MOTOR INSURANCE - FLEET RATING

Summary

A working party has been examining the practices of parts of the fleet market in the United Kingdom and considering the logical basis for the procedures adopted. The members of the party were:

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We must first acknowledge the considerable assistance we have had from various companies especially the Commercial Union, Co-operative, Cornhill and Royal, both in regard to descriptions of their practices and in the provision of data based on very large numbers of vehicle years exposure covering much of the period 1970 to 1977. For reasons of confidentiality, this report contains little original data and makes no references to the practice of individual companies: the consistency of data and information from different companies however leads us to think that the figures we quote are reliably established and form a better guide to the inherent variability of experience data than would be available to any individual insurer.

It is quite clear that fleet rating requires the exercise of individual underwriting judgement based on knowledge of the fleet operator and other matters, often of a subjective nature. The main message of the report, however, is the very limited extent to which that judgement ought to be influenced by the actual claims experience of an individual fleet, unless some thousands of vehicle years of exposure are available. In particular the provision of claims experience for small fleets based on three years data, of which the last year has not been fully exposed, and for none of which years have the larger claims been settled, and where at least one-third of the final cost is based on estimates on an unverifiable basis, is an extremely flimsy peg on which to hang a calculation of risk premium.

No mention is made of expenses. They form an emotive subject, and it is a matter for managerial judgement whether a given fleet, or fleets generally, should be charged expenses on some fixed basis or whether, given a reliable calculation of risk premium, the underwriters should be expected to quote office premiums, so as to aim for a total contribution to overall expenses that is acceptable. This subject was extensively discussed in the paper presented by Mr. I.L. Rushton, to the Students Society in the 1977/78 Session.

The main part of the report is brief and has been written in non-technical language in the hopes that it may be circulated among motor underwriters. It is followed by two appendices written by J.R. Hooson, dealing with the underlying statistical theory: some knowledge of mathematical statistics is required and whilst these appendices form the theoretical basis of the figures quoted in the main report, it is not necessary to understand them in order to follow our argument. It should be noted that in some cases where theory would allow little notice to be taken of experience data, we have proposed that rather more allowance should be made than is theoretically indicated, since it seemed commercially undesirable to make very small adjustments.

FLEET RATING

Introduction

This report considers current practice in regard to the rating of fleets of motor vehicles insured in the United Kingdom. It examines the theoretical justification for having regard to the experience of an individual fleet in recent years and some of the practical problems in measuring that experience.

Suggestions are made for improving the basis on which premiums for fleets are based. The note is intended for both underwriters and actuaries and therefore contains a number of explanations for the benefit of one group which will, of necessity, be extremely elementary to the other group. Appendices deals with the mathematical theory underlying the report and with the application of credibility theory.

Definitions

FLEET

For the purpose of this report a fleet comprises a group of several vehicles and may include any type or types. Some fleets will consist partly or wholly of private cars, but most will be dominated by commercial vehicles; these may range from pedestrian controlled vehicles (milk floats) to 40-ton trucks used for inter-continental haulage or to special vehicles, such as mobile cranes, tankers or to buses and coaches. The feature that will distinguish a fleet from a collection of vehicles is that the premium will depend partly or wholly on the past experience of that individual fleet. Some companies may treat a fleet of mixed vehicles as if it were two or more separate, but more homogeneous, fleets.

BOOK RATE

This is the rate (usually before any no claim discount) that would be charged for a single vehicle individually insured. The rate may, but need not, be loaded for factors other than the actual vehicles, for example place of garage, excesses, young drivers or occupation.

EXPERIENCE

In connection with fleets, experience is commonly taken over a period of three years and measures total estimated claim costs. These may be related to total book rate premiums or to the number of vehicles or to some other measure of exposure. Since the three years are normally those three consecutive years ending at the date of the next renewal the data for the last year will be incomplete, quite apart from IBNR, and there will be considerable uncertainty in regard to the actual ultimate cost, quite apart from statistical fluctuations in both the number and amount of claims incurred, and from estimates that differ from the ultimate cost.

CURTAILING

Because of the disturbing effect of individual large claims it is usually found necessary to disregard the amounts of claims which are in excess of some fixed amount (say £2,000) and to make an allowance for the expected amount of payments over that sum. The curtate total is the total claim amount subject to any claim over the fixed amount being treated as equal to the fixed amount. The grossing-up factor (GUF) is the ratio of the actual claim total to the cutate total. It will usually be estimated from the pooled experience of many fleets and the problems involved in estimating it are discussed later.

It is understood that some underwriters may use different excess points for different types of vehicle. Unless the claims size distribution is very unusual we can see little value in doing so: the effect of varying the cut-off point over the range say £1,000 to £5,000 (in terms of 1978 currency) is not very great.

CREDIBILITY

This is a technical term and is a measure of the weight one should give to the actual results of one or more fleets. A very large fleet (involving say 20,000 or more claims) might be regarded as fully credible. A fleet of under 100 vehicles will have very low credibility. For example if we have a fleet with the following characteristics, insured comprehensively:

Vehicles	100
Claims Frequency (claims per vehicle year)	0.25
Average claim amount	£250
Years of Experience Available	3
Claims Curtailed At	£1,500
GUF (1978)	1.3

Then the "expected" claims cost per vehicle year (averaged over a large number of such fleets) will be £62.50 but there is an even chance that the results of any single fleets experience will lie outside the range £45-£85 quite apart from errors in estimating outstanding claims. It will be seen that the actual results of one such fleet, even if they are accurately known, will be of very little use in assessing the true risk. For non-comprehensive cover the range is at least twice as big. This will be discussed in detail later and in the appendices.

Current Practice

Information

The variety of market practices almost defies any attempt to describe them. Recently however there has been a move in the London market to standardise the information to be presented to an underwriter and most forms now require at least the following:-

Insured's name and address.
Current cover and variations, if any, during the last three years.
Number of vehicle/years or number insured at start of each year.
Total number of accidents/claims.
Claims paid (AD and TP separately).
Claims outstanding (AD and TP separately).

Total paid and outstanding.

The last five lines are to be given for the current (incomplete) year and the two previous years. There is no mention of IBNR.

Forms in use by some companies tend to ask for more information, for example a breakdown of the vehicles currently operated into classes such as:-

Private cars.
Goods carrying vehicles (in 2-4 weight or class groups).
Special type vehicles.
Others (to be specified).

Sometimes details of individual claims including both payments and estimates and brief particulars of the claims are called for although for a large fleet the latter would clearly be an onerous undertaking. In fact if there are enough claims to give even modest credibility this list would be so long that it would probably never be supplied - whereas if the list is short it will be realised that its value as a measure of experience is very limited indeed.

The remarks above apply primarily where a fleet is being offered to an insurer who has no previous experience of it. For those fleets, and they will be quite numerous, that remain with an insurer for many years, the problem is somewhat easier. We give prominence to the procedure where competitive quotations are required since this seems to us the area where the underwriter is most lacking in information and where statistical guidance is most urgently needed. We must make quite clear at the outset however that there are many facets that can be sensed only by an underwriter with flair - and the actuary or statistician will never be able to replace that. Flair is not enough, however, and the need to guide underwriters on the fallibility of the so-called "factual" information seems to us to be urgent. It is perhaps unfortunate that facts in the shape of an analysis of past experience may not always be a reliable guide to the future and a measure of the uncertainty inherent in any claims experience should be the first thing an underwriter asks for and is given before considering the results of the experience.

In the case of an existing fleet, an underwriter may have many years experience available, and it is difficult to say how much regard should be had to the claims experience earlier than the last three years. Enquiries we have made in regard to the practice of some leading fleet insurers suggest that its main use is to give the underwriter a "feel" for the fleet and any changes that may have occurred. It is very difficult to separate fact and opinion here and we suspect that it is easy to form an unjustifiable opinion on the basis of a good or bad experience. For example an underwriter once remarked that a holder of a comprehensive private car policy who made three claims in ten years must be a bad risk. Not so: whilst he <u>is</u> a little more likely to be bad than one who had made only one claim in the period it is more likely that he is among the good risks for even if these risks <u>average</u> only one claim in ten years, about one in 16 of them will have three claims and about 1 in 50 will have four or more in that ten year period arising purely from random fluctuation. Random processes produce random results and it is a salutary exercise for statisticians as well as underwriters to be frequently reminded of the fallibility of even quite large samples or large experiences.

Examination of the Experience

It seems to be a common practice to calculate a claims cost per vehicle year for each of the three years of experience, or for all years if more are available. If all the vehicles are of similar type and size and if the conditions of use do not change, this cost is likely to be a useful figure, subject to projection for inflation, and subject to the limitations to which we refer elsewhere.

Where however several types of vehicles are included, it is likely that there will be a change in their relative proportions, and this will have its influence on average cost per vehicle year. The underwriter must then make use of any other information to hand, for example the relative office premiums for the various types of vehicle if they were insured individually. If the changes are gradual they can be allowed for by a simple "guesstimate" which will probably be good enough not to increase significantly the other errors inherent in the process.

If changes are abrupt, as for example where a fleet is increased following a merger, or if more precision is required, then the extent of the change must be measured.

One way of doing this, which seems to be used whenever enough information is available is to calculate total "book rate" premiums for the fleet. These will take into account vehicle type and weight at least and may also include place of garage or other relevant factors. The total book rate will give a much more reliable picture than a mere vehicle count, even if the book rates do not correspond too closely to the proper risk premiums.

Given the book rate premiums and the claim costs for the usual three years we can calculate a claims ratio and if necessary project it to the coming year: unless book rates have not reflected true costs accurately we can then obtain a measure of the fleet's experience. For example we may find ratios of say 50%, 80%, 65%, and require 10% each for commission and expenses. On a premium of 100 we could then say that claim costs were 65 leaving after expenses and commission a margin of 15.

However, unless it were a <u>very</u> large fleet it would be most unwise to do any such calculation. If our "target" claim costs are 80 and the actual 65 we would be most unwise to work on a figure less than 75 unless there were at least 500 vehicles in the fleet or under 70 unless there were 2,000 or more. These numbers are very approximate and are affected by errors in the claim reporting and estimating process and the incidence of large claims and IBNR. They are however very definitely minima and should serve as a strong warning against using claim costs per vehicle year where there is no book rate or other statistic to compare those costs with.

Two warnings must be given over the use of book rates. The first is that they should be reasonably closely related to the risk premium: If the rate book is distorted for commercial reasons or from lack of knowledge of some specialised vehicles or from widely differing expense ratios, then book rates may be unreliable where a fleet has an abnormal proportion of vehicles in the affected groups.

The other and potentially more serious point is to see that book rates and experience run together. This is best made clear by a reductio ad absurdam.

Suppose our book rates have always been just right, and our experience in line with expectation. Then we shall have a claim ratio of (say) 80%. If we apply this to a new book rate that is still correct we shall get the right premium. However, if the new book rate is double what it ought to be then we shall be charging double. Now anything as extreme as this would not happen in practice, but the same effect does occur on a smaller scale if:

1. The new book rates are wrong.
2. The old book rates were wrong (unless they have all been wrong in the same way and to the same extent, and the error is perpetuated in the new book rates).
3. Rates have not been revised at annual intervals (which is really a special case of (1) and (2).

One company is proposing to overcome these problems by using "special" book rates, based generally on its ordinary commercial rates. These special rates will however be raised annually on the same day each year. The rate of increase will not necessarily be the same as that applying to ordinary rates, even if the latter are revised at the same time. It is however intended, we understand, that the rate of increase applied will be roughly that expected to apply to claim settlements during the period to which it relates.

The special rates are intended to apply to renewals about the middle of the year to which they apply. For example if rates are revised on 1st January they should aim to be the correct monetary amounts for a renewal year staring on 1st July that year. This will give an expected claim ratio equal to that obtained for a 1st July renewal but in inflationary times will lead on average to a slightly higher claim ratio for December renewals and slightly lower for January. Because the company will apply a credibility formula this means that the smaller fleets will gain or lose if their renewal dates differ much from 1st July. However, unless rates of inflation are very high the differences will be small. If more accuracy were required we could revise the table every one, three or six months, but this seems a needless complication and we doubt whether it would be commercially justifiable or, in many quarters, understandable.

Credibility

If we find that the claims ratio of a fleet is 50% averaged over three years and we aim at 80%, what is a reasonable ratio to apply for charging? The answer is obviously somewhere between 50 and 80 but how near to 50 or to 80? To calculate the answer requires some mathematical knowledge and it also requires us to make some assumptions about the variability and reliability of our claim costs and possibly also our way of measuring exposure. A rough working rule is that for a fleet of 50 vehicles insured comprehensively (unless there is a big excess, say over £500) then we can allow about one seventh of the difference between 50 and 80, that is accept 76% as the best estimate, for the time being, of the true cost of this fleet. In order to justify a claim ratio as low as 65% we ought to require a fleet of at least 2,000 vehicles exposed for at least three years.

The mathematical theory is examined in the appendix. However, when we come to apply our theory we have to make a number of guesses about the variability of, and any bias in, our data so that a fairly rough and ready table will be adequate. One company aims at applying the table below based on a target 75% claim ratio on book rates, but it must be recognised that in the market place as it exists in the United Kingdom it is easier to get more premium than the formula would allow if there has been a big claim and less easy if there has not. But it is vitally important to spread far and wide the fact that actual results from fleets are surprisingly poor guides until there are thousands of vehicle years exposure available and that from fairly recent experience.

PERCENTAGE ADJUSTMENT TO "STANDARD" PREMIUMS

Claims Ratio % Over Last 3 Years

No. of Vehicles	From To	- 40	40 50	50 60	60 70	70 80	80 90	90 100	100 110	110 +
up to 20		-4	-3	-2	-1	0	1	2	3	4
21 - 50		-8	-6	-4	-2	0	2	4	6	8
51 - 100		-12	-9	-6	-3	0	3	6	9	12
101 - 200		-16	-12	-8	-4	0	4	8	12	16
201 - 400		-20	-15	-10	-5	0	5	10	15	20
401 - 800		-24	-18	-12	-6	0	6	12	18	24
over 800		-28	-21	-14	-7	0	7	14	21	28

This table is based on a target 75% claims ratio, and allows rather more to the experience of small fleets than is strictly justifiable theoretically.

The figures above relate to comprehensive cover with, at the most, a small excess. Where there is a large excess or the cover is non-comprehensive the size of fleet required to give a similar credibility is anything from 5 to 10 times as big - in other words except for a fleet so big as to justify self-insurance plus an excess of loss treaty, the actual experience of such a fleet is of very little value indeed. If any underwriter wishes to challenge that view he should produce properly documented results: it is not something that can be determined properly without some fairly extensive and careful statistical analysis.

Large Claims

We have obtained distributions of numbers of claims by size from several large portfolios. For recent years the numbers and amounts of large claims are still uncertain and it is thought that many have been estimated on a very conservative basis. We have therefore combined our data and produced a distribution of claims that seems likely to apply in 1979 for a fleet insured comprehensively with a small excess. The numbers of small claims will be very dependent on the amount of the excess and the practice of the fleet owner in regard to minor incidents. For example one fleet may produce an average of one claim per vehicle year with an average of £70 whereas another quite similar fleet with a small excess may produce only one claim for every four vehicle years but with an average of £230. We have chosen our claim distribution to fit the case where there is one claim every three years with an average of £180 so that with a three year experience the expected number of claims is equal to the number of vehicles in the fleet. This is convenient for the reader and is sufficiently typical of real life to permit a clear demonstration of the principles. Out of 10,000 claims in the year 1979 we might expect them to be distributed as follows:-

AMOUNT OF CLAIM £	NO. OF CLAIMS	APPROXIMATE COST £000	% OF TOTAL COST IN BAND	CUMULATIVE % TOTAL COST
Nil	1350	0	0	100
0-50	3000	90	5	100
50-250	4000	500	28	95
250-1000	1350	455	25	67
1000-2500	265	375	21	42
2500-10,000	28	95	5	21
10,000-50,000	5	125	7	16
Over 50,000	2	160	9	9
	10000	1800	100	

It will be seen that over 20%* of the total cost in 1979 will probably arise from claims over £2,500. However, if our fleet comprises only 100 vehicles then we can expect on average only one claim of that size in every third fleet in a three year experience. Obviously we cannot have one third of a claim in any one fleet: There must be an integral number of them - and if we have a lot of fleets we find that roughly:

7 out of 10 give rise to no claims of this size.
1 in 4 give rise to one claim of this size.
1 in 20 give rise to two or more claims of this size.

It is clear therefore that whilst the contribution of these large claims to the experience is substantial our three year experience of a fleet of 100 tells us very little about them.

There is a simple and fairly effective way out. It is to disregard any amount over some fixed limit and to take the curtate total as defined earlier. We then need to multiply this curtate total by a grossing-up factor.

We have had data from several companies in regard to grossing-up factors based on the experience in all of many millions of vehicle years and the factors in the table below can be accepted as being suitable for 1979. One thing is certain, namely that any allowance significantly less than this for the ultimate cost of large claims is likely to involve the underwriter in loss. The grossing-up factors will require adjustment from year to year, those for comprehensive cover will grow at the rate of probably .01 to .03 each year whilst for non-comprehensive the increases will be of the order of .05 to .1.

* This is an average figure, but is fairly sensitive to the incidence of the odd claim or so over £50,000 where the amount is likely to vary greatly from year to year and may occasionally involve a serious catastrophe.

GROSSING UP FACTORS FOR USE IN 1979

COVER	Limit at which claims are curtailed £ 1,000	1,500	2,500	5,000
Comprehensive no excess	1.34	1.26	1.19	1.14
Non-comprehensive	2.8	2.5	2.2	1.8

The non-comprehensive figures are taken from a much smaller experience and will be affected by the incidence of claim sharing and Knock-for-Knock Agreements. There is no doubt however about their being of the right order of magnitude.

Conclusion

We think it should be clear from our analysis, which we must emphasise is based on the results of some very large fleet accounts and on other data relating to large claims, that an analysis of fleet records with fewer than say 1,000 vehicle years exposure is of very little value indeed as an indication of the likely future cost of that fleet, even if its use and composition have been and continue to be stable. Where cover is non-comprehensive a much larger exposure would be needed.

If the fleet can be measured against some standard, such as book rates, preferably designed for this purpose and adjusted annually in inflationary times, then we have shown the extent to which regard may be had to the individual fleet results. It is emphasised however that many features of a risk can be allowed for only by the skill and experience of an underwriter. What we hope we have shown is that he should pay very little regard in most cases to claim data.

We append, for the mathematician (only), a note explaining the theory underlying this report.

APPENDIX 1.

Truncation of Large Claims

1. It is inevitable that many small fleets will provide insufficient loss experience to establish a stable loss rate, because of the very skew nature of the claims amount distribution. Actual experience will then be characterised by many fleet cases giving experience over quite an extended period which is below the true loss rate for the fleet, together with a small number of cases where the experience is very bad simply because one or two very large claims have occurred.

2. In order to get a truer average experience, some effort must be made to reduce the effect of the skewed claims distribution and one way of doing this is to truncate the observed experience at a chosen claim size. By truncation we mean that all claims above the cut off size will be included at the cut off value instead of their true value. The overall result for a fleet must then be scaled up to balance out the reductions on average.

3. If this process is to be worth undertaking, we need to understand what gains will accrue, and we need to know what size of multiplier we will be using typically. We have tackled this in two ways, firstly by giving an example on a theoretical distribution, and secondly by showing that this lines up well with results from real data.

Mean and Variance of Claims Cost per Vehicle Year

4. For the purposes of planning, it is not too unreasonable to regard the claim process as being a Poisson process. If the claim frequency per vehicle year is ℓ and the second moment about the origin of the claim size distribution f (x) is m_2, then the variance of average claim cost per vehicle year is given by

$$V = \ell m_2$$

$$\text{with } m_2 = \int_0^\infty x^2 f(x)\, dx$$

The mean claim cost is lm_1 where m_1 is the average claim

$$m_1 = \int_0^\infty x f(x)\, dx$$

5. If, now, we truncate the distribution at a point L, the mean claim of the resulting distribution is given by

$$m_1(L) = \int_0^L x f(x)\, dx + L\,[1 - F(L)]$$

and the multiplier K to apply to the truncated result (i.e. the GUF) is given by

$$K = \frac{m_1}{m_1(L)}$$

The variance of the adjusted claims cost per vehicle year is

$$V(L) = K^2 \ell \left\{ \int_0^L x^2 f(x)\, dx + L^2 [1 - F(L)] \right\}$$

and the adjusted co efficient of variation is

$$C(L) = \frac{\sqrt{V(L)}}{\ell m_1}$$

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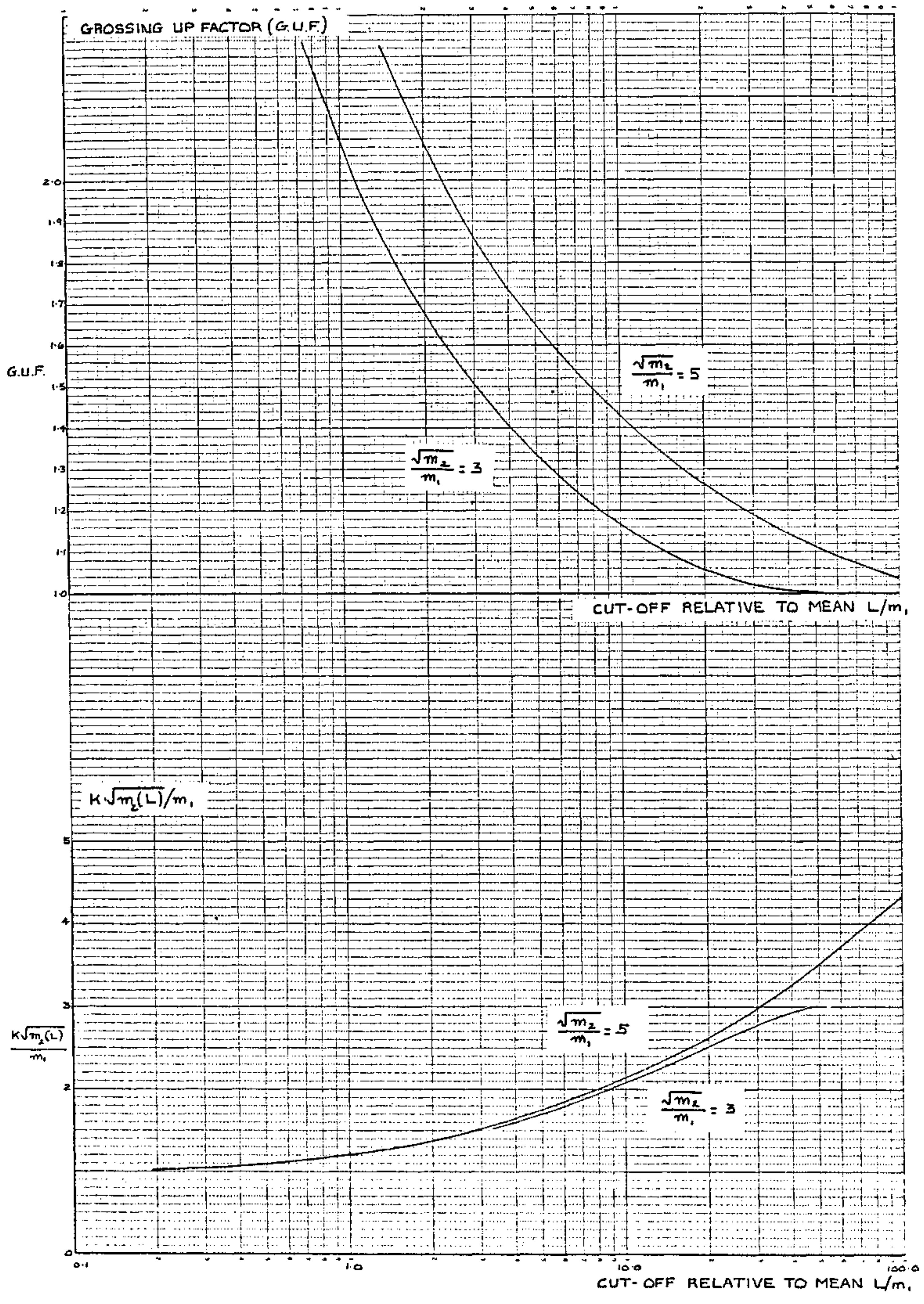
GROSSING UP FACTOR and $K\sqrt{m_2(L)}/m_1$ for the CLIPPED PARETO EXAMPLE



6. By reducing L, we will reduce V(L) until in the limit, the variance is determined just by the Poisson variability. However, in practical terms, values of L less than a few times the mean would not be used since other questions would arise, such as the bias in estimating K, the heteregeneous mix of claims types destroying the representation of claims by a single function f(x), and so on. We prefer instead, simply to plot out the behaviour of the adjusted co efficient of variation C(L), so that this can be discussed in practical terms.

Pareto Example

7. In order to be reasonably typical, a theoretical distribution should have a long tail with a high co efficient of variation and a Pareto curve may suit. In order to produce an m_2 near 3 to 5 observed in practice, however, we need the exponent to be near the value which produces infinite variance. While the algebra is still tractable for such a case, we have preferred to use the infinite variance case with the top then cut to give the required co efficient of variation.

Thus we choose

$$F(x) \triangleq \begin{cases} \left(1 - \frac{a}{x}\right) \Big/ \left(1 - \frac{a}{A}\right) & a < x < A \\ 0 & \text{otherwise} \end{cases}$$

where a, A are the lower and upper limits respectively. For this distribution:

$$m_1 = \frac{a}{\left(1 - \frac{a}{A}\right)} \log \frac{A}{a}$$

and $m_2 = aA$

and $$\frac{\sqrt{m_2}}{m_1} = \frac{\left(1 - \frac{a}{A}\right)\sqrt{\frac{A}{a}}}{\log \frac{A}{a}}$$

8. Typical values of a/A are 0.0034 for $\frac{\sqrt{m_2}}{m_1} = 3$ and 0.0008 for $\frac{\sqrt{m_2}}{m_1} = 5$.

If we now truncate this distribution at L, we must use as a multiplier:

$$K = \frac{\log \frac{A}{a}}{\left[\log \frac{L}{a} + \left(1 - \frac{L}{A}\right)\right]}$$

and claims cost variance per vehicle year is given by

$$V(L) = \frac{\ell K^2 a}{\left(1 - \frac{a}{A}\right)} \left[2L - a - \frac{L^2}{A}\right]$$

9. For the typical values quoted, graphs of the multiplier K and the factor $\frac{\sqrt{m_2}}{m_1}$ (which represents C(L) apart from the multiplier $\sqrt{\ell}$) are plotted in the facing figure. The abscissae scale is the ratio of truncation point to original mean i.e. L/m_1 in order to be able to relate more easily to real figures.

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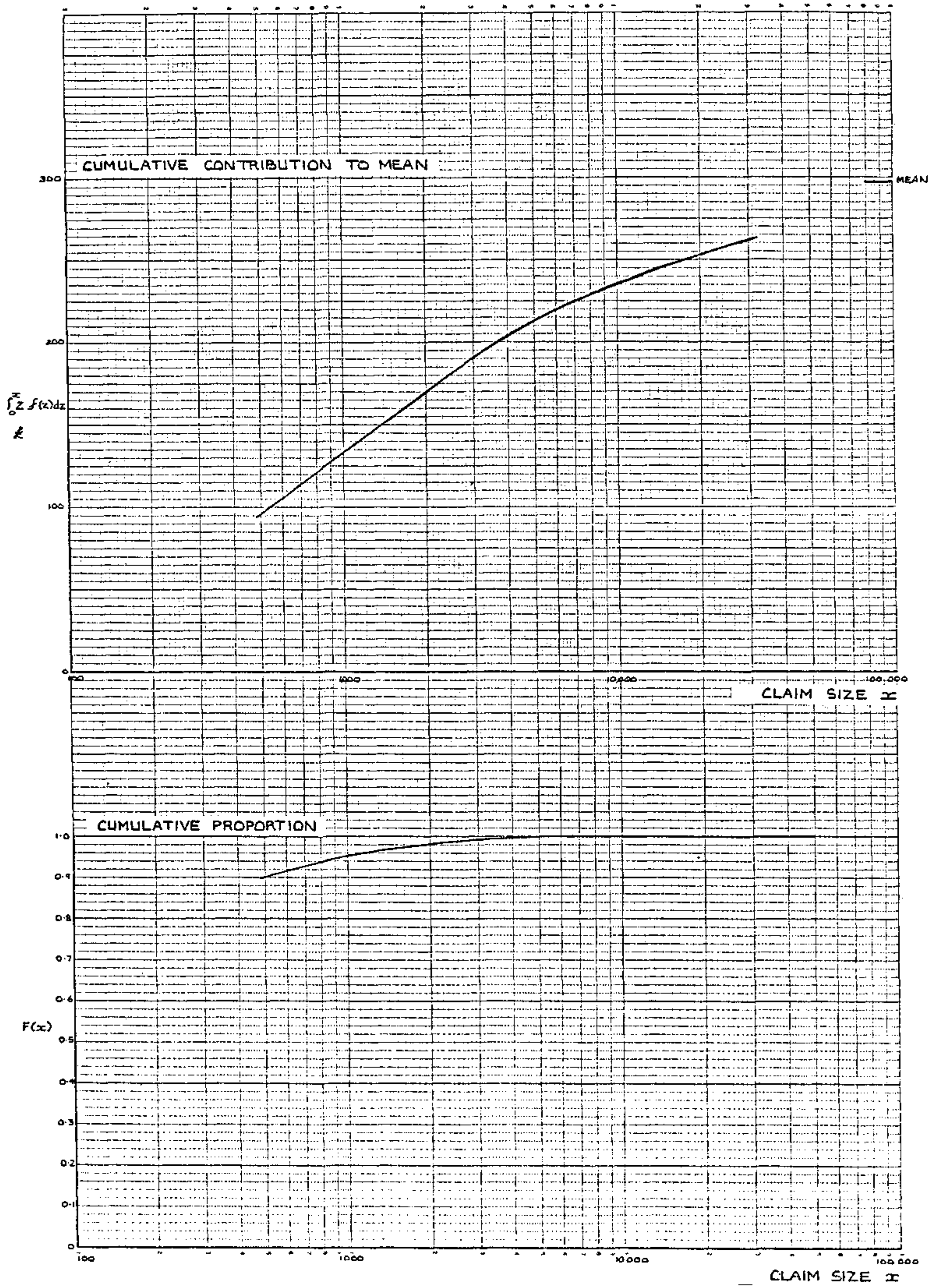
10. It is clear that a steady fall in L/m_1 steadily reduces $\frac{\sqrt{m_2(L)}}{m_1(L)}$ towards unity. In practical terms, we might choose to truncate at 2 to 10 means and the necessary multiplier would be in the region 1.2to 2. The improvement in stability is useful but not dramatic in aggregate terms. The main point to recognise is that individual small fleets which do not experience claims above the truncation point need a substantial loading.

11. In view of the size of the effect, it is essential to test out the position on real data. Theoretical curves, such as the Pareto are very useful for sketching out possibilities if the parameters chosen are reasonable, but the true shape of the tail could have a substantial effect on the size of multiplier needed and the reduction in variance achieved for a given truncation level. There may be a great difference between a reasonably shaped curve and a statistically established one.

12. We have therefore carried out calculations on the claim size distribution for Fleets for one Company for one year of origin, as a further illustration. The particular case chosen was for a mix of Comprehensive and non-Comprehensive business; it might be expected that comprehensive cover would produce somewhat lower multipliers and non-comprehensive would produce somewhat higher than this data indicates. The data was for a recent year so that many claims carry substantial outstanding estimates which may be expected to produce some ultimate saving in aggregate although some individual ones may be revised upward, in the light of revised medical prognosis, for example. This particular year also contained more claims over £30,000 than usual; while the numbers are small, the proportions in the extreme tail are a little high.

13. While these points about the quality of the data have to be borne in mind, and absolute values would be in need of adjustment for some purposes, the broad shapes are well founded. Indeed figures for earlier years developed to the same date give a very similar picture when due allowance is made for monetary inflation. The best composite information currently available is given in the main text.

CUM. CONTRIB. to MEAN and CUM. PROPORTION for FLEET DATA for ONE COMPANY, ONE YEAR

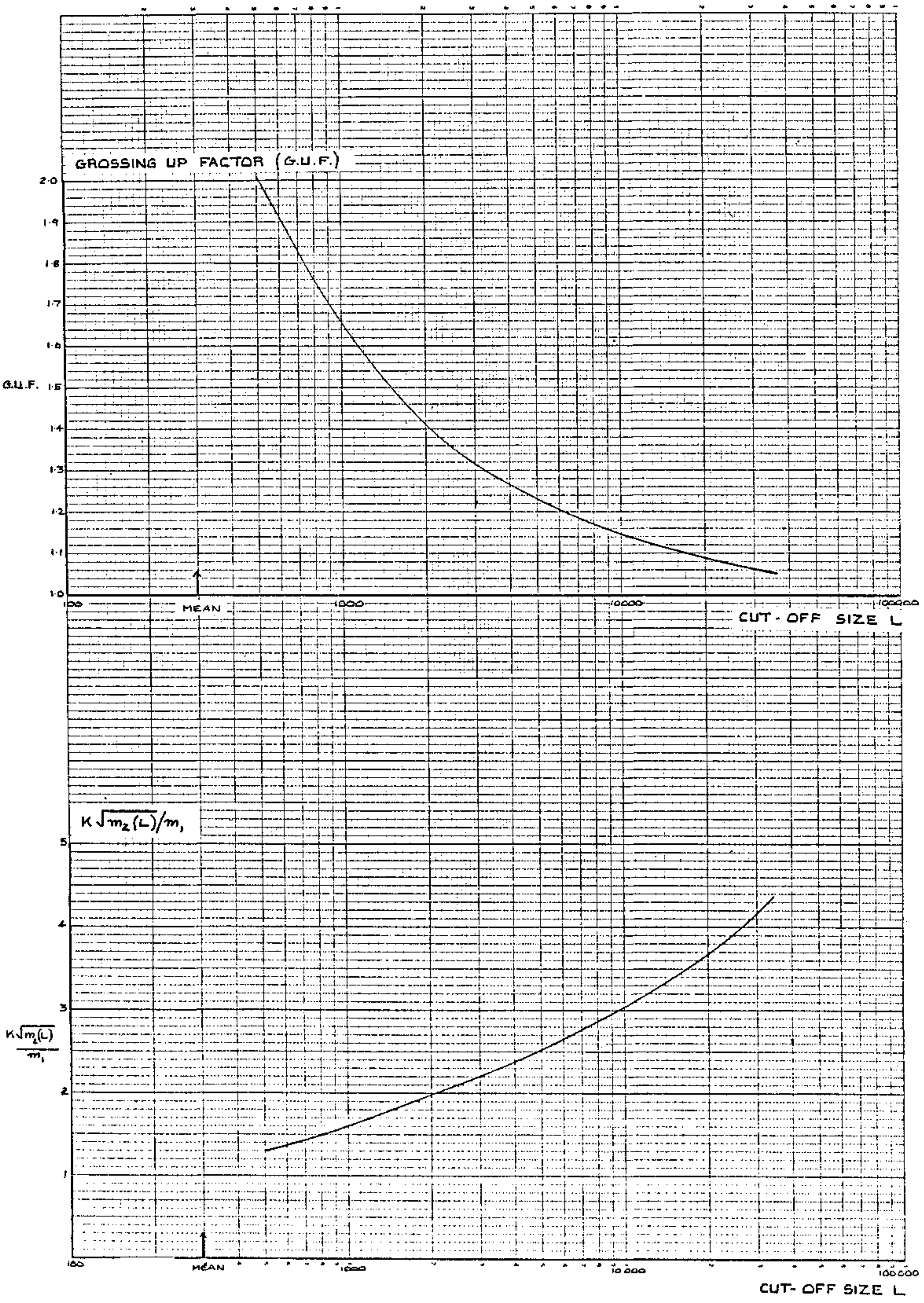


14. We thought that it would be helpful to give a pictorial view of the 'long tail'. Unfortunately this is not easy to do, since the tail is so long that it cannot be plotted on linear graph paper, while a log scale conceals from the eye the point we are trying to make. We settled for the form of presentation opposite showing in two graphs the cumulative proportion by number and the cumulative contribution to the mean as claim size increases. These illustrate the following significant points.

15. About 90% of all claims were below £500 (values being related to 1977 prices) but these only contributed about one third of the overall mean claim of just under £300. Almost all of the remaining 10% lay in the range £500 - £5,000 and these contributed a further £125 or so. The remaining tiny proportion (0.6%) of claims above £5,000 contribute a further £85 or so. We can easily calculate by subtraction the contribution to mean between any two claim sizes which may interest us.

16. These graphs confirm in broad terms points made in the main text. Taking a different illustration of a large fleet of 1000 vehicles with 4 years available experience and a frequency of 0.3, we would expect to have about 1200 claims on which to base our judgement of experience. About 28% of the cost would be determined by the occurrence and size of 7 claims during the 4 year period. Another 120 claims would determine another 40% or so.

GROSSING UP FACTOR and $K\sqrt{m_2(L)}/m_1$ for FLEET DATA for ONE COMPANY, ONE YEAR



17. Turning now to the question of curtailing the distribution, we show opposite two graphs plotted against cut-off size of claim. The upper one shows the grossing up factor (GUF) necessary to return the overall average to the original uncurtailed average. The lower shows the ratio of the second moment about the origin of the curtate distribution as a ratio to the original mean; (The uncurtate distribution has a ratio above 5). Suitable cutting will reduce this as near as we like to unity. A cutting point of £10,000, for example, reduces the figure to 3 while a cutting point of £1,000 (3 times the mean roughly) gets the figure to 1.6.

18. As we have remarked, cutting at too low a level becomes dangerous because the assumptions on which the theory is built start to become dubious, and the upper graph shows that we may be talking about a GUF well above 2.

19. A practical level may be around £2,000 where most of the reduction in variability has been gained and at this level, we need a GUF around 1.4.

20. A split of GUFs between Comprehensive and non-Comprehensive for a range of cut-offs is given in the main text. The consequences of such cutting on the weight which may be placed on the experience data is discussed in the second Appendix.

Bias introduced by the Cutting Process

21. There are certain theoretical problems associated with the curtate distribution which we discuss briefly here. We do not believe that these are sufficient to throw doubt on the GUF's recommended at the present state of the art.

22. The GUF is calculated from an overall claim size distribution. This implies a model where individual vehicle claim size distributions are identical. Furthermore, the GUF is biased in the statistical sense, since increasing the sample size available does not alter K. To an extent, this latter point might be taken care of, at least in theory, by using a cut-off point which made the co efficient of variation of claims cost constant for all fleets; thus the cut-off point would increase for a given fleet as more vehicle years of experience became available and the GUF would gradually reduce towards unity.

23. However in practical terms we have to consider the cost and complication of doing this against any perceived gain in accuracy. When one considers the size of fleets we are talking about in most cases, their changing composition from year to year, the incomplete nature of the information available, particularly our deficient knowledge of the exact shape of the tail of the claims distribution for a given case, we conclude that a simple to apply rule which is based on an established overall distribution is appropriate. Where a clear division of the overall information appears necessary and data is available, then obviously segmentation should be carried out, and we have taken the split of comprehensive and non-comprehensive cases to be the one essential split at this stage of our knowledge.

24. Insofar as our GUFs are biased for some cases, then we are using transfer pricing between fleets, in effect, since any small overall absolute affect would be taken out over time by rating considerations. We doubt whether such transfers would be any larger than other non-detectable transfers inherent in fleet rating.

APPENDIX 2.

Credibility Rating

1. In attempting to rate individual fleets on a rational basis, there are effectively three ingredients we would like to take into account.

   (1) Experience of similar risks in non-fleet circumstances.

   (2) Underwriters knowledge of the individual fleet, particularly quality of management and special features of business undertaken.

   (3) Actual experience.

   With sufficient quantities of (3), we could theoretically manage without (1) and (2) at all, but we cannot get anywhere near this in practice. This means that we have to find a way of formalising the three ingredients in such a way that we may decide how to give appropriate weights to each for a given fleet.

2. This is the realm of credibility theory, and of subjective probability in the case of (2), and the general background to this is explored briefly in this Appendix. However, we must say at the outset that, given the size of fleets we are dealing with and the quality of information on which decisions have to be based, we would regard the best approach available currently to be that set out in the main report i.e. a simple premium percentage adjustment table based on fleet size and claim ratio experienced. We would see any more elaborate solution as being some way into the future, and then only for the largest fleets.

3. Our approach in this appendix is first to obtain variances for use in one of the commonly used credibility formulae, mainly as a background to what follows and we give some idea of the fleet magnitudes involved. We then mention 'deemed' credibility and what is meant by 100% credibility. Then we discuss how knowledge of book rates can be brought into account. In order to bring in underwriting judgement, subjective probability has to be considered and we give a section on how this might be attempted (some day) in practice. Throughout, we illustrate with some fairly typical figures for fleets which point us to the conclusion of our main report.

4. Statistical Derivation

   The statistical derivation of one commonly known credibility formula is based on the assumption that our entire knowledge is that provided by loss experience for a number of years for each fleet. In particular, we make no use of book rates or underwriting flair. We work on the hypothesis that the average claim cost per policy year for a particular fleet will in part reflect the figure for all fleets, but this will be modified by the experience we have available for the particular fleet. The compromise we take will be dependent in principle on the within-fleet variance from policy year to policy year and the residual variance. These variances need to be established from the composite data available, and we may use a standard Analysis of Variance approach.

Cont/....

5. The basic unit on which distributions are established is the vehicle year. However, we will not normally have recorded losses for each individual vehicle in a fleet and we assume that calculations will have to be based on fleet loss totals in each policy year. However, we assume that the number of vehicle years in each fleet in each year is known and we ignore between-year variation. This therefore amounts to a hierarchic analysis of variance with no information available to compute directly the error sum of squares; instead, the error sum of squares is estimated from the within-fleet between-years sum of squares.

6. We can visualise the data for m fleets as follows:-

Fleet No.	Loss in the year and (No. of vehicles in the year)	Loss total for the fleet and (No. of vehicle years) for all available years
1	C_{11} (n_{11}), C_{12} (n_{12}), ...C_{1n_1} (n_{1r_1})	C_1 (n_1)
2	C_{21} (n_{21}), ...	C_2 (n_2)
.	.....	
.	.....	
m	C_{m1} (n_{m1}), ...C_{mn_m} (n_{mr_m})	C_m (n_m)

We also write $C = C_1 + C_2 + \dots C_m$ for the grand loss

$N = n_1 + n_2 + \dots n_m$ for the total No. of vehicle years

and $R = r_1 + r_2 + \dots r_m$ for the total No. of fleet years

We can then calculate the required quantities as follows:-

Source of Variation	Sum of Squares	Degrees of Freedom	Mean square estimates
Between Fleets	$\sum_{i=1}^{m} (C_i^2/n_i) - \frac{C^2}{N}$	$(m-1)$	$M_2 = \sigma_e^2 + \bar{n}\,\sigma_f^2$
Within Fleets	$\sum_{i=1}^{m} \sum_{j=1}^{r_i} (C_{ij}^2/n_{ij}) - \sum_{i=1}^{m} (C_i^2/n_i)$	$(R-m)$	$M_1 = \sigma_e^2$

Where $\bar{n} = \left(N^2 - \sum_{i=1}^{m} n_i^2\right) \Big/ \left(\overline{m-1}\;\; N\right)$

Cont/....

7. For use in our credibility formula, we may therefore estimate σ_e^2 by M_1 and σ_f^2 by $(M_2 - M_1)/\bar{n}$. In so doing, we must clearly recognise that between-year variation has been taken to be zero. If there were, in fact, significant variation due to the weather, say, then σ_e^2 would be over-estimated while the precise effect on σ_f^2 is not easy to determine, since in subtracting M_1, we are subtracting an estimate which has been pooled over years.

8. *The credible average cost per vehicle year for the ith fleet is then estimated* by:-

$$\hat{C}_i = \left(\frac{n_i\,\hat{\sigma}_f^2}{n_i\,\hat{\sigma}_f^2 + \hat{\sigma}_e^2}\right)\cdot \overline{C}_i + \left(\frac{\hat{\sigma}_e^2}{n_i\,\hat{\sigma}_f^2 + \hat{\sigma}_e^2}\right)\cdot \overline{C}$$

where $\overline{C}_i$ is the observed marginal average for the ith fleet and $\overline{C}$ is the observed grand average

i.e. $$\overline{C}_i = \frac{C_i}{n_i}\,, \quad \overline{C} = \frac{C}{N}$$

9. Stability of Experience Data

The residual error σ_e^2 can be considered further in theoretical terms if we consider that the claim generation process is that one vehicle year of risk will generate a random number of claims each of which then produces on independent claims amount from an identical size distribution F(x).

10. For planning purposes, it is sufficient to regard the claim generation process as being Poisson with intensity ℓ per annum and if m_2 is the second moment about the origin of F(x), the variance of the claims cost per vehicle year is given by

$$V = \ell\, m_2$$

It is convenient to scale m_2 with respect to the mean claim m_1 by writing

$$\sqrt{m_2} = C_0 m_1$$

so that $$V = \ell C_0^2 m_1^2$$

If we have N vehicle years of exprience available, the variance of the total amount is $N\ell C_0^2 m_1^2$ which can be compared with the total claim cost $N\ell m_1$ to give a resulting coefficient of variation for the experience

$$\frac{C_0}{\sqrt{N\ell}}$$

11. This simple formula gives us enough to get an idea of the amount of experience we need to be able to place a given degree of reliance on the claim cost. Typical observed claim size distributions have values of C_0 in the order 3 to 5. Basing our calculations on a Normal distribution (90% certainty equivalent to 1.645 *SD from mean)*, if we wanted to be 90% sure that our observed average was within 5% of the true average, we would need

$$0.05\ \sqrt{N\ell} = 1.645\ C_0$$

Cont/...

12. Taking, for example C_0 to be 3, we would need the number of claims N1 to be 9742. Taking a claim frequency of say 0.25 p.a., we would need no less than about 40,000 vehicle years of experience. Of course, if we were prepared for only 10% accuracy, this would be reduced to 10,000 vehicle years, for example. A fuller discussion of this point is given by Hansen: Proceedings of CAS VOL LIX 1972 and cited references.

13. As we indicated in Appendix 1, this problem can be alleviated by truncating the claim size distribution. If we wanted to restrict the G.U.F. to be no more than 1.3, this would only allow C_0 to fall to about 2.4: in the example given in the main text, this would still mean that 100 vehicle years of experience would have an even chance of being 30% out. If we allowed the multiplier to increase to a figure in the range 1.5 to 2 so that C_0 could be brought down to 1.5 say, the 100 vehicle years of experience could still be 20% out on an even chance.

14. Thus the gains from truncation, are useful but not spectacular. They reduce the amount of data required for a given level of accuracy by the square of the reduction in C_0 which may be by 2 to 15 times depending on the G.U.F. selected, but this still leaves one needing hundreds of vehicle years to achieve any sort of accuracy.

Using Book Rates

15. The difficulty with the approach so far given is that the between fleet variance σ_f^2 will be large in most Companies because of the mix of vehicle types, districts, etc. Nevertheless, if there are only a few vehicle years of experience available for a particular fleet, the credibility formula will cause the credible mean claim cost to be near the overall fleet average. The rate so indicated would be unlikely to accord with the Underwriter's feeling about the likely needed rate. This is not so much a shortcoming of the method as such as a recognition that the data has to be further segmented into appropriate risk groups, leading in principle to a multiway analysis of variance. In practice, there is insufficient data to contemplate other than the most rudimentary of splits. What we must do instead is assume that such differentials as are necessary are the same as those experienced on normal Commercial Vehicle business.

16. The establishment of book rates for single Commercial Vehicles itself requires a multiway analysis using data along the lines used by MRSB for Private Car analysis, and now being established for Commercial Vehicles. However, we go into this point no further here but simply assume that suitable book rates have been established on a suitable volume of business. (Several strictures apply to the use of such rates, and these are referred to in the main text).

17. With a reasonably stable portfolio, one may then simply use the normal credibility formulae but replacing the observed grand average by the claim cost implied by the book rate obtained by aggregating over the rates on each category of vehicle in the fleet, and replacing the between fleet variance by a variance based on the presumed volume of business on which the book rate is based.

18. One simple version of this is derived by putting into the credibility formula:

$$\sigma_f^2 = \frac{\sigma_e^2}{\lambda N_p}$$

where Np is the assumed number of vehicle years on which the book rates are based. We then obtained for $\hat{C}$, the credible average claim cost per vehicle year:

$$\hat{C} = \frac{N_p}{N + N_p} C_p + \frac{N}{N + N_p} C$$

where Cp is the book rate, C is the observed average cost, and N is the number of vehicle years of experience available.

19. This formula may be rearranged to show Q, the percentage adjustment to Cp:

$$Q = \left(\frac{N}{N_p + N}\right)\left(\frac{C}{C_p} - 1\right) \times 100$$

If, for example, the book rate is based on 2000 vehicle years worth of experience with Cp = £75 we have

$$Q = \left(\frac{N}{2000 + N}\right)\left(\frac{C}{75} - 1\right) \times 100$$

A fleet of 600 vehicles with three years experience producing losses of £45 p.v.y. would then be allowed a discount of

$$Q = \frac{1800}{3800} \times \frac{30}{75} \times 100 = 19\%$$

If the fleet were only 75 vehicles, the discount would be

$$Q = \frac{225}{2225} \times \frac{30}{75} \times 100 = 4\%$$

20. One Company is aiming to apply just such a table, although the discounts given for small Fleets are too high in relative terms (they are in any case small in absolute terms) for commercial reasons.

The main text also gives rough working rules for small and large fleets; the rule for small fleets is again relatively very generous since the small number of claims generated in three years (i.e. 35 to 40) will give an insecure loss ratio.

21. Where the fleet make up has not been steady over the available experience years, the past claim cost experience will not be so valuable without some modification. One way of achieving this modification is to use as the base for exposure, not the vehicle year, but the book premium. Assuming that book rates do reasonably reflect experience, claims costs for different years will then tend to line up. For example, if the portfolio has been changed from 40 vehicles of a type costing £45 p.v.y. to 20 of a type costing £60, the office rates on the former should be say £60 and the latter £80 so that the loss ratio for both is 75% and we may use a composite experience of 60 vehicle years. If there have been office problems with expense or other loadings, or effects from timing of rating changes, the appropriate corrections would have to be made; indeed, whenever we refer to book rates, we are really referring to the pure premium (i.e. £45 and £60 in the above example) rather than the office premium.

'Deemed' Credibility

22. The usual form of the credibility formula will give the book rate if there is no experience available, and would give the experience rate only if there were an infinitely large sample of experience available. There is a body of opinion and a great deal of practical usage, particularly in North America, which uses the idea of 100% credibility. An amount of experience which gives a chosen accuracy with a chosen probability is deemed to be 'fully credible' and an alteration is made to the credibility formula to 'fair in' the approach to full credibility as exposure increases. (See e.g. the General Insurance text book chapter 8, page 180). It is clear from figures we have given that say 2000 vehicle years experience using curtate claims would be a minimum, and a shaky one at that.

23. The difficulty with such a development is that the relationship with objectivity gets lost and the fairing in process becomes a mixture of theory, market practice, and market pressures. This has led to an enormous growth in the U.S.A. literature and 'actuarial practice'.

24. However, just to illustrate the argu ments, our formula

$$\hat{C} = \frac{N_p}{N + N_p} C_p + \frac{N}{N + N_p} C$$

would never quite charge the rate C. If Np and N were both 2000, we would bisect Cp and C. Suppose we decide that full credibility for a fleet is to be achieved by Nc vehicle years. Then one possible faired-in formula would be

$$\hat{C} = \frac{N_p (N_c - N)}{N_c (N + N_p)} C_p + \frac{N (N_c + N_p)}{N_c (N + N_p)} C$$

Cont/....

25. If full credibility is taken to be 2000 vehicle years, and the book rates are also based on 2000 vehicle years, we get

$$\hat{C} = \frac{(2000 - N)}{(2000 + N)} \quad Cp \quad + \quad \frac{2\ N}{(2000 + N)} \quad C$$

For a fleet of 500 vehicles, for example, we would give the experience a weight of 40% as opposed to the original formula weight of 20%.

26. Another possible faired-in formula would be

$$\hat{C} = \frac{Np(Nc^2 - N^2)}{NN_c^2 + N_pN_c^2} \quad Cp + \quad \frac{N(Nc^2 + NpN)}{NN_c^2 + N_pN_c^2} \quad C$$

and with the same figures

$$\hat{C} = \left(1 - \frac{N}{2000}\right) \quad Cp \quad + \quad \frac{N}{2000} \quad C$$

In this case a fleet of 500 vehicles would give 25% weight to the experience, more nearly in line with the original formla. Obviously, one can design any number of formulae which agree with the original formla for small N to any degree of accuracy but which arrive at unity when N = Nc.

27. Since different fairings, or indeed different assumptions about the number of vehicles available for determining the book rate, can give different weights to experience, use of such formulae depends on a market consensus to accept the deeming. Conversely, if a rating basis is accepted by the market, based on credibility theory or not, orderliness of the market ultimately results.

28. Our feeling is that 'deemed' credibility formulae may very well be satisfactory tools for use in individual cases with individual insured where a smooth formula which both parties accept is required. For the purpose of illustrating our points for the general run of fleets, we prefer to stick to the derived credibility formula. In the main report, we are suggesting rates which are based on derived credibility with commercial allowances for small fleets; while no doubt a formula could be invented to fit, it would not add anything to our credence (in the true sense of the word).

29. It may be appropriate to add at this point that, even in the derived sense, there are other possible criteria for credibility. Those interested are referred to 'Mathematical Methods in Risk Theory' by Buhlmann.

Approach Using Underwriting Judgement

30. A natural consequence of any credibility formla is to place the credible average loss close to the book rate where data is sparse. The shortcoming of this, and a possible reason for its low utilisation in the U.K. is that the result may not accord with the Underwriters own feelings about the risk. He will know of important features, particularly quality of management, special features of business undertaken and so on, which will cause him to want to rank a set of fleets differently from the results of applying credibility formulae. If he also has Commercial pressures upon him, he will rightly yield to his judgement. Since we are in the business of providing information which will actually be used, we ought to find a way of coping with this while still allowing the statistical evidence to have its proper say.

31. One solution to this would be to establish an approximate prior probability distribution for each fleet based on discussion with the underwriter, rather than on the between-fleet observations or the book rate. The mean and variance of this prior could then be used in the credibility formula.

Establishing the Prior

32. In order to establish the prior, we discuss with an underwriter with good knowledge of the group of fleets being rated what his own feelings about the average loss for a given fleet might be. The question and answer session for the fleet go something like this.

Q. Would you be surprised if this fleets average were at or near norm?
A. Somewhat surprised but not very.

Q. Would you be surprised if it were double?
A. Yes, very surprised.

Q. Could it be 50% higher?
A. Possible but unlikely.

Q. Could it be 20% higher?
A. Easily.

and so on.

33. It is easy to see that, by the above procedure, one quickly establishes a bell shaped curve which both locates the relative average loss the underwriter would place on this risk and also indicates the uncertainty of his judgement. This curve is achieved without using any numerical information at all. Indeed, it is important to avoid doing so; we do not ask if the average loss could be £50, we ask could it be a certain percentage from book rate. In other words we make use of intuitive ability to rank without risking blurring the result by relating to actual average losses.

34. An apparent weakness of this approach is that we have to establish the variance by making a numerical evaluation of 'very surprised' and other judgemental and even emotive phrases, although this can be overcome by a calibration process if there are sufficient fleets available to make it worthwhile. First of all a broad concensus is agreed. For example, a grading for placement on a normal curve tried in one office was (as overleaf).

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Very surprised	:	5% in tail
Unlikely	:	5% - 15% in tail
Possible but unlikely	:	15% - 25% in tail
Possible	:	25% - 35% in tail
Completely neutral	:	35% - 50% in tail

In another office, however 'very surprised' would be placed at 75% to 80%. Clearly, an understanding with the underwriter and technique would have to be developed, the most important feature being to achieve consistency of application to all fleets.

35. Provided that this consistency were achieved, the composite effect of all priors would produce a result which could be checked against the observed inter-fleet mean and variance and our judgemental scale could be re-calibrated to achieve finally usable prior means and variances.

Calibration

36. We assume that the average claim per vehicle year for the ith of m fleets for the jth year for the kth vehicle is drawn from a distribution f_i with mean μ_i and variance σ_e^2 (i.e. independent of vehicle or year). The prior distribution of μ_i is assumed normal with mean α_i and variance $K^2 \zeta_i^2$ where α_i and ζ_i are provided by discussion with the underwriter and the scaling factor K has to be established. The probability of the data is then given by

$$P = \prod_{i=1}^{m} \left[\prod_{jk} f_i\left(q_{ijk}\right)\right] \frac{1}{K\zeta_i\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{K_i-\alpha_i}{K\zeta_i}\right)^2}$$

This is minimised for variation in K by

$$K^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\mu_i-\alpha_i}{\zeta_i}\right)^2$$

37. This expression cannot be used directly since μ_i values are not known and we must use $\hat{c}_i$ instead. Consider the estimator

$$\hat{K}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\bar{c}_i-\alpha_i}{\zeta_i}\right)^2$$

then :

$$E\left\{\hat{K}^2\right\} = \frac{1}{m}\sum_{i=1}^{m} E\left\{\left(\frac{\bar{c}_i-\mu_i+\overline{\mu_i-\alpha_i}}{\zeta_i}\right)^2\right\} = K^2 + \sigma_e^2 \ \frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{n_i\zeta_i^2}\right)$$

if it can be assumed that

$$E\left\{\frac{(C_i-\mu_i)(M_i-\alpha_i)}{\zeta_i^2}\right\} = 0$$

38 Subject to the given assumptions, a satisfactory estimator of K is

$$\hat{K} = \sqrt{\left\{\frac{1}{m}\sum_{i=1}^{m}\left(\frac{\bar{c}_i-\alpha_i}{\zeta_i}\right)^2 - \sigma_e^2 \ \frac{1}{m}\sum_{i=1}^{m}\frac{1}{n_i\zeta_i^2}\right\}}$$

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Credibility Formula with a calibrated Prior

39. If the a_{ijk} are drawn from a normal population, then for a particular fleet, the posterior probability is minimised for variation in μ_i by setting

$$\sum_{jk}\left\{\left(\frac{a_{ijk}-\mu_i}{\sigma_e^2}\right)-\left(\frac{\mu_i-\alpha_i}{K^2 \zeta_i^2}\right)\right\} = 0 \qquad \text{whence}$$

$$\hat{C}_i = \frac{n_i K^2 \zeta_i^2}{(n_i K^2 \zeta_i^2 + \sigma_e^2)}\,\bar{C}_i + \frac{\sigma_e^2}{(n_i K^2 \zeta_i^2 + \sigma_e^2)}\,\alpha_i$$

40. When estimates for σ_e^2 , ζ_i^2 and K^2 are inserted in this formula, its direction correspondence with the previous credibility formla can be seen, the inter-fleet variance and overall average being replaced by the underwriters calibrated prior variance and mean for each fleet.

Residual Variance

41. If the credibility formla is used, the residual variance in the credibility estimate is given by:

$$V(\hat{C}_i) = \frac{\sigma_e^2 K^2 \zeta_i^2}{(n_i K^2 \zeta_i^2 + \sigma_e^2)}$$

This formula exhibits the properties we would expect. If M_i is small the expression is near to the prior variance $K^2 \zeta_i^2$ while it m_i is large, the expression is close to σ_e / n_i

Use of Residual Variance in discussions with the Underwriter

42. The residual variance is an important feature of the rating and negotiation process. We can envisage a particular case of small fleet where information is sparse but the underwriter is also unsure of himself, resulting in a credibility formula where the two are given roughly equal weights. Those who draw comfort from applications of formulae would be tempted to use the credible average as though it conferred a sacrosanct answer, unless they made reference to the residual variance.

43. The alternative would be to quote the credible average and the residual standard deviation (or a confidence interval) to the underwriter so that he was fully armed for his negotiations. It could be argued that there would be a temptation in tight market circumstances to under-rate too many cases if wide confidence intervals were quoted; this may well be so, but it is not a matter for the actuary unless he is also the negotiator responsible for the whole fleet portfolio. The job of the actuary per se is to make clear the basis on which the decision has to be made.

Some Objection to Method

44. Use of priors

Many objections have been raised against Bayesian methods where there is no justification for the choice of prior, and we would agree. In this case we do have abasis for each prior, albeit a subjective one.

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45. Use of Subjective Belief

The arguments on subjective belief are too big to encompass here. Our justification would be that underwriters we have spoken to both understand what we would be trying to do and accept its validity, while they are suspicious of the usual 'objective' credibility approach because they are aware of the wide heterogeneity of the risk portfolio. In addition, the common sense behind their ranking seems clear, while calibration then removes too much subjectivity.

46. A second objection sometimes made to subjective probability is that different results will be obtained by different underwriters:

47. Firstly, let us consider the approach where the re-calibration process is not applied. It would be possible to imagine a very determined underwriter who believed he knew the underlying differentials almost exactly; this would produce a narrow prior distribution and even a weight of statistical evidence would have little effect on the credible average. If this same underwriter were responsible for the rates, then one can argue that the results produced would nevertheless be correct in the circumstances. Any others would not be used. If several underwriters were involved, each producing different priors, then a weighted composite prior would have to be used. While one could certainly point to inconsistences, one cannot regard this as an objection to method if best judgement is being given. Whichever underwriter got the greatest weight for his prior presumably would carry the greatest weight in rating decisions, so again the credible results produced would be right in the sense of being acceptable for use. The above scenario is in any case somewhat unrealistic since we believe that in practice a reasonable compromise prior would be reached without much difficult in most cases.

48. If the calibration process is applied, then the determined underwriter who is right will be proved so (since the calibration will hardly change his prior); the determined underwriter who is wrong will find the prior widened out since his assumed averages will not agree closely enough with observations. Conversely a diffident underwriter who was, in fact, right would find the prior narrowed, and so on. Of course, this calibration process would need to be understood and accepted but again we do not see that as a practical difficulty.

49. Note that in the above method, the underwriters prior distribution is not necessarily centred on the book rate. For example, if he thinks that a fleet is badly managed, he will place his 'very surprised', 'unlikely', etc in such a way that his prior mean comes out above book rate. If he were pretty uncertain about his feelings, however, the book rate would usually lie within a standard deviation or two.

50. One other difficulty is that some Underwriters might find great difficulty in giving a fair prior estimate since they would be aware of coming commercial pressures. It is difficult to see how much could be done about this since it is not detectable, and the only course might be to restrict more the confidence intervals on the subjective scale.

Note also that the variance used in the credibility formula is the sum of the Underwriters prior variance and the book rate variance since uncertainty in either book data or the underwriters judgement adds to the uncertainty of the credibility rate.

Credible Fleet Size

52. It is of interest to do some rough calculations based on the previous theory to get an idea how the numbers of vehicle years of experience necessary for rating might be affected by bringing in the underwriter's intuition.

53. On the basis of the claims distribution given for one Company with a mean claim of £296, we might choose to cut at £1,500, which implies a multiplier of 1.5. To achieve say 10% accuracy with 90% probability, we would need about 900 claims per annum. If we assume a frequency of 0.25 p.a. This gives 3600 vehicle years of experience on this criterion. The average claim cost per vehicle year is £74 with SD of £270 after curtailing.

54. Now let us suppose that, for a particular fleet, the underwriter believes experience to be 10% worse than average (i.e. £81 pvy), he thinks 20% worse to be possible but unlikely and 30% to be very unlikely. Ignoring the calibration process and using the judgemental scale set out earlier, we would take 3σ to be very roughly 20% of £81, or £16, and the prior standard deviation would be a third of this, or say £5.

55. In our credibility formula we then insert

$$\begin{aligned} \zeta^2 &= 5^2 &= 25 \\ \sigma_e^2 &= 270^2 &= 72900 \\ \alpha &= 81 \times 1.1 &= 89 \end{aligned}$$

56. The credibility mean is then given by

$$\hat{c} = \frac{25N}{25N + 72900} \; C + \frac{72900}{25N + 72900} \times 89$$

If we take, for example, a sample mean c around £81, we get

$$\hat{c} = \frac{2025N + 6488100}{25N + 72900}$$

We need samples around 2900 vehicle years for the statistical experience to count as much as the underwriters judgement, and at that size, the credible mean would be £85, the average of $\bar{c}$ and £89.

57. If we had available 1000 vehicle years in fact, we would get a credible mean of £87. The residual variance is

$$\frac{\sigma_e^2 \, \zeta^2}{N\zeta^2 + \sigma_e^2} = \frac{72900}{N + 2916} = 18.6$$

The 3σ accuracy of the credibility mean is thus about £13, only a little less than the underwriters prior. Thus in this particular example, even 1000 vehicle years worth of data affects the rate or its precision only a little.

Cont/....

58. On the other hand, if the underwriter had a much less secure view of the fleets characteristics, the prior S.D. might be 30. Leaving other parameters the same, we get

$$\hat{C} = \frac{72900N + 6488100}{900N + 72900}$$

59. Only 81 vehicle years are now needed to be comparable with the underwriters judgement. With 1000 available, the credibility mean is £81.6, very close to the experience mean. The residual variance is given by

$$\frac{72900}{N + 81} = 67.4$$

The 3σ precision is about £25, much reduced on the prior but uncomfortably large in absolute terms. We are here faced with the limited best that experience and judgement together can do in these circumstances.

Summary

60. To sum up, our need for a way of balancing off the three elements - book rate, underwriting judgement, and experience can be tackled theoretically. While subjective probabilities may seem a flight of fancy, they are the only reasonable approach to 'scientific' underwriting.

61. Our reservations arise not from deficiencies of the theories but from the small purchase they are likely to gain in the particular case of fleets, where there is often likely to be insufficient experience to influence the book rate modified by underwriting judgement, where there may be too few fleets to calibrate the judgement, and where the ultimate residual variance may be uncomfortably large. Some exploration of the subjective side would be valuable in the future and could possibly command support by underwriters for practical use. For the present, it would be a sufficient gain if the consequence of high claim variance, small fleets and the need for large GUFs were recognised by the market.

Fleet Rating

A brief outline of two methods used in calculating the premium for fleet risks is included in the report. It is considered that during the session which discusses the report it would be useful if delegates could provide additional information on the methods used by their own company in estimating the premiums to be charged.

The information may include :

(a) The number of years of claims information of an individual fleet that are used.

(b) The treatment of large claims.

(c) The treatment of inflation.

(d) Different treatment of payments for property damage and bodily injury.

(e) Whether different treatment is given to held risks compared with new business.