribution of parameters
- Many new data sets are created by sampling with replacement from the observed data
- Result is a “simulated” distribution of parameters

#### Observed Data

<table>
<thead>
<tr>
<th>Mean</th>
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<tbody>
<tr>
<td>8</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>9</td>
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<td>5</td>
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<tr>
<td>8</td>
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<td>4</td>
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<tr>
<td>8</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>6</td>
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</tbody>
</table>

#### Bootstrap Samples

<table>
<thead>
<tr>
<th>Bootstrap</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>3.9</td>
</tr>
<tr>
<td>Sample 2</td>
<td>5.7</td>
</tr>
<tr>
<td>Sample 3</td>
<td>7.3</td>
</tr>
<tr>
<td>Sample 4</td>
<td>3.0</td>
</tr>
<tr>
<td>Sample 5</td>
<td>5.9</td>
</tr>
<tr>
<td>Sample 6</td>
<td>7.5</td>
</tr>
<tr>
<td>Sample 7</td>
<td>9.3</td>
</tr>
<tr>
<td>Sample 8</td>
<td>3.3</td>
</tr>
<tr>
<td>Sample 9</td>
<td>5.9</td>
</tr>
<tr>
<td>Sample 10</td>
<td>7.5</td>
</tr>
</tbody>
</table>

10,000 Bootstrap standard error: 0.68

### Predictive Distribution

Assuming Poisson process

#### Forecast

<table>
<thead>
<tr>
<th>Forecast</th>
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<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
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<tr>
<td>7</td>
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</tbody>
</table>

Bootstrap standard error 2.54

#### Prediction error

2.54 Prediction error
Predictive Distribution

Frequency: Forecast

Stochastic Reserving and Bootstrapping

- Define and fit statistical model
  - Overdispersed Poisson Model
  - Mack
    - or any other model than can be clearly defined
- Obtain residuals and pseudo data
- Refit statistical model to pseudo data
- Obtain forecast

Bootstrapping the Chain Ladder (ODP)

- Fit chain ladder model
- Obtain Pearson residuals
  \[ \epsilon = \frac{O - E}{\sqrt{V}} \]
- Resample residuals
- Obtain pseudo data
  \[ \epsilon^* = \epsilon + \sqrt{V} \]
- Use chain ladder to re-fit model, and estimate future incremental payments
Bootstrapping the Chain Ladder (ODP)

- Simulate observation from process distribution assuming mean is incremental value obtained at Step 5
- Repeat many times, storing the reserve estimates
- Prediction error is then standard deviation of results

What is being modelled?

- For a given model and for a given data set:
  - uncertainty in a forecast around an assumed development pattern due to observed historic variability taking account of
    - parameter risk
    - process risk
  - assuming
    - development pattern is the same for all origin periods
    - origin periods are independent

What is not being modelled?

- Model risk
  - if the underlying model is wrong, the results will be wrong
- Risks that do not appear in the data
- Other risks
  - some operational risks
  - regulatory risk
  - future changes in legislation
  - etc.
Use of Models at Allianz

- Additional Management Information
- Preparation for Solvency II and IFRS Phase II
- Internal Risk Capital Assessment (not yet fully implemented)
- Additional Insight into Traditional Reserving Process

Section 3: Beazley – useful models?

**Beazley’s objectives**

- Business planning – Efficient capital use
  - Capital cost over lifetime of policy
  - Risk adjusted returns on capital
- Reserve setting – Prudential risk margins
  - Is the level of prudence in our reserves changing?

Section 3: Beazley Background

**Market cycles**

- Are London Market swings more severe?
Section 3: Beazley Background

Medium

Short

Tail Length

Stamp growth

1997 1999 2001 2003 2005

£0.0bn £0.2bn £0.4bn £0.6bn £0.8bn £1.0bn

Section 3: Review objectives

We need to produce the following chart

By class and team for interest

Company level is key

0% 20% 40% 60% 80% 100% 120% 140%

2001 2002 2003 2004 2005 2006

Upper NLR

Plan NLR

Premium buffer

Capital buffer

Section 3: Beazley approach

Marked Cycle

Next year

Claims Volatility

This year

MC

CV

Last year

etc

MC

CV
Section 3: Claims volatility

- $62.8M to $66.0M
- $58.5M to $61.7M
- $54.3M to $57.5M
- $50.0M to $53.2M
- $67.1M to $70.3M
- $71.4M to $74.6M
- $75.6M to $78.8M
- $79.9M to $83.1M

- 0.6 - 0.75
- >0.95
- 0.75 - 0.95
- 0.5 - 0.6
- 0.4 - 0.5
- 0.3 - 0.4

Section 3: Claims volatility

- $45.7M to $48.9M
- $41.4M to $44.7M
- $37.2M to $40.4M
- $32.9M to $36.1M
- $28.6M to $31.8M
- $24.4M to $27.6M
- $20.1M to $23.3M
- $15.8M to $19.0M
- $11.5M to $14.7M
- $7.3M to $10.5M

- 0.2 - 0.3
- 0.002 - 0.004
- 0 - 0.002

Section 3: Market cycle

- 60%
- 70%
- 80%
- 90%
- 100%
- 110%
- 120%
- 130%
- 140%

- ULR
- Plan LR
Section 3: Risk reduction

- Upper NLR
- Plan NLR

Inurred as %
Ultimate

0% 20% 40% 60% 80% 100% 120% 140% 160%

2001 2002 2003 2004 2005 2006

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Section 3: Combination (e.g. 2006)

Class 1
Prem $100m
Div 40% - $40m
UnDiv 20% - $20m

Total
Prem $300m
Div 15% - $45m
UnDiv 13% - $40m

Class 2
Prem $200m
Div 5% - $10m
UnDiv 10% - $20m

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Section 3: Uses - Reserve strength

Gives a reserve strength index over time

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Example of the range of possible outcomes for one class and year
Questions?