Summary / Introduction

- Modelling many Lines of Business (LOBs); need results consolidated
  - Could be for Capital adequacy, RI purchase (eg stoploss)
- How best to model the fact that the LOB’s aren’t independent
  - Standalone LOBs and estimate combined
  - Marginals & Correlation / Copula
  - Shared events & Drivers
- Operational issues
  - Aim to get some discussion over practicality of driver approach
  - Are the benefits worth the extra effort

Why of Interest?

- Choice of method to implement correlations can have impact on an integrated liability model
- This could be relevant for regulatory capital (ICA)
- But more importantly whether or not you can use your model in the real world
  - If you don’t know what drives your risk you can’t explain your model output!
  - No large losses => cannot look at Risk UL Surplus
  - No cat model => cannot look at cat ri purchase
  - No inflation => cannot look at hedging with inflation linked assets
  - Model not integrated properly => harder to look at more interesting ri solutions such as agg step loss, structured QS etc.
Comparison

<table>
<thead>
<tr>
<th>Segment</th>
<th>Pros</th>
<th>Cons</th>
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<tbody>
<tr>
<td>Residual Risk / Softer issues</td>
<td>Closer to reality of what is being modelled</td>
<td>Less reliance on statistical theory</td>
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<td>What drives the risk</td>
<td>Requires more work - need to understand drivers</td>
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<td>Harder to explain</td>
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<td>Less industry comfort with parameters used</td>
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<td>Harder to calculate results</td>
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<td>Not restricted to linear correlation</td>
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<td>Copula can be hard to explain to non-statisticians</td>
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<td>So no tail dependency</td>
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<td>Often used by actuaries; intuitive understanding of various correlation levels</td>
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<td>Assumes linear correlation</td>
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<td>Can be applied using many software packages</td>
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<td>Correlation Matrix Not much use for RI pricing</td>
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<td>No &quot;hidden&quot; statistical effects</td>
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<td>Easy to calculate</td>
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<td>Combine Marginal Capital by hand</td>
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Example: change correlation

- Example for LOB correlation only
- One LOB: look at Gross UW Result
- Losses reasonably volatile
- Standalone capital calculated using VaR at 99.5%
- How much for 5 LOB?
- Somewhere between 1000 and 5000?

Linear Correlation and VaR / Capital

- Graph shows how choice of correlation parameters can drive capital requirements
- Same results for 5, 10, 20 LOBs
Linear Correlation and VaR / Capital

- So what – isn’t this what you expect
  - Does show that results are sensitive to choice of correlation parameters
    - Especially as you aggregate many LOB
  - Same result for different risk measures
  - And for different distributions
  - Try & compare with same 5 LOBs but use Student T copula

T Copula and VaR / Capital

- Graph shows how choice of copula parameters can drive capital requirements
  - Results for different choices of \( \rho \) and \( t \) (degrees of freedom)

- Tail dependence here means higher capital required at all correlation levels

Parameter Estimation

- Estimating correlation coefficients for linear correlation from data can be hard
  - Harder for copulas with tail dependence - for example T copula
    - In theory can estimate \( t \) from tail dependence
    - But needs to look at say 95% or 99% point of distributions
    - Hard to do even if you have >100 data points
  - Model sample correlation coefficients given sets of data generated from joint distribution with known correlation structure and parameters
    - Example using linear correlation & T copula (t=3)
    - Look at possible ranges if we have 10, 100 and 1000 data points to estimate from
Parameter Estimation

Alternatives?

- Correlation / Dependence modelling easy to do
  - But not necessarily that helpful
- Alternative is to think about what drives shared loss behaviour
  - Impact of shared economics
    - Severity inflation / event frequency
    - Shared events (cat, clash losses, latent claims, new legislation)
    - Softer issues such as shared management, pricing teams and underwriter philosophy; common risk mitigation and control environment
- And what drives premium behaviour (the underwriting cycle)
  - More understood so will focus on loss behaviour
Example: Common Shock Model

- Can be thought of as an overall inflation adjustment – for example applies to aggregate distribution.
- For our example with all LOB identical \( Y_i = (1 + b) X_i \)
  - \( X \) is base aggregate distribution for the LOB based on some expected inflation
  - \( b \) is the shared inflation / common shock parameter
  - In this case \( b \) has mean 0 and is normally distributed.
- For a "real" model \( b \) might have mean 0 but would have different variance scalar for each LOB.
  - \( Y_i = (1 + b_i) X_i \)
- Choice of distribution a matter of care (probably not Normal / skew / Fat tail ?)
- Probably easier to model actual assumptions about inflation and apply directly to loss payments – captures sensitivity to the length of the tail.

Common Shock: inflation

- Model 2 LOBs as per last example
  - use LogNormal for (uninflated) aggregate losses
  - Have common inflation across 2 LOB
  - Target overall CV 37.5% for inflated losses and sample correlation at 25%, 50% and 75%
- What does the common shock do for the joint pdf?
- Look at what these correlation levels mean in terms of inflation.

Output: Dependency Structures

- Plots of joint loss distributions, based on ranks
- Three approaches, all at 50% "correlation", marginal CV 37.5%
  - Common Shock
  - T Copula, \( t = 3 \)
  - Linear correlation
Output: Implied inflation

- High correlation means huge shared shock – if this is all from loss severity, can translate into inflation terms
- More volatility in the underlying class required more volatility in the shared shock component for a given correlation level

Common Shock Model: Pros / Cons

- Can get the right effects (implied correlation at various levels, tail dependency)
- Reduces the need to estimate all cross-correlation parameters
  - With correlation matrix across 20 LOB need to estimate 190 parameters
  - Looking at relationship each LOB has with a shared driver reduces this
- Shared inflation drives correlations across years (runoff & new business)
  - Can use this to understand standard correlation assumptions
    - Are standard correlation parameters too high?
- Downsides: must recalibrate marginals
  - Extract inflation from data first & fit
  - New distribution $Y = (1+b)X$ won't be from the same family as original distribution $X$
  - Also need to choose a model for the shock / inflation
  - And do the extra modelling

Frequency and Severity

- Common shock (inflation) for large losses
- Shared Frequency driver for large losses and / or attritional
- Could be thought of as
  - Economic climate adjustor (GDP linked)
  - Parameter uncertainty
- Not sure if want to link the shared severity with attritional losses also
- Pros:
  - This implied correlation can be explained
  - Can be used for other purposes (e.g., to price shared RI)
- Cons:
  - Now have to estimate the freq & severity distributions plus common shock parameters
Frequency and Severity – Joint distribution

- Aggregate distribution has CV 37.5%
- Aggregate model split into freq / severity
- Use shared driver for freq & severity to target 25% correlation

Case Study: Non-unique solutions

- Looking at efficiency of XoL programme across MTPL and GTPL
- Parameters provided from capital model
  - Defines the attritional, large loss freq & severity distributions
  - And the correlation coefficient for aggregate losses across 2 LOB \( \rho = 0.3121 \)
- To model this we wanted to consider correlations across
  - Attritional loss model
  - Large loss frequency
  - Large loss severity
- and make sure we maintained the overall correlation for the aggregate distribution

Case Study: Non-unique solutions

- Sticking to linear correlations across the 3 components separately gives us 2 free parameters
  - \( \Rightarrow \) an infinite number of possible solutions
- Not just academic: the reinsurance pricing was dependent on choice of parameters used
  - Technical price for lowest layer changed 25% in value just from different correlation choices
- Moral of this story: important to drill into what’s driving the (aggregate) correlation of 0.3121
Shared Events

- Using drivers is common place though — shared events?
- Nowadays no-one would model the effect of cats across different LOBs using a copula, even a Gumbel copula
- Shared events typical have a single source but inflict losses across several LOBs
  - Could be a cat loss, ie US Hurricane causing losses to household & commercial property
  - Or liability related - collapse of major corporation triggers losses across D&O, PL and Financial Institutions
- Can be modelled using output from commercial cat models
  - Model event frequency and severity for each LOB relating to each event
  - Or use own / underwriters understanding of likely shared risks
  - RDS style scenarios
- Can be tricky associating frequencies & severities with events though

Compare Output

- Density plot of joint losses for 2 LOBs exposed to European storm losses
- Modelled using RMS event set for shared losses: Independent LogNormals for attritionals
- Second set shows density plot when using marginals with the same correlation coefficient, using a Gumbel copula to combine

Softer Issues

- In reality the biggest "driver" behind correlated losses across LOBs might be shared management and/or underwriting skill
- Underwriting cycle
- Insolvencies not driven by mis-estimation of pricing frequency and severity assumptions
- But usually by eg:
  - rapid growth (as knowingly and repeatedly undercharging)
  - or a massive lack of understanding of the exposures written (US liability losses)
  - Ineffective controls
- Do we include these factors while modelling UW risk as correlated drivers across LOBs, or as operational risk?
  - If we have capital for operational risk and high correlations across LOBs are we double counting?
Conclusions

- Use of just correlation / copulas & guessing parameters ⇒ guessing capital
  - It adds little value to your understanding of the real risk or dependency
  - Results cannot be explained to a non-technical audience
  - Some apply for stop-loss pricing etc
- Better to guess events / drivers as at least these can be explained to management and underwriters and so can be challenged
  - Can also look at the drivers for those scenarios that drive your risk
- Can calibrate new events as understanding evolves
  - Can challenge / understand traditional correlation assumptions
  - Driver-based model can be used to look at "what-if" analysis
- Cons:
  - Harder to do — requires more modeling & analysis
  - Underlying parameters probably still guessed
  - Complaint that drivers do not fully cover all shared risk
  - Operational Risk Drivers?!