The Actuarial Profession
making financial sense of the future

33rd ANNUAL GIRO CONVENTION
Top down / Bottom up Correlation

Hilton Vienna Hotel, Am Stadtpark, 28 September 2006

Colin Kerley / Simon Margetts
Benfield ReMetrics / Ernst & Young LLP

Summary / Introduction

- Modelling many Lines of Business (LOBs); need results consolidated
  - Could be for Capital adequacy, RI purchase (eg stoploss)
- How best to model the fact that the LOB's aren't independent
  - Standalone LOBs and estimate combined
  - Marginals & Correlation / Copula
  - Shared events & Drivers
  - Operational issues
- Aim to get some discussion over practicality of driver approach
  - Are the benefits worth the extra effort

Why of Interest ?

- Choice of method to implement correlations can have impact on an integrated liability model
- This could be relevant for regulatory capital (ICA)
- But more importantly whether or not you can use your model in the real world
  - If you don't know what drives your risk you can't explain your model output !
  - No large losses => cannot look at Risk XL / Surplus
  - No cat model =>
    - cannot look at cat r/i purchase
    - cannot quantify aggregation risk
  - No inflation => cannot look at hedging with inflation linked assets
  - Model not integrated properly => harder to look at more interesting ri solutions such as agg stop loss, structured QS etc

Comparison

Approach	Pros	Cons
Combine Marginal Capital by hand	Easy to calculate No "hidden" statistical effects	Hard to justify the answer Not much use for RI pricing
Correlation Matrix	Can be applied using many software packages Often used by actuaries; intuitive understanding of various correlation levels (?)	Assumes linear correlation So no tail dependency Can be hard to explain to non-statisticians
Copula	Not restricted to linear correlation	Harder to calculate results Less industry comfort with parameters used Harder to explain
Drivers	Easily explained Less reliance on statistical theory Closer to reality of what is being modelled	Requires more work - need to understand what drives the risk Residual Risk / Softer issues

The Actuarial Profession
making financial sense of the future

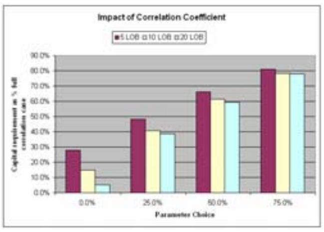
Example: change correlation

- Example for LOB correlation only
- One LOB : look at Gross UW Result
- Losses reasonably volatile
- Standalone capital calculated using VaR at 99.5%
- How much for 5 LOB ?
- Somewhere between 1000 and 5000 ?

Single LOB Assumptions	
GWP	1,000
Losses	850
UW Result	150
StDev Losses	319
CoV	0.375
Detail - Gross Underwriting Result	
Prob < 0	26.5%
Result at 99%	-851
Result at 99.5%	-1,027
Capital Required *(Standalone)*	1,000

The Actuarial Profession
making financial sense of the future

Linear Correlation and VaR / Capital



Impact of Correlation - 5 LOB					
Correlation	0.0%	25.0%	50.0%	75.0%	100.0%
Capital	1,422	2,472	3,383	4,139	5,113
Capital As % max (100% correlation)	27.8%	48.4%	66.2%	80.9%	100.0%

- Graph shows how choice of correlation parameters can drive capital requirements
- Same results for 5, 10, 20 LOBs

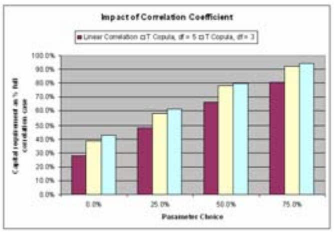
The Actuarial Profession
making financial sense of the future

Linear Correlation and VaR / Capital

- So what – isn't this what you expect
  - Does show that results are sensitive to choice of correlation parameters
  - Especially as you aggregate many LOB
- Same result for different risk measures
- And for different distributions
- Try & compare with same 5 LOBs but use Student T copula

The Actuarial Profession
making financial sense of the future

T Copula and VaR / Capital



- Graph shows how choice of copula parameters can drive capital requirements
- Results for different choices of ρ and t (degrees of freedom)

- Tail dependence here means higher capital required at all correlation levels

The Actuarial Profession
making financial sense of the future

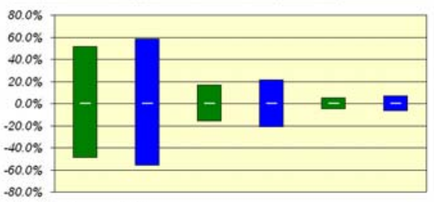
Parameter Estimation

- Estimating correlation coefficients for linear correlation from data can be hard
- Harder for copulas with tail dependence - for example T copula
  - In theory can estimate t from tail dependence
  - But needs to look at say 95% or 99% point of distributions
  - Hard to do even if you have >100 data points
- Model sample correlation coefficients given sets of data generated from joint distribution with known correlation structure and parameters
  - Example using linear correlation & T copula (t=3)
- Look at possible ranges if we have 10, 100 and 1000 data points to estimate from

The Actuarial Profession
making financial sense of the future

Parameter Estimation

90% Range of Outcomes for Sample Correlation Coefficient : Known parameter = 0%

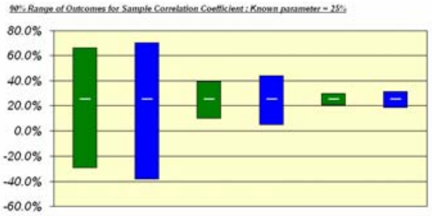


	Linear	T Copula				
Number of data points	10	10	100	100	1,000	1,000
High	51.7%	58.8%	16.6%	21.4%	5.4%	6.8%
Expected	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Low	-49.0%	-56.1%	-16.1%	-21.3%	-5.1%	-6.9%

The Actuarial Profession
making financial sense of the future

Parameter Estimation

90% Range of Outcomes for Sample Correlation Coefficient : Known parameter = 25%



	Linear	T Copula				
Number of data points	10	10	100	100	1,000	1,000
High	66.5%	70.4%	39.2%	44.0%	29.9%	31.1%
Expected	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%
Low	-29.5%	-38.9%	9.3%	4.2%	20.3%	18.3%

The Actuarial Profession
making financial sense of the future

Alternatives ?

- Correlation / Dependence modelling easy to do
  - But not necessarily that helpful
- Alternative is to think about what drives shared loss behaviour
  - Impact of shared economics
    - Severity Inflation / event frequency
  - Shared events (cat, clash losses, latent claims, new legislation)
  - Softer issues such as shared management, pricing teams and underwriter philosophy; common risk mitigation and control environment
- And what drives premium behaviour (the underwriting cycle)
  - More understood so will focus on loss behaviour

The Actuarial Profession
making financial sense of the future

Example : Common Shock Model

- Can be thought of as an overall inflation adjustment – for example applies to aggregate distribution
- For our example with all LOB identical $Y_i = (1 + b) X_i$
  - X is base aggregate distribution for the LOB, based on some expected inflation
  - b is the shared inflation / common shock parameter
  - In this case b has mean 0 and is normally distributed
- For a "real" model b might have mean 0 but would have different variance scalar for each LOB
  - $Y_i = (1 + b.\sigma_i) X_i$
- Choice of distribution a matter of care (probably not Normal ! skew ? Fat tails ?)
- Probably easier to model actual assumptions about inflation and apply directly to loss payments – captures sensitivity to the length of the tail

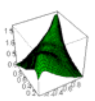
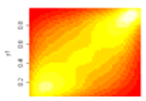
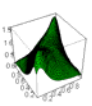
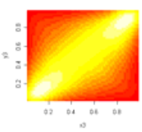
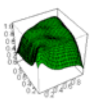
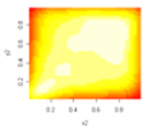
The Actuarial Profession
making financial sense of the future

Common Shock : inflation

- Model 2 LOBs as per last example
  - use LogNormal for (uninflated) aggregate losses
  - Have common inflation across 2 LOB
  - Target overall CV 37.5% for inflated losses and sample correlation at 25%, 50% and 75%
- What does the common shock do for the joint pdf
- Look at what these correlation levels mean in terms of inflation

The Actuarial Profession
making financial sense of the future

Output : Dependency Structures

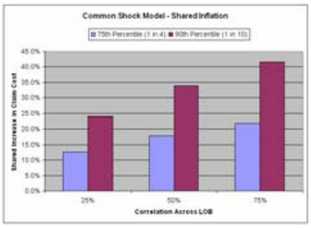
- Plots of joint loss distributions, based on ranks
- Three approaches, all at 50% "correlation", marginals CV 37.5%
  - Common Shock
  - T Copula, t=3
  - Linear correlation

The Actuarial Profession
making financial sense of the future

Output : Implied inflation

Common Shock Model - Inflation Assumptions

Target Correlation	25%	50%	75%
Normal StDev	0.188	0.265	0.325
Percentile	*Increase in Total Claims*		
75%	12.7%	17.9%	21.9%
90%	24.1%	34.0%	41.7%
95%	30.9%	43.6%	53.5%
99%	43.7%	61.6%	75.6%
Claim Increase as annual excess inflation			
Long Tail - Payout over 10 years, 5 years mean term			
75%	2.4%	3.3%	4.0%
90%	4.4%	6.0%	7.2%
95%	5.5%	7.5%	8.9%
99%	7.5%	10.1%	11.9%
Short Tail - Payout over 5 years, 2.5 years mean term			
75%	4.9%	6.8%	8.3%
90%	9.0%	12.4%	14.9%
95%	11.4%	15.6%	18.7%
99%	15.6%	21.2%	25.3%



- High correlation means huge shared shock – if this is all from loss severity, can translate into inflation terms
- More volatility in the underlying class required more volatility in the shared shock component, for a given correlation level

Common Shock Model : Pros / Cons

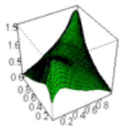
- Can get the right effects (implied correlation at various levels, tail dependency)
- Reduces the need to estimate all cross-correlation parameters
  - With correlation matrix across 20 LOB need to estimate 190 parameters
  - Looking at relationship each LOB has with a shared driver reduces this
- Shared inflation drives correlations across years (runoff & new business)
- Can use this to understand standard correlation assumptions
  - are standard correlation parameters too high ?
- Downside : must recalibrate marginals
  - extract inflation from data first & fit
  - New distribution Y = (1+b) X won't be from the same family as original distribution X
- Also need to choose a model for the shock / inflation
- And do the extra modelling

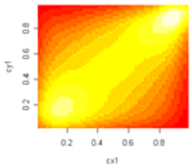
Frequency and Severity

- Common shock (inflation) for large losses
- Shared Frequency driver for large losses and / or attritional
- Could be thought of as
  - Economic climate adjustor (GDP linked)
  - Parameter uncertainty
- Not sure if want to link the shared severity with attritional losses also
- Pros :
  - this implied correlation can be explained
  - can be used for other purposes (eg to price shared RI)
- Cons :
  - now have to estimate the freq & sev distributions plus common shock parameters

Frequency and Severity – Joint distribution

- Aggregate distribution has CV 37.5%
- Aggregate model split into freq / severity
- Use shared driver for freq & severity to target 25% correlation





Case Study : Non-unique solutions

- Looking at efficiency of XoL programme across MTPL and GTPL
- Parameters provided from capital model
  - Defines the attritional, large loss freq & severity distributions
  - And the correlation coefficient for aggregate losses across 2 LOB [ρ = 0.3121 !]
- To model this we wanted to consider correlations across
  - Attritional loss model
  - Large loss frequency
  - Large loss severity
- and make sure we maintained the overall correlation for the aggregate distribution

Case Study : Non-unique solutions

- Sticking to linear correlations across the 3 components separately gives us 2 free parameters
  - => an infinite number of possible solutions
- Not just academic : the reinsurance pricing was dependent on choice of parameters used
  - Technical price for lowest layer changed 25% in value just from different correlation choices
- Moral of this story : important to drill into what's driving the (aggregate) correlation of 0.3121

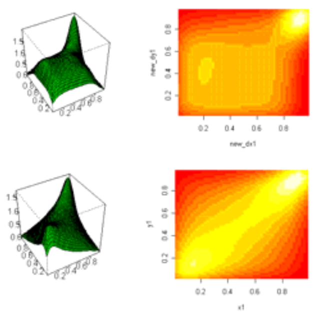
Shared Events

- Using drivers is common place though – shared events ?
- Nowadays no-one would model the effect of cats across different LOBs using a copula, even a Gumbel copula
- Shared events typical have a single source but inflict losses across several LOBs
  - Could be a cat loss, ie US Hurricane causing losses to household & commercial property
  - Or liability related : collapse of major corporation triggers losses across D&O, PI and Financial Institutions
- Can be modelled using output from commercial cat models
  - Model event frequency and severity for each LOB relating to each event
- Or use own / underwriters understanding of likely shared risks
  - RDS style scenarios
- Can be tricky associating frequencies & severities with events though

The Actuarial Profession
making financial sense of the future

Compare Output

- Density plot of joint losses for 2 LOBs exposed to European storm losses
- Modelled using RMS event set for shared losses; independent LogNormals for attritionals
- Second set shows density plot when using marginals with the same correlation coefficient, using a Gumbel copula to combine



The Actuarial Profession
making financial sense of the future

Softer Issues

- In reality the biggest "driver" behind correlated losses across LOBs might be shared management and/or underwriting skill
- Underwriting cycle
- Insolvencies not driven by mis-estimation of pricing frequency and severity assumptions
- But usually by eg:
  - rapid growth (ie knowingly and repeatedly undercharging)
  - or a massive lack of understanding of the exposures written (US liability losses)
  - Ineffective controls
- Do we include these factors while modelling UW risk as correlated drivers across LOBs, or as operational risk ?
  - If we have capital for operational risk and high correlations across LOBs are we double counting ?

The Actuarial Profession
making financial sense of the future

Conclusions

- Use of just correlation / copulas & guessing parameters ⇔ guessing capital
  - It adds little value to your understanding of the real risk or dependency
  - Results cannot be explained to non-technical audience
  - Same applies for stoploss pricing etc
- Better to guess events / drivers as at least these can be explained to management and underwriters and so can be challenged
  - Can also look at the drivers for those scenarios that drive your risk
- Can calibrate new events as understanding evolves
- Can challenge / understand traditional correlation assumptions
- Driver based model can be used to look at “what-if” analysis
- Cons :
  - Harder to do – requires more modelling & analysis
  - Underlying parameters probably still guessed
  - Complaint that drivers do not fully cover all shared risk
- Operational Risk Drivers!?

The Actuarial Profession
making financial sense of the future

Any Questions ?

“We expect senior management to ‘own’ the assessment and understand what it means for how they run their business”
[FSA : “ICAS – one year on”]

The Actuarial Profession
making financial sense of the future