

GENERAL INSURANCE STUDY GROUP

REPORT OF SOLVENCY WORKING PARTY

1. The Working Party on Solvency was established at the end of 1982, following discussions at the Stratford Seminar of the GISG, with the following terms of reference:

- (a) To review the lessons to be learnt from the Finnish report on Solvency of Insurers and Equalisation Reserves, and to suggest specific investigations which might be carried out in the U K in order to develop the Finnish work.
- (b) To consider the extent to which the variability of a company's results should be reflected in the methods and bases used for the valuation of the assets and liabilities.

2. A first report was presented to the Bristol Seminar in November 1983 and this led to a paper being presented to a sessional meeting of the Institute in February 1984. The Working Party was then reconstituted with its current membership and work began in earnest on setting up a simulation model of a general insurance company which could be used to explore further the problems of variability in both assets and liabilities and to develop work along similar lines to that carried out in Finland. The first results of this work were presented to the Cheltenham Seminar in October 1985.

3. The comprehensive simulation model has now been completed and formed the basis for papers presented to the International Conference on Insurance Solvency in Philadelphia, U S A in June 1986 and the ASTIN Colloquium in Tel Aviv in September 1986. The attached paper carries the work one stage further, with some modifications and refinements to the model and a more extended treatment of the issues raised in the consideration of solvency and the financial strength of a general insurance company. This paper is due to be presented to a sessional meeting of the Institute of Actuaries on 27 February 1987.

4. It will not come as a surprise to members of the GISG that the Working Party has not found it possible to suggest a simple formulation for solvency control which would rectify all the shortcomings of the present régime. Instead, the recommendation is that far more emphasis should be placed on the role of professional reporting within general insurance companies, ideally by actuaries. This is seen both

as a contribution to more effective management and as a possible way forward to a system of solvency supervision which would be more sensitive to the needs and circumstances of individual companies.

5. Some might say that this would simply pass the responsibility to the professional person who was required to report on the financial strength of the company. It cannot be denied that such reporting would raise many difficult issues in regard to the interpretation of the available information and the assumptions to be made for the future. However, we do not see the role of the professional report as being to provide a simplistic resolution of the complex of problems that underlie the operation of a general insurance company but rather to provide an improved structure in which essential management decision making can take place on a more informed basis.

6. A similar objective would be in mind in the application of such an approach to solvency control by the supervisory authorities, since the availability of a professional report on the financial strength of the company would permit a more informed dialogue to take place between the supervisory authority and the management of the company than is at present possible on the basis of the vast array of unsubstantiated numbers that appear in the current D T I returns. The situation would become much more akin to that which already exists in respect of life companies, where the principal basis for supervision is the report by the appointed actuary on the valuation of the long term liabilities.

7. The Working Party would welcome views on these proposals and in particular the suggestion that actuaries would be well placed to perform such a reporting role. A draft NORP which the Institute might issue to assist actuaries working in this area has been incorporated at Appendix 7 of the paper to encourage further discussion of the professional issues that would be involved.

8. It is one thing to suggest that an actuary should report on the financial strength of a general insurance company and quite another to say how he should do it. Our concern throughout has been to emphasize the importance of variability and uncertainty as they affect the dynamics of a general insurance operation and we would hope that all actuarial reporting in this area would emphasize these aspects. To this end, and without any intention of ruling out other alternative approaches, we have put forward the concept of emerging costs and the use of simulation as providing tools for tackling the very complex problems involved. We have shown how a simulation model can be used to explore the sensitivity of a company's financial position

to the various aspects about which there may be uncertainty and to the elements over which management may be able to exercise a significant degree of control.

Our model was designed primarily for the purposes of our research, but it would not be difficult to adapt it for use in the particular circumstances of an individual company. Indeed, it is our current intention to develop the program along these lines, so that it can be made more widely available and encourage practical application of the ideas by individual actuaries in their companies.

There will, of course, be room for considerable further research in exploring different ways of tackling these problems and modelling the operations of general insurance companies. We hope that the ideas we have put forward are both sufficiently well formulated in themselves to provide a basis for immediate practical application and also sufficiently far ranging in their scope to encourage further work by others to refine and develop appropriate methods and techniques. Such development will play an important part in determining whether the actuarial profession can establish a major role in the assessment of the financial strength of general insurance companies.



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23 September 1986

ASSESSING THE SOLVENCY AND FINANCIAL STRENGTH OF A GENERAL
INSURANCE COMPANY

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ABSTRACT

After reviewing some general issues concerning solvency and the problems associated with establishing the financial strength of a general insurance company using the traditional balance sheet concept, the authors put forward an emerging costs approach for examining the strength of a company. This enables the true nature of the assets and liabilities to be taken into account, including their essential variability. Simulation is suggested as a powerful tool for use in examining the financial strength of a company and in exploring the impact of alternative scenarios. A particular example of such a simulation model is then presented and used to explore the resilience of a company's financial position to variations in a wide variety of parameters. The model enables the user to quantify the probability that the assets will prove adequate to meet the liabilities with or without an assumption of continuing new business. This in turn permits an appropriate asset margin to be assessed individually for any particular company in the light of the strategy that the company intends to follow. Some of the implications of this approach for the management and supervision of general insurance companies are explored. The suggestion is made that the effectiveness of statutory supervision based on the balance sheet and a crude solvency margin requirement is limited. More responsibility should be placed on an actuary or other suitably qualified professional individual to report on the overall financial strength of the company, both to management and to the supervisory authorities.

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1. Introduction

1.1 This paper represents the culmination of some four years of deliberation by the Working Party on Solvency of the General Insurance Study Group. The Working Party was established at the end of 1982, following discussions at the Stratford Seminar of the GISG, with the following terms of reference:

- (a) To review the lessons to be learnt from the Finnish report on Solvency of Insurers and Equalization Reserves, and to suggest specific investigations which might be carried out in the UK in order to develop the Finnish work.
- (b) To consider the extent to which the variability of a company's results should be reflected in the methods and bases used for the valuation of the assets and liabilities.

1.2 The Working Party reported to the Bristol Seminar of the GISG in November 1983¹ with a review of the uncertainties affecting general insurance companies and some suggestions for ways in which the problem of setting standards of adequacy for technical reserves might be approached. A paper was presented to a sessional meeting of the Institute in February 1984².

1.3 After this the membership of the working party was substantially changed and work began on developing appropriate ways of translating the Finnish solvency study into a form appropriate for the UK general insurance environment. The first stage of work concentrated on modelling the run-off of a general insurance company writing no further new business. This built on the ideas put forward in the Working Party's earlier papers. It sought to quantify the impact of run-off uncertainties as they affect the liabilities and to model the effect of changes in the value of the assets on the ability of the insurer to meet its liabilities. The resulting model was described in papers presented to the ASTIN Colloquium in Biarritz³ and the Cheltenham Seminar of the GISG⁴, in October 1985.

1.4 The report of the Finnish solvency study⁵, edited and brought together by Professor Teivo Pentikäinen and Dr Jukka Rantala, was an inspiration for the work and the members of the Working Party had a number of valuable discussions with Professor Pentikäinen. Neither the asset risk nor the uncertainties inherent in the technical reserves had been covered very fully in the existing risk theory literature (see for example, Beard, Pentikäinen and Pesonen⁶) but this omission began to be rectified as a result of the Anglo-Finnish discussions⁷.

1.5 The Working Party proceeded to develop a more comprehensive simulation model of a general insurance company. Although designed primarily for solvency research purposes, it soon became clear that such a model could become a valuable analytical tool in a commercial environment for assessing the financial strength of a general insurance company and for investigating issues extending far beyond the narrow confines of establishing current solvency.

1.6 The new model was presented in a paper to the International Conference on Insurance Solvency in Philadelphia, USA in June 1986⁸ and to the ASTIN Colloquium in Tel Aviv in September 1986.

1.7 These later papers showed a change in emphasis as compared with the paper² presented to the Institute in February 1984. This reflected a number of factors. In the first place, the 1984 paper had met with some criticism in general insurance circles for propounding the idea that technical reserves should contain enough margins to ensure that they are adequate with a reasonably high degree of probability. Many felt that it would not be acceptable to introduce an actuarial concept of prudent reserving for solvency control purposes, when normal industry practice was to use the same figures for reserves in the returns to the Department of Trade and Industry as for provisions shown in the Companies Act accounts.

1.8 Although accountants were also guided by a concept of prudence in establishing appropriate provisions, this concept differed from actuarial prudence in being more concerned with ensuring that liabilities were not underestimated or omitted than with requiring cautious margins in the estimates in order to ensure a high probability of adequacy. Indeed many would argue that incorporating prudent margins could compromise the ability of the accounts to show a "true and fair view".

1.9 A further relevant factor was the knowledge that the Association of British Insurers (ABI), in response to pressure from the accounting profession, was preparing a draft Statement of Recommended Practice (SORP) for accounting for insurance companies. A discussion on what was desirable in this area took place in Staple Inn in May 1985 at a joint meeting between the Institute of Actuaries Students' Society and the Young Chartered Accountants Group of London. A first draft of the SORP was distributed to member companies of the ABI in December 1985 and was also sent to the Institute for comments.

1.10 The issuance of an agreed document is still awaited. However, it seems likely that the SORP will reinforce the idea that provisions should not contain contingency margins. Indeed it may suggest that it would be appropriate in certain circumstances to discount the provisions to allow for the effect of future investment income, a practice which is not currently widely adopted, at least in explicit form.

1.11 Reaction was also somewhat mixed to the proposal in the 1984 paper that a technical provision should be established to make "appropriate provision against the effects of possible future changes in the value of the assets on their adequacy to meet the liabilities". Many felt that this was a task for the solvency margin to perform and that it would be difficult to reconcile this approach with what might be thought appropriate for provisions in a Companies Act sense.

1.12 A major task for the Working Party in following up the 1984 paper was to develop an appropriate methodology for establishing "mis-matching reserves" of the type envisaged and for avoiding double-counting solvency margins by assuming that the solvency margins required to cover different risks were additive. An approach which commended itself was to examine the emerging pattern of income and outgo for a company and to introduce asset modelling techniques similar to those used by the Maturity Guarantees Working Party⁹. Such an approach had already been proposed by Coutts, Devitt and Ross¹⁰ in a paper for the Sydney Congress. The Working Party built on this conceptual base. Coutts and Devitt have developed their ideas further in terms of a generalized emerging costs model for life and non-life insurance and for pensions.^{11,12}

1.13 The Working Party also moved from attempting to retain a traditional actuarial concept of technical "reserves" with prudent implicit margins, designed to be adequate in the great majority of circumstances, to an approach where uncertainty in all its forms is taken care of at the level of the totality of the assets of the company. It is then of less importance how one draws the arbitrary dividing line between technical provisions and solvency margin and whether cautionary margins should be held implicitly in technical provisions or explicitly as solvency margin.

1.14 One consequence of this is that the rest of this paper does not talk about technical reserves but about technical provisions, using this in the accounting sense. We take as our starting point technical provisions assessed on a basis which we believe might be regarded as acceptable in this context, without implying that an actuary would necessarily want to certify such provisions as adequate, and consider the margin of assets in excess of technical provisions (the asset margin) which might be needed to ensure adequacy with a high degree of probability. We start by looking at a run-off situation and go on to look at the additional asset margin which might be regarded as necessary to support the continuation of new business for a limited period. The model itself is quite flexible and can be used in the situation of continuing business, although we do not consider the more general case here.

1.15 We present the basic structure of a simulation model of a general insurance company which can be used to examine the effects of uncertainty and to model the behaviour of both assets and liabilities. However, our main objective is to put forward a way of approaching the problem of assessing the financial strength of a general insurance company. The advantage of the modelling approach is to make explicit the assumptions which are being made and to enable management to explore some of the consequences of alternative strategies within a clearly defined framework. The structure of the model has many ingredients and others will no doubt suggest alternative ways of handling the different aspects. We look forward to seeing further research carried out in this area and to seeing practitioners finding ways of tackling the many issues which will arise on applying the approach to specific circumstances.

1.16 The paper also discusses solvency control , as a subset of the problem of assessing financial strength, and reviews some alternative ways of looking at solvency and of dealing with the problem of supervision. This is a subject of considerable interest internationally, but where there are many differences in underlying philosophy.

1.17 One of our main conclusions is that the traditional balance sheet presentation in the accounts of a general insurance company provides only very limited information about the financial strength of a company. The size of the margin of assets over liabilities has no real meaning unless the level of adequacy of the technical provisions is defined and unless the assets can be relied upon to produce the values shown. Partly for this reason, but also because it fails to take into account the circumstances of individual companies and the risks to which they are subject, the EEC solvency margin régime has serious shortcomings. Although some improvements could perhaps be made, we see difficulties in making the balance sheet presentation satisfactory, either for supervisory purposes or to assist management and shareholders in their understanding of the situation.

1.18 In our view a more promising way forward would be for the accounts of the company and the returns to the supervisory authority to be accompanied by a professional report on the financial strength of the company, taking into account both assets and liabilities and encompassing a proper assessment of uncertainty. The actuarial profession could be well-placed to provide such reports.

2. The Nature of Solvency

2.1. Solvency is a concept which is often referred to but rarely defined. When it comes to definition most people would readily agree that being solvent implies having assets sufficient to meet any liabilities. However, assets and liabilities may be valued in different ways and attention must also be paid to the timing of the various items. There are also questions about what should be taken into account and the extent to which future events are assumed to be relevant.

2.2. One distinction which may be drawn is between a static assessment of solvency, based purely on interpretation of the position at a point of time, for example as presented in a balance sheet, and a dynamic assessment, which takes into account in some way the effect of the continuing activity of the company.

2.3 This distinction was drawn by Campagnel³ some 30 years ago in the work on solvency which he prepared for OECD and which was later used as a basis for the EEC Directives.¹⁴ Although the EEC and a number of other supervisory authorities adopted a static approach, actuarial opinion has moved in favour of the dynamic concept, whether in respect of general insurance, life

insurance or pension funds. Among many papers presented to the Sydney Congress on the subject of solvency, Humphrys¹⁵, Snee¹⁶ and Wootton¹⁷, for example, all approached solvency from a dynamic point of view. Humphrys suggested ways of improving the balance sheet presentation but argued that "what is needed is a forward spread of cash-flow so a clear picture can be presented of what funds will be available from time to time in future and what cash will be needed to meet claims and expenses as they emerge". We also believe that this is the right way forward.

2.4 The insolvency of a general insurance company may be a matter of company law, as in the case where a company discloses that it has insufficient assets to meet its liabilities and its creditors petition to the Courts for the company to be wound up.

2.5 More often perhaps, the solvency of an insurance company is regarded as a matter for the relevant supervisory authorities. In most countries the supervisory authorities set solvency requirements which go beyond a straight assessment of the adequacy of the assets to meet the liabilities. They may require margins to be taken in valuing either assets or liabilities (or both) and they usually require assets to be held which exceed the value of the liabilities by a prescribed amount. In practice, therefore, it may be true to say that a company is solvent if the supervisor says that it satisfies his requirements¹⁸. Solvency in this sense is perhaps best described as "meeting the statutory solvency requirements."

2.6 Three main approaches to the measurement of solvency can be distinguished: winding-up, run-off and going concern. The winding-up basis is that which would be used by the Courts, although this is in fact a relatively rare occurrence for an insurance company. On this approach the liquidator may seek to agree figures for outstanding claims on the basis of estimates or he may attempt to find another insurer who will accept a transfer of the business or will take over responsibility for the outstanding claims in return for a payment of a reinsurance premium. The liquidator will normally seek to realize any investments fairly quickly for as good a price as can be obtained.

2.7 At the other extreme is the going concern basis of assessment. This is the conventional approach for accounts prepared for shareholders. The aim is to give a "true and fair view" of the affairs of the company and the natural assumption is that it will continue in business. For this purpose provision has to be made for the outstanding liabilities, including claims arising in respect of unexpired risks, and a commonly held view, in the UK at least, is that these provisions should not be deliberately over-estimated or contain cautious margins, although in practice many companies' provisions are more than sufficient if investment income is taken into account. It is assumed that the company continues to write business and that the costs of administration can be spread over new business as well as old. The only expenses for which provisions need to be established are the directly attributable expenses of settling

3. Sources of uncertainty

3.1. Insurance is a risk business and uncertainty is a fundamental characteristic. The solvency margin is intended to provide some security in the face of that uncertainty. However, there is not only uncertainty about what events might occur which will give rise to a claim. There is also uncertainty about the cost of claims which have already occurred and about when they will be settled. Three broad categories of uncertainty may be distinguished:

- * uncertainty about the true amount and incidence of the existing liabilities
- * uncertainty about the adequacy of the assets to meet the existing liabilities as they fall due
- * uncertainty about the profitability (or unprofitability) of future premiums and unexpired portions of past premiums in relation to insured events that have yet to occur.

3.2 The various elements of uncertainty and the risks to which an insurance company is subject have been extensively discussed elsewhere. A full treatment may be found in the report of the Finnish Working Party⁵ and a discussion in the UK context in the 1984 paper of the UK Solvency Working Party². More recently, papers presented to the Philadelphia Conference by Pentikäinen¹⁹ and by Buchanan and Taylor²⁰ have explored the subject in some detail.

3.3 The first type of uncertainty referred to in paragraph 3.1 above may be characterized as an uncertainty of measurement. The events giving rise to claims have already occurred but it is impossible to estimate precisely what the cost of the claims will be or when they will be settled. Apart from having less than a complete knowledge about the claims themselves, there will be uncertainty about future inflation and the impact which this will have on the claims settlement process and uncertainty about future developments in the Courts. There will often be a category of claims which have occurred but not been reported (IBNR) at the date of drawing up any accounts or returns, in respect of which no detailed information will be available.

3.4 Asset uncertainty may fall into two categories. The first relates to the realizable value of the assets. This may depend on when the assets have to be realized and could well be significantly more or less than the market value (or other value) shown in the accounts. This will not usually be of any concern with an ongoing business but is of some importance if a run-off basis of assessment is used.

3.5 Of greater significance in most cases is uncertainty about the rate of return which is to be obtained on the assets and the extent to which this offsets the effect of inflation on the liabilities.

3.6 Another factor in relation to the existing liabilities may be the security of the reinsurance arrangements and the extent to which full recovery of reinsured amounts may be anticipated.

3.7 A major source of uncertainty is the profitability or otherwise of new business (and the unexpired portions of premiums received). This will depend on a great many factors, although all may be characterized in terms of the difficulty of setting the right price in advance for the assumption of risks. The adequacy of the premiums will be affected by market pressures, which determine what rates can be charged, and by what happens to the risks that are insured. Part of this may be a purely stochastic variation, whereas other elements of the risk process may be identifiable as reflecting long term trends or cycles of some sort.

3.8 Another unknown is the future level of expenses, both the expenses of running the business and the claims settlement expenses. These will depend on a number of factors, including inflation, the volume of business written and management control.

3.9 In addition there will be a whole range of miscellaneous risks which may range from fraud to incompetent management. These are essentially unquantifiable but probably account for quite a high proportion of companies which have been in difficulties in the UK in the last thirty years.

4. Solvency control in the UK

4.1 At present the financial position of a general insurance company in the UK is disclosed through annual Companies Act accounts for shareholders and through returns to the Department of Trade and Industry (DTI). Solvency in the Companies Act sense is demonstrated by showing that the assets exceed the liabilities, on bases which, subject to regulations, are chosen by the company. For supervisory purposes the assets must exceed the liabilities by a specified margin. The solvency margin requirements follow those of the EEC Non-Life Establishment Directive²¹ and are embodied in the Insurance Companies Act 1982.

4.2 In life assurance the regulations require a report by the actuary on the valuation of the life fund in which details of the basis adopted have to be set out. By contrast the basis on which general insurance liabilities have been assessed is not usually stated. Furthermore, whereas in life assurance the appointed actuary takes account of the assets and effectively advises on the total financial strength of the life fund, there is no one with this role in a general insurance company.

4.3 In principle, the balance sheet represents no more than the Directors opinion about the financial position of the company. There is considerable uncertainty about the true amount of the liabilities and the realizable value of the assets. The auditors may place some restraints on how the Directors present the position but their role is largely confined to ensuring that what the Directors have done is reasonable.

4.4 The EEC Directive lays down a two-stage solvency margin trigger. The higher level is referred to as the required solvency margin and the lower is termed the guarantee fund. The origins of the EEC requirements have been described by Daykin¹⁴. If an insurer fails to maintain its required solvency margin it must provide to the supervisor a plan for the restoration of a sound financial position, which may include demonstration that on a properly drawn up business plan, and with realistic assumptions about profitability, the solvency margin will be restored within a reasonably short space of time. Only if the company fails to maintain the guarantee fund, set at one-third of the solvency margin, with a specified minimum in absolute terms, is immediate action to inject additional capital required in order to stave off withdrawal of authorization.

4.5 The object of the statutory solvency margin is two-fold. It reduces the probability that the assets will prove inadequate to meet the liabilities and it provides a buffer against further deterioration in a company's financial position which can occur in the period before its authorization to write new business can be withdrawn.

5. Alternative approaches to solvency control

5.1 It may not be self-evident that solvency control by a supervisory authority is desirable. Some would argue that the market should be allowed to determine solvency and that policyholders should not be protected from the consequences of insuring with a weak company.

5.2 Most would feel that such a régime would be too harsh, bearing in mind the difficulty for policyholders in making any reasonable assessment of the relative financial strengths of competing insurers. An alternative may be to provide a system of guarantee funds, to which all insurers would pay premiums in advance, assessed on the basis of their financial strength. This would refer the problem of solvency assessment to those operating the guarantee funds. There would then be a question of who should insure the guarantee fund. In practice there would probably have to be reinsurance by the State.

5.3 Most countries seem to regard some measure of supervision as desirable. Some rely mainly on requiring companies to provide information about their activities for the supervisory authorities to analyse. Others adopt a policy of regular inspection of companies to look over their records in detail. Some exercise control over premium rates, which may assist control of solvency, or, in some cases, conflict with it. Different philosophies of supervision correspond in large measure to the degree of openness of the respective markets.

5.4 The EEC requirements are generally regarded as fairly modest. However, their impact depends very much on the particular company to which they are applied and the way in which rules for valuing the assets and liabilities are laid down. Germany, for example, adopts a very conservative method of asset

valuation, whereby assets are held at the lowest value they have ever attained since purchase. Furthermore, the Germans also appear to require the liabilities to be assessed on a conservative basis.

5.5. Finland likewise adopts a conservative approach to the liabilities. However, their solvency requirements are more sophisticated than the EEC requirements, relying on a risk-theoretical approach and being effectively tailored to the risk profile of the company, in the first instance by a simplified formula but with a back-up provision for borderline cases which involves analysis by an actuary.

5.6 Discussions of solvency tend to be complicated by arguments over the acceptability or otherwise of implicit margins. Should liabilities be set up on the basis of best estimates or on a prudent basis? Life assurance in the UK has traditionally been operated with prudent reserves and largely implicit margins. EEC requirements now mean that some explicit margins have to be demonstrated but the role of implicit margins is formally recognized, not by direct quantification but by allowing a crude estimate of future profits to be counted towards the solvency margin.

5.7 The papers to the 22nd International Congress of Actuaries in Sydney 1984 highlighted the different views on these issues. Papers presented covered both life insurance and general insurance. Support for cautious assumptions in the calculation of reserves or the valuation of assets came from Limb²² (life), De Hullu²³ (life and general) and Daykin¹⁴ (general). Best estimates were favoured for actuarial reporting by Skerman²⁴ (life) and Sleet¹⁶ (general).

5.8 Our own view is that in considering the overall solvency of a general insurance company it should be a matter of indifference as to whether the total asset margin is set up specifically as a solvency margin or included as prudent margins in the provisions for liabilities. The apparent transfer of funds between shareholders and those interested in the liabilities, i.e. policyholders and third party claimants, may, however, incline companies to approach one or the other. If implicit margins are used, they should be capable of being quantified and taken into account in assessing overall solvency. However, this presents problems with general insurance liabilities as it is very difficult to know how strong the provisions are.

6. Financial strength

6.1 Solvency as such is a limited concept. Abbott suggested in his contribution to the discussion at the International Congress in Sydney that: "the meaning of solvency and solvency control be restricted to legislative aspects, leaving other words or phrases for the management-oriented view of conducting business in a prudent, but commercial fashion. In this context we should talk not of solvency but of financial soundness or strength". This broader concept of financial soundness or strength is appropriate

in balancing the conflicting interests of many of the parties reliant on or concerned with the soundness of insurance companies. These encompass:

- legislators
- regulators
- policyholders
- third party claimants
- intermediaries
- company creditors
- shareholders
- management
- employees

6.2. The insurance policyholder purchases a product that involves a promise of a benefit which is to be met in the future in monetary terms. Buyers of other goods and services, outside the financial field, do not generally suffer major financial loss from the insolvency of a company, which in such cases primarily affects shareholders, creditors, management and employees. It is the nature of the promise built into the insurance product, and its impact on unconnected third party claimants, that ensures that there will continue to be public interest in the financial soundness of insurance companies and that governments will feel the need to regulate the industry.

6.3 All those mentioned in paragraph 6.1 have an interest in the continuing survival and financial soundness of one or more insurance companies. In consequence they have a priori an interest in a high asset margin. The one exception is the shareholder for whom a low asset margin increases operational gearing and hence expected return on capital.

6.4 Legislators and regulators of insurance companies have a principal aim of protecting the interest of policyholders and third party claimants. Their own interests are co-incident with those whom they are protecting since any failure in this protection exposes them to public criticism.

6.5 The mutuality of interest between the above groups breaks down when premium levels are considered. Consumer pressure may incline legislators to seek low premium levels. This may create particular difficulties when different government agencies or departments are responsible for control of pricing and of solvency. Third party claimants may be assumed to be indifferent to premium levels other than through their dynamic effect on continuing solvency. Thus the public at large may have a slight bias in favour of low premium levels. Regulators should require adequate premium levels to ensure ongoing dynamic solvency. Shareholders, intermediaries, management and employees are all likely to have rational preferences for high or at least adequate premium levels.

6.6 Policyholders seek protection but at a price which provides value for money. Economic theory considers a rational purchaser who will accept increasing premium levels for greater security. Less obvious, but equally justifiable is that the rational

purchaser accepts that companies that are less sound should charge lower premiums. In these circumstances the policyholder bears more of the risk of meeting part or all his claims himself because of the higher probability of insolvency of his insurer. This may be more theoretical than practical since it depends upon a degree of knowledge and awareness of risk levels and personal utility which is unlikely to be common. It may be present in the largest commercial organisations, but even where well served by intermediaries it is unlikely to be present in the personal sector. Thus freedom with publicity can only work so far.

6.7 Collett²⁵ suggests five types of protection that can exist to provide security to the policyholder. These are:

- (a) Buyer awareness or buyer self-protection (buyers' evaluation of risk of seller);
- (b) Management quality and fidelity;
- (c) Professional review of insurance company's actions and financial picture;
- (d) Government regulation; and
- (e) Industry-wide or governmental benefit guarantee.

Other suggestions for control can in one way or another be incorporated within these five.

6.8 The key problem goes beyond that specified by De Hullu²³, that a policy of insurance contains an implicit promise of long term solvency but that it is impossible to look ahead with any certainty for more than a few years. It may be seen as the problem of balancing the conflict of interests in premium rate levels and asset margin between the affected parties listed in paragraph 6.1. All parties, however, will benefit from increased efficiency and effectiveness of the insurance industry. Regulation and solvency considerations should, therefore, not restrict competition or control the operations, pricing etc. of companies to an extent that reduces commercial pressure to become more efficient.

6.9 Although much of the research in solvency has been carried in the actuarial field (see, for example, the excellent summary by Kastelijn and Remmerswaal²⁶), new ideas are being introduced reflecting developments in other financial fields. Kahane, both alone²⁷ and with others²⁸, has made use of the Capital Asset Pricing Model, from the world of investment and finance. With a definition of solvency based on a minimum level of return on capital he develops a band strategy for the regulation of insurance companies. Cummins²⁹ has applied the developing theory of option pricing to calculate the rate of contribution necessary to support a nationwide solvency guarantee fund. Doherty³⁰ and Derrig³¹ have used similar methods to examine appropriate capital structures and premium risk loadings.

7. Emerging costs

7.1 In the statutory returns for the DTI, assets are required to be shown at market value. This has the advantage of being an objective value to be placed on the company's investments, but it is of doubtful relevance to the ability of the company to meet its liabilities, even in the context of the run-off basis.

7.2 The assets will not in practice have to be realized on a particular date and, in any case, by the time the accounts or returns have been prepared, the market value at the date to which those accounts relate is a matter of no more than historical interest. What is important is whether the proceeds of the assets, both capital and income, will prove sufficient to meet the liabilities as they emerge. This is what solvency is really about.

7.3 The concept of projecting the emerging costs of the liabilities to which an enterprise is subject and placing them alongside the expected pattern of income is one which is familiar to actuaries in the life assurance and pensions contexts and is also fundamental to investment appraisal by economists in many other spheres of industry. However, little work seems to have been done on the application of the concept to general insurance companies.

7.4 Actuarial concepts of looking at the company as a whole were applied to general insurance in a paper by Benjamin³² and the use of emerging costs was implicit in two papers by Ryan^{33,34} on the use of simulation techniques in general insurance. This was also the basis of the Finnish solvency study⁵, although the treatment of assets was simplified and no account was taken of the settlement pattern for the liabilities. There has been some consideration from the viewpoint of financial economics by Kahane³⁵ and others which is not widely known among UK actuaries. Coutts, Devitt and Ross¹⁰ set out explicitly the application of the concept of emerging costs to a general insurance company. Somewhat similar approaches have been adopted by Paulson and Deekshit³⁶ in the US and by the Faculty Working Party on Life Assurance Solvency³⁷.

7.5 The concept is a simple one. It involves analysing the inflows and outflows of actual cash in each successive year. The inflows may consist of some or all of the following:

- * premium income
- * interest and dividends on assets
- * reinsurance recoveries in respect of claims

The outflows may consist of the following:

- * claims settled or amounts paid on account
- * reinsurance premiums
- * expenses
- * tax
- * dividends

7.6 The effect of the various items in each year will be either a net amount available for investment or a shortfall. In the latter case assets need to be sold. So long as there are sufficient assets available to enable all the outflows to be met as they arise, the company is solvent in an absolute sense, whatever the balance sheet may have shown. If all the assets have been realized but net liabilities still remain, the situation is one of de facto insolvency.

7.7 An emerging costs analysis should be carried out on the totality of the assets and liabilities of the company. For this purpose the dividing line between technical provisions and asset margin is of no real importance although estimates of future claims payments are necessary. The uncertainties of general insurance are such that it will not generally be sufficient to use deterministic values for the liabilities and the assets. Some measures of variability need to be introduced. However, this should not be allowed to detract from the essential simplicity of the concept. It only means that some or all of the items listed above should be treated as random variables. To handle this the emerging costs can be examined using simulation.

7.8 A single simulation is one realization of a random process in which each of the required quantities is assigned a value. By examining a large number of simulations a picture can be obtained of the likely pattern of development resulting from the interaction of the various variables. Simulation permits the use of stochastic models for the investment processes and allows the uncertainty in the outstanding claims and in the profitability of new business to be taken into account. The approach has much in common with the ideas developed by the Finnish Solvency Working Party⁵ and extended to cover run-off risk by Pentikäinen and Rantala⁷, although they did not use a stochastic approach for the investments.

7.9 In practice the various elements may be modelled in a variety of different ways. For some purposes very complex models may be desirable; for others a simpler model may suffice. The important principle is that the totality of the company's operations is being considered.

7.10 The procedure is very flexible. It might enable, for example, questions to be asked about the impact of writing different lines of business and of alternative investment strategies and about the effect of possible adverse claims development or failure to recover from reinsurers. It provides a management tool which may offer a way forward for exploring a balance in the conflicts of interest referred to in paragraph 6.8 above and perhaps for more rational supervision. This would involve the submission to the authorities of a report on total financial strength by an actuary or other suitably qualified expert, as a supplement to balance sheet requirements. The result would be a system better able to take account of the true position of each company, having regard to the specific risks to which it is subject and the inherent uncertainties of both assets and liabilities.

8. The simulation model

8.1 General structure

8.1.1 In order to demonstrate the potential of the emerging costs approach we present here a model which provides a representation of the dynamics of a general insurance operation. In order to be reasonably realistic the model is quite complex but, however complicated the model, it is essential that the concepts should be capable of being put across in a straightforward way and the results must be capable of being presented in ways that can be directly related to management concerns such as corporate strategy and decision-making.

8.1.2 At its most basic, the model is a projection of cash flow, bringing together income from premiums and from assets and outgo in respect of expenses, tax, dividends and claims, determining the net balance for each year, investing or disinvesting as the case may be and proceeding similarly for as many years into the future as one needs. It may be considered more fully in terms of three separate components:

- * liabilities arising from existing business
- * future premiums and the liabilities resulting from the risks underwritten
- * asset returns and asset value movements

8.1.3 A mathematical formulation of the model is given in Appendices 1 and 2 and a description of the computer program in Appendix 3.

8.2 Existing liabilities

8.2.1 The existing liabilities, as shown in the balance sheet, consist of estimates of outstanding claims, including IBNR, and unearned premium reserves (including any additional amount for unexpired risks). Unearned premiums can be dealt with along similar lines to new written premiums (see section 8.3) since the uncertainty includes uncertainty about the adequacy of premium rates in relation to events which have not yet occurred.

8.2.2 As far as outstanding claims are concerned, there is uncertainty about the amounts of claims and about when they will be settled. The model needs to provide an adequate representation of this uncertainty. We make the simplifying assumption that the variability in rates of settlement can be subsumed into a variation in the amount of claims settled in each period. We also restrict our attention to claims net of reinsurance (see section 10 for a brief discussion of reinsurance recoveries).

8.2.3 The first stage is to estimate the expected claims payments in each successive year for each year of origin. In order to do this, fixed settlement patterns have to be specified

in real (constant money) terms. The model permits different run-off patterns to be assumed for different types of business. Inflation then has to be allowed for. Future inflation is generated by a stochastic model and this is combined with the expected settlements in real terms to give the expected development of claim amounts. The inflation model is an integral part of the model used for the assets (see paragraph 8.4.4).

8.2.4 The variability of claim amounts payable in each period can be dealt with in a variety of ways. In an earlier paper describing the application of a similar model to a run-off situation, Daykin and Bernstein³ proposed that the actual outstanding claims settled in each year in respect of each year of origin should be varied. They assumed that each separate entry in the run-off triangle was distributed about the mean estimate of claims settled at that particular duration for that year of origin in accordance with a log normal distribution. This was attractive as a means of simulating the interaction between different years of origin and different classes of business, but it resulted in a somewhat lengthy simulation process.

8.2.5 In order to simplify the model and allow account to be taken directly of different sizes of company the model presented here uses an aggregate approach, whereby the amount that is varied is the total amount of claims settled in a particular period, for all years of origin combined. This aggregate figure is assumed to vary according to a normal distribution with a standard deviation of the type:

$$aX + b\sqrt{X}$$

where X is the mean estimate of total claim payments in the year and a and b are suitably chosen constants. We understand that a similar formula is used by the Finnish supervisory authority for their statutory minimum solvency margin (see Appendix 5 for discussion of this formula which may be seen as a practical approximation to the more rigorous formula developed by Buchanan and Taylor²⁰).

8.2.6 The amounts payable in future years in respect of risks arising from future written premiums and from unearned premium reserves are included with the amounts payable in respect of existing liabilities before applying the overall variability formula. The extent of the assumed variability can be adjusted by varying the constants a and b in the formula above. For the standard basis we have assumed that they take initial values 0.15 and 75 respectively, with claims amounts being expressed in 1986 pounds, but that b is allowed to increase year by year once the company has ceased to write business and begins to run off. The amounts payable in successive years are assumed to vary independently of each other. The variability is intended to cover not only stochastic variability of claim amounts, but also uncertainty about the expected run-off model in constant money terms. Uncertainty about future inflation is dealt with separately.

8.2.7 Two typical run-off patterns have been assumed, characterized as short and long-tailed. Details are given in Appendix 3. In order to place a value on the technical provisions which would be established at the outset in respect of the outstanding claims, it has been assumed that inflation would be allowed for at 5% a year and that the resulting outstanding claims would not be discounted. (For further discussion on the interaction between the reserving basis and the solvency margin, see paragraph 9.2).

8.2.8 In practice an actual outstanding claims portfolio could be used as the basis for the input to the model in respect of existing liabilities. It would need to be expressed as an expected run-off in real terms. For illustrative purposes, however, we have assumed that the outstanding claims have been generated as a result of a past pattern of business. The existing liabilities at the base date have been generated in a similar way to the liabilities in respect of future written premiums, by specifying a rate of real premium growth and claim ratios. For the purpose of generating the outstanding claims at the base date no variability was assumed in the historic claim ratios, in contrast to the process described in section 8.3. In conjunction with the specified run-off patterns and the inflation model, the liabilities generated in this way give rise to estimates of outstanding claims payable in each future year in respect of each past year of origin.

8.3 Future written premiums

8.3.1 Future premiums are generated from an assumed initial premium level and an assumed real annual growth rate. The effects of inflation are then built in explicitly. Although the existing portfolio of business is generated by assuming a past pattern of premium growth, as described in paragraph 8.2.8 above, a different growth rate assumption may be made for the future. The proportions of written premiums which are assumed to relate to different types of business can be specified. The written premiums are taken to be net of commission and initial expenses and net of any outwards reinsurance premiums.

8.3.2 For each year for which additional premiums are assumed to be written, a ratio of claims to premiums net of commission and expenses is generated for each type of business. The ratio is assumed to be normally distributed with mean and standard deviation to be specified. The assumption of a normal distribution of claim ratios about the mean is a not unreasonable approximation, bearing in mind the large numbers of claims involved. The resulting ratio is applied to the assumed net written premiums to produce an initial estimate of total claims in respect of that business, without any allowance for future inflation or for discounting. This ratio is such that a value of 100% implies break-even if future investment income exactly balances inflation. The assumed proportions of claims settled in each future year are then applied to obtain uninflated estimates of expected claims payments. Future inflation, as generated by

the model described below (paragraph 8.4.5), is incorporated when the expected claim payments in terms of constant money have been aggregated with the corresponding estimates in respect of the existing liabilities. The combined estimates are then varied as described in paragraph 8.2.5 above.

8.3.3 Since the claim ratios generated are ratios of claims to written premiums net of commission and expenses, no explicit allowance needs to be made for these items of outgo. Expenses of claims settlement are assumed to be included in the costs of settled claims.

8.3.4 This relatively simple approach has been used as a practical expedient in view of the complexity of the underlying risk process. An alternative approach, described by Beard, Pentikäinen and Pesonen⁶ and developed in the Report to the Finnish Solvency Working Party⁵, would be to treat the basic claims process as a Poisson process and then build on a series of "structure variables" to take account of:

- * trends of claims frequency
- * long term variations in premium rate adequacy
- * year to year fluctuations in mean claims frequency

Further assumptions then have to be made about the claims size distribution.

8.3.5 Whilst it is clearly possible to specify a model which takes explicit account of each of these, the added complexity can be justified only if the parameters of the model can be satisfactorily determined. We have not as yet been able to assemble data in a suitable form for calibrating such a model. The problem of calibration still arises with the simpler model, but it is intuitively more accessible and enables judgement to be applied in the area which is probably of the greatest importance, i.e. changes in the relationship between premium levels prevailing in the market and the underlying risk premium. This is the factor described as "long-term cycles" by Pentikäinen and Rantala⁵.

8.3.6 Although the adequacy of premium rates does exhibit the characteristics of a business cycle, experience seems to show that the variation does not have a regular periodicity or a constant amplitude. A considerable degree of judgement is needed to decide where in the "cycle" the industry finds itself at any particular moment. Our model allows for the user to give explicit consideration to this and requires the mean claim ratio for the next couple of years to be estimated. If the model were to be used to examine the effects of future written premiums over a longer period than 2 years, further consideration would need to be given to modelling this component.

8.4 Asset variability

8.4.1 The variability inherent in the asset portfolio of a company depends on the nature and distribution of the assets. The realizable value of many assets will vary from day to day as market conditions change. In our model, the initial distribution of the assets by category has to be specified and the various components of the asset distribution are then analysed separately, simulating the income generated and the capital value of each type of asset for each future year. Rules need to be specified for investment and disinvestment.

8.4.2 Three different types of asset are assumed: cash, medium-dated Government securities and ordinary shares. Allowance has been made for the fact that a proportion of the assets is effectively non-interest-bearing (eg agents' balances) whilst a company is open to new business. The balances are assumed to run down as soon as a company ceases to write business.

8.4.3 The development of the various components of the asset distribution has been represented by a series of interrelated stochastic processes, suggested by Wilkie^{38,39,40}, which generate future scenarios for the values of different types of asset and the income from them.

8.4.4 Wilkie's models are autoregressive and are based on a detailed analysis of the behaviour of inflation, interest rates, share dividends and yields over a period of some 65 years from 1919 onwards. The models were developed for the purposes of exploring the effects of inflation and asset behaviour over reasonably long periods. They were not designed for very short-term forecasting. In the short term the autoregressive behaviour is not so evident and market movements can be characterized in terms of a model more akin to a random walk.

8.4.5 The time scale with which we are concerned is not as long as that envisaged in some applications of the Wilkie model (eg the Faculty of Actuaries Working Party on Life Assurance Solvency³⁷). However, it is long enough for the Wilkie models to be more appropriate than short-term forecasting models. We have spent some time examining the behaviour of the Wilkie models and are satisfied that the results which they produce are intuitively reasonable and accord with behaviour which the respective parameters have been seen to exhibit historically.

8.4.6 The models require appropriate starting conditions, which may be generated by inputting data for a number of recent years. This would be a necessary process in using the model in a practical application, but for our purposes we have assumed so-called "neutral" starting values as being appropriate to investigations which are not intended to be based on any particular time-period.

8.4.7 The models are described in detail in Appendix 2. In addition to the models of asset returns and asset values, the Wilkie models include a model for inflation and this has been used where it is needed in the simulation of the liabilities.

8.4.8 The initial asset mix is based on assets covering the technical provisions and assets representing the asset margin. Different proportions may be specified for each. A variety of different investment and disinvestment strategies may be applied to the total funds.

8.5 Results of the simulations

8.5.1 The number of potential combinations of variables is vast, even allowing each variable to take only three or four different values. We have limited our considerations by adopting a standard set of parameters and normally varying only one parameter at a time.

8.5.2 The simulation process involves sampling scenarios from an infinite set and the results are necessarily subject to statistical error. For any particular case which is of interest more simulations can be carried out in order to improve the accuracy of the estimate. In order to illustrate the results on a large number of scenarios, we have limited our considerations to 1000 simulations for each. The same 1000 sets of random numbers have been used for each different parameter combination, so that the comparisons are not significantly affected by any bias in the particular sets of random numbers chosen.

8.5.3 Figure 1 shows, for illustrative purposes, the results of 100 simulations, assuming no new business. This demonstrates the general shape of the results, which is common to all the scenarios, although the variability differs greatly. The graph shows the assets of the model company year by year throughout the run-off of the business.

8.5.4 When a line goes below the x-axis, this implies that all the assets have been exhausted on that particular simulation. If that should occur before the end of the run-off, true insolvency has occurred. In describing the results of the simulations, insolvency is used in this sense, without regard to the way in which the financial position of the company might be presented in the accounts or statutory returns at the base date or at any later date.

8.5.5 We thus define:

- * an insolvency occurs when the assets run out before all the liabilities have been met (on an emerging costs basis)

In the simulations a realization which runs into insolvency is allowed to continue by borrowing (at the rate of interest on cash plus a margin of 3 per cent); this permits one to see how insolvent it becomes.

8.5.6 On the standard basis insolvency in this sense occurred in 9 cases out of 1000 with no new business and 36 cases out of 1000 with 2 years' new business. The distributions of assets at the end of the run-off, deflated to the date of assessment using the retail prices index, are shown in Figures 2 and 3. The *written premiums*

Figure 1 Run-off of assets assuming no new business (100 simulations)

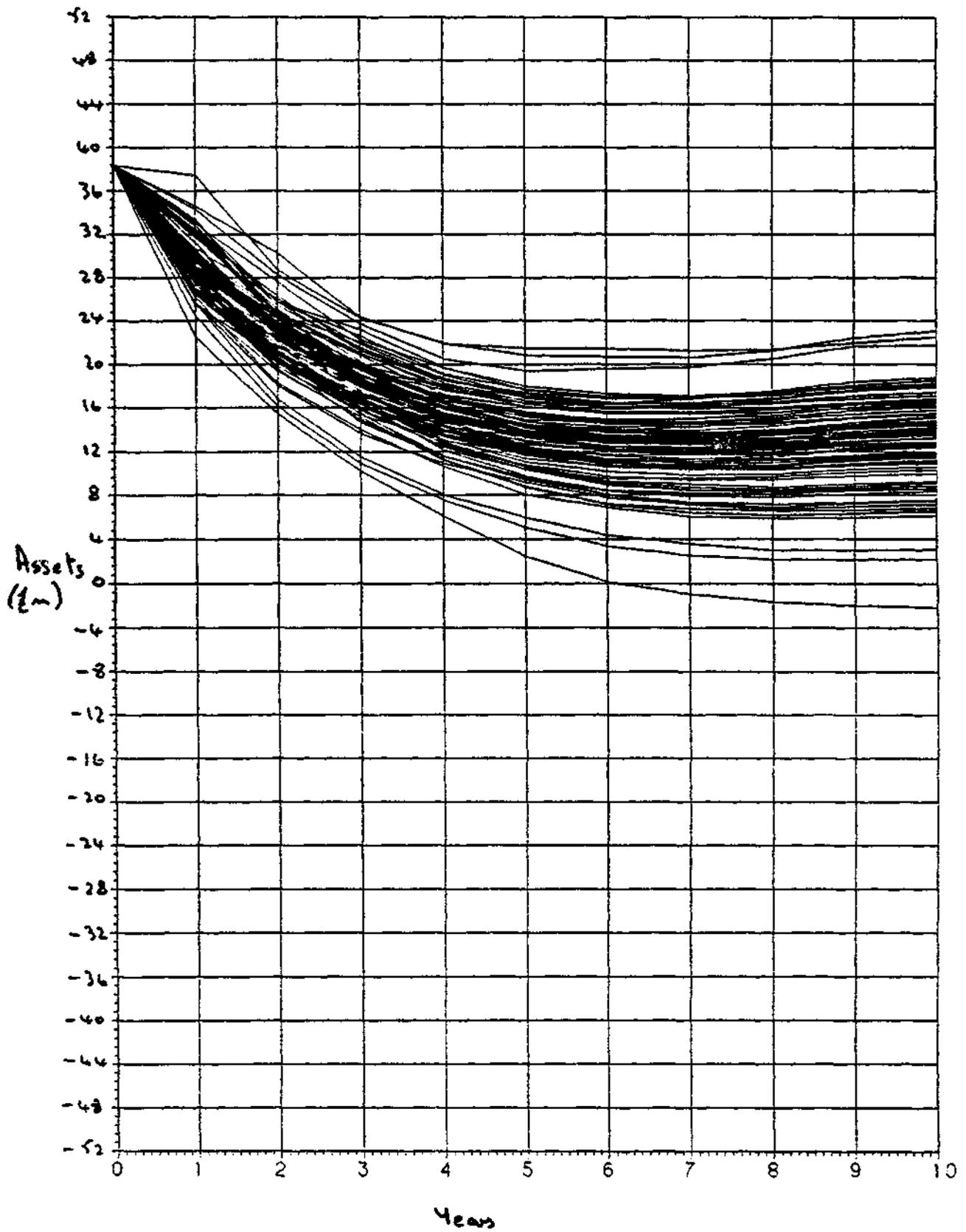


Figure 2

Distribution of assets at end of run-off for 1000 simulations on standard basis

No future business

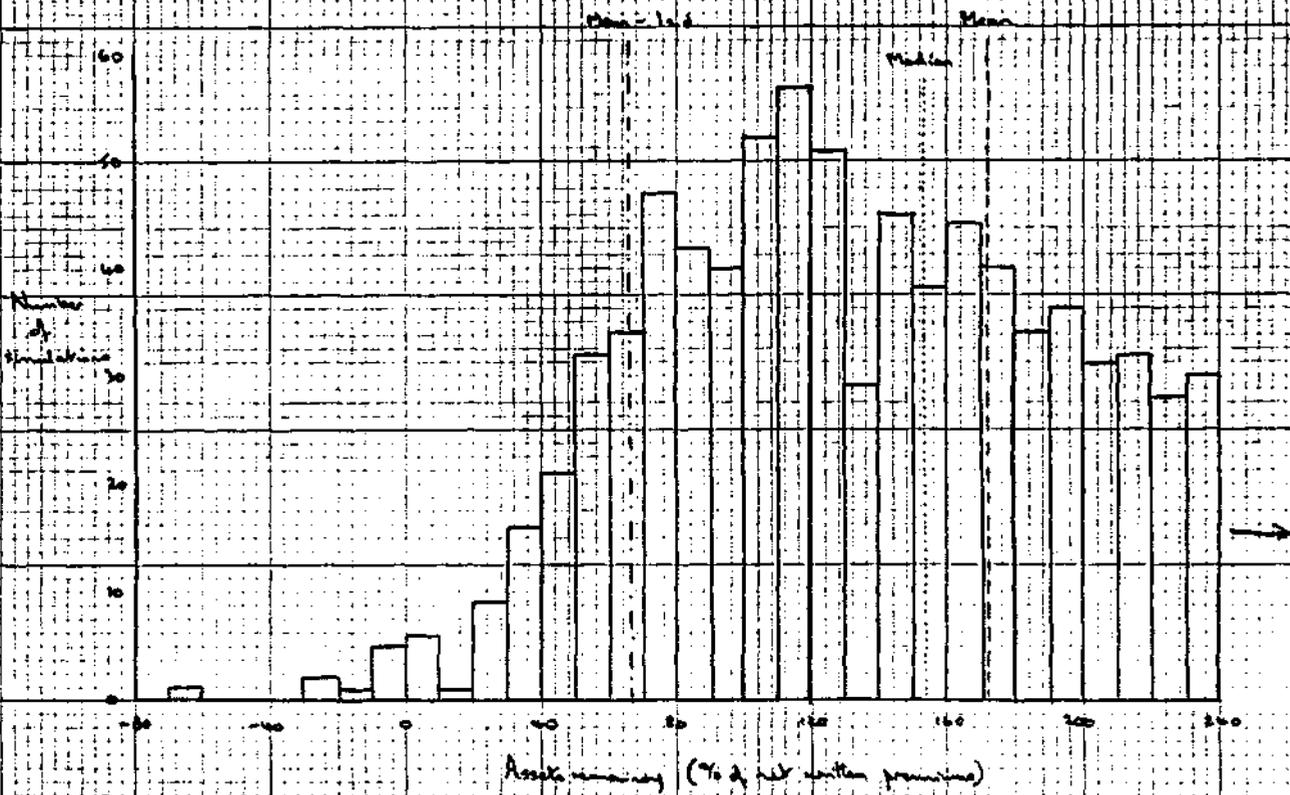


Figure 3

Distribution of assets at end of run-off for 1000 simulations on standard basis

2 years' future business



in question are those in the year before the base date. It should be recalled that the written premiums are net of commission and expenses. Results expressed as percentages of net written premiums can be rated down (say, by applying a factor of 75% or 80%, depending on the type of business) to obtain comparable results in terms of gross written premiums. The mean level of remaining assets for the 1000 simulations was 172% of net written premiums with no new business and 205% with 2 years' new business, with standard deviations of 106% and 157% respectively of net written premiums.

8.5.7 Full details of the assumptions underlying the standard basis are given in Appendix 4. However, we will return to the results after commenting on the application of the model.

8.6 Application of the model

8.6.1 A simulation model of an insurance company, based on the emerging costs concept, provides a powerful and flexible tool for examining the dynamics of an insurer's operation, for exploring the effects of uncertainty and for developing the financial aspects of corporate strategy within a logical framework. This should be of value both to management and to the supervisory authorities. Crucial to this process would be the presence of a suitably qualified actuary or other expert within the company, or acting as consultant, who could develop a suitable model and apply the necessary judgement to the use of the model in the circumstances of the particular company. The responsible expert would report to management on the financial strength of the company, taking all relevant factors into account.

8.6.2 The simulation approach would also enable the actuary to advise management on the potential effects of different new business and investment strategies, the risks involved and the return on capital which might be expected if additional capital is injected to enable a particular strategy to be adopted.

8.6.3 A report on the financial strength of the company could accompany the statutory returns to the supervisory authorities. It would be desirable for this to be on public record as with the rest of the returns, although some of the assumptions about future business might have to be treated as confidential to the supervisor. The actuary would be answerable to the supervisor on the details of this report. One could envisage this leading to an informed dialogue between the supervisory authority and the company under scrutiny on the nature of the proposed corporate strategy, whether in relation to investment policy, growth or premium levels. The supervisor could then ask for an assessment of the effect of alternative strategies and seek agreement with the company on appropriate changes to its strategy as a condition for being permitted to continue writing business.

9. Solvency considerations

9.1 The results of the simulations can be presented in terms of numbers of insolvencies out of a given number of simulations. This is an estimate of the probability of ruin. Each result

derives from an assumption about the excess of assets over technical provisions (the "asset margin") and a specified basis for calculating the latter. Given a basis for the technical provisions, the process can be used to derive the required initial asset margin in order to achieve a specified probability of ruin in a particular case.

9.2 The required asset margin will clearly differ according to differing definitions of the technical provisions. Table 2 illustrates this point. The table shows the technical provisions on the standard basis described above and the technical provisions on alternative bases as to inflation and discounting, but for the same set of outstanding claims. The table shows what asset margins would be necessary, expressed both as a percentage of technical provisions and as a percentage of net written premiums, in order to achieve the same degree of overall security as the technical provisions on the standard basis. Technical provisions on the standard basis are calculated assuming 5% inflation and no discounting which can be regarded as including some implicit solvency margin, as may be the case with the provisions adopted by many companies. Thus if the reserves do not allow for any future inflation, or have been discounted using a rate of interest equal to the assumed rate of inflation, an asset margin of 38% of net written premiums in the year before the base date or 13% of technical provisions would be needed to produce the same level of total assets as the technical provisions alone on the standard basis. The figures in this table underline the arbitrary nature of a statutory solvency requirement unless standards of technical provisions can be adequately specified.

Table 2 Technical provisions and asset margins

Reserving basis (net inflation assumed) %	Technical provisions*	Asset margin to achieve same security as standard†	
		% of net written premiums**	% of technical provisions
-5	25645	70	27
0	28796	38	13
5	32627	0	0
10	37328	-47	-13
15	43147	-105	-24

* based on 40% long tail business and 60% short tail

** premiums net of commission and initial expenses

† i.e. total assets of 32627

9.3 First we give some results for a pure run-off, i.e. with no future premiums assumed to be written. The outstanding claims and unexpired risks are allowed to emerge and the adequacy of the total assets (technical provisions and asset margin) is examined.

Table 3 shows the number of insolvencies and the mean assets remaining at the end of the run-off and the standard deviation of the assets remaining on some alternative bases. Table 4 gives a similar set of results with the inclusion of 2 further years' written premiums. Appendix 4 gives details of all the assumptions and a full set of results.

9.4 As would be expected in real life, the assumption described in paragraph 8.2.5 ensures that, other things being equal, the probability of ruin falls as the volume of business increases and with increasing initial asset margin. The probability of ruin increases:

- * as the proportion of long-tailed business increases
- * as the mean claim ratio of the future business increases
- * as new business is included
- * as the rate of growth of the business increases

The results do not change significantly for different levels of variability of the claim ratio. A higher proportion of equity investment gives rise to higher mean assets remaining but a greater spread of results and a higher probability of ruin. The selling rule for assets does not make a lot of difference but the best rule appears to be sell gilts first, then cash and hold on to the equities will last. The worst rule is the exact reverse of this, selling equities first and gilts last. The general shape of the results can be seen in Figure 4.

9.5 For any given set of assumptions the initial asset margin required to achieve a particular probability of ruin can be determined. Figure 5 shows how the probability of ruin varies by different initial asset margins for different levels of mean claim ratio for long-tailed business. The other assumptions are as for the standard basis. Figure 6 shows the effect of different initial asset portfolios.

9.6 Tables 5 and 6 show the explicit asset margins required to achieve a probability of ruin of 1 in 100 for each of the combinations of assumptions in Tables 3 and 4 respectively, assuming that the technical provisions are established on the standard basis of 5% inflation and no discounting. The asset margins are shown in terms of both net written premiums in the year before the base date and as a percentage of technical provisions at the base date. The results can be expressed in terms of net written premiums even for the pure run-off case, since these are the premiums in the year before the base date when premiums are assumed to cease. As described in paragraph 8.2.8, we have in fact generated the outstanding claims from past premiums. The difference between Tables 5 and 6 provides a measure of the additional capital needed in order to go on writing business for two more years (we have not included any additional costs of run-off in the case of no future business). Figure 7 shows some of the results graphically.

9.7 It is clear that the results obtained depend critically on the models used and the parameters assumed. More work is needed

Table 4

Summary of results with 2 further years' business (with 1000 simulations)

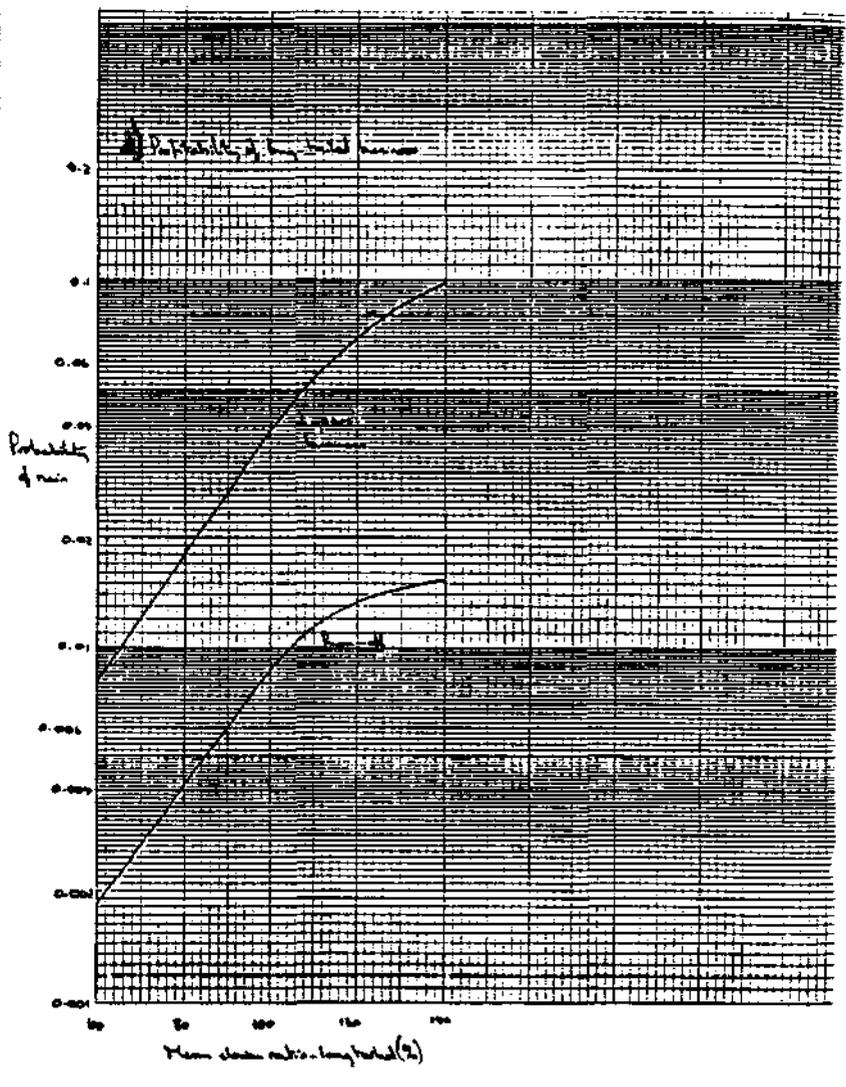
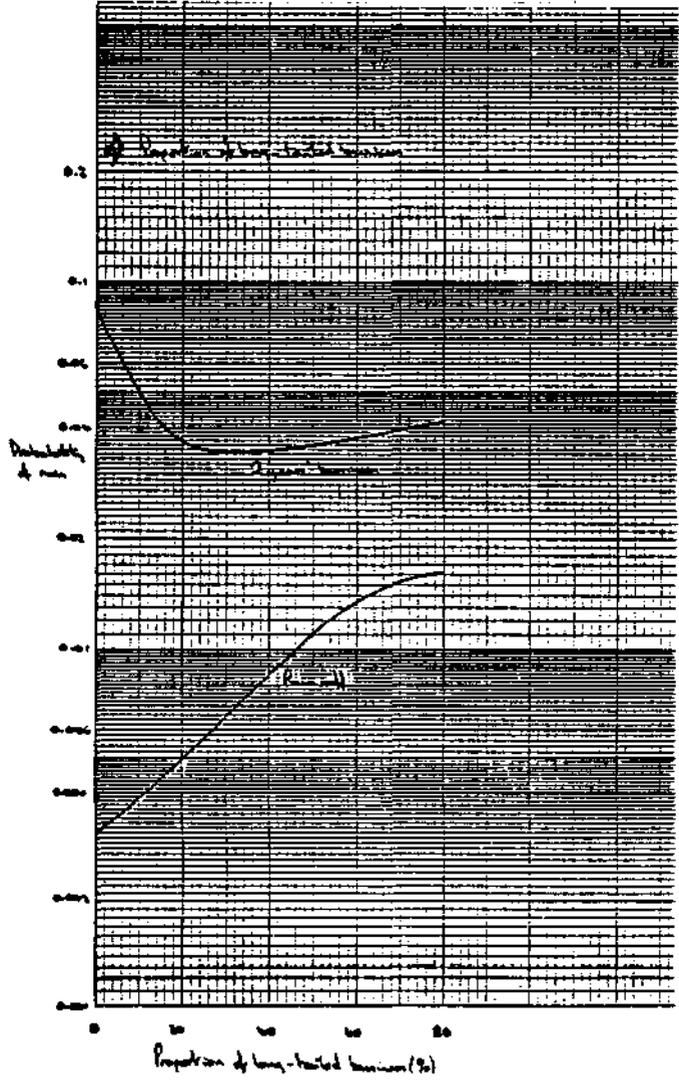
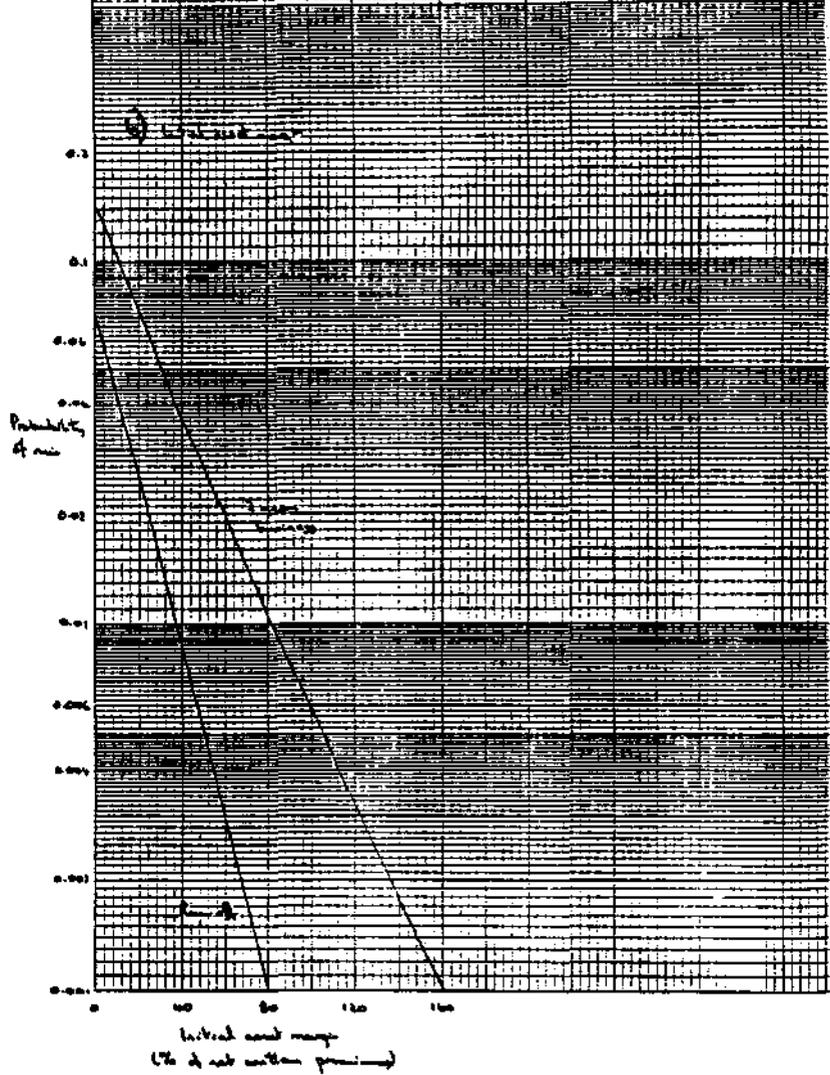
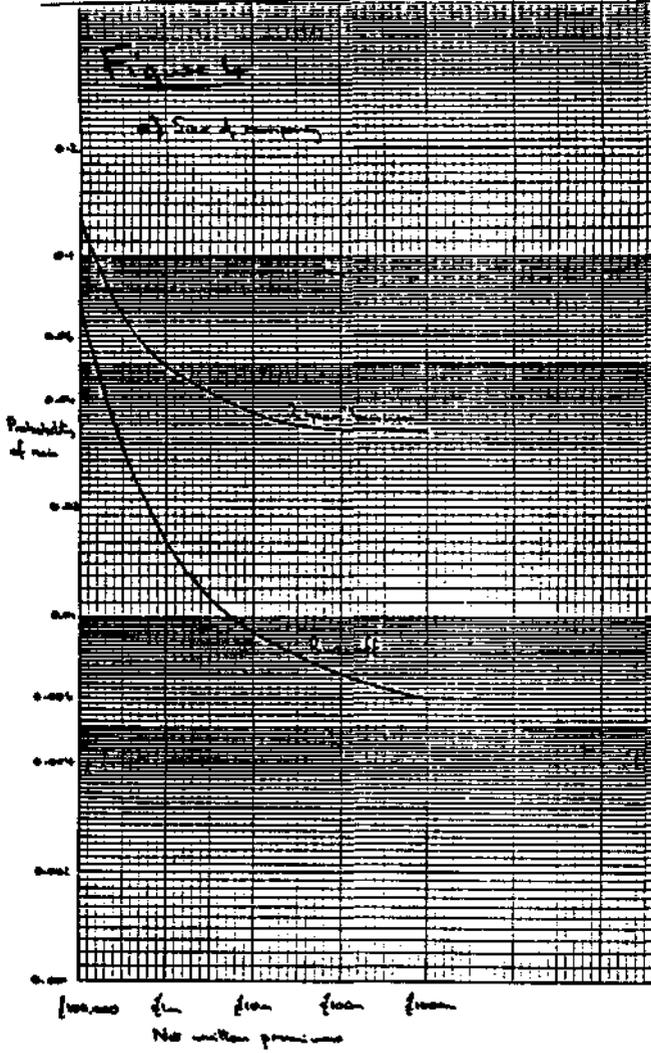
Assumptions	No of insolvencies	Mean assets remaining* £	Standard deviation of assets remaining* £
Standard basis	36	205	157
1. Net written premiums:**			
(a) £1m a year	49	204	166
(b) £10m a year (s)	36	205	157
(c) £100m a year	32	206	155
2. Proportion of long-tailed business:			
(a) 0% of net written premiums	88	99	90
(b) 40% of net written premiums (s)	36	205	157
(c) 80% of net written premiums	42	310	246
3. Real growth rate (past and future):			
(a) -20% a year	38	291	231
(b) No growth (s)	36	205	157
(c) +20% a year	49	185	146
4. Mean claim ratio† (short-tailed):			
(a) 80% of net written premiums	14	257	167
(b) 100% of net written premiums (s)	36	205	157
(c) 120% of net written premiums	94	153	149
5. Variability of claim ratio (short-tailed):			
(a) Standard deviation 5% NWP	32	206	157
(b) Standard deviation 10% NWP (s)	36	205	157
(c) Standard deviation 15% NWP	38	204	157
6. Mean claim ratio† (long-tailed):			
(a) 80% of net written premiums	17	219	147
(b) 100% of net written premiums (s)	36	205	157
(c) 120% of net written premiums	73	192	168
7. Variability of claim ratio (long-tailed):			
(a) Standard deviation 10% NWP	35	205	156
(b) Standard deviation 15% NWP (s)	36	205	157
(c) Standard deviation 20% NWP	40	205	158
8. Initial asset distribution:			
	Cash	Gilts	Equities
(a) TP+AM	-	-	48
(b) -	TP+AM	-	55
(c) -	-	TP+AM	76
(d) ‡TP	‡TP	AM (s)	36
			171
			208
			300
			205
			138
			188
			306
			157
9. Initial asset margin:			
(a) 0% of net written premium	142	118	138
(b) 40% of net written premiums (s)	36	205	157
(c) 80% of net written premiums	13	295	183

* deflated to the date of assessment using the retail prices index and expressed as a percentage of net written premiums** in the year before the date of assessment (see App 3.5.8).

** here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses.

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

(s) indicates the assumption made for the standard basis.



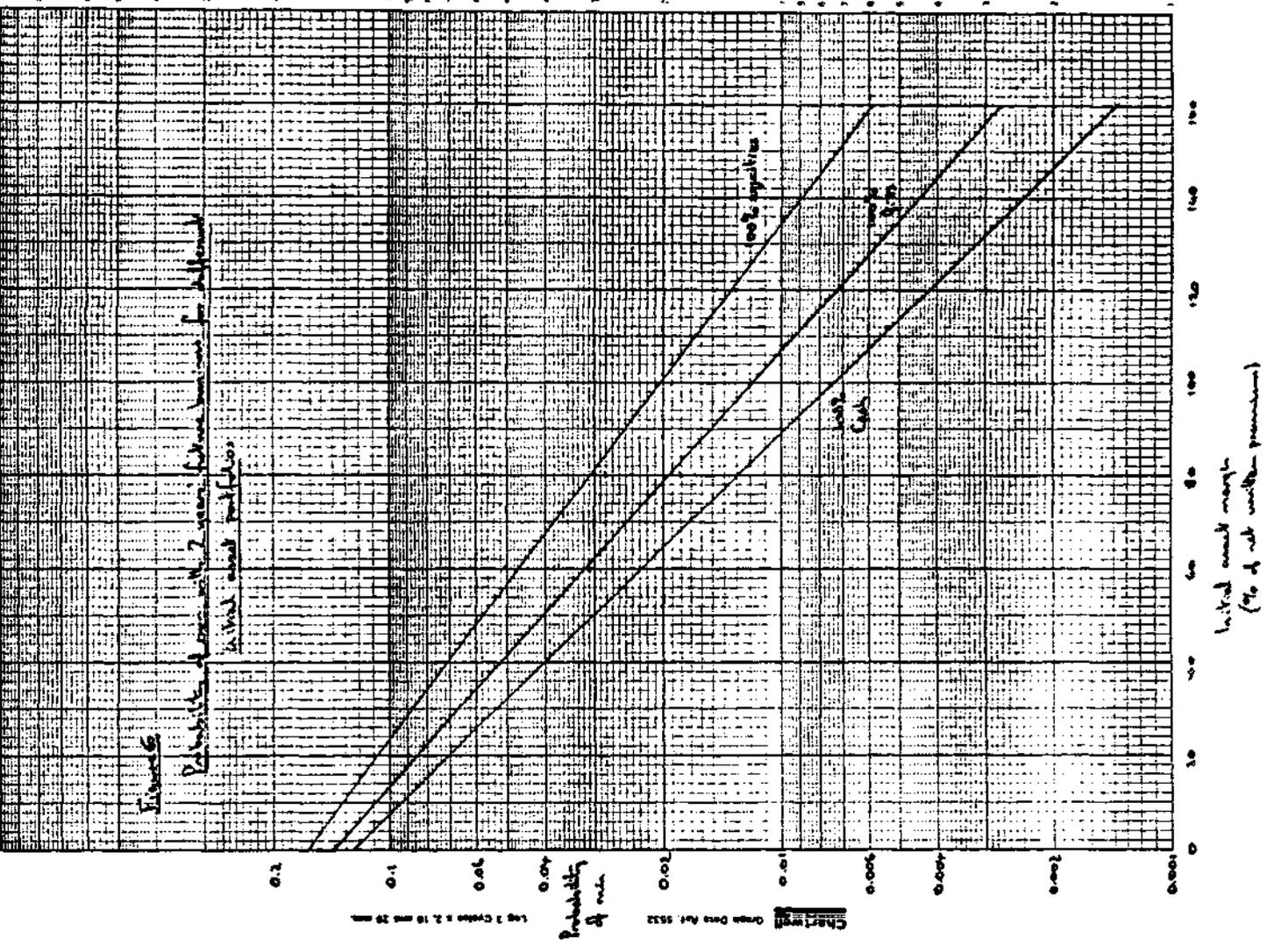
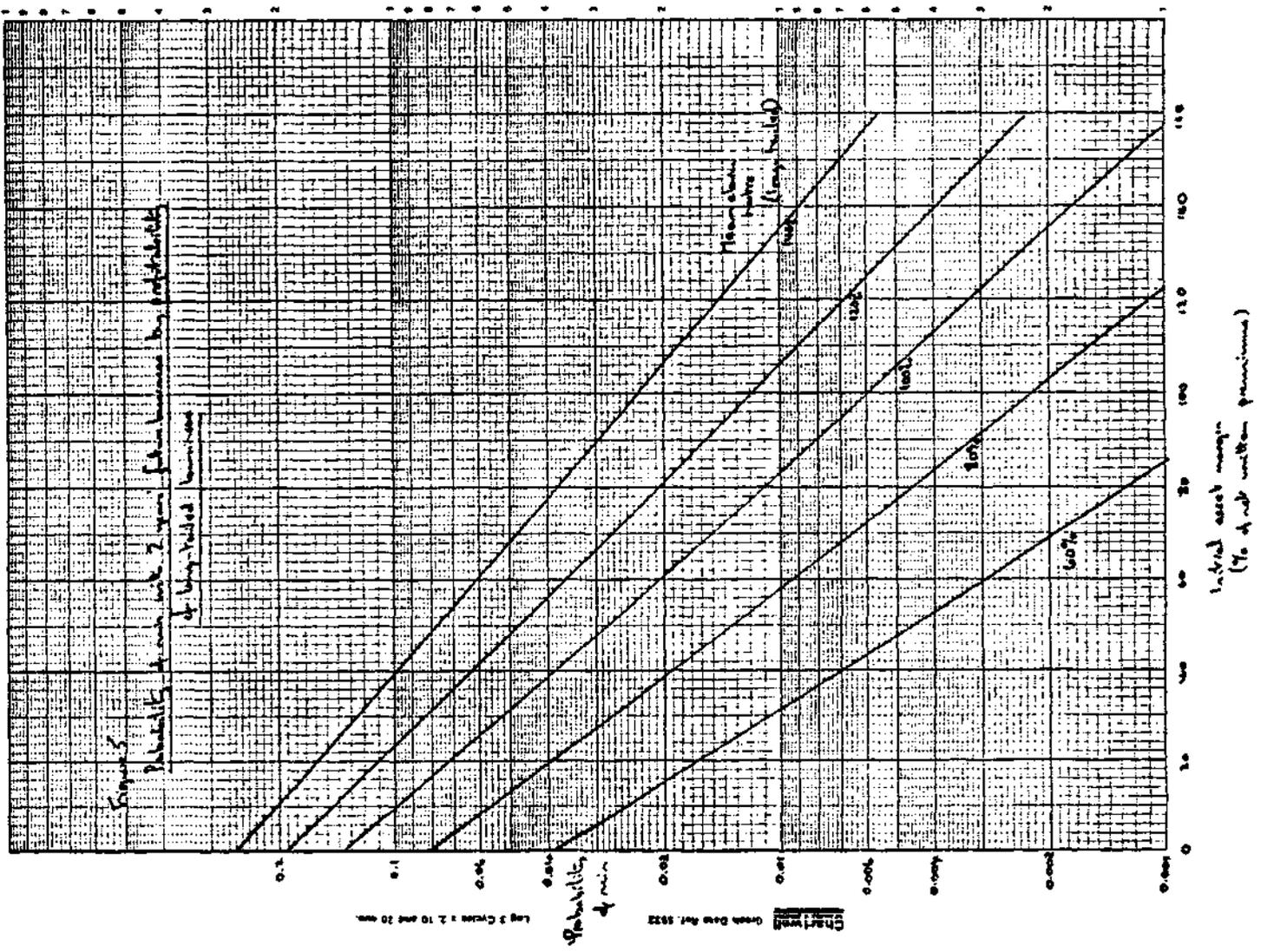


Table 5 Asset margins required to achieve 1/100 probability of ruin - no future new business

Assumptions	Asset margin as % of NWP*	Asset margin as % of technical provisions
Standard basis	35	10
<hr/>		
1. Net written premiums:*		
(a) £1m a year	50	15
(b) £10m a year (s)	35	10
(c) £100m a year	35	10
2. Proportion of long-tailed business:		
(a) 0% of net written premiums	35	25
(b) 40% of net written premiums (s)	35	10
(c) 80% of net written premiums	55	10
3. Initial asset distribution:		
Cash Gilts Equities		
(a) TP+AM - -	45	15
(b) - TP+AM -	50	15
(c) - - TP+AM	85	25
(d) ½TP ½TP AM (s)	35	10
4. Asset selling rules:		
(a) Equities; cash; gilts	50	15
(b) Gilts; cash; equities	30	10
(c) In proportion to holdings (s)	35	10
(d) Sell best performer first	45	15

* here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses.

(s) indicates the assumption made for the standard basis.

Table 6 Asset margins required to achieve 1/100 probability of ruin - two years' new business

Assumptions	Asset margin as % of NWP*	Excess asset margin as compared to run-off (as % of NWP*)		
Standard basis	85	50		
1. Net written premiums*				
(a) £1m a year	105	55		
(b) £10m a year (s)	85	50		
(c) £100m a year	85	50		
2. Proportion of long-tailed business:				
(a) 0% of net written premiums	80	45		
(b) 40% of net written premiums (s)	85	50		
(c) 80% of net written premiums	105	50		
3. Real growth rate (past and future):				
(a) -20% a year	110	40		
(b) No growth (s)	85	50		
(c) +20% a year	90	60		
4. Mean claim ratio (short-tailed):				
(a) 80% of net written premiums	50	20		
(b) 100% of net written premiums (s)	85	50		
(c) 120% of net written premiums	125	80		
5. Variability of claim ratio (short-tailed):				
(a) Standard deviation 5% NWP	85	50		
(b) Standard deviation 10% NWP (s)	85	50		
(c) Standard deviation 15% NWP	85	50		
6. Mean claim ratio (long-tailed):				
(a) 80% of net written premium	65	35		
(b) 100% of net written premiums (s)	85	50		
(c) 120% of net written premiums	115	70		
7. Variability of claim ratio (long-tailed):				
(a) Standard deviation 10% NWP	85	50		
(b) Standard deviation 15% NWP (s)	85	50		
(c) Standard deviation 20% NWP	85	45		
8. Initial asset distribution:				
	Cash	Gilts	Equities	
(a) TP+AM	-	-	-	85
(b) -	-	TP+AM	-	105
(c) -	-	-	TP+AM	130
(d) †TP	†TP	-	AM (s)	85

* here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

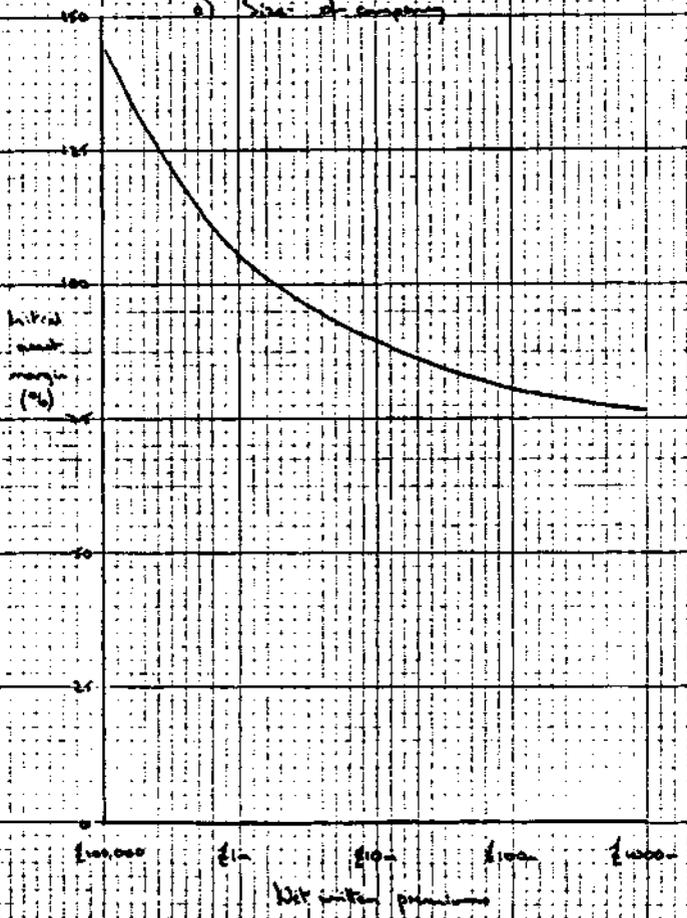
(s) indicates the assumption made for the standard basis.

Figure 1

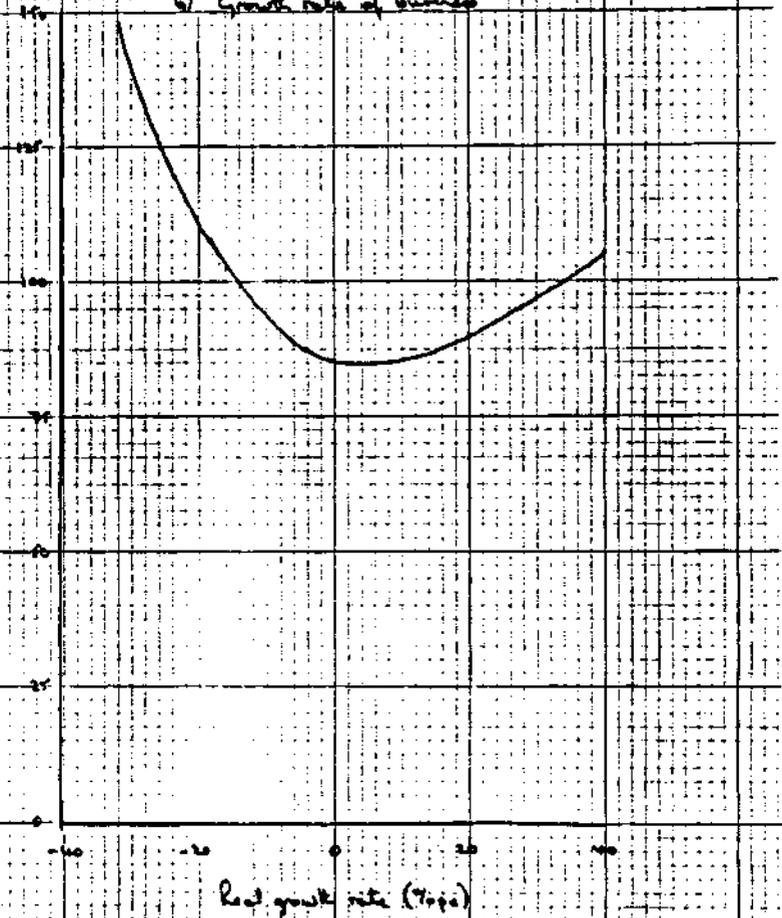
Initial asset margin as percentage of net written premium to achieve

1/100 probability of ruin with 2 year future business

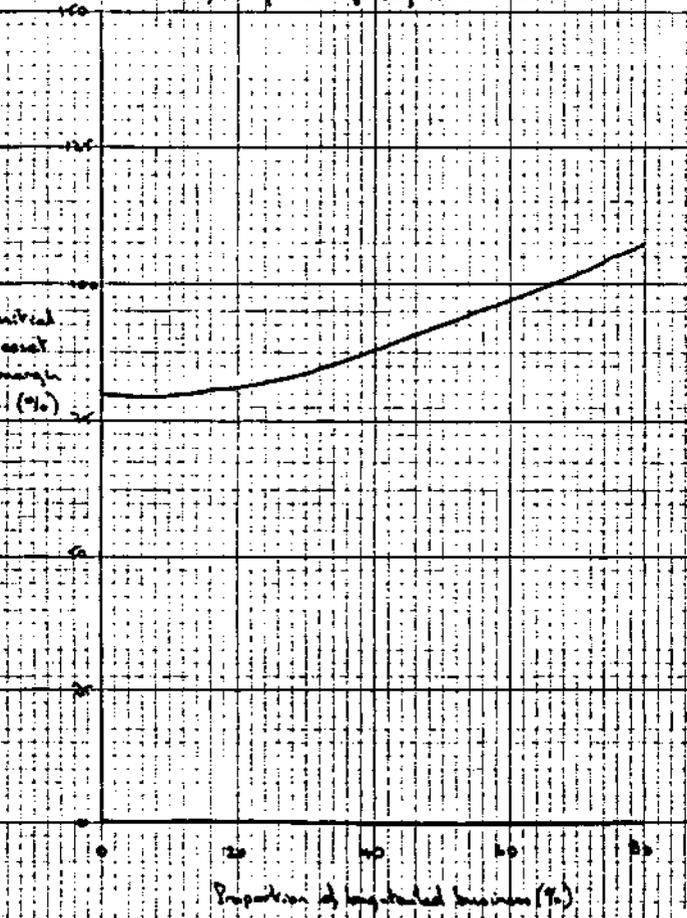
a) Size of company



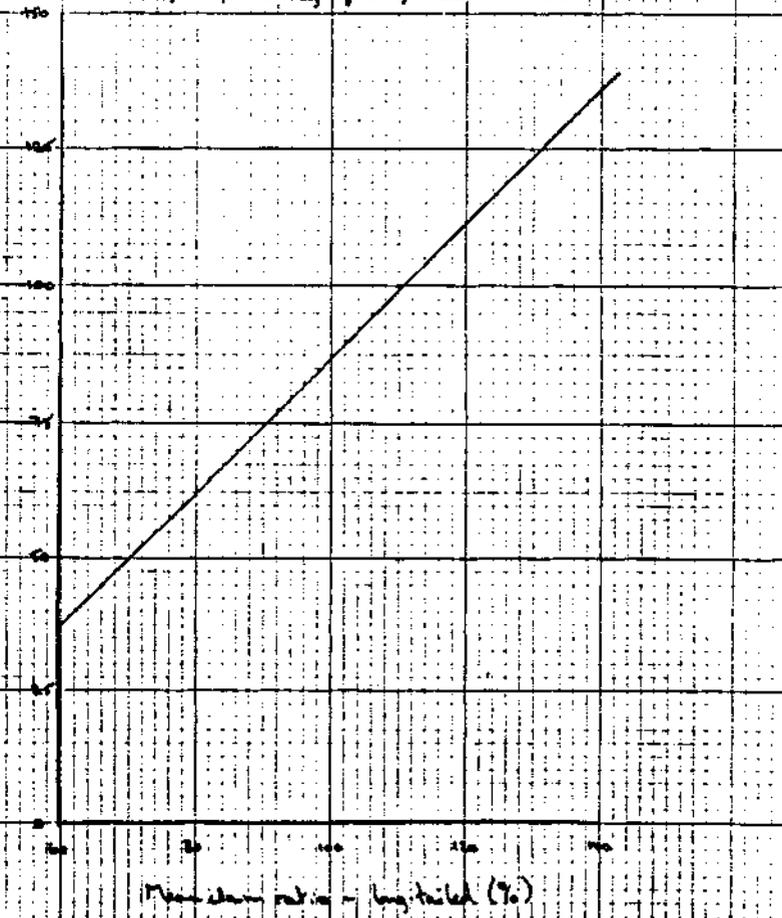
b) Growth rate of business



c) Proportion of long-tailed business



d) Profitability of long-tailed business



on a number of different aspects. However, the results presented do appear consistent and sensible and variations in relation to changing parameter values conform with general reasoning.

9.8 It is difficult from these results to draw conclusions about an appropriate level for a minimum statutory solvency margin. In fact we have avoided using the term solvency margin in this section because of its special significance in statutory terms and have referred to the necessary margin as the asset margin. Our asset margins relate to particular assumptions about the basis for the technical provisions and provide a defined degree of security in relation to specified scenarios on the basis of our model. A statutory solvency margin, in the sense in which it is usually used, provides a general level of security, independent of the particular circumstances of the company, against all possible future scenarios, including the effect of unquantifiable risks such as fraud, mismanagement and the failure of reinsurers.

9.9 A starting point for consideration of an appropriate level of statutory solvency margin might be to look at the asset margin for a company with a fairly standard distribution of business, a moderate growth rate and a cautious investment policy. In some ways it would be more logical for the resulting margin to be in two parts:

- * a percentage of the technical provisions at the assessment date;
- * a percentage of written premiums.

The former would represent the margin required in respect of the run-off risks and the latter the margin required in respect of writing up to two years' further new business. To the new business margin might be added a contingency loading to cover other unquantifiable risks.

9.10 This would provide a basic safety net for an average company, assuming that technical provisions were at least up to the standard envisaged. Statutory reserving standards might be necessary to achieve this, since it has to be recognized that a solvency margin requirement based on technical provisions has a similar weakness to one based on written premiums. If the provisions are understated the requirement is reduced, whereas it should in fact be higher.

9.11 However, such a basic level of solvency margin requirement would not deal with the problem of different asset backing or of other differences between the risks to which companies are subject. Alongside this requirement, therefore, there would be a requirement for a report by an actuary or other expert on the overall financial strength of the company. This would transcend the arbitrary dividing line between technical provisions and solvency margin and would take specific account of the nature of the business written by the company, the proportions of different types of business, the assets held, and all other relevant factors, including the nature of and the security of the reinsurance programme.

9.12 If a requirement for an actuarial report is not introduced, then further consideration would need to be given to whether the solvency margin requirement should include components relating to the assets held and the reinsurance recoveries expected. Regard should also be had to the nature of the outstanding claims portfolio and the type of business being written. However, such a solution would be far from ideal.

10. Reinsurance

10.1 Reinsurance business accepted may be regarded as another class of business, which is often particularly volatile and unpredictable. Appropriate reserving levels for casualty reinsurance business are likely to present particular problems, since it can take many years for the liabilities (including IBNR) to develop fully. Solvency margins certainly ought to have regard to this uncertainty. In principle there seems no reason why the simulation approach should not also provide some insights in this area of an insurer's portfolio.

10.2 Much more difficult to handle in the context of the assessment of financial strength is the security of reinsurance cessions. Many insurers are critically dependent on their ability to recover from reinsurers, since the size of the risks they write is such as to bankrupt or cripple them if they had to bear the liability alone. One safeguard against reinsurance failure is to spread reinsurance cessions widely, so that there is not any great dependence on particular reinsurers. However, this does not remove the need to look carefully at the security of individual reinsurers chosen for the programme.

10.3 From the reserving point of view, a decision has to be made on the extent to which reinsurance recoveries can be relied on. Extreme caution might point towards reserving for the full gross liability but this is not a practical commercial possibility in most cases. Clearly recoveries from reinsurance companies already known to be in trouble should be ignored or heavily discounted, but it is more difficult to know what should be done when there are no specific known problems. In accounting terms it may be difficult to set up a provision against an unquantifiable possibility of reinsurance failure. On the other hand the accountancy concept of prudence would preclude taking credit in advance for receipts which are uncertain, so it would be possible to justify taking only partial credit for reinsurance recoveries, depending on an assessment of the viability of the reinsurers.

10.4 The issue is of particular importance in considering the overall financial strength of the company. This would be one aspect which the actuary would need to cover in his report. Further work is clearly needed in this area to develop ways of modelling reinsurance recoveries. It has been assumed in our model that all claims are net of reinsurance. This may be good enough for many companies, with relatively little dependence on

reinsurance. However, it will be far from adequate for other companies for which the possibility of failure to recover from reinsurers is a significant one and the potential impact disastrous in solvency terms. Some tentative ideas of a possible way of tackling this using the simulation approach are set out in Appendix 6.

10.5 A detailed examination of the reinsurance programme can hardly be practicable for the supervisory authorities and here again it seems that an actuary's report could help. No general solvency requirement can be a substitute for this. The practice adopted for the EEC solvency margins of reducing the solvency margin requirement calculated on the basis of gross written premiums to allow for reinsurance based on actual recoveries in the past three years, but with a maximum reduction of 50%, is a very rough and ready solution and does not have any regard to the actual dependence on reinsurers for future recoveries. With non-proportional reinsurance the premium can be very small in relation to the potential liability, so no simple percentage of premium is likely to make sense as a solvency margin. A percentage of anticipated recoveries from reinsurers would have a stronger rationale, but it would be difficult to find a logical basis for any particular percentage and the amount of anticipated recoveries is itself often very difficult to estimate.

11. Professional Considerations

11.1 One of the important aspects of what we are proposing is that it would give rise to a professional report on the financial strength of the company. The background of our consideration is the financial soundness of general insurance companies, but it need not be limited to that. Simulation using an emerging cash concept provides a powerful analytical tool which enables the investigation of issues extending beyond solvency to the consequences of alternative management strategies.

11.2 Current statutory reporting for general insurance companies in the U.K. largely involves presentation of sets of figures, without any accompanying description of the assumptions or methodology. Use of a model such as the one we have presented would enable the methodology to be made explicit and the assumptions set out in a form suitable for presentation to management, or to the DTI, as appropriate. Although there will certainly be many different models which might be deemed to be appropriate and substantial scope for professional judgement about the assumptions used, such an approach would open up the way for a discussion of the underlying issues in a way that is not possible using current accounts and returns to the DTI.

11.3 Section 4 above discusses solvency control in the U.K. In addition to reporting and solvency margin requirements, current regulations pay some attention to assets and reinsurance. The level of liabilities and the adequacy of current premium rates are not regulated except in terms of a general statement that liabilities should be dealt with in accordance with generally accepted accounting principles. Valuation and admissibility

rules for assets may in effect incorporate some implicit margins into a balance sheet view of solvency. Whether there are margins on the liabilities side depends on the interpretation of generally accepted accounting principles. Only a broad description of reinsurance arrangements is required and the regulations relate to disclosure rather than control, although the DTI may bring pressure to bear on a company in the light of the disclosure. We have not attempted in this paper to provide a detailed critique of current statutory regulations, but we believe that standardized reporting, although it has a role to play, is not capable of dealing adequately with the many individual differences that occur between companies operating in a free commercial market.

11.4 Government regulation of solvency is only one of the five levels of protection described by Collett (paragraph 6.7). The other levels of protection required to provide security to the policyholder receive less complete consideration in the U.K. Management fidelity is partly, but inadequately, covered by the "fit and proper" rules for the approval of controllers, directors and managers of insurance companies. Quality of management however, is not attended to except indirectly for public companies through the results reported. Buyer awareness is low as regards solvency and probably heavily influenced by companies' own advertising. Intermediaries and newspapers provide the principal alternatives to self-advertisement in making buyers aware of the degree of security provided by particular companies, but the level of analysis offered is not very deep. A government benefit guarantee for personal lines insurance is provided by the Policyholders' Protection Act 1975 but clearly absent is any requirement for a professional review of a general insurance company's actions and its overall financial position.

11.5 As regards life insurance, the ingredients of a report by the appointed actuary on the valuation of the life fund of a long-term business company are laid down in Schedule 4 of the Insurance Companies Act (Accounts and Statements) Regulations 1983 and follow a pattern which has been established over many years in an area where the actuary has traditionally had a major reporting role. The requirements of valuation regulations are supplemented by guidance notes issued by the Institute and Faculty of Actuaries (GN1 and GN8). Although there are no comparable regulations for the valuation of pension funds, professional guidance notes (GN9) have been issued to assist members.

11.6 The situation in general insurance is somewhat different. In view of the absence at present of a formal reporting role for the actuary, or for that matter for anyone else other than the Directors and the auditors, such reports as are prepared follow individual circumstances and respond to the particular task requested. In these circumstances it would, in our view, be desirable for the Institute of Actuaries to issue guidance notes for actuaries practising in this area, with the objectives both of introducing some measure of standardization into the scope of

such reports and of emphasizing the profession's commitment to general insurance. We believe that practising actuaries would be glad to have the force of professional guidance behind them in carrying out their duties in a new and relatively undeveloped area. This would apply whether the reports are designed to cover the overall financial position of a company or are more restricted, for example covering the level of provisions for liabilities, the adequacy of premium rates or the suitability of the rating structure.

11.7 The precise status of such guidance notes would perhaps depend upon whether there was any formal requirement being placed on companies to have a report of the sort envisaged. If this were to become a requirement laid down by the DTI, then we might expect to see regulations along the lines of Schedule 4 for long term business companies and formal guidance from the Institute of Actuaries along the lines of GN1 and GN8. If there were no legislative requirement to have a report on the financial strength of the company, but only a general move by insurance companies in this direction, then guidance of a different form might be appropriate. This might be along the lines of GN9 for pension fund valuations or take the form of "Notes on Recommended Practice" (NORP), as suggested by Abbott in a paper presented to the Institute in March 1986⁴¹.

11.8 Abbott indicated that at present the actuary has to prepare reports without the benefit of an agreed reference framework. The NORP would set out an appropriate reference framework with which actuaries would feel more comfortable and which would relate, where relevant, to the broader reference framework of accounting standards or a Statement of Recommended Practice (SORP). Until recently it has been difficult for the actuarial profession to lay down a reference framework in the absence of a specific accounting standard or SORP and in view of the widely differing stances taken by different companies. The preparation of a SORP, issued by ABI, with the approval of the accountancy bodies, should serve to clarify the position somewhat, although some areas are likely to remain unclear. Actuaries may find the reference framework of a SORP to be one with which they find themselves uncomfortable, but this should not prevent the actuary from being able to offer advice within that accounting framework, although he or she will want to make clear the framework within which his advice is given and emphasize that the advice does not extend to certifying that the provisions are adequate in a normal actuarial sense.

11.9 Although the provisions may be established under the reference framework of the SORP, together with the statutory minimum solvency requirement under EEC Directives and regulations from the DTI, the actuary would be free, in reporting on the financial strength of the general insurance company, to adopt an actuarial reference framework, in which both assets and liabilities are taken into account and proper regard paid to uncertainty. The proposed NORP would assist in defining this framework. A first draft of such a NORP is set out in Appendix 7. It picks up from section 6 of the draft NORP

proposed by Abbott, which is at present being discussed by Council with a view to its early promulgation. It does not repeat the sections on provisions for outstanding claims and unexpired risks, which might be treated as separate NORPs or covered by separate Institute Guidance Notes.

12. Conclusions

12.1 We have outlined the weaknesses in the traditional balance sheet concept for describing the true financial strength of a general insurance company. Assets and liabilities should not be treated as independent aspects and much more attention needs to be focussed on their variability. Appropriate techniques have been developed by actuaries for dealing with these problems in the life and pensions areas and similar principles can be used to begin to tackle the general insurance problem. The parallels are drawn out in the paper by Coutts and Devitt¹².

12.2 However, there are also differences, arising mainly from the greater uncertainty and volatility of claim amounts in general insurance. The problem of variability can be explored by means of simulation. A simulation model of a general insurance company provides a powerful tool for analysing the impact of all types of uncertainty and assessing the true financial strength of the company.

12.3 A solvency margin requirement expressed in terms of a simple percentage of written premiums (or in terms of a percentage of technical provisions, which might be more appropriate to cover the run-off risk) cannot have proper regard to the risks to which each company is subject, whether as regards the assets or liabilities. It must, therefore, be seen as a general underlying safety net, providing a margin against the effects not only of stochastic variations but also of mismanagement, fraud or simply error, and permitting the statutory authority to operate a satisfactory control system.

12.4 Despite our strong belief that the solvency margin should relate to the various risks affecting the financial position of an insurance company, we accept that there will be interest in the use of our model to provide a rationale for a minimum statutory solvency margin. Some thought needs to be given to the implications of adopting a particular probability of ruin for this purpose and to the assumptions which might be appropriate, particularly in regard to the assumed standard of reserves, the starting conditions for the asset and inflation models and the appropriate level for mean claim ratios and the level of variabilities assumed for claims outgo. Our standard basis was chosen to provide a suitable basis for comparison and it should not be assumed that it is appropriate as a basis for setting a solvency margin requirement.

12.5 Simply by way of illustration, however, it can be seen how some conclusions might be drawn from the results of our model. At the level of security provided by a probability of ruin of one

per cent, Table 5 shows that, for a company on our standard basis the margin necessary to cover the run-off risks would be about 10% of technical provisions, assuming that the provisions for outstanding claims are set up on an undiscounted basis with allowance for inflation at 5% (the mean value used in the Wilkie model) which may be taken to imply some implicit margin. Such a margin may be a little stringent as a minimum for larger companies but Table 5 indicates that it should be higher for smaller companies. A similar standard to this 10% margin would be obtained for the mix of business considered here by setting up provisions for outstanding claims allowing for inflation at 10% with no discounting.

12.6 Care has to be taken in interpreting the extra margin implied as being necessary to allow for the risks contingent on writing new business for two years. The margins to cover the run-off risk have been expressed for this purpose in terms of net written premium and these margins (for the respective sets of assumptions) have then been subtracted from the margins obtained assuming two further years in business. It could be argued that if the risks of new business and run-off are to be provided for independently, then the model should be run with no past business in order to assess the appropriate margin for new business risks. We have not done this as we do not believe that the two issues are independent, there being interactions in regard to both assets and the variability of the run-off of claims. Assuming that the margins expressed as a percentage of net written premiums are additive, Table 6 indicates a margin of 50% of written premium net of commission and expenses for a company on our standard basis and somewhat less for a company with investment entirely in cash. This might be equivalent to 35-40% of actual gross written premiums.

12.7 Such a solvency margin requirement appears rather high and it is worth considering briefly some of the major factors which give rise to it. A significant part arises from the effect of simulated future inflation and the possibility that returns on cash will not be adequate to compensate for it. This suggests that the risks might be reduced with greater use of index-linked stocks.

12.8 Much of it also arises from the assumption on the standard basis of a mean claim ratio of 100% of net written premiums. As described in paragraph 8.3.2, this implies break-even if future investment income exactly balances inflation. Thus the assumption is that business is written on a basis where the only profit on an expected value basis is to the extent that a positive real rate of return can be obtained. This might be perceived as too stringent for a minimum solvency margin requirement, although it may not be unrealistic for certain types of business in current conditions. The requirement could be reduced by about 1% of actual written premiums for every percentage point by which the expected claim ratios are reduced below 100%.

12.9 It may be that for some purposes a security level as high as 99% (probability of 1%) would be regarded as excessive. The required asset margin depends critically on the level chosen. To illustrate this, Table 7 shows the margins necessary on our standard basis to give different security levels.

Table 7 Asset margins by security level on standard basis

Asset margins required*			
Security level	No future business	2 years' future business	Excess as compared to run-off
%	%	%	%
90	-10	15	25
95	5	30	25
98	25	60	35
99	35	85	50
99.5	55	110	55
99.9	70	140	70

* as percentage of written premium net of commission and expenses

12.10 Any general solvency requirement will have its limitations. Apart from the points mentioned in paragraph 12.3, there is also the problem of relating the requirement to written premiums or to technical provisions, which may themselves be more adequate for some companies than for others. The adequacy of the technical provisions is of particular importance (cf paragraph 9.2), since they determine what assets are apparently available as a margin. There is, therefore, a need for consistent standards to be applied in setting technical provisions, suggesting that there would be considerable advantages in requiring the provisions to be established on the basis of advice from an actuary or other claims reserving expert, acting within the framework of an appropriate professional standard. However, it has to be acknowledged that there is always likely to be some uncertainty about the strength of technical provisions.

12.11 We have also argued that a crude minimum solvency margin requirement cannot adequately have regard to the true level of risk for a particular company. The supervisory authority is not well-placed to assess each company's risk situation in detail on an individual basis and the answer would seem to be to rely on an appointed actuary or other similarly qualified person within the company (or acting as a consultant to the company). The actuary would be responsible for reporting both to management and to the supervisory authority on the financial strength of the company, taking all relevant factors into account. A summary of the actuary's report could appear in the statutory returns, with full details being available to the supervisory authority on request. The supervisory authority would be able to question the actuary on the effects of alternative assumptions and could then discuss with management an appropriate strategy for reducing the risk profile to an acceptable level.

12.12 To perform his duties effectively, the actuary would need to use simulation techniques. There is plenty of scope for developing appropriate simulation models for this task and one such model is presented here as an example of what can be done. Apart from providing a framework for analysing the existing position of the company, such models could be powerful tools for answering a wide variety of "what if?" questions, such as:

- * is the investment strategy too risky with the present asset margin?
- * what additional capital would be needed to pursue a particular growth strategy?
- * will the strategy give a reasonable expected return on the additional capital?

13. Acknowledgements

13.1 The Working Party wishes to acknowledge a debt of gratitude to the authors of the Finnish Solvency Study⁵ for starting them off along the lines of this paper and, in particular, to Professor Teivo Pentikäinen for his advice and encouragement. Our thanks are also due to many UK actuaries who commented on the work at different stages.

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Appendix 1 DESCRIPTION OF SIMULATION MODEL OF GENERAL INSURANCE
COMPANY

A1.1 In standard risk theory the year to year transition formula is of the form:

$$\Delta U = B + I - X - C - T$$

where ΔU is the change in the solvency margin U
 B is the earned premium income, including safety and expense loadings but net of reinsurance premiums
 I is the income from investments
 X is incurred claims net of reinsurance recoveries
 C is the cost of administration, reinsurance etc
 T is dividends, tax, etc

By implication, incurred claims includes changes to estimates of outstanding claims generated in previous years and included in the technical provisions at the start of the year in question. This formulation is also deficient in that changes in the values of investments are ignored.

A1.2 GENERAL FORMULA

More generally, we define:

$$\begin{aligned} \Delta A(j) &= A(j) - A(j-1) \\ &= \sum_k A_k(j) - \sum_k A_k(j-1) \\ &= \sum_k A_k(j-1) [(1+y_k(j-1))(1+g_k(j-1)) - 1] + \{B(j) - C(j) - T(j) - \sum_{i \leq j} X(i;j)\} \end{aligned}$$

where $A(j)$ is the total value of the assets at the end of year j
 $A_k(j)$ is the total value of component k of the asset portfolio at the end of year j (In our model $k=1$ for cash, 2 for redeemable government securities, 3 for ordinary shares, 4 for non-interest bearing assets)

$y_k(j)$ is the yield on asset component k at the end of year j . In particular, in our model:

$$y_1(j) = c(j) - .01$$

$$y_2(j) = c(j) - .01 + .0005n$$

$$y_3(j) = y(j)$$

$$y_4(j) = 0$$

where $c(j)$ is the yield on 2.5% Consols

$y(j)$ is the dividend yield on the Financial Times Actuaries All-Share Index.

$n = \min [m, 10]$ and m is the number of years to the end of the run-off

$P(j;n) = 9a_{\overline{n}|} + 100v^n$ at rate of interest $c(j) = .01 + .0005n$

$g_k(j)$ is the proportionate change in capital values between the end of years j and $(j+1)$. In particular, in our model:

$$g_1(j) = 0$$

$$g_2(j) = y_2(j)a_{\overline{n}|} + v^n$$

$$g_3(j) = \frac{d(j+1)y(j)}{d(j)y(j+1)}$$

$$g_4(j) = 0$$

where $d(j)$ is an index of share dividends (= dividend yield x price index) corresponding to $y(j)$

$a_{\overline{n}|}$ and v^n are calculated at rate of interest $y_2(j+1)$

$B(j)$ is the written premium income in year j including safety and expense loadings

$C(j)$ is the cost in year j of administration, commission, reinsurance etc.

$T(j)$ is the amount paid out in dividends and tax in year j

$X(i;j)$ is the amount settled in year j (net of reinsurance recoveries) in respect of claims arising in year i

We now define $B'(j) (= B(j) - C(j))$ as the written premiums in year j net of commission and all expenses other than claims settlement expenses and $X(i;j)$ as including claims settlement expenses.

A1.3 ASSET AND INFLATION MODELS

The asset components $A_k(j)$ can be defined in a variety of ways relative to the total $\sum A_k(j)$. For example, if investment or disinvestment is proportional to the value of assets brought forward to the end of the year from the previous year-end,

$$A_k(j) = \frac{A(j)}{A(j-1)} A_k(j-1)$$

If proportions p_k ($\sum p_k = 1$) are specified such that p_k of any new investment is invested in component k :

$$A_k(j) = A_k(j-1)[1+y_k(j-1)][1+g_k(j-1)] + p_k\{B(j)-C(j)-T(j)-\sum_{i \leq j} X(i;j)\}$$

We also define $q(j)$ as the retail price index at the end of year j and $r(j)$ as the price growth in year j :

$$r(j) = \frac{q(j) - 1}{q(j-1)}$$

The variables $q(j)$, $d(j)$, $y(j)$ and $c(j)$ are defined by an interrelated set of autoregressive models, described in detail in Appendix 2.

A1.4 TAX AND DIVIDENDS MODEL

The dividends and tax term is expressed in terms of the investment income and an input parameter (t), representing the proportion of investment income absorbed by tax and dividends paid to shareholders, by the following:

$$T(j) = t \cdot \sum_k A_k(j-1)y_k(j-1)$$

A1.5 MODEL OF CLAIMS GENERATION PROCESS

We define written premiums in the year prior to the date of assessment (taken as the time $j = 0$) as B_0 and the rate of growth of written premiums before and after that date as e_1 and e_2 . Then:

$$B'(j) = B'(0) (1 + e_1)^j \quad (j < 0)$$

$$B'(0) (1 + e_2)^j \quad (j \geq 0)$$

$$\text{and} \quad B'_k(j) = f_k B'(j) \quad \text{for } k=1,2$$

where f_k is the proportion of written premiums in respect of type of business k ($k=1$ for short-tailed, 2 for long-tailed).

Claims are assumed to be generated from written premiums by means of a variable claims ratio and specified proportions settled in each year of the run-off. Thus the estimated payment in year j in respect of premiums written in year i is given by:

$$X(i;j) = \sum_k s^k(j) R_k(i) B'_k(i) \prod_{l=i+1}^j (1+r(l))$$

where $R_k(i)$ is the uninflated, undiscounted claims ratio in year i , assumed to be normally distributed with mean R_k and standard deviation σ_k . For $i \leq 0$, $R_k(i) = R_k$

$s^k(j)$ is the proportion of uninflated, undiscounted claims from type of business k that are assumed to be settled in development year j .

A1.6 MODEL OF CLAIMS SETTLEMENT

Claims settled in each year of development are aggregated from all the separate years of origin, whether before or after the date of assessment. The total amount of claims settled in year j ($X(j)$) is assumed to be normally distributed with mean $X(j)$ and standard deviation $a \bar{X}(j) + b \sqrt{\bar{X}(j)}$ where a and b are specified constants and $\bar{X}(j)$ is defined as:

$$\bar{X}(j) = \sum_{i \leq j} \sum_k s^k(j-i) R_k(i) B'_k(i) \prod_{l=i+1}^j (1+r(l))$$

TECHNICAL PROVISIONS

The technical provisions ($TP(0)$) at the date of assessment are calculated from the estimates of claims to be settled in future years arising from premiums earned prior to the date in question. They allow for inflation at a specified rate (r) and discounting at a specified rate (d). They can be expressed as follows:

$$TP(0) = \frac{1}{2} \sum_k B'_k(0) R_k \sum_{j=0}^{\infty} s^k(j) (1+r)^j (1+d)^{-j} + \sum_k R_k \sum_{i=0}^{\infty} B'_k(i) \sum_{j=i}^{\infty} s^k(j) (1+r)^j (1+d)^{-j} + \frac{1}{2} B'_k(0)$$

The initial solvency margin ($SM(0)$) is defined as a function of written premiums in the year before the date of assessment:

$$SM(0) = \alpha B'(0)$$

The initial assets are thus given by:

$$A(0) = TP(0) + SM(0)$$

Appendix 2 DESCRIPTION OF STOCHASTIC MODELS USED FOR ASSETS AND INFLATION

A2.1 The investment and inflation models used are those proposed by Wilkie^{38,39}. A summary of the specification of the model is given below.

The variables used are:

$q(t)$ The UK retail prices index

$d(t)$ An index of share dividends.

$y(t)$ The dividend yield on these same share indices, that is, the dividend index at the specified date divided by the share price index at that date.

$c(t)$ The yield on 2.5% Consols (irredeemable), which is taken as a measure of the general level of fixed interest yields in the market.

A2.2 The model used for $q(t)$ is:

$$\nabla \ln\{q(t)\} = \mu_q + \alpha_q (\nabla \ln\{q(t-1)\} - \mu_q) + \sigma_q \cdot z_q(t)$$

where the backwards difference operator is defined by

$$\nabla x(t) = x(t) - x(t-1)$$

and $z_q(t)$ is a sequence of independent identically distributed unit normal variates.

The values adopted for the parameters are:

$$\mu_q = 0.05, \alpha_q = 0.6, \sigma_q = 0.05.$$

A2.3 The model for $y(t)$ is:

$$\ln\{y(t)\} = \omega_y \cdot \nabla \ln\{q(t)\} + y_n(t)$$

where $y_n(t) = \ln\{\mu_y\} + \alpha_y (y_n(t-1) - \ln\{\mu_y\}) + \sigma_y \cdot z_y(t)$

and $z_y(t)$ is a sequence of independent identically distributed unit normal variates.

The values adopted for the parameters are:

$$\mu_y = 0.04, \alpha_y = 0.6, \omega_y = 1.35, \sigma_y = 0.175.$$

A2.4 The model for $d(t)$ is:

$$\nabla \ln \{d(t)\} = \omega_d \left(\frac{\delta_d}{1-(1-\delta_d)B} \right) \nabla \ln \{q(t)\} + \alpha_d \nabla \ln \{q(t)\} \\ + \beta_d \sigma_y z_y(t-1) + \sigma_d z_d(t) + \gamma_d \alpha_d z_d(t-1)$$

where the backwards step operator B is defined by

$$\bar{B}x(t) = x(t-1)$$

and hence $\bar{B}^n x(t) = x(t-n)$

and $z_d(t)$ is a sequence of independent identically distributed unit normal variates.

The term in parentheses above involving δ_d represents an infinite series of lag effects, with exponentially declining coefficients:

$$\delta_d, \\ \delta_d(1-\delta_d), \\ \delta_d(1-\delta_d)^2, \text{ etc}$$

The sum of these coefficients is unity, so this part of the formula represents the lagged effect of inflation, with unit gain. This means that if retail prices rise by 1 per cent this term will also, eventually, rise by 1 per cent. We can alternatively describe it as the "carried forward" effect of inflation $m(t)$, where

$$m(t) = \delta_d \nabla \ln \{q(t)\} + (1-\delta_d) m(t-1),$$

from which we see that the amount that enters the dividend model each year is δ_d times the current inflation rate, plus $(1-\delta_d)$ times the amount brought forward from the previous year, and that this total is then carried forward to the next year.

The values adopted for the parameters are:

$$\omega_d = 0.8, \delta_d = 0.2, \alpha_d = 0.2, \beta_d = -0.2,$$

$$\gamma_d = 0.375, \sigma_d = 0.075.$$

A2.5 The model for $c(t)$ is:

$$c(t) = \omega_c \left(\frac{\delta_c}{1 - (1 - \delta_c)\bar{B}} \right) \sqrt{\ln\{q(t)\} + n(t)},$$

where $\ln\{n(t)\} = \ln\{\mu_c\}$

$$+ (\alpha_c \cdot \bar{B} + \beta_c \cdot \bar{B}^2 + \gamma_c \cdot \bar{B}^3) (\ln\{n(t)\} - \ln\{\mu_c\}) \\ + \varphi_c \cdot \sigma_y \cdot z_y(t) + \sigma_c \cdot z_c(t),$$

where $z_c(t)$ is a sequence of independent identically distributed unit normal variates.

The term in parentheses in δ_c has a similar form to the δ_d term in the dividend model, though the parameter value is different. It represents the current value of expected future inflation as an exponentially weighted moving average of past rates of inflation.

The values adopted for the parameters are:

$$\omega_c = 1.0, \delta_c = 0.045, \mu_c = 0.035, \alpha_c = 1.20,$$

$$\beta_c = -0.48, \gamma_c = 0.20, \varphi_c = 0.06, \sigma_c = 0.14.$$

A2.6 Interested readers are referred to Wilkie's paper³⁹ for interpretation of what the model implies and how it can be used. A fuller description of the derivation of the model is given in another paper by Wilkie³⁸.

A2.7 There is no specific provision in Wilkie's model for cash as an investment. We have assumed that the return on cash for any year is the Consols yield at the start of the year less one percentage point.

- A3.1.1 In order to simulate the run-off of an insurance company it is necessary to make decisions in regard to a large number of parameters. The program is written in such a way as to allow for a range of values of each of the parameters. As there are at least 20 parameters or values that may vary, and several may have up to 8 or 9 values, it would not be sensible to provide for every possible combination of values of the parameters. The program would take too long to run and the volume of output could not be assimilated.
- A3.1.2 The program is written, therefore, to allow each of the parameters to vary in turn over its whole range, whilst the others are kept constant at a "normal" or standard level. It also permits an analysis by two parameters at a time, for every possible combination of the various levels of those two parameters.
- A3.1.3 The same set of random numbers has been used for each different parameter combination, so that the comparisons are not affected by any bias in the particular sets of random numbers chosen.

The basis of the simulations

- A3.2.1 The program works from a series of written premiums, going back sufficiently far into the past to include every year for which claims are still to be run off. Provision is made for three alternative bases for the future.
- 1 A wind-up - an assumed return of the unearned premium reserve (UPR) as the policyholders claim on the liquidator for the unearned part of their premiums.
 - 2 A run-off - the UPR is translated into a pattern of future claims payments and included with payments in respect of the outstanding claims.
 - 3 A continuing business - the future period of writing premiums can be selected and after that there is a run-off as in 2 above.
- A3.2.2 It is necessary to generate claim ratios for each type of business and for provision to be made for the claims ratios to vary stochastically. The classes of business are characterized by the length of run-off period and settlement pattern and the proportions of business written in each category of tail are set by a parameter.
- A3.2.3 The investment model is that given by Wilkie³⁹. The investment mix may be varied according to the nature of the business and the initial mix is specified separately for the technical provisions and asset margin. The rules for selling and buying investments may be selected. Buying is likely to occur where there is a continuing business and written premiums are growing but it can also arise in the later years of a run-off where the income from the assets is large, particularly in the case of larger initial asset margins.

- A3.2.4 Since the volume of written premiums affects the ratio of outstanding claims to the latest year's written premiums, premiums can be allowed to grow or diminish in real terms over the years.
- A3.2.5 Corporation tax is payable by a general insurance company in the UK on its profits, which include capital gains as well as income from assets and exclude any allowance for indexation of the purchase price of securities. However, such "income" is not subject to tax if it is used to pay claims and expenses and it seems likely that with a company that is in any danger of becoming insolvent there will be past losses carried forward, as well as future claims outgo, that will probably absorb most, if not all, of the income. This will mean that the effective rate of tax on interest will be very low. Provision is made for notional rates of tax for the first five years, at rates well below the current rates of corporation tax. The "tax" is assumed also to include the payment of dividends to shareholders. This will result in an overstatement of the outcome in scenarios where the company remains solvent, but this is not the main feature of the results with which we are concerned. The tax and dividend treatment in the model could clearly be made more sophisticated.

Future statutory solvency

- A3.3.1 For a continuing company it is necessary to examine the financial position at the end of each year, if not more often. Accounts and returns have to be presented and a simulation of the future development of the company for management purposes would need to have regard to how the position might appear in presentational terms at each future reporting date.
- A3.3.2 For a company that is already being run-off or to test what would happen in such circumstances, the reporting constraint is less relevant and our aim has been to look at "true" solvency, rather than the position as constrained by reporting conventions. The model simply looks at the adequacy of the assets to meet the liabilities as they are simulated to arise during the run-off. It does not check the solvency position as it might be reported to shareholders or to the supervisory authority at points during the run-off. Such a factor could be introduced if a procedure for deciding on appropriate bases for the technical provisions in future years were to be defined.

The choice of parameters and their values

- A3.4 Every parameter is allowed to have at most 9 values, but need not be given more than 1. The parameters are numbered 1 to 13 and their levels 1 to 9. The value for level 5 is the standard and a value must be inserted for this parameter in every case, even if it is not included in the list of parameters to be analysed, since the program requires a value to be assigned for every parameter. A detailed list of the parameters and the factors underlying their choice is given below:

1 Written premiums The values used are £100,000, £1 million, £10 million, £100 million and £1000 million a year. The written premiums are taken as being net of initial expenses and commission.

2 Variability of run-off - secular factor The parameter (a) if the first term of the formula $ax + b\sqrt{x}$ (described in paragraph 8.2.5 and Appendix 5) is given the values .05, .10, .15, .20 and .25, with .15 as the standard value.

3 Variability of run-off - stochastic factor The second parameter (b) in the formula $ax + b\sqrt{x}$ is given the initial values 25, 50, 75, 100, 125 with 75 as the standard value. Once the written premiums cease this parameter is increased by 5 in each successive year, whatever the initial value.

4 Claim ratio - long This is the claim ratio for future business of a long-tailed nature, i.e. with a run-off period of 16 years. Claim ratios are assumed to include the expenses of claim settlement as well as actual claim costs but they are related to written premiums net of commission and expenses (cf paragraph 8.3.2). We have used values from 60% up to 140%, with 100% as the standard.

5 Standard deviation - long We have used 5%, 10%, 15%, 20% and 25% of written premiums, with a standard value of 15%.

6 Claim ratio - short This is the claim ratio for future business of a short-tailed nature. We have used values from 60% up to 140%, with a standard value of 100%.

7 Standard deviation - short We have used 5%, 10%, 15% and 20% of written premiums, with a standard value of 10%.

8 Growth rates Separate real growth rates may be assumed before and after the assessment date. Rates varying from -40% a year to +40% a year have been used and the effect of changes in the rate of growth at the date of assessment have been examined. Inflation is automatically allowed for in the program so that the growth assumptions relate to growth in real terms. The standard basis assumes no real growth before or after.

9 Proportions of business These are the proportions of written premiums represented by long-tailed business and short-tailed business, each varying from 0% to 100%, with a standard basis of 40% long-tailed and 60% short-tailed.

10 Asset mix - asset margin Different proportions of equities, gilts and cash are considered, including 100% in each and 50% in each possible pair. The standard basis is 100% in equities.

11 Asset mix - technical provisions The proportions are specified to link with the assumptions for the asset margin. The standard basis is 50% cash and 50% gilts.

12 Asset margin This is expressed as a percentage of the net written premiums in the last year before the date of assessment. The margin is allowed to range from nil to 160%. The normal value has been taken as 40%. Different reserving strength, arising from the assumptions made in calculating the outstanding claims, allowing for inflation and for discounting, can be studied by looking at different asset margins (cf paragraph 9.2). On the standard basis the technical provisions are established using 5% inflation and no discounting, which may be regarded as incorporating some implicit margin, in line with the practice of many companies.

13 Selling rules There are 8 alternative rules, namely:

- a. Sell equities until they are exhausted, then cash and finally gilts
- b. Equities, gilts, cash
- c. Gilts, equities, cash
- d. Gilts, cash, equities
- e. Cash, equities, gilts
- f. Cash, gilts, equities
- g. Sell rateably (ie in proportion to the current value of holdings)
- h. Sell each year whatever has performed best since the start of the run-off

Investment (where there is a surplus of income over outgo) is always done in proportion to the current value of holdings.

It is also necessary to specify:

1. The number of future years. This is limited to the range 1 to 10 but the parameter can take the values 0 or -1 meaning that we are assuming no new business written and that we have either a run-off (0) or a wind-up (-1).
2. The number of simulations.
3. The number of parameters to be analysed, that is 1 or 2.
4. Which parameters are to be allowed to vary.

The program plan

A3.5 The program has been written to permit it to be run on FORTRAN IV (otherwise known as FORTRAN 66). In particular we have avoided the use of negative values in arrays. For this purpose we have assumed that the past is represented by years 1 to 20 and the future by years 21 to 42. Whilst this means that some arrays have to be larger than they would otherwise need to be, the simplification is worthwhile. The program is divided into sections:

1 Initialization This sets out the values of the parameters, dimensions the arrays and sets some initial values. The values of the parameters could be inserted by lead cards if preferred. This section also includes the values for the number of future years, the parameters to be analysed and the number of simulations. This section also contains some data manipulation and checking to avoid time-consuming operations later in the program.

2 The random number generator This generates the necessary number of random normal variates and stores them in an array for use by the later stages of the program. This ensures that the same numbers are used for every parameter combination in respect of each realization of random numbers. They are recalculated for each further realization. The random number generator of the machine has been used to generate uniform random variates in the range 0 to 2. After subtracting 1 these are used in Marsaglia's polar method to generate the corresponding random normal variates. This method requires pairs of uniform randoms and produces normal variates if, and only if, the sum of the squares of the two variates is less than 1. The program counts the number of useful pairs and stops when it has enough to fill the array.

We have tested this process and found that a distribution of 3 million variates was very closely normal, using 9-figure tables of the normal integral for the test. This however does not test that they come in a random order and we have further tested them to count the number of cases where there is a run of 1 increase, 2 increases and so on up to 7 increases. It is not difficult to calculate theoretically the expected number of such runs both upwards and downwards and their expected size. The results are within expected limits. The methods will, it is hoped, be described in detail in a paper to be written by two of the authors of this report, together with notes on the times taken to make the calculations. These seem to vary considerably from one method to another. It is perhaps worth mentioning that what we require are representative sequences rather than purely random ones. Kendall and Babington Smith noted in 1938 that a sequence of 10^{10} random numbers is almost certain to contain a sequence of a million zeros (or, for that matter any other sequence you care to specify). This might be a random sequence but it is not very useful in practice for simulation..

3 Investment values The program now calculates the investment values for up to 26 future years, depending on the particular run-off period involved. The values are of:

1. The retail price index
2. Equity dividends
3. Equity yield
4. Equity price
5. Irredeemable gilt yield
6. Dated gilt yield
7. Cash yield
8. Borrowing rate
9. Relative gilt price
10. Absolute gilt price
11. A net income multiplier (see below)
12. An equity price ratio over half a year
13. A gilt price ratio over half a year
14. A mean retail price index

Note: Items 12, 13 and 14 are required in order to permit calculation of sales and purchases at an assumed mid-year point.

Redeemable gilts are assumed to have a maximum term of 10 years, and to have a gross redemption yield $\frac{1}{2}\%$ lower than the yield on irredeemables generated by Wilkie's model with the yield falling linearly to 1% below the yield on irredeemables as the term to maturity falls to zero. "Cash" is assumed to be money on deposit or very short term gilts. It is assumed that the cash yield is 1% below that of irredeemable gilts and that when cash becomes negative and we have to borrow, it is at a rate 2% higher than the irredeemable gilt rate. The gilts held at the start are assumed to be 10-year stocks. At the end of each successive year the stocks (by then 9-year stocks) are sold and the proceeds used to purchase more 10-year stocks. Once the period left to the end of the run-off reaches 10 years the 10-year stocks then held are held to maturity, or until they need to be sold. Purchases are assumed to be of stocks at par, that is with coupon at the 10-year yield then ruling.

An alternative strategy would be to assume a constant coupon for all dated gilts (eg $8\frac{1}{2}\%$ as the "normal" redeemable gilt yield with the Wilkie model). When the yield fluctuates violently the latter method gives a more stable result, but tests we have carried out suggest that the two methods give very similar answers with a large number of simulations.

The equity price ratio is the square root of the ratio of the equity price at the end of the relative year to its value at the start of the year. Its purpose is to revalue equities from the year-end value, on which the income is based, to the mid-year value at which it is assumed that sales take place or purchases are made. The gilt price ratio performs a similar function for dated gilts. Cash is assumed not to vary. After the mid-year transactions the remaining values of gilts and equities are updated to the year-end by a further multiplication by the equity (or gilt) price ratio. Although interest is calculated on the values at the start of the year, allowance is made for the loss of income on selling during the year by multiplying the net outgo by a factor of 1 plus half the average yearly yield on the investments. Whilst this assumes that the values of all three classes are equal, the effect of differences is likely to be too small to be of any consequence in practice.

4 Best investment The next section is really a continuation of section 3 in that it calculates which of the three classes of investment has performed best since the start of year 21 and stores this information for use later in the program.

5 Outstanding claims The program now calculates the outstanding claims at the end of year 20. For each earlier year the program calculates the claims according to the mean claim ratios and then, using the run-off rates shown below, calculates the amounts, in real (constant money terms), which it expects to pay out in each future year.

Duration from year of origin	Proportion of claims settled (%)	
	Short-tail*	Long-tail**
0	61.2	2.0
1	24.1	8.0
2	5.2	12.0
3	3.7	14.0
4	2.7	14.0
5	2.2	13.0
6	0.9	11.0
7	-	9.0
8	-	6.0
9	-	4.0
10	-	3.0
11	-	2.0
12	-	1.0
13	-	0.5
14	-	0.3
15	<u>100.0</u>	<u>0.2</u>
		100.0

* taken from Abbott et al⁴²

** based on a Craighead curve ⁴³ with B = 6.

These are stored in an array by year of expected payment and the total is accumulated, allowing for 5% future inflation, in a variable TOTOS which is the total provision for claim amounts outstanding at the end of year 20. By using these run-offs we have automatically taken into account the IBNR claims. If we have a wind-up situation then TOTOS is the technical provision.

In the case of a run-off or a continuing business the assets will include half of the premiums written in year 20. However, since for most insurers about 3 months' premiums are held by brokers or in some form of asset that does not bear interest, we add a quarter of the premiums for year 20 to determine the initial assets. The other quarter (representing the balance of the UPR) is brought in as a cash receipt in year 21 together with three-quarters of the written premiums for year 21 and so on until written premiums cease.

6 Future premiums The program adds into the arrays of future payments the expected contribution to claims outgo arising from future written premiums and from the unearned premium reserve for the last year, to give the expected claims outgo in real (constant money) terms.

7 Emerging costs The program now has the information to enable it to calculate the expected payments in each future year. The claims outgo is adjusted for inflation according to the Wilkie model and is allowed to vary stochastically. We assume a normal distribution and a formula of $ax + b\sqrt{x}$ as the standard deviation for the total claim outgo in any year. The square root factor is dominant for the smaller amounts and the smaller companies but for the larger companies the stochastic variation is negligible and it is only realistic to assume some sort of overall secular variation (see Appendix 5).

We take the values of the assets at the beginning of the year and calculate the income on each type of asset, reducing the total income for the year by the tax factor where appropriate. We then have the outgo, adjusted to allow for inflation and stochastic variation, less the income and less any written premiums for a continuing business. As mentioned in A3.5.3 we adjust for the loss of part of the year's investment income as a result of net selling during the year (or vice versa in a net buying situation). Investment or disinvestment is assumed to take place at mid-year values. If there is net investment, it is assumed to be carried out proportionately to the existing values of the three classes of investment. Where there is net outgo, the specified selling rule is applied.

8 Final assets This process continues until the last year's claims outgo has been paid. The final assets are in the currency of the final year as a result of the application of the investment model which revalues the assets, combined with the models for income and outgo in each year which allow implicitly for future inflation. In order to bring the final asset value into the currency of the start of the run-off, it is divided by the ratio of the retail price index in the final year to that at the date of assessment. The result is then expressed as a percentage of the written premiums in year 20 (the year before the date of assessment). These values from the 1000 simulations are grouped into ranges and output as a distribution, together with their mean and standard deviation.

APPENDIX 4RESULTS OF SIMULATION

Full details of the results of 1000 simulations on a variety of different bases are set out in Tables A4.1 to A4.4. Tables A4.1 and A4.2 show summary distributions of the simulations by the assets remaining at the end, as well as the number of insolvencies and the mean and standard deviation of the distributions. Results are also given for a few additional variants not tabulated in Tables 4 and 5. Tables A4.3 and A4.4 also include a number of additional variants and Table A4.4 shows the additional asset margin required in the case of 2 years' new business as compared to the pure run-off with the same assumptions (in so far as these are applicable).

The tables show the standard basis at the top and also in each of the groups of alternative assumptions (marked (s)). The variants examine the effect of varying the one assumption referred to, whilst leaving all the other assumptions the same as in the standard basis.

The assumptions underlying the standard basis are as follows:

Net written premiums*	£10m a year
Proportion of long-tailed business	40% of net written premiums
Past growth	In line with inflation
Future growth	In line with inflation
Mean claim ratio** (short-tailed)	100% of net written premiums*
Standard deviation of C.R.** (short-tailed)	10% of net written premiums*
Mean claim ratio** (long-tailed)	100% of net written premiums*
Standard deviation of C.R.** (long-tailed)	15% of net written premiums*
Initial asset distribution	Technical provisions: 50% cash; 50% gilts Asset margin: 100% equities
Asset selling rule	Proportionate to holdings
Asset margin (for Tables A4.1 and A4.2)	40% of net written premiums

* premiums net of commission and expenses

** ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

Table A4.1

Summary of results for pure run-off of business (with 1000 simulations)

Assumptions	No of insolvencies	No of simulations with remaining assets* of					Mean assets remaining* £	Standard deviation of assets remaining* £
		0%-40%	40%-80%	80%-120%	120%-160%	Over 160%		
Standard basis	9	32	134	190	163	472	172	106
1. Net written premiums:**								
(a) £100,000 a year	75	84	120	144	127	450	165	144
(b) £1m a year	16	48	142	165	170	459	170	113
(c) £10m a year (s)	9	32	134	190	163	472	172	106
(d) £100m a year	7	28	131	194	169	471	172	105
(e) £1000m a year	6	28	130	195	169	472	172	105
2. Proportion of long-tailed business:								
(a) 0% of net written premiums	4	124	369	299	126	78	87	49
(b) 20% of net written premiums	5	45	226	239	206	279	130	76
(c) 40% of net written premiums(s)	9	32	134	190	163	472	172	106
(d) 60% of net written premiums	14	25	86	129	161	585	214	136
(e) 80% of net written premiums	16	19	65	99	111	690	256	171
(f) 100% of net written premiums	17	21	47	77	82	756	298	204
3. Future real growth rate (in constant money terms):								
(a) -40% a year before and after	85	13	22	22	15	843	911	876
(b) -20% a year before and after	26	28	64	94	116	672	261	188
(c) No growth (s)	9	32	134	190	163	472	172	106
(d) +20% a year before and after	3	31	190	227	203	346	144	82
(e) +40% a year before and after	2	36	220	249	214	279	131	72
4. Mean claim ratio (short-tailed):								
(a) 60% of net written premiums	3	18	83	166	192	538	189	108
(b) 80% of net written premiums	6	21	115	175	177	506	181	107
(c) 100% of net written premiums(s)	9	32	134	190	163	472	172	106
(d) 120% of net written premiums	13	49	145	196	165	432	163	106
(e) 140% of net written premiums	19	66	161	190	166	398	155	105
5. Variability of claim ratio (short-tailed):								
(a) Standard deviation 5% NWP	8	32	136	189	164	471	172	106
(b) Standard deviation 10% NWP(s)	9	32	134	190	163	472	172	106
(c) Standard deviation 15% NWP	9	32	138	191	158	472	172	107
(d) Standard deviation 20% NWP	10	35	132	190	160	473	172	107
6. Mean claim ratio (long-tailed):								
(a) 60% of net written premiums	2	24	164	233	207	370	150	82
(b) 80% of net written premiums	3	30	148	213	178	428	161	94
(c) 100% of net written premiums(s)	9	32	134	190	163	472	172	106
(d) 120% of net written premiums	13	34	125	165	158	505	183	119
(e) 140% of net written premiums	15	43	113	138	154	537	194	131
7. Variability of claim ratio (long-tailed):								
(a) Standard deviation 5% NWP	8	30	134	193	165	470	172	106
(b) Standard deviation 10% NWP	8	31	137	187	166	471	172	106
(c) Standard deviation 15% NWP(s)	9	32	134	190	163	472	172	106
(d) Standard deviation 20% NWP	9	31	139	186	161	474	172	106
(e) Standard deviation 25% NWP	9	30	139	187	164	471	172	106

Table M.1 (continued)

Summary of results for pure run-off of business (with 1000 simulations)

Assumptions	No of insolvencies	No of simulations with remaining assets* of					Mean assets remaining* %	Standard deviation of assets remaining* %		
		0%-40%	40%-80%	80%-120%	120%-160%	Over 160%				
Standard basis	9	32	134	190	163	472	172	106		
8.Variability of outgo (a)										
(a) a = .05	4	26	118	183	193	476	174	102		
(b) a = .10	5	28	125	191	181	470	173	104		
(c) a = .15 (s)	9	32	134	190	163	472	172	106		
(d) a = .20	13	41	139	178	170	459	171	116		
(e) a = .25	23	44	143	171	165	454	170	115		
9.Variability of outgo (b)										
(a) Initial value of b = 25	7	29	131	193	167	473	172	105		
(b) Initial value of b = 50	7	33	134	187	164	475	172	106		
(c) Initial value of b = 75 (s)	9	32	134	190	163	472	172	106		
(d) Initial value of b = 100	9	33	139	185	165	469	172	107		
(e) Initial value of b = 125	11	31	139	185	170	464	171	106		
10.Initial asset distribution:										
	Cash	Gilts	Equities							
(a) TP+AM	-	-	13	50	167	209	202	359	149	94
(b) -	TP+AM	-	17	59	129	189	149	457	175	127
(c) -	-	TP+AM	45	67	102	114	118	554	225	195
(d) †TP	†TP	AM (s)	9	32	134	190	163	472	172	106
(e) †TP+†AM	†TP+†AM	-	14	51	145	210	168	412	162	109
(f) †TP+†AM	-	†TP+†AM	7	34	110	158	148	543	193	124
(g) -	†TP+†AM	†TP+†AM	8	35	101	148	126	582	206	135
(h) TP	-	AM	7	30	149	211	173	430	161	94
(i) -	TP	AM	12	38	120	170	157	503	183	121
11.Initial asset margin:										
(a) 0% of net written premiums	69	197	235	189	115	195	96	87		
(b) 20% of net written premiums	23	93	198	204	166	316	134	96		
(c) 40% of net written premiums (s)	9	32	134	190	163	472	172	106		
(d) 60% of net written premiums	3	14	55	148	173	607	211	118		
(e) 80% of net written premiums	0	9	20	86	146	739	250	136		
(f) 100% of net written premiums	0	4	13	38	111	834	291	143		
12.Asset selling rules:										
(a) Equities; cash; gilts	15	48	142	195	165	435	167	114		
(b) Equities; gilts; cash	13	48	164	203	172	400	159	106		
(c) Gilts; equities; cash	10	40	140	188	167	455	167	103		
(d) Gilts; cash; equities	4	20	116	195	181	484	173	95		
(e) Cash; equities; gilts	9	48	123	157	156	507	179	113		
(f) Cash; gilts; equities	5	27	106	178	159	525	184	110		
(g) In proportion to holding (s)	9	32	134	190	163	472	172	106		
(h) Sell best performer first	14	44	146	204	170	422	164	105		

* deflated to the date of assessment and expressed as a percentage of net written premiums** in the year before the date of assessment (see App 3.5.8).

** here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses.

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

(s) indicates the assumption made for the standard basis.

Table A4.2

Summary of results with 2 further years' business (with 1000 simulations)

Assumptions	No of insolvencies	No of simulations with remaining assets* of					Mean assets remaining* %	Standard deviation of assets remaining* %
		0%-40%	40%-80%	80%-120%	120%-160%	Over 160%		
Standard basis	36	52	96	120	143	551	205	157
1. Net written premiums:**								
(a) £100,000 a year	125	61	96	88	111	519	198	209
(b) £1m a year	49	57	95	128	132	539	204	166
(c) £10m a year (s)	36	52	98	120	143	551	205	157
(d) £100m a year	32	53	95	125	141	554	206	155
(e) £1000m a year	33	51	92	129	143	552	206	154
2. Proportion of long-tailed business:								
(a) 0% of net written premiums	88	159	194	211	145	203	99	90
(b) 20% of net written premiums	36	92	144	177	153	398	152	117
(c) 40% of net written premiums(s)	36	52	98	120	143	551	205	157
(d) 60% of net written premiums	37	40	65	89	112	657	258	201
(e) 80% of net written premiums	42	30	55	67	69	737	310	246
(f) 100% of net written premiums	45	25	44	54	65	767	363	293
3. Future real growth rate (in constant money terms):								
(a) -40% a year before and after	86	14	9	20	16	855	956	955
(b) -20% a year before and after	38	43	52	69	99	699	291	231
(c) No growth (s)	36	52	98	120	143	551	205	157
(d) +20% a year before and after	49	63	112	138	141	497	185	146
(e) +40% a year before and after	64	69	111	139	125	492	181	152
(f) +40% a year (no past growth)	65	48	90	106	120	571	220	187
(g) -40% a year (no past growth)	19	52	95	143	159	532	194	134
4. Mean claim ratio† (short-tailed):								
(a) 60% of net written premiums	5	8	24	52	83	828	309	179
(b) 80% of net written premiums	14	23	49	95	112	707	257	167
(c) 100% of net written premiums(s)	36	52	98	120	143	551	205	157
(d) 120% of net written premiums	94	100	131	149	123	403	153	149
(e) 140% of net written premiums	209	143	133	137	94	284	100	145
5. Variability of claim ratio (short-tailed):								
(a) Standard deviation 5% NWP	32	56	93	123	139	557	206	157
(b) Standard deviation 10% NWP(s)	36	52	98	120	143	551	205	157
(c) Standard deviation 15% NWP	38	59	87	128	142	546	204	157
(d) Standard deviation 20% NWP	42	59	90	128	138	543	204	159
6. Mean claim ratio† (long-tailed):								
(a) 60% of net written premiums	8	14	51	100	141	686	234	138
(b) 80% of net written premiums	17	35	70	124	144	610	219	147
(c) 100% of net written premiums(s)	36	52	98	120	143	551	205	157
(d) 120% of net written premiums	73	57	108	137	119	506	192	168
(e) 140% of net written premiums	98	91	112	117	114	468	178	175
7. Variability of claim ratio (long-tailed):								
(a) Standard deviation 5% NWP	37	45	99	133	134	552	205	156
(b) Standard deviation 10% NWP	35	51	95	126	140	553	205	156
(c) Standard deviation 15% NWP(s)	36	52	98	120	143	551	205	157
(d) Standard deviation 20% NWP	40	53	93	124	142	548	205	158
(e) Standard deviation 25% NWP	40	52	90	127	139	552	205	158

Table A4.2 (continued)

Summary of results with 2 further years' business (with 1000 simulations)

Assumptions	No of insolvencies	No of simulations with remaining assets* of					Mean assets remaining* %	Standard deviation of assets remaining* %		
		0%-40%	40%-80%	80%-120%	120%-160%	Over 160%				
Standard basis	36	52	98	120	143	551	205	157		
8. Variability of outgo (a)										
(a) a = .05	25	44	94	124	145	568	208	148		
(b) a = .10	33	42	96	126	148	555	206	152		
(c) a = .15 (s)	36	52	98	120	143	551	205	157		
(d) a = .20	46	59	91	130	130	544	204	164		
(e) a = .25	59	61	96	131	111	542	202	173		
9. Variability of outgo (b)										
(a) Initial value of b = 25	32	53	97	123	143	552	206	155		
(b) Initial value of b = 50	33	52	100	119	143	553	205	156		
(c) Initial value of b = 75 (s)	36	52	98	120	143	551	205	157		
(d) Initial value of b = 100	38	54	94	121	144	549	205	158		
(e) Initial value of b = 125	39	54	100	118	143	546	205	159		
10. Initial asset distribution:										
	Cash	Gilts	Equities							
(a) TP+AM	-	-	48	73	133	156	142	448	171	138
(b) -	TP+AM	-	55	67	97	143	116	522	208	188
(c) -	-	TP+AM	76	37	75	74	98	640	300	306
(d) †TP	†TP	AM (s)	36	52	98	120	143	551	205	157
(e) †TP+†AM	†TP+†AM	-	49	73	110	146	132	490	189	161
(f) †TP+†AM	-	†TP+†AM	37	31	84	95	119	634	242	187
(g) -	†TP+†AM	†TP+†AM	40	26	75	89	114	656	261	205
(h) TP	-	AM	30	57	109	138	147	519	188	138
(i) -	TP	AM	41	57	81	117	129	575	222	180
11. Initial asset margin:										
(a) 0% of net written premiums	142	132	165	143	118	300	118	138		
(b) 20% of net written premiums	78	84	139	142	139	418	161	146		
(c) 40% of net written premiums (s)	36	52	98	120	143	551	205	157		
(d) 60% of net written premiums	18	32	51	100	115	684	250	169		
(e) 80% of net written premiums	13	13	38	57	100	779	295	183		
(f) 100% of net written premiums	6	9	20	39	67	859	340	199		
12. Asset selling rules:										
(a) Equities; cash; gilts	55	60	105	136	130	514	197	166		
(b) Equities; gilts; cash	52	68	115	143	126	496	191	163		
(c) Gilts; equities; cash	34	66	97	117	132	554	202	154		
(d) Gilts; cash; equities	28	43	82	124	151	572	208	149		
(e) Cash; equities; gilts	37	60	85	107	130	581	213	162		
(f) Cash; gilts; equities	29	45	78	112	148	588	218	158		
(g) In proportion to holding (s)	36	52	98	120	143	551	205	157		
(h) Sell best performer first	49	63	101	145	124	518	196	163		

* deflated to the date of assessment and expressed as a percentage of net written premiums** in the year before the date of assessment (see App 3.5.8).

** here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses.

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

(s) indicates the assumption made for the standard basis.

Table A4.3 Asset margins required to achieve 1/100 probability of ruin - no future new business

Assumptions	Asset margin as % of NWP*	Asset margin as % of technical provisions
Standard basis	35	10
1. Net written premiums*		
(a) £100,000 a year	100	30
(b) £1m a year	50	15
(c) £10m a year (s)	35	10
(d) £100m a year	35	10
(e) £1000m a year	30	10
2. Proportion of long-tailed business:		
(a) 0% of net written premiums	35	25
(b) 20% of net written premiums	30	15
(c) 40% of net written premiums(s)	35	10
(b) 60% of net written premiums	45	10
(e) 80% of net written premiums	55	10
(f) 100% of net written premiums	65	10
3. Future growth rate (in constant money terms):		
(a) -40% a year before and after	N/A	N/A
(b) -20% a year before and after	70	10
(c) No growth (s)	35	10
(d) +20% a year before and after	30	15
(e) +40% a year before and after	25	15
4. Mean claim ratio (short-tailed):		
(a) 60% of net written premiums	25	10
(b) 80% of net written premiums	30	10
(c) 100% of net written premiums(s)	35	10
(d) 120% of net written premiums	45	15
(e) 140% of net written premiums	50	15
5. Variability of claim ratio (short-tailed):		
(a) Standard deviation 5% NWP	35	10
(b) Standard deviation 10% NWP(s)	35	10
(c) Standard deviation 15% NWP	35	10
(d) Standard deviation 20% NWP	40	10
6. Mean claim ratio (long-tailed):		
(a) 60% of net written premiums	25	10
(b) 80% of net written premiums	30	10
(c) 100% of net written premiums(s)	35	10
(d) 120% of net written premiums	45	10
(e) 140% of net written premiums	50	10
7. Variability of claim ratio (long-tailed):		
(a) Standard deviation 5% NWP	35	10
(b) Standard deviation 10% NWP	35	10
(c) Standard deviation 15% NWP(s)	35	10
(d) Standard deviation 20% NWP	40	10
(e) Standard deviation 25% NWP	40	10

Table M.3 (continued) Asset margins required to achieve 1/100 probability of ruin - no future new business

Assumptions	Asset margin as % of NWP*	Asset margin as % of technical provisions		
Standard basis	35	10		
8. Variability of outgo (a)				
(a) a = .05	25	10		
(b) a = .10	30	10		
(c) a = .15 (s)	35	10		
(d) a = .20	45	15		
(e) a = .25	55	15		
9. Variability of outgo (b)				
(a) Initial value of b = 25	35	10		
(b) Initial value of b = 50	35	10		
(c) Initial value of b = 75 (s)	35	10		
(d) Initial value of b = 100	40	10		
(e) Initial value of b = 125	40	15		
10. Initial asset distribution:				
	Cash	Gilts	Equities	
(a) TP+AM	-	-	-	45
(b) -	TP+AM	-	-	50
(c) -	-	TP+AM	-	85
(d) $\frac{1}{2}$ TP	$\frac{1}{2}$ TP	-	AM (s)	35
(e) $\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	-	45
(f) $\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	-	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	35
(g) -	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	35
(h) TP	-	-	AM	35
(i) -	TP	-	AM	45
11. Asset selling rules:				
(a) Equities; cash; gilts				50
(b) Equities; gilts; cash				45
(c) Gilts; equities; cash				40
(d) Gilts; cash; equities				30
(e) Cash; equities; gilts				40
(f) Cash; gilts; equities				30
(g) In proportion to holdings (s)				35
(h) Sell best performer first				45

* here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

(s) indicates the assumption made for the standard basis.

Table A4.4

Asset margins required to achieve 1/100 probability of ruin - two years' new business

Assumptions	Asset margin as % of NWP*	Excess asset margin as compared to pure run-off (as % of NWP*)
Standard basis	85	50
1. Net written premiums*		
(a) £100,000 a year	145	45
(b) £1m a year	105	55
(c) £10m a year (s)	85	50
(d) £100m a year	85	50
(e) £1000m a year	80	50
2. Proportion of long-tailed business:		
(a) 0% of net written premiums	80	45
(b) 20% of net written premiums	80	50
(c) 40% of net written premiums(s)	85	50
(b) 60% of net written premiums	100	55
(e) 80% of net written premiums	105	50
(f) 100% of net written premiums	115	50
3. Future growth rate (in constant money terms):		
(a) -40% a year before and after	N/A	N/A
(b) -20% a year before and after	110	40
(c) No growth (s)	85	50
(d) +20% a year before and after	90	60
(e) +40% a year before and after	105	80
(f) +40% a year (no past growth)	120	85
(g) -40% a year (no past growth)	70	35
4. Mean claim ratio (short-tailed):		
(a) 60% of net written premiums	15	-10
(b) 80% of net written premiums	50	20
(c) 100% of net written premiums(s)	85	50
(d) 120% of net written premiums	125	80
(e) 140% of net written premiums	145	95
5. Variability of claim ratio (short-tailed):		
(a) Standard deviation 5% NWP	85	50
(b) Standard deviation 10% NWP(s)	85	50
(c) Standard deviation 15% NWP	85	50
(d) Standard deviation 20% NWP	90	50
6. Mean claim ratio† (long-tailed):		
(a) 60% of net written premiums	35	10
(b) 80% of net written premiums	65	35
(c) 100% of net written premiums(s)	85	50
(d) 120% of net written premiums	115	70
(e) 140% of net written premiums	130	80
7. Variability of claim ratio (long-tailed):		
(a) Standard deviation 5% NWP	85	50
(b) Standard deviation 10% NWP	85	50
(c) Standard deviation 15% NWP(s)	85	50
(d) Standard deviation 20% NWP	85	45
(e) Standard deviation 25% NWP	90	50

Table M.4 (continued) Asset margins required to achieve 1/100 probability of ruin - two years' new business

Assumptions	Asset margin as % of NWP*	Excess asset margin as compared to pure run-off (as % of NWP*)		
Standard basis	85	50		
8. Variability of outgo (a)				
(a) a = .05	60	35		
(b) a = .10	65	35		
(c) a = .15 (s)	85	50		
(d) a = .20	110	65		
(e) a = .25	125	70		
9. Variability of outgo (b)				
(a) Initial value of b = 25	85	50		
(b) Initial value of b = 50	85	50		
(c) Initial value of b = 75 (s)	85	50		
(d) Initial value of b = 100	90	50		
(e) Initial value of b = 125	95	55		
10. Initial asset distribution:				
	Cash	Gilts	Equities	
(a) TP+AM	-	-	-	85
(b) -	TP+AM	-	-	105
(c) -	-	-	TP+AM	130
(d) $\frac{1}{2}$ TP	$\frac{1}{2}$ TP	-	AM (s)	85
(e) $\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	-	90
(f) $\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	-	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	85
(g) -	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	$\frac{1}{2}$ TP+ $\frac{1}{2}$ AM	-	90
(h) TP	-	-	AM	85
(i) -	TP	-	AM	95
11. Asset selling rules:				
(a) Equities; cash; gilts				95
(b) Equities; gilts; cash				95
(c) Gilts; equities; cash				85
(d) Gilts; cash; equities				85
(e) Cash; equities; gilts				90
(f) Cash; gilts; equities				85
(g) In proportion to holdings (s)				85
(h) Sell best performer first				90

* here and elsewhere in the table net written premiums are taken to be premiums net of commission and expenses

† ratio of claims (including claims settlement expenses), without allowance for future inflation or for discounting, to premiums net of commission and expenses (see paragraph 8.3.2).

(s) indicates the assumption made for the standard basis.

- A5.1 An earlier paper by the authors³ assumed that the amount of the payments made in each development year for each year of origin varied log-normally. This meant that a payment amount that was to be varied stochastically was multiplied by $\exp(R \cdot S + M)$ where R is a random normal variate, S the standard deviation and M the mean. In order that the overall mean should be correct the value of M has to be equal to minus half the square of the standard deviation. This formula is suitable for a single payment, but in most cases the payment amounts considered were the totals of several or many individual amounts. Furthermore different values would need to be adopted for funds of different sizes if account was to be taken of the fact that variation is not the same for a small fund as for a large one.
- A5.2 This was cumbersome and not entirely satisfactory, so an alternative approach was sought. The formula should reflect the number of payments involved and, if possible, the ratio of the standard deviation to the mean (the coefficient of variation). Consideration was given to the estimation of the numbers of claims (or claim payments) in each year's totals. We were unable to obtain any figures from actual portfolios but information from returns to the supervisory authority and from other sources suggested that for short-tailed business an average payment rising from £500 in the year of occurrence by multiples of 2 to £16,000 in the last year of development was not unreasonable. For long-tailed business the average payments rose over 10 years from £800 to £15,000.
- A5.3 We assumed that coefficients of variation were in the range of 2 to 10, increasing at later durations as fewer, larger claims are settled. We were then able to estimate both the numbers of claims and their average amounts for different mixes of business by year of development. For this purpose it was assumed that claims were identical with payments, and whilst this is clearly not the case, it is not thought that it would make much difference if we were able to make more detailed assumptions. These calculations suggested that the formula for standard deviation should be a multiple of the square root of the number of claims, or its deemed equivalent, the total amount of payment. For convenience we used the amount of money, even though inflation would involve a change in the multiplier over time.
- A5.4 It must be realized that precision was out of the question since we could not take into account all the possible variations in the make-up of a portfolio. It was also necessary to have regard to the fact that the bulk of the outstanding claims are paid in the first two or three years of run-off and relate primarily to the latest two or three years' business. Calculations showed that out of total outstandings of £1 million about one-half was paid

in the first year and a quarter in the next year. By year 7 the payments were under £20,000, so that variation in these later years was less significant in the overall context. What is more, for many insurers the later payments, if they turn out to be large, may well be recoverable from reinsurers and so not form part of the problem for net run-off patterns. It simply moves the problem to another area. Further consideration would need to be given to the variability of the tail in the case of a company with a lot of long-tailed business and relatively high retentions.

- A5.5 Experiment suggested that a multiplier of about 50 to 100 times the square root of the amount (in pounds sterling in 1986) was of the right order of magnitude. However, it was clear that whilst this gave a reasonable amount of variation for the smaller insurer it was wholly inadequate for a large one. In present conditions most of the variation for the larger fund arises from secular change and this is more likely to be proportional to the actual amount to be paid than to its square root. The problem is to choose a multiplier to give a realistic variation. Experience over recent years suggests that it must be at least 0.1, to give a variation of 20% in 95% of all cases. We finally adopted the formula

$$SD = aX + b\sqrt{X}$$

using values of 0.15 for a and 75 for b, with the value of b increasing by 5 each year once written premiums have ceased.

- A5.6 This formula is similar to one which we understand was introduced by the Finnish supervisory authority in 1952 and is referred to by Byrnes⁴⁴. Whilst we are well aware of the approximations and assumptions involved in its derivation, we think it is adequate for the purpose. It also greatly simplifies the calculations. As indicated above, the earlier paper calculated the outgo for each future year for each year of occurrence and for each length of tail separately and applied the stochastic factor to each such amount. The main effect of this was to reduce the overall variation compared with applying the same formula to the total and this effect can be achieved by adjusting the overall level of the variation. It was decided, therefore, to calculate the total outgo in each year, including that from future business where appropriate, and apply the variability factor to the total.
- A5.7 It is interesting to compare the values produced by the formula with those from the exponential basis. The comparisons, with values of R corresponding to the 5%, 25% and 50% points, are shown in Table A5.1. The correspondence between the two formulae, coupled with the size of variation by insurer, suggests that the new formula is in line with the old but more realistic in its relation to the actual amounts of payments.

Table A5.1

Stochastic multiplier (1 + R * S/x) for different values of R
and standard deviation (s)

	Random normal variate (R)				
	-1.96	-.675	0	.675	1.96
<u>log-normal</u>					
s = 0.3	.53	.78	.96	1.17	1.72
s = 0.5	.33	.63	.88	1.24	2.35
<u>square root formula</u> (S = 0.15X + 75/x)					
100,000	.24	.74	1.0	1.26	1.76
1,000,000	.56	.85	1.0	1.15	1.44
10,000,000	.66	.88	1.0	1.12	1.34

- A6.1 It is not possible to simulate reinsurance recoveries in our model in any very precise way, firstly because it is too complicated and secondly because the model simulates claims only in aggregate. It would in principle be possible to think in terms of a specified number of reinsurers, each bearing a share of the anticipated reinsurance recoveries, and find a way to model the failure of reinsurers. Rather easier, and probably no less realistic, would be to go directly to the proportion recovered. One way of approaching the problem is set out below.
- A6.2 Reinsurers would be allocated to say, three categories - strong, average and weak. For any class of business the proportion of reinsurance recoveries anticipated from each of the three categories of reinsurer would be input as data. The model would then be to apply a process, defined separately for each category, to determine the proportion not recovered in respect of any particular year's estimated gross claim payments. There remains, of course, the problem of estimating gross claims payments and simulating their outturn, so that there would be considerable practical problems in implementing an approach of this sort.
- A6.3 The probability of recovery would be assumed to be related to the gross claims outturn for the ceding company. This is simply a proxy for deterioration in results generally in the market. One might take the estimate of gross claims paid in the year in question to be the mean estimate of claims paid, based on proportions expected to be settled in the year, the rate of inflation assumed in setting the technical provisions and, in the case of claims arising from future business, the mean claim ratio. There would then be a set of formulae, one for each category of reinsurer, to define the proportion of gross claims paid in the year which is assumed not to be recovered, based on the ratio of gross claims outturn to estimated gross claims for the year. For year j we might, for example, define the proportion not recovered $Y(j)$ by:

Weak

$$Y(j) = \frac{k(j)}{400} \quad [0 < k(j) < 400]$$

Average

$$Y(j) = \frac{k(j) - 100}{600} \quad [100 < k(j) < 700]$$

Strong

$$Y(j) = \frac{k(j) - 200}{800} \quad [200 < k(j) < 1000]$$

$$\text{where } 1 + \frac{k(j)}{100} = \frac{X(j)}{\sum_{i:j} \hat{X}(i:j)}$$

$X(j)$ = actual total gross claims settled in year j

and $\sum_{i:j} \hat{X}(i:j)$ = expected gross claims settlement in year j in respect of year of origin i on basis of mean claims ratio, assumed settlement pattern and expected inflation.

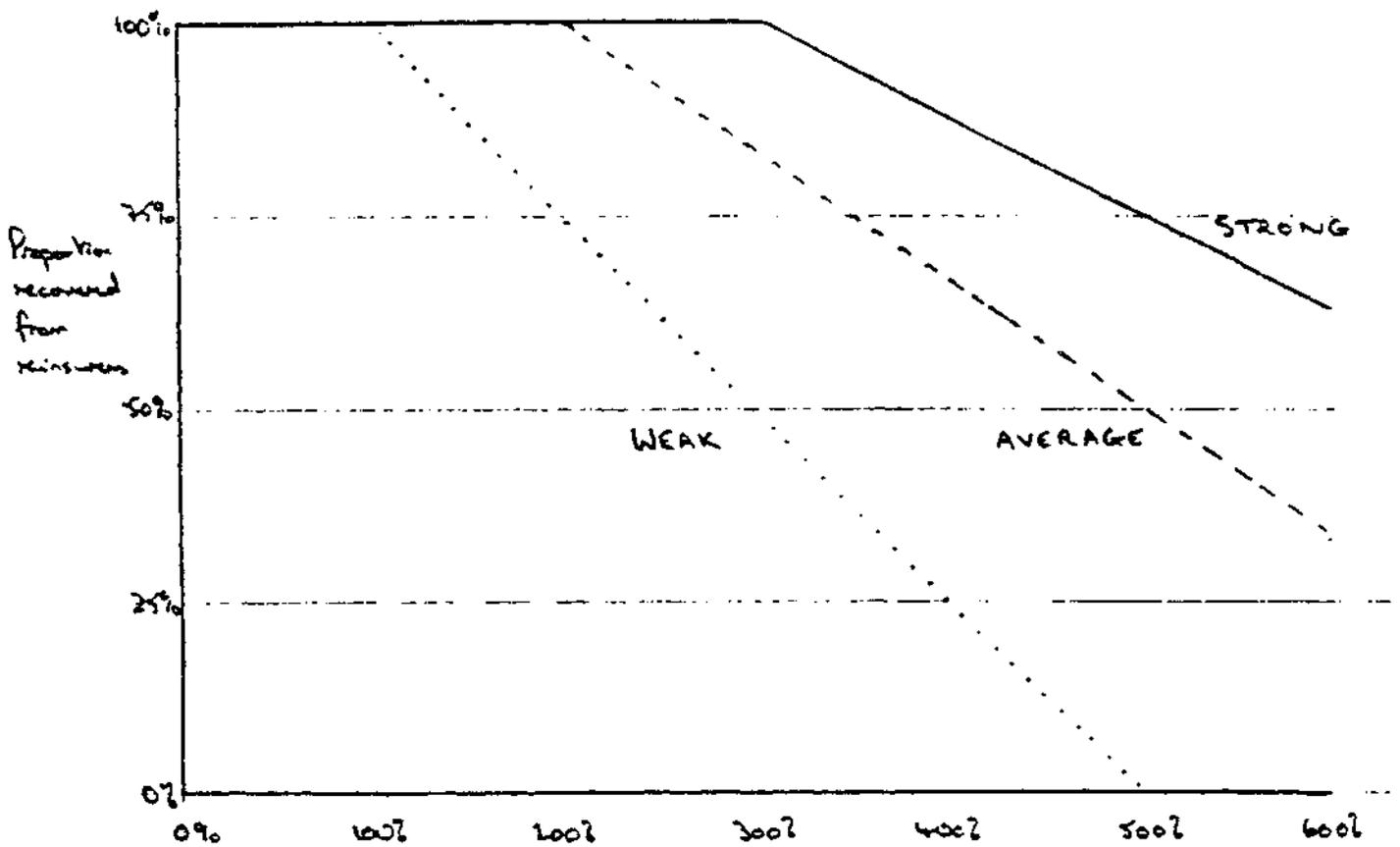
In terms of the notation of Appendix 1

$$X(i;j) = \sum_k s^{k(j-1)} R_k B_k(i) (1+r)^{j-i}$$

- A6.4 The formulae can obviously be adapted to reflect one's ideas of a plausible model for reinsurance recoveries. These particular illustrative formulae are shown graphically in Figure A6.1. The general principle is that one would expect higher proportions not to be recovered from weaker reinsurers and lower proportions not to be recovered from stronger reinsurers. Above a certain threshold, higher claims relative to the expected level of claims imply a worsening market situation and hence a higher proportion not recovered. These formulae do not attempt to distinguish between high claims as a result of high initial loss ratios, high inflation or adverse development. In principle one could also develop some form of cumulative trigger so that failure to recover increased with a series of high claims payments rather than simply on the basis of a single year. This approach is very crude, based as it is on the experience of the business of the ceding company rather than the business as a whole that might be affecting reinsurance companies. However, it might help to give a broad indication of the potential impact of reinsurance failure on the situation.
- A6.5 Consideration would also need to be given to whether to apply the formulae to all classes together or each class separately. Possibly the most realistic would be to apply it to the total claims on those classes of business where significant amounts are reinsured.
- A6.6 The simple approach suggested here may not be sufficiently realistic for some companies for whom reinsurance recovery is a major issue. Further development of these ideas is clearly needed. However, it is suggested that it may be possible to obtain a useful indication of the role of reinsurance in a particular case by the use of straightforward models.

Figure A6.1

Proportion recovered from reinsurers as function
of deterioration of claims experience



Actual gross claims outgo in year
as % of expected gross claims outgo

APPENDIX 7

DRAFT NOTES ON RECOMMENDED PRACTICE (NORP) - ASSESSING FINANCIAL SOUNDNESS

1. INTRODUCTION

[Note: This introduction might be considerably shortened if a previous general NORP has been issued.]

- 1.1 General insurance business in the UK is carried on by companies subject to the provisions of the Insurance Companies Act 1982 and the Companies Act 1985. It is also carried on by underwriting syndicates at Lloyd's, which is controlled by self-regulation subject to the terms of the Lloyd's Act 1982. Actuaries have no statutory responsibility for general insurance under these Acts; nevertheless, from time to time actuaries are called upon to report on the financial soundness or solvency of a general insurance business. These Notes on Recommended Practice give guidance to actuaries in such circumstances.
- 1.2 Actuaries are also called upon to report on premium rating, reinsurance programmes, corporate strategy, expense allocation, rating structure and other areas of general insurance business. These Notes do not cover reports in these areas.
- 1.3 The Notes apply whether the actuary is acting as an individual, as a partner, as a director in a corporate body or in the course of his employment.
- 1.4 The actuary who signs the report should ensure that, as soon as possible after completion, it reaches the person for whom it was prepared, without interference or amendment.
- 1.5 The Notes have been prepared with United Kingdom requirements and conditions in mind. Where a member is practising outside the United Kingdom, and the Council of the Institute or the Faculty as the case may be has agreed, the Notes may be replaced by guidance from an actuarial body of the country in which the actuary practises.

2. PURPOSE OF NOTES

- 2.1 The purpose of these Notes is to ensure that reports cover such aspects of general insurance business as may affect financial soundness and contain details of the information used to support the conclusions.
- 2.2 The actuary needs to bear in mind that his advice may be made available to third parties who can reasonably be expected to rely on it.

3. THE REPORT - FRAMEWORK

- 3.1 Financial soundness will normally be assessed within a framework of guiding accounting principles and these must be clearly stated in the report.
- 3.2 The report may be prepared under terms of reference which postulate one of three distinct business situations. These are:
1. Liquidation (or winding up).
 2. Run-off of a closed fund.
 3. Continued existence as a going concern for a stated period.

The methodology used by the actuary must take account of the appropriate business situation.

- 3.3 Various methodologies may be used in assessing financial soundness. These Notes are not intended to restrict the actuary's choice of whether to use a balance sheet or emerging costs methodology. The methodology used should enable the actuary to consider the uncertainty inherent in general insurance.

3.4 A proper appreciation of financial soundness cannot be made if there are implicit but unquantifiable margins taken in the valuation of asset or liabilities. In general reliance on unquantifiable implicit margins should be avoided by the actuary.

3.5 There is no universally accepted terminology for general insurance and the actuary must ensure that the words used are clearly understandable by the recipient of the report. The word solvency should generally be used in its statutory sense, e.g. an insolvent company is one which has failed to satisfy minimum statutory requirements. Further clarification of the term may sometimes be necessary. The concept of financial strength/soundness for a going concern will usually cover not only the sufficiency of the excess of assets over liabilities to give a high probability that the claim payments in respect of existing business can be made as they fall due but also the ability of the insurer to remain solvent in statutory terms as further business is written.

4. THE REPORT - ITEMS FOR INCLUSION

4.1 The items discussed below are normally to be regarded as essential components of any report. Other information may often be desirable and suitable explanations of some features or trends may be very important.

4.2 BASIC INFORMATION

4.2.1 An opening statement showing:

- a) Who has commissioned the report
- b) The addressee of the report, where this is different from (a)
- c) The terms of reference given
- d) The extent to which the report meets those terms of reference

- e) Where appropriate, the date of the last similar report and any changes in the basis used
- f) Where the actuary is acting in a professional capacity, the name of the actuary and that he/she is a Fellow of the Institute of Actuaries (or the professional actuarial body which is appropriate).

4.2.2 A brief summary of the data on which the investigation is based. The report should also indicate the steps taken to verify the accuracy of the data and, where appropriate, a reference to any written assurances as to the correctness and completeness of the data obtained from the insurer. If the actuary has any reservations as to the reliability of the data, an explanation or qualification should be given.

4.2.3 A brief summary of such discussions as are relevant that were held by the actuary with the insurer's directors, staff and auditors and the reliance placed on the information so obtained.

4.3 FINANCIAL SOUNDNESS

4.3.1 The financial soundness of a general insurer depends upon its assets, liabilities and the expected profitability of new and renewed business. If the terms of reference do not incorporate all three aspects then the report should indicate the limitations that result from not considering one or more of these factors.

4.3.2 Currency movements may affect the valuation of assets and liabilities and premium rate adequacy. The report should describe the assumptions made about exchange rates and indicate the likely impact of significant currency movements.

4.4 ASSETS

4.4.1 A statement of the basis of valuation of assets and any assumptions made.

4.3.2 The assumed rate of return on assets, including income and capital appreciation or depreciation.

4.4.3 A comment on the variability of the value and the potential future cash proceeds from assets. Consideration should be given to the mix of assets and how they relate to the nature of the liabilities. As a minimum, assets should be shown separately by country or currency for the following classes:

- ordinary shares
- fixed interest securities - by term
- index linked securities
- property
- other assets - interest
 - non-interest-bearing

4.4.4 The nature of the asset portfolio can be changed very quickly. In assessing the financial soundness of the company the actuary should pay regard to the past investment policy pursued, any expected changes in it and whether it could become inappropriate having regard to the nature and term of the insurer's liabilities. In this case the report might refer to any constraints on investment policy which might be necessary to ensure the future soundness of the insurer.

4.4.5 A comment on whether there is a potential liability to capital gains.

4.4.6 For classes of business where there is significant delay in receiving premiums, an asset may be included in respect of premiums due on business already written, subject to any allowance necessary for bad debts.

4.4.7 Where Unexpired Premium Reserve (UPR) is shown gross of initial expenses, a deferred acquisition cost should be shown as an asset.

4.5 LIABILITIES

4.5.1 A statement of those classes of business accounted for on a funded basis and those treated on an annual accounting basis.

4.5.2 A statement whether the provisions are established net of reinsurance or gross with a separate provision for reinsurance recoveries.

4.5.3 A statement of the methods and assumptions made in valuing the following liabilities:

- known (i.e. reported) outstanding claims
- incurred but not reported (IBNR) claims
- settled claims that may be reopened
- future claims handling expenses
- unearned premium reserve (UPR)
- any additional amount for unexpired risks not covered by the UPR

4.5.4 The actuary should give consideration to the categorisation of the business into homogeneous groups by geographic area, currency, class of business or other relevant factor. The report should include a statement of the basis used for classifying claims and a comment on the consequences of any lack of homogeneity.

4.5.5 A statement of the assumptions made in respect of the impact of future inflation and the escalation of claims costs.

4.5.6 A statement of the rate or rates of interest used to discount provisions. Comment should be made on the appropriateness of the rates used in relation to the assets held.

4.5.7 A clear statement of other liabilities including any reserve necessary for bad debts from reinsurers or intermediaries. On a winding up or run-off basis, allowance may need to be made for statutory payments to redundant staff and for other costs associated with the transition to a run-off situation.

4.6 PREMIUM RATES

4.6.1 A clear statement of the main distinct categories of business and the expected profitability on new and renewed business.

4.6.2 A statement of any significant change of current rating practices and in sources and classes of business written.

4.6.3 The adequacy of the premium rate basis with reference to potential impact of:

- the homogeneity of changing mix of data
- the effect of large claims
- trends in claim frequency and claim costs
- cyclical movements
- expenses
- growth of business

4.6.4 Attention should be drawn to any difference in principles or assumptions used in evaluating premium rate adequacy from those used in evaluating the rate of return on assets and the level of liabilities for outstanding claims and unexpired risks.

4.7 REINSURANCE

4.7.1 A statement of the nature of the reinsurance protection in place, its appropriateness for the business being written and a broad comment on the security of the reinsurers.

4.8 OTHER FACTORS

- 4.8.1 The report should identify wider issues considered although the actuary should recognise that there may be aspects of a subjective nature which are not necessarily within his/her competence.
- 4.8.2 A brief note of any external factors outside of the control of the company, which could significantly impact upon its financial soundness.

5. THE REPORT - CONCLUSIONS

The conclusions will need to relate to the terms of reference given. They may, however, be expected to draw attention to the sensitivity of the conclusions to any of the assumptions made which may be particularly uncertain. An indication should also be given of areas where a change in management policy could significantly affect the conclusions reached.