Extreme Events and Portfolio Construction

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Agenda

- Analysing fat-tailed behaviour
- What causes fat-tailed behaviour?
- Selection effects – an example of model risk
- Portfolio construction in the presence of fat tails

- Talk based on material in:
Extreme events: Robust portfolio construction in the presence of fat tails

- Chapters:
  1. Introduction
  2. Fat tails – in single (i.e. univariate) return series
  3. Fat tails – in joint (i.e. multivariate) return series
  4. Identifying factors that significantly influence markets
  5. Traditional portfolio construction techniques
  6. Robust mean-variance portfolio construction
  7. Regime switching and time-varying risk and return parameters
  8. Stress testing
  9. Really extreme events
- Plus Principles (Chapter 10) and Exercises (Appendix)
- Toolkit available through www.nematrian.com

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- Talk based on material in:
Analysing fat-tailed behaviour

- There are various ways of visualising fat tails in a single return distribution. Easiest to see in format (c) below
- By ‘fat tail’ we mean probability of extreme-sized outcomes (returns / movements / events) seems to be higher than from (log) Normal distribution

![Example Probability Density Function](image)

![Example cumulative probability distribution plot](image)

![Example quantile-quantile plot](image)

Source: Nematrian (illustrative)

QQ-plots

- Largest divergences relate to extreme events
  - Usually what we want
  - However, could wrongly emphasise extreme events
    - Under-emphasise: VaR vs TVaR
    - Over-emphasise: fat tails can add rather than subtract value
Tail behaviour dependent on time-scale (1)

- Higher frequency data
  - Typically viewed as more fat-tailed than lower frequency data
- Period analysed below: June 1994 to December 2007

Tail behaviour dependent on time-scale (2)

- Higher frequency data
  - More data points => QQ-plot is naturally further into the tail
  - For these data sets, daily data not much more fat-tailed than weekly data
  - But note e.g. Oct 1987
Skew(ness), kurtosis and Cornish-Fisher

- Fat tails involve deviation from Normality
  - Hence some higher cumulants (moments), aka semi-invariants, e.g. skew and (excess) kurtosis, deviate from zero (Normality)
- Cornish-Fisher (4th moment version) estimates distributional form from merely the first 4 moments, i.e.
  \[
  \text{mean} = \mu = E(x) \quad \text{standard deviation} = \sigma = E\left(\frac{x - \mu}{\sigma}\right)
  \]
  \[
  \text{skew} = \gamma_1 = E\left(\frac{x - \mu}{\sigma}\right)^3 \quad \text{(excess) kurtosis} = \gamma_2 = E\left(\frac{x - \mu}{\sigma}\right)^4 - 3
  \]
  - Regularly appears in risk management academic literature
- Standardised QQ-plot estimated via a cubic equation:
  \[
  y_{x,4}(x) = x + \frac{1}{6} x^3 - 3x^2 + 2x^1 - 6x^0 - 1
  \]

Flaws in Cornish Fisher (and hence in skew/kurtosis)

- Doesn’t model index return distributions particularly well
  - Particularly parts risk managers might be most interested in, i.e. downside tails
- Computation gives less weight to tail observations (most observations are in middle of the distribution)
- Lacks a desirable stability criterion
  - Applying CF twice can lead to a more extreme distribution
- Fit QQ-plot directly, e.g. with cubic (or other weightings)?
Joint fat-tailed behaviour

- Usually split between
  a. Marginals
  b. Copula
- Facilitates Monte Carlo simulation
- But some disadvantages
  - Fat-tailed characteristics difficult to see (copulas akin to joint pdf / cdf)
  - Many problems depend on (a) and (b) in tandem
- Kemp (2010) proposes a multi-dimensional variant of QQ-plots to circumvent these difficulties

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- Talk based on material in:
What causes fat-tailed behaviour?

- Time varying volatility (and other distributional characteristics)
- Regime switching
- Crowded trades and leverage

Time-varying volatility

- Very widely observed phenomenon
  - Fits our intuition – sometimes markets more turbulent than at other times
- Distributional mixtures of Normal distributions
  - E.g. draw $X_1$ with probability $p$ from $N_1$, draw $X_2$ with probability $(1-p)$ from $N_2$
  - Quite different behaviour to linear combination mixtures, i.e. $aX_1 + bX_2$
- If $N_1$ and $N_2$ have same mean but different standard deviations then distributional mixture fat-tailed (if $p \neq 0$ or 1) but not linear combination mixture
- Time-varying volatility creates an analogous effect
  - Because drawing from different distributions at different times
Explains some market index fat tails, particularly on upside

**Raw Data**
Daily returns (End Jun 1994 to end Dec 2007)

**With Short-term Volatility Adjustment**
Daily returns (End Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)

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### Average extent to which tail exceeds expected level (average of 6 most extreme outcomes)

<table>
<thead>
<tr>
<th></th>
<th>Downside (%)</th>
<th>Upside (%)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Unadj</td>
<td>Adj for vol</td>
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<tr>
<td>FTSE All-Share (in GBP)</td>
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<tr>
<td>S&amp;P 500 (in USD)</td>
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<td>FTSE Eur ex UK (in EUR)</td>
<td>48</td>
<td>53</td>
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<tr>
<td>Topix (in JPY)</td>
<td>54</td>
<td>72</td>
</tr>
</tbody>
</table>

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Not just a developed market phenomenon

**Raw Data**
Daily returns (End Jun 1994 to end Dec 2007)

**With Short-term Volatility Adjustment**
Daily returns (End Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)

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Source: Nematrian, Threadneedle, FTSE, Thomson Datastream
A longer term phenomenon too

Raw Data

With Short-term Volatility Adjustment

Tail analysis for S&P 500 and FTSE All-Share price movements 31 December 1968 to 24 March 2009

Tail analysis for S&P 500 and FTSE All-Share price movements (vol adj, by trailing 50-day vol, early 1969 to 24 March 2009)

Expected (rescaled to zero mean, unit standard deviation)

Source: Nematrian, Threadneedle, S&P, FTSE, Thomson Datastream

Time-varying volatility

• Also known as heteroscedasticity
• Closely allied with GARCH modelling
  – E.g. s(t) = a.s(t-1) + c, where s = volatility (if using AR(1) model)
  – The C in GARCH is because we are talking about the volatility conditional on the current time and/or on volatility at earlier times
• Why not incorporate time-varying behaviour in distributional parameters including means and correlations (covariances)?
• More commonly then called regime switching
Regime switching

- Idea: two or more ‘regimes’ (each e.g. characterised by a complete $N(\mu, \Sigma)$ distribution, say $R_1$ and $R_2$.
- World is in one of these states at time $t$.
- Switches from $R_i$ to $R_j$ with probability $p_{ij}$ at time $t$.
  - Usually adopt a ‘simple’ Markov chain formulation, in which $p_{ij}$ does not depend on what regimes the world was in before the last time period.
- Can be generalised to continuously varying distributions, and continuous time.
  - If latter then typically solved using stochastic calculus.
  - Numerical solution typically reintroduces time grid.

Regime switching (continued)

- Adds complexity and therefore sophistication.
  - And risk of over-fitting, i.e. lack of parsimony.
- Regimes might be Normal but have different means e.g. ‘normal’ and ‘bear’ regimes of Ang and Bekaert (2004).
  - Can introduce fat tails and conditional tail correlation effects.
- In general, risk-return trade-off dynamics are altered.
  - Optimal (i.e. efficient) portfolios then regime dependent.
  - Also time dependent (and hence more sensitive to transaction costs).
  - Also utility dependent, both re. fat tails and re. inter-temporal utility.
Crowded trades

- Some fat tails still seem to come “out of the blue”
  - E.g. Quant funds in August 2007
  - Too many investors in the same crowded trades? Behavioural finance implies potentially unstable
  - For less liquid investments, impact may be via an apparent shift in price basis
- Portfolio and system-wide equivalents via leverage?
  - Leverage introduces/magnifies liquidity risk, forced unwind risk and variable borrow cost risk

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- Talk based on material in:
Selection effects, see e.g. Kemp (2010a, 2010b)

• ‘Selection’ effects are a common problem in finance
  – E.g. Individuals buying annuities typically have longer life expectancies than individuals who don’t
• Can also apply to portfolios being analysed by risk models
  – Many risk models assume behaviour that is (approximately) Gaussian, i.e. multivariate (log) Normal, akin to lots of different sources of random noise
  – Can decompose multiple series return data into principal components, the most important of which contribute the most to the aggregate variability exhibited by securities in the relevant universe
• But what if portfolios are structured to seek ‘meaning’ (e.g. if they are actively managed!) and ‘meaning’ is (partly) associated with non-Normality?
  – Both meaning and magnitude are important

<table>
<thead>
<tr>
<th>Component</th>
<th>PCA, only StdDev (c = 0)</th>
<th>Blended (1 in 200 quantile level, CF4)</th>
<th>ICA, Only Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>StdDev</td>
<td>Kurt</td>
<td>Criterion</td>
</tr>
<tr>
<td>1</td>
<td>10.6%</td>
<td>3.1</td>
<td>10.6%</td>
</tr>
<tr>
<td>2</td>
<td>6.5%</td>
<td>2.1</td>
<td>6.5%</td>
</tr>
<tr>
<td>3</td>
<td>5.6%</td>
<td>1.7</td>
<td>5.6%</td>
</tr>
<tr>
<td>4</td>
<td>4.8%</td>
<td>1.4</td>
<td>4.8%</td>
</tr>
<tr>
<td>5</td>
<td>4.2%</td>
<td>0.4</td>
<td>4.2%</td>
</tr>
<tr>
<td>6</td>
<td>3.7%</td>
<td>1.1</td>
<td>3.7%</td>
</tr>
<tr>
<td>Av (top 6)</td>
<td>5.9%</td>
<td>1.6</td>
<td>5.9%</td>
</tr>
<tr>
<td>Av (all 23)</td>
<td>3.2%</td>
<td>1.2</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

• (a) Principal components analysis – focuses on standard deviation, (b) independent components analysis – focuses on, say, kurtosis, or (c) blend
• Sizes of ‘1 in 200’ events potentially underestimated several-fold by PCA (and hence traditional risk systems), if factors expressed are selected for fat-tailed characteristics
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Portfolio construction

- Traditional (quantitative) approach involves portfolio optimisation
  - Typically mean-variance optimisation
  - Identify expected return (‘alpha’) from each position
  - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
  - Maximise $\mathbf{a} \cdot \mathbf{r} - \lambda \mathbf{a}^\mathbf{Va}$
- Time-varying parameters add realism and complexity
  - alpha + beta ‘separation’
Portfolio construction – sensitivities

- Output results notoriously sensitive to input assumptions
- Possible responses:
  - Treat quant models with scepticism (the fundamental manager’s approach?)
  - Use ‘robust’ approaches, Bayesian priors/anchors, e.g.:
    - Black-Litterman
    - ‘Shrinkage’
    - Position limit ‘priors’ (e.g. 1/N, long-only, etc.)
  - Focus on reverse optimisation

Portfolio construction – impact of fat tails (1)

- If all return opportunities (and combinations of them) ‘equally’ fat-tailed, then end results the same, if risk budget adjusted appropriately
- If different combinations exhibit differential fat-tailed behaviour then in principle adjust portfolio construction to compensate:
  - If we can reliably estimate these differentials
  - And if investors do not have solely quadratic utility functions
Solution A - simplest

- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
  - Calculate covariance matrix between return series after stripping out effect of time-varying volatility
  - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
  - Adjust risk aversion/risk budget appropriately
  - Then unravel time-varying volatility adjustment
  - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed

Solution B – more sophisticated

- Model with a mixture of multivariate Normal distributions
- Time-stationary? Maybe not realistic?
- Time-varying?
  - (Discrete) regime switching, and/or
  - (Continuous) parameterisation (and continuous time?)
- However:
  - Even a mixture of just two multivariate Normal distributions involves estimation of twice as many parameters
  - Making parameter estimation correspondingly less reliable
  - Results very sensitive to input assumptions
  - Time varying => dynamic => sensitivity to transaction costs
Solution C – lower partial moments

- Any return = threshold + upside + downside
- Non-quadratic utility will typically give greater weight to downside and will in general also depend on higher moments
- Single series, define as: \( \text{lpm}(K,m) = \mathbb{E}[\min((r-K)^m,0)] \)?
- Multiple series, define as: \( \text{lpm}_{ij}(K,m,n) = \mathbb{E}[\min((r_i-K)^m(r_j-K)^n,0)] \)?
  - Or max
  - E.g. co-skewness, co-kurtosis
  - Or symmetric alternatives
- Substantially increased numbers of parameters, and few observations in tail
  - Specify candidate distributional form and fit this?

Summary

- Fat-tailed behaviour
  - Very common in practice
  - Several intrinsic reasons for its existence, including time-varying world
  - QQ plots focus more on extremes than pdf / cdf
- Active management may 'select' fat-tails
  - Potentially major implications for risk modelling
- Portfolio construction can be refined to cater better for extreme events
  - But refinements potentially complex, especially in a time-varying world
References


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