Claims Reserving Working Party Paper

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1. Summary

The purpose of the paper is to provide practical guidance on general insurance claim reserving from the following viewpoints:

- Providing a framework or working guide to the execution of reserving. Section 2 gives guidance on the factors which should be considered as background to the reserving process. These factors should help to determine the methodology to be used and help in assessment of parameters where judgement is involved in the reserving process.

- Practicalities of the chain ladder projection method. The chain ladder method is the most widely used method for projecting results. Section 3 considers the practicalities involved, which are highly contingent on the amount, type and reliability of the data available.

- Survey of reserving methodology used in practice. As an aid to providing guidance we arranged for the Institute of Actuaries to send out two questionnaires to reserving practitioners, the first on reserving in general (that is methods actually used and how parameters selected) and the second on stochastic reserving methods used, reserve ranges and risk margins and discounting. The replies to the survey have been analysed and the results of the analysis are shown in section 4.

- Provision of ranges using stochastic or other methodology. Section 5 investigates the practical provision of a range of results, rather than a theoretical approach based on models which may bear little resemblance to the more practical method used to obtain the underlying reserve figure.

- Review and description of new methods, that is currently used methods which have not been presented in the Claims Reserving Manual. These methods include the Generalised Cape Cod Method in section 6.

- Chain Ladder or Link Ratio methods. These are described in section 7 within a framework that can be extended to cover most actuarial projection techniques, the focus being on practical reserving methodology rather than mathematical rigour.

- Provision of reserving software. The software generated in investigation of the reserving methods in the paper will be posted to the Institute web-site. These include Linear Reserving, Cape Cod, Thomas Mack and Practical Stochastic Reserving.

2. Framework

2.1 Factors to be Considered as Background

The following is a check-list of factors to be considered when conducting any reserving exercise:

A. Claim Settlement Pattern

a. Timing of Claim Occurrences
b. Allocated Loss Adjustment Expenses
c. Changes in Settlement Procedures
d. Large Claims
e. Claim Frequency
f. Partial Settlements
g. Special Settlements
h. Nil Claims/ Precautionary Advices
i. Judicial Awards

B. Nature/Mix of Business

a. Changes in Portfolio Volumes
b. Change in Mix of Business
c. Change in Policy Conditions
d. Aggregate Deductibles
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C. Data Constraints

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b. Catastrophe Covers/ Large Claims

2.2 Amplification of Check-list

The following discussion amplifies the above check-list.

2.2.1 A. Claim Settlement Pattern

Stability of the claim payment pattern is an assumption that is used in many projection methods. In practice this ideal is rarely realised and the following are factors which should be examined before using or modifying this assumption.

a. Timing of Claim Occurrences

For property classes, adverse weather experience late in an origin year would increase the proportion of payments and settlements made in the first development quarter following the end of the year. For a household account with short development patterns, distortion could be severe.

b. Allocated Loss Adjustment Expenses

If any specific allocated expenses are included in payment data, changes in the method of allocation (such as between classes of business) or in the timing of allocation (for example: date of payment, date of settlement) will distort the payment data.

c. Changes in Settlement Procedures

These could arise in a number of ways, such as policy decisions to press for early settlements, changes in claims handling efficiency, staffing levels and closing-off exercises on outstanding claims.

d. Large Claims

Large claims usually have a longer settlement pattern than small ones. Any change in the mix of severity of claims or random variations in number, amount and date of payment will change the overall settlement pattern.

e. Claim Frequency

A change in frequency without a change in the mix or type of claims will not affect most projection methods, but a change resulting from increased claim awareness or the introduction of bonus protected motor policies may affect the settlement pattern.

f. Partial Settlements

The increasing use of partial settlements would have an effect.

g. Special Settlements

Changes in policy towards ex gratia payments or the attitude to borderline cases may affect settlement patterns.

h. Nil Claims/ Precautionary Advices

The effect on any method using the average claim size is obvious. The volume of precautionary advices can be changed by publicity about the need for such advices.

i. Judicial Awards

The level of settlements for injury may not follow any price or earnings index, but have sharp upward movements after particular judicial awards, followed by a period of stability.

2.2.2 B. Nature/Mix of Business

a. Changes in Portfolio Volumes

With a significant change in portfolio volume, it is unlikely that the nature and mix of the business would remain similar over time. Inevitably there would be a change in underwriting standards and the type of risk brought about either explicitly, by extending cover to certain policyholders previously declined cover, or implicitly, by starting to charge below-market premium rates. This fundamental change in the underlying nature of the "average" risk and the difficulty in quantifying the effect on the claims process should immediately introduce an air of caution into the statistical calculation of the outstanding claims reserve.

Even in the unlikely event of similar before and after portfolios the effect of random fluctuations (which are more prevalent with a low-volume portfolio) must be allowed for when projecting the outstanding reserves for a larger portfolio using historical low-volume statistics. Similarly, when going from a high-volume to a low-volume situation, the stability which might have been experienced in the past because of the minimal effect of such random fluctuations, could well disappear in the low-volume portfolio. This could lead to uncertainty about the accuracy of the statistical reserve.

On an even more practical side, it would be necessary to be aware of the effect on the claims staff of a large increase in portfolio volume. Inevitably the numbers and experience of such staff would not keep pace with the changing portfolio, at least initially, so the claims handling procedures would change. This could lead to delays in computer notifications, less thorough investigations and delays in settlements with potentially higher ultimate costs. The inherent nature of the claims could have changed and certainly the statistics of the emerging cohort of business (associated with the increased volume) would be different from previous low volume cohorts.

b. Change in Mix of Business

If a class of business is very narrowly defined with all risks which fall under this classification being similar in claim characteristics, then there is little problem. Alternatively, consider a company with a single private motor classification. If it experiences a swing away from predominantly non-comprehensive business to comprehensive policies then not only will the claim size distribution be significantly different, but also the reporting and settlement patterns will change.

c. Change in Policy Conditions

Some changes can, in practice, have little effect. For example, increasing a policy excess by £50 should cause the average net claim to subsequently decrease by a somewhat similar figure. In reality the before and after situations might well not be too dissimilar, making projections less uncertain. However, for a large change in deductible (for example changing an excess from zero to £1,000) the average claim cost may rise as a result of there being a lower proportion of claims that are just larger than the deductible.

Other changes can have a significant effect. For example, extending private motor cover to broken windscreens without application of an excess and without any effect on the NCD or granting protected NCD's for a minimal arbitrary premium could cause the incidence of claims to increase sharply and have a significant effect on the claim size distribution. In this case a projection of the current outstanding claims based on past averages is likely to give an over-inflated reserve.

An even more significant change would be where an NCD scale was changed or where the rules for determining position on the scale were amended. Such a change could have a significant effect on the type of claim subsequently reported.

d. Aggregate Deductibles

Contracts are common, particularly in the London Market, which provide coverage excess of underlying deductibles. In such cases there is often a time lag before the insurer is notified of claims or books claims to the relevant layers, or makes claims payments. Triangulation methods applied to claims at the insurer's level would not provide reasonable estimates in such situations. Such contracts should be extracted from triangulations and considered separately using appropriate techniques, which may include stochastic methods to reflect the fact that only excess claims are payable by the insurer. Work would be required on understanding the policy structure, developing a model and collecting additional data.

e. Premium Rates

Rate changes will affect any projection methods such as Bornhuetter-Ferguson or (Generalised) Cape Cod which use initial expected loss ratios. Also, if premium rates are inadequate then there would be a need to calculate an unexpired risk reserve ("URR").

2.2.3 C. Data Constraints

Data can be inadequate or erroneous in content for a number of reasons and these are examined below:-

a. Computer Systems

A company's computer system is usually the main, if not the only, source of information open to the claims reserver. Computer systems capable of handling large volumes of diversified policy and revenue information are necessarily complex. In order to interpret computer generated statistics, it is essential to have a sound knowledge of the computer's database. Data can be processed incorrectly owing to a misunderstanding of how the computer system works. The statistician must always be aware of current processing policy if data processing errors of this nature are to be avoided. Accurate statistics are essential. A close association with the database engenders a sense of awareness of processing errors. The sooner errors are highlighted and corrected the cleaner the data become.

Inception-to-date statistics show revenue development by contract or group of contracts and should tie in exactly with revenue details. Extraneous factors can sometimes hinder this relationship. Statistics must always be reconciled with revenue accounts. Otherwise, distortions will appear in the projection of outstanding reserves.

The importance of knowing one's computer system may be illustrated by the following questions, the answers to which must be known:-

- are premiums gross or net of commissions?
- does the term commission include acquisition costs such as premium taxes, fire brigade taxes, profit commission?
- Do paid losses incorporate provision for settlement costs such as legal fees, court costs?
- Does the term "outstanding losses" represent only the amount advised by the broker or are there additional elements to cover outstanding court costs, legal fees, additional reserves assessed by the claims manager or even IBNR estimates (rare but does occur such as for some US Auto risks written at Lloyd's)?

b. Availability of Data

A computer system is only as good as the information it contains and the ease with which the information can be accessed. It is pointless storing information which cannot be accessed or reported in the format required. A statistician can find himself in a situation where certain information is required, but no programs exist to extract it in the format required.

Sometimes, information essential to claims reserving is absent from the computer system. Certain reinsurances, recorded manually, may never have been processed. In such cases, allowance must be made when projecting net statistics to compensate for the known deficiency. Availability of all data, be they manual or computer records, is essential for proper reserving.

c. Reliability/ Credibility of Data

The "garbage in – garbage out" scenario is highly relevant in claims reserving. The statistician must always watch out for processing errors, (for example from input errors, incorrect currency codes or exchange rates, non-processing of reinsurances).

Data can be processed wrongly owing to a processing technician's lack of training or understanding.

Statistics net of reinsurance may be overstated if certain reinsurance amounts have not been processed owing to omission or timing lags. Unless allowances are made, projection of such data will generate an overstatement in reserves.

Clean, credible data are all-important when establishing the best estimate of the reserves to be carried in the company accounts.

d. Processing Backlogs

These can arise, for example, from staff shortages, strikes, an increasing portfolio or holidays.

If the backlog is seasonal, for example owing to holidays, then this may be a regular occurrence and statistical developments may not be distorted. If however, the backlog has arisen for reasons which had not happened in the past, then subjective allowance must be made to accommodate any statistical distortion which results.

Growth of a business without sufficient growth in backroom staff is a common example of a non-regularly recurring backlog. In this case, particular care is needed in assessing the data since the size of the processing backlog would likely increase over time.

e. Heterogeneity of Data

A commonly used technique in claims reserving is to divide the database up into homogenous sub-sets. Taken to the extreme, we would end up with hundreds of data sets having little if any statistical stability. This is impractical and undesirable. The statistician must therefore restrict himself to broader sub-sets such as by FSA class. Depending on the size of a reserving class, a further sub-division by business type may be appropriate.

Changes in characteristics over time will affect claims development. Unless these are known and allowed for, incorrect reserve estimates will result. In practice, however, it may be very difficult to identify changes in business profile from computer data alone.

The underwriter is probably the best person to approach for guidance as to the change in business type by year. Taking this into account, we can subjectively recognise and make allowance for the heterogeneity of the data when projecting claims reserves. To be more scientific is impossible.

Data constraints often prevent the statistician from dividing his database into homogenous sub-sets. If heterogeneity cannot be avoided, the statistician must apply subjective analysis when establishing outstanding claims reserves.

(There is more on homogenity et cetera in section 3.3 in reference to chain ladder considerations).

2.2.4 D. Exogenous Influences

Such influences are to a large extent outside the control of the insurer. Two of the most important of these influences are inflation in the general level of prices and earnings (to the extent that these affect claim settlements) and uncertainty of investment yields.

Other exogenous influences affecting the claims experience may be revealed by a gradual trend in the figures being analysed, but in many cases the exogenous influences will cause a sharp discontinuity in the experience. Where a discontinuity is known to have occurred and its effects can be reasonably quantified, observed experience should, of course, be adjusted to eliminate the effects of the discontinuity before making an assessment of the provision necessary for outstanding claims. Since most types of exogenous influence are not amenable to statistical measurement, it is not possible to include in the provision for outstanding claims a scientific assessment of reserves for exogenous influences which may or may not arise in future. All that can be done is to include an arbitrary margin in the provision made. Even for those exogenous influences where some statistical assessment could in theory be made (such as the risk of catastrophes), the limited nature of the data available to the individual insurer may render a statistical approach inappropriate. These features are illustrated in the following paragraphs.

a. Changes in Legislation

Changes in legislation, whether fiscal or otherwise, are clearly factors beyond the control of individual insurers. For example, an increase in the rate of value-added tax could result in increased claim costs for motor car repairs effected after the relevant date, whether or not the damage was inflicted before the relevant date. Provided the proportion of claim costs subject to

VAT is known, the effect of a change in the rate of VAT can be quantified and past experience can be adjusted to produce consistent figures for assessment of the outstanding claims provision. It is not possible to make allowance on a statistical basis for any future changes in the rate of VAT. Landmark judicial decisions may have a similar effect.

b. Social Environment

Apart from legislative changes, there may be changes in the social environment which lead to uncertainty in the valuation of outstanding claims. For example, a more sympathetic attitude towards disabled claimants may be reflected in higher compensation payments awarded by courts, particularly where the payment is determined by a jury. This feature would normally be revealed as a gradual trend in settlement costs, although not necessarily on a steady basis, and may be indistinguishable from other features affecting the claim settlement process discussed in ‘a’ above. However, occasionally, this type of change may produce a sharp discontinuity in the experience.

c. Weather Conditions

The vagaries of the weather can result in a fluctuating incidence of claims between different accounting years, which may give rise to different settlement patterns. The problem is greatest when the most recent year has experienced abnormal weather conditions. It is unlikely that the data available on past experience would be sufficiently credible to allow any adjustment to the assumed future run-off pattern for the latest year other than on the basis of informed subjective judgement. Use of weekly or monthly delay tables could help with certain classes.

Infrequent climatic events, such as typhoons, hurricanes and other catastrophes, can also complicate the analysis of past experience. The preferred course of action is likely to be to eliminate catastrophe claims from past experience, to use these adjusted claims figures to assess the outstanding claims provisions, and then to incorporate a further provision for any outstanding catastrophe claims known at the valuation date. The difficulties arise in deciding which claims (or which parts of claim payments) are attributable to a catastrophe.

d. Currency Movements

It is clearly desirable for the sake of homogeneity that experience in different territories should be examined separately, provided there is a sufficient volume of data. Although the figures in the UK supervisory returns are normally shown in sterling, the analysis of the experience should ideally be made in original currencies to reduce the distortion caused by fluctuating exchange rates. However, it is not always possible to segregate the data completely on the basis of currency. For some risk groups, particularly those risks that are of an international nature, the currency in which a claim is made may not be known in advance. Even where the policy conditions prescribe payment in a particular currency, the amount of claim ultimately paid may effectively be linked to some other currency, for example depending on the location of the claim event or on the country in which court action was pursued. The currency may also depend on the nature of the claim. For example, energy claims are always in US dollars even when the policy currency is not.

If records are available of the proportion of claim payments made (or effectively made) in each currency within the particular risk group being analysed, then adjustments to the experience figures can be made to compensate for past changes in exchange rates. However, it is likely that only approximate adjustments would be possible in practice.

There could also be problems when claims are in a different currency to premiums, giving a mismatch, such as when premiums are used as the exposure measure on Bornhuetter-Ferguson or Generalised Cape Cod methods.

e. Miscellaneous

This paragraph considers a further set of distorting influences which are not outside the control of the insurer, but which are extraneous to the claims experience. For example, some methods of estimating outstanding claims involve the calculation of ratios of claims paid to premiums received. In such cases, it is important to be aware of any changes in the general level of premium rates and to adjust the ratios for such changes so that the figures examined are on a consistent basis. Similarly, previous changes in reserving techniques may make earlier figures not directly comparable with later figures. It may also happen that pressure from management in their desire either to maximise or to stabilise company profits could have had a variable effect on the assessment of reserves in previous years. The specialist within the insurance company who is charged with responsibility for assessing outstanding claims ought to be aware of all these factors, but the specialist outside the company, relying only on published information, may not be. In that case, a greater degree of uncertainty may be expected in estimating the outstanding claims provision.

2.2.5 E. Outwards Reinsurance

a. Net Liability Calculations

There are in effect two ways in which net liabilities may be calculated:

- ❖ By calculating the gross liability and the outwards reinsurance liability separately and hence calculating the net liability as the difference between these two figures.
- ❖ By using data which is net of outwards reinsurance and so calculating the net liability directly.

Both these methods can be considered, although different results are likely as the valuation methods are not usually additive models. If the second is used then the gross liability still needs to be calculated and problems may be caused by using this order of calculation in obtaining results which are consistent. In general it would appear to be more logical to consider gross and reinsurance as two separate entities, particularly where the proportion reinsured is substantial.

b. Catastrophe Covers/ Large Claims

The treatment of large claims and catastrophe covers can produce distortion in the results as there are a number of ways in which allowance is made for these. Also, problems are caused by whole account and other forms of catastrophe cover which are used to protect more than one account as it is then very difficult to obtain the amounts recovered for each account separately. Also, any allocation to account may be arbitrary.

3. Chain Ladder Practical Considerations

The following describes how chain ladder development factors are selected in practice.

3.1 Chain Ladder Development Factors: Judgmental Selection

Using the observed historical loss development experience, the following are reviewed: (i) report-to-report incremental development factors (RTR factors), (ii) volume-weighted RTR factor averages, and (iii) simple average RTR factors. Development factors are selected based on the developmental experience, the RTR factors and the RTR factor averages which are examined for the following characteristics:

- Smoothness of RTR factors and factor averages, with the ideal patterns showing steadily decreasing incremental development from evaluation to evaluation, especially in the later evaluations;
- Stability of RTR factors, with the ideal case being a relatively small range of RTR factors or a small variation within each column;
- Credibility of the experience, based on the volume of losses for a given origin year and age;
- Changes in patterns of loss developments such as (i) an increase or decrease in RTR factors within a column or (ii) a shortening or lengthening pattern of development over time; and
- Applicability of the historical experience in projecting for all origin years, based on qualitative descriptions regarding changes in the book of business over time.

3.2 Benchmark Development Factors

Development factor selection is based upon actual historical experience of a class of business, supplemented by benchmark patterns which are constructed drawing upon available relevant sources of loss development data. Benchmarks should be revised periodically as new information and trends emerge. While the actual development of a class of business can be expected to vary from the benchmark because of individual circumstances, the benchmark may be considered to be an appropriate supplement to the analysis of triangle data, as it represents the current judgement as to the typical payment patterns that can be expected for a type of business. It is, of course, possible that this historical data will not be predictive of future loss experience of the business concerned.

3.3 Homogeneity versus Paucity of Data

With the use of more sophisticated computer systems it is often the case that data can be subdivided down to a very finely defined level. A very large number of triangles of data can be produced, each of which can be considered to be more homogeneous than the combined data. For example, Aviation data could be split into airlines, airports, products, general aviation, war, deductible cover, satellites and helicopters with some of these split into hull and liability. Figures converted to one currency can be split into US dollars, Sterling, Canadian Dollars and Euros. Here Sterling could include all other currencies converted to sterling (which describes common analyses at Lloyd's, in order to comply with syndicate reporting requirements) or these could all be shown separately. However, while splitting the data could give more homogeneous data, the data can be so sparse that stability is lost and reliable projections cannot be made.

A balance has to be made between the two extremes. Other considerations that need to be made are:

- What is the minimum requirement? For valuation of Lloyd's syndicates, there are some sub-divisions that have to be made. For example, US dollar and Canadian dollar business needs to be split out for trust fund purposes. Without this requirement Canadian dollar business, in particular for smaller syndicates, would often not be treated separately when it was only a small portion of the whole. In practice the development factors used for small classes denominated as Canadian dollars are through necessity usually based on the US Dollar selected values.

- Whether there are particular reasons why sub-divisions are required, such as over-riding requirements by management that ultimate values for particular sub-divisions are required.

- There are cases where particular sub-sections of data disturb the overall development and should be considered separately. Obvious examples are environmental losses (asbestos, pollution, health hazards) and large losses (in particular the 11 September 2001 events).

- Changes in the mix of business need to be considered. If the sub-divisions of business have different development patterns there is less need to sub-divide if the proportions of business written and relative loss ratios within each sub-division have remained relatively constant for succeeding origin years.

- For combined currencies projections are likely to be more accurate if data are converted all at the valuation date rates of exchange. If data are converted at historic rates of exchange there can be distortions, particularly in times of rapid changes or revaluations of currencies.

3.4 Triangle Data: Missing Cells

Data may be missing from triangles of claim data because of problems with systems. In particular, the data from the top left part of the triangle may be missing as historical records were not maintained. Options available are as follows.

- The most obvious option is to try to restore the missing data, possibly by examining paper records or off-line computer systems.

- If cumulative-from-inception data are available then the standard methods are still applicable. The chain-ladder methods can be used with fewer years available on which to base development factors. This may in fact be more appropriate in practice even when all data are available as development for the missing top left triangle of data may no longer be appropriate with the lapse of several years.

- If cumulative data are not available, one option is to re-create the data based on later development. For example, if ultimate values are normally based on incurred claim projections, the missing paid claim data can be estimated based on development data for subsequent development periods. This will then give cumulative incurred claim data that can be used as normal.

- There are methods which can be used, such as development methods based on incremental data, curve fitting methods or the least squares method. Depending on the quantity of data missing, these methods can be highly unstable.

3.5 Chain Ladder: Interpolation

One example where interpolation is required is for a non year-end valuation, particularly at the end of the third quarter of a year when an early indication of the year-end results is required or when more time can be spent on preparing the forecast compared with the year-end which is often subject to severe time constraints. The data available are often end-year data for all of the triangle apart from the most recent diagonal, which would be as at the valuation date (or third quarter for our example).

The first problem is that standard development factor estimation cannot be done. The RTR factors in the triangle for the latest development period are based on three-quarters of a year rather than a whole year. One option is to ignore these ratios when calculating average factors, particularly if it is considered that the development to third quarter is less reliable than to year-end for various reasons such as that year-end figures are subject to more rigorous audit. A second option is to include the values in the averages, after grossing up to year-end values, usually assuming linear development over the year (possibly with less weight used in the averages for the grossed-up factor).

The second problem is that the averages derived are values to progress from one year-end to the next whereas the current data is at third quarter. This is usually solved by linear interpolation between the derived year-end percentage of ultimate figures. Linear interpolation is unlikely to be precise. The hope is that it is close enough to be used in practice. This is likely to be true for later development periods but may be inappropriate for the first few development periods. One consideration is whether the RTR factors are being selected at the third quarter in order for them to be used at year-end, when fourth quarter data are available, without further analysis or if they are required to project third quarter data when more accuracy would be required.

If the historic data are available as at third quarter for previous years, more accurate interpolation formulae can be derived. Alternatively, as described by Sherman (1984), a curve could be fitted to the selected end-year development data and used to determine the interpolation formulae (although this may not be applicable for the latest origin year with just three quarters development).

If complete quarterly development data are available then there are various options:

(a) Triangles based on end-year data for all apart from the current diagonal can be used, as above.

(b) Quarterly development factors can be derived and used to project to ultimate. This has the advantage of using all the data and no interpolation is required. The disadvantages are:

- considerably more work is involved;
- historic quarterly development will likely be more subject to fluctuation than annual development and hence, selection/smoothing of development factors will likely be more difficult;
- quarterly data other than year-end may be less accurate as they may not be subject to audit;
- true underlying development factors may not reduce smoothly, for example if there are annual reviews of reserves
- presentation of triangles requires much larger space.

(c) Triangles with annual data being that applicable to the current diagonal. For example, if the valuation is at third quarter then the historic data would all be at third quarter in the respective calendar years. The advantage of this approach is that it is straightforward and no interpolation is required (unless a forecast as at the end of the year is needed). The disadvantages are:

- Non year-end data is likely to be less reliable than year-end data for insurers who do not issue audited financial reports at the valuation date;

- The development factors selected at previous year-end or previous quarter cannot be used as the framework for the current quarter as is possible with (a) above.

3.6 Chain Ladder: Irregular Reporting Dates

The data for the triangle may be at irregular dates, that is the diagonals for the triangle may be at various, non year-end, dates. The options available, depending on the degree of difference from standard triangle development, includes:

- Ignoring the fact that data are not annual and making ad-hoc adjustments as required.

- Interpolating between values in the data to derive annual data (using linear interpolation or non-linear if warranted by the data and if this is feasible).

- Curve-fitting using standard development curves with appropriate adjustment to a non-annual basis.

- Using a more frequent development period than annual, with appropriate adjustments if there is no unique choice.

- Using benchmark development factors with interpolation as appropriate.

3.7 Bornhuetter-Ferguson

3.7.1 Initial Expected Loss Ratios

In order to use the Bornhuetter-Ferguson ("BF") method initial expected loss ratios (IELRs) need to be determined. The BF method is heavily dependent on the quality of the IELRs. If a poor IELR is used this will result in a poor estimate. The following are methods used in practice:

- If very little information is available about the class of business a fixed value of 100% as a break-even value (as was suggested in the original paper by Bornhuetter and Ferguson), or allowing for practical issues as 100% less various values such as commission (if relevant) and profit margin. This approach should only be used as a last resort. If there is any information which suggests that 100% may not be appropriate then this information should be used.

- An alternative to the previous is to start with such an IELR for the first valuation and then at subsequent valuations (either quarterly or year-end) to use the selected ultimate loss ratio rolled forward from the previous valuation for each origin year as the IELR for the new valuation. It should be noted that different results would be derived based on how frequently valuations are made. Using the results from the previous valuation gives results that are not very different to the previous valuation. This has the advantage that projection results are fairly stable over time. However, a serious disadvantage is that an inappropriate result from the previous valuation would not be corrected.

- Underwriters' estimates can be used based on their experience and judgement (but allowance may need to be made for undue optimism!)

- If rate adequacy changes for the class of business being valued, either based on the actual business or using market knowledge, including information about the underwriting cycle, are available, these can be used (adjusted for inflationary influences) to determine an IELR for an origin year relative to the IELR or ultimate selected loss ratio for the preceding origin year.

3.7.2 Large Loss Adjustment

The chain ladder method can be adjusted by:

- Removing large losses

- Projecting excluding large loss data using development factors assumed pertaining to data excluding large losses.

- Individual projection of the individual large losses to give IBNER (using curve-fitting, graphical analysis, market knowledge, exposure information etc).

- Calculation of pure IBNR for large losses (which would usually be zero if data are on an accident year basis as relevant large losses should be known at the time the valuation is made, although this may be less true for casualty than for property).

For the Bornhuetter-Ferguson method, either one of the following applies:

- The initial expected loss ratio is assumed to be for data including large losses. In this case the formula does not necessarily need adjustment. For the incurred BF method, the formula for IBNR of $(1-c).\chi P$ is applicable where c is the expected proportion of ultimate for claims including large losses, χ is the initial expected loss ratio and P is the premium.

- The initial expected loss ratio is assumed to be for data excluding large losses. In this case the IBNER for large losses needs to be added to the above formula for IBNR, together with pure IBNR for large losses if the data are not on an accident year basis.

3.8 Inflation

We are now in a low-inflation era, but this does not mean that we can ignore inflation, particularly as claims inflation may be much higher than anticipated compared with RPI. Should future inflation be allowed for explicitly or implicitly? There are some methods which allow for inflation explicitly such as the inflation adjusted chain ladder method and the separation method. The standard chain ladder method assumes that future inflation will be similar to the inflation inherent in the historical data. This may be more or less accurate than using an explicit inflation method for which assumptions need to be made about future rates of inflation. Consideration should be given to ways of adjusting the standard chain ladder method without using the full inflation-adjusted method to allow for changing levels of inflation.

3.9 Discounting

When discounting results consideration needs to be given as to what interest rates should be used. Depending on circumstances these could be related to the company's actual investment portfolio, at fixed rates, or at rates related to standard risk-free bond rates. Different rates may be used for short-tail and long-tail classes. Rates may be adjusted to allow for non-invested assets. Section 4.2.2.3 suggests that in practice the discount rate used is a risk-free rate, with or without a margin.

3.10 Gross versus Net Projections

This is mentioned in Section 2.2.5. It should be noted that projection techniques rely on the past being representative of the future. This is invalidated by changing reinsurance structures and therefore the reinsurance programme applicable to each origin period should be considered before selecting an approach.

One approach used is to project the gross data and then to work out how the reinsurance programme applies in order to calculate the net results. How easy this is to do depends on a number of factors:

- How complex the reinsurance programme is. The programme may comprise a large number of contracts with complex arrangements, each covering various classes of business with various inuring previous reinsurances.
- Levels of deductibles. If the excess of loss cover is at high levels then large losses can be projected at the gross level to determine recoveries. For lower level cover it may be impractical to project losses directly and in particular to determine IBNR claims.
- Some reinsurance contracts may require special consideration, in particular those with aggregate deductibles (including but not limited to stop loss contracts).

If reinsurance is small relative to gross then net-to-gross or reinsurance-to-gross ratios may be used, possibly based on incurred amounts by origin year and applied to ultimate amounts or directly to reserves.

Alternatively, net projections can be made directly. In some cases the development is smoother at the net level as the peaks from the gross development have been smoothed out. Factors that need to be considered are current and possible future exhaustion of reinsurance contracts and how the reinsurance programme has changed over time. Also, net figures may be projected at levels net of proportional and some excess of loss programmes but before multi-class covers and stop loss covers which would be estimated subsequently.

3.11 Large Losses

Judgement is required as to what is defined as a large loss if these are excluded from the standard projection methods and treated separately. As intimated above, one consideration is at what level claims enter the reinsurance programme. Also, too low a threshold would involve considerable work. For the larger losses there is information available from the market to help determine ultimate losses.

4. Survey of Reserving Methodology

4.1.1 As part of the process for this year's working party, we decided to conduct a survey of reserving methodology. There is a huge amount of background material available on reserving methods, apart from the Institute and Faculty's Claims Reserving Manual, including a large number of papers written in this country and abroad, including USA, Canada, Australia and the rest of Europe. For anyone embarking on a claims reserving exercise the main questions for which they require answers are:-

- What method(s) should I use?
- What results should I produce, for example: reserve estimate, range around estimated reserve, discounted reserve?

4.1.2 As an aid to answering these questions it would be helpful if we knew what was done in practice. The survey, conducted through the auspices of GIRO, aims to identify:

- What can be considered to be normal practice among UK reserving actuaries.
- The range of methods used (and found useful) in practice, so that others may consider using methods they might not otherwise have used.

4.1.3 The survey looks at reserving in general and three specific areas:

- The use of stochastic methods in practice
- Reserve ranges in practice
- Risks margins and discounting

The questions asked in the survey were as follows:

4.1.4 Background

- What percentage of your working time over the course of a year relates to reserving?
- To which market areas does your reserving work generally relate?
- Type of organisation worked for (insurer; consultancy; other, to be specified)
- How many years of reserving experience do you have?

4.1.5 Reserving Survey 1: Reserving in General

Q1. Identify those projection methods that you use in practice: regularly, less frequently, once in a "blue moon" (chain ladder; inflation-adjusted chain ladder; average cost per claim; simple loss ratio applied to ultimate premiums; ratio applied to case reserves to estimate IBNR; exposure based method; other, to be specified).

Q2. Identify the main method(s) you regularly use to allow for reinsurance (push gross losses through actual reinsurance programme; apply net-to-gross ratios or reinsurance-to-gross ratios; projecting gross and net data separately; apply same method and same assumptions to net as were selected for gross; other, to be specified).

Q3. Identify the main methods you regularly use when rolling projections forward, rather than re-projecting, for example when rolling forward to the next quarter. (No change in ultimates, that is reduce reserves by payments in the quarter; look at "actual vs. expected" movements and use judgement; mechanically apply same method and assumptions, with one quarter removed; apply same method and assumptions, but review of necessary changes; Bornhuetter-Ferguson approach, using previous ULR as IELR; other, to be specified).

Q4. Identify the main method(s) you regularly use to select development factors for chain ladder or similar methods (calculated average; manual selection; curve fitting; benchmarks; other, to be specified).

Q5. Identify the main method(s) you regularly use to select tail factors for chain ladder or similar methods (manual selection; curve fitting; benchmarks; other, to be specified).

Q6. Identify the main method(s) you regularly use to select initial expected loss ratios for Bornhuetter-Ferguson or similar methods (ULR from previous underwriting years, adjusted where necessary; IELR from previous underwriting years, adjusted where necessary; underwriter's estimate; ULR from previous valuation; default break-even ratio, for example 100%; manual selection; benchmarks; other, to be specified).

Q7. Identify the associated reserves that you normally cover; whether included explicitly or implicitly (ALAE; ULAE, UPR; (additional) URR; reinsurance bad debt).

Q8. If where you would like to be is different from where you are in respect of your general reserves approach, what are the main differences?

Q9. If where you would like to be is different from where you are, what are the main reasons for this?

4.1.6 Stochastic Reserving Survey 2:

Q1. Have you used stochastic reserving methods in a real reserving situation? (No; yes, but only occasionally; yes, sometimes; yes, frequently).

Q2. In which situation, do you use them? (Estimating reserve requirements; identifying reserve ranges; DFA/asset-liability modelling).

Q3. Identify methods you have used and how useful you found them (Bootstrap; random walk model for paid loss development; Zehnwith; Wright; Mack; Verral; Hoel curve and GAMs; other, to be specified).

Q4. Indicate where you would like to be (compared to where you are), that is whether you would ideally like to use stochastic methods in your reserving work (No; yes, but only occasionally; yes, sometimes; yes, frequently).

Q5. If where you would like to be is different from where you are, principal reasons for this (Too time-consuming; too complicated; need to learn more about them; other, to be specified).

4.1.7 Reserve Ranges

Q6. Do you identify reserve ranges when performing reserve work? (No; yes, but only occasionally; yes, sometimes; yes, frequently)

(If answer “No”; Q7 to Q10 ignore).

Q7. How would you describe the range(s) you usually produce? (Range of outcomes with specific associated probabilities; range of likely outcomes; range of possible outcomes excluding very unlikely outcomes; range of reserve estimates that I would consider reasonable/acceptable; other, to be specified)

Q8. What principal method(s) do you normally use to identify reserve ranges? (Stochastic methods; varying projection assumptions; rules–based framework; judgement; other, to be specified)

Q9. If where you would like to be is different to where you are, what are the main differences?

Q10. If where you would like to be is different to where you are, what are the main reasons for this?

4.1.8 Risks Margins and Discounting

Q11. How would you describe your undiscounted reserve estimates in general? (Mean; median; mean plus a margin; median plus a margin; other, to be specified).

Q12. Do you produce discounted estimates when performing reserving work? (No; yes, but only occasionally; yes, sometimes; yes, frequently)

Q13. (If yes to Q12)
Identify your usual approach(es) to determining the discount rate(s) to use: (Risk-free rate; risk-free rate, reduced by a margin; rate based on actual investment portfolio; rate based on actual investment portfolio, reduced by a margin; a number of rates, for others to select from; other, to be specified)

Q14. Do you make explicit allowance for risk margins when performing reserving work? (No; Yes, but only occasionally; Yes, sometimes; Yes, frequently)

Q15. (If Yes to Q14)
Identify your usual approach(es) to risk margins (Subjective addition to reserve estimates; calculated addition to reserve estimates, method to be specified; subjective reduction in discount rate; calculated reduction in discount rate, method to be specified; other, to be specified)

Q16. If where you would like to be is different from where you are, what are the main differences?

Q17. If where you would like to be is different from where you are, what are the main reasons for this?

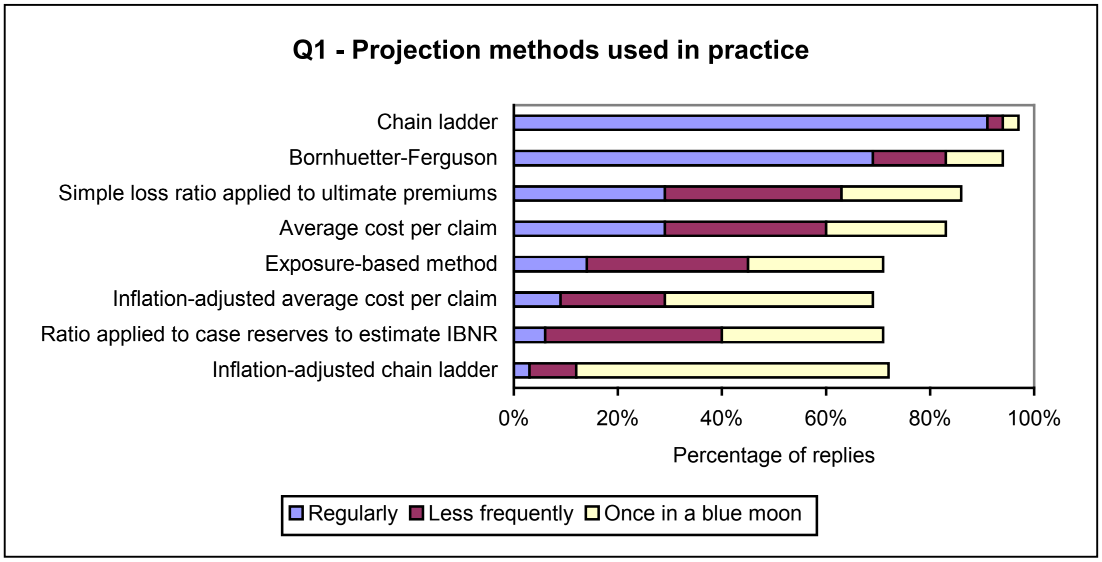
4.2 Results

We are grateful to all those who filled in and returned the surveys. A good cross-section of actuaries responded, balanced between those working for insurers and for consultancies and including those involved in a variety of business lines. The results of the survey are summarised below.

4.2.1 Reserving Survey 1: Reserving in General

Projection methods that are used regularly were, unsurprisingly, dominated by the chain ladder and Bornhuetter-Ferguson methods. These are not universal preferences though – one person felt strongly that we should be moving away from these deterministic methods, believing that they produce misleading results.

Only in a few cases did people indicate that they regularly used projection methods other



than those listed in the survey question. The most popular of these other methods were the projected case estimate method and variations on the average cost per claim method.

Similar methods were used by those working for insurers and those working for consultancies. Those working for insurers tended to use average cost per claim methods more often, perhaps linked to the fact that more of them indicated that they worked in personal lines. In general the consultants used a greater variety of methods and more often mentioned methods not on the list in the survey question. This may simply be owing to a tendency to look at a greater variety of types of business.

In question 2, the main methods regularly used to allow for reinsurance by those responding were: Push gross losses through actual reinsurance programme; Apply net-to-gross ratios (or reinsurance-to-gross ratios); Project gross and net data separately. Each of these methods was regularly used by about 50% of those responding. The fourth option, Apply same method and same assumptions to net as were selected for gross, was less popular (15%). A small proportion of those responding said that they regularly used a stochastic approach.

Question 3 asked which methods were regularly used when rolling projections forward (rather than re-projecting), such as when rolling forward to the next quarter. The most popular methods were: Apply same method and assumptions but review for necessary changes (regularly used by about 50%) and Look at 'actual vs expected' movements and use judgement (50%), closely followed by No change in ultimates – just reduce reserves by payments in the quarter (35%). In fact, No change in ultimates was more often used by those working for insurers, probably because they are likely to assess reserves more frequently.

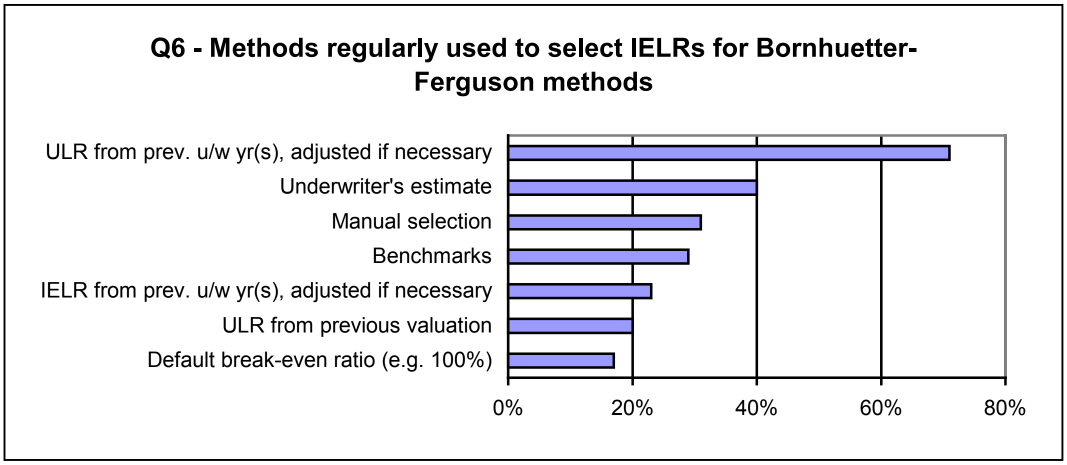
When it comes to selecting development factors for chain ladder or similar methods, by far the most popular approaches are manual selection and calculated averages. In each case, about 75% of replies indicated that this was one of the main methods used. Curve fitting was regularly used by 50% and benchmarks by 30%.

Following on from this, manual selection was also by far the most regularly used method for selecting tail factors (80%), following by curve fitting and benchmarks (about 45% in each case).

Question 6 asked how Initial Expected Loss Ratios are selected in practice for use in the Bornhuetter-Ferguson method. The most popular approach was to use the Ultimate Loss Ratio from previous underwriting years, adjusted where necessary (see below).

In each of the three questions above, the most noticeable difference between those working for consultancies and those working for insurers was that consultants were more likely to incorporate benchmarks (as might be expected).

In question 7 the survey asked which associated reserves actuaries normally cover in their reserving work. The results were: ALAE 66%, ULAE 49%, UPR 46%, Additional URR 51%, Reinsurance bad debt 34%.



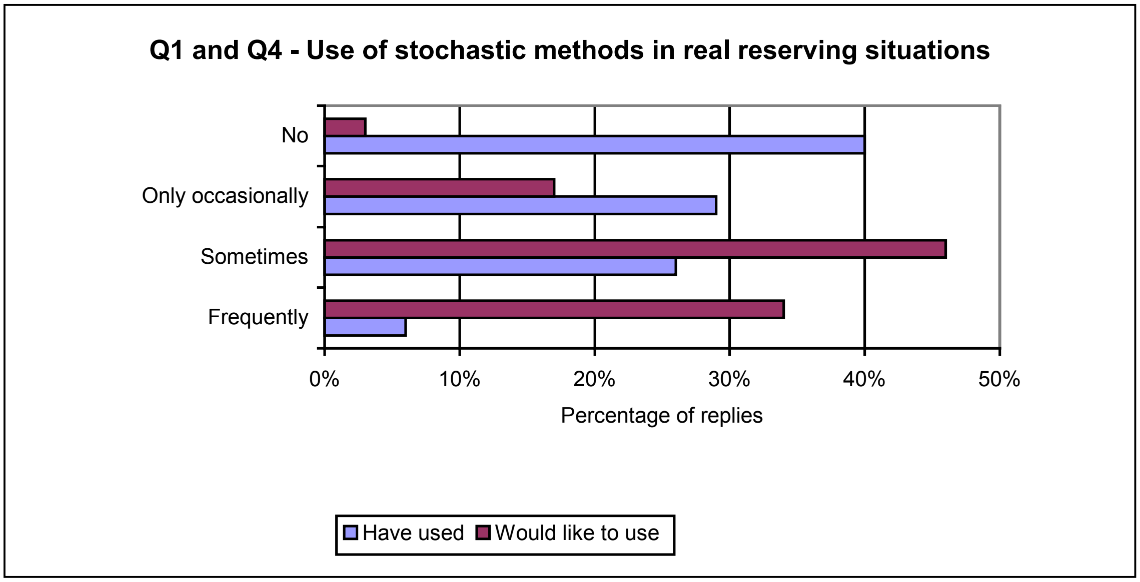
The first seven questions were designed to identify current normal practice among reserving actuaries. The survey then went on to ask how actuaries would ideally like to approach their reserving work, if this is different to what they do at the moment. They were also asked to indicate the main reasons for any differences. The most common comment was a desire to use stochastic methods or to use them more often (23%). A variety of reasons were given as to why this is not done at the moment. The main reasons were a lack of time or resources (including a few people who felt the extra time required was not justified by the value added) and a lack of familiarity with stochastic methods.

4.2.2 Reserving Survey 2: Stochastic Reserving Methods, Reserve Ranges, Risk Margins and Discounting

The second survey was designed to address three aspects of reserving that we felt were of particular interest.

4.2.2.1 Stochastic Reserving Methods

The results of the survey showed that the majority of people have not used stochastic methods in a real reserving situation or have done so only occasionally (about 70%). Those working for consultancies were more likely to have used stochastic methods.



However, a large proportion of people indicated that ideally they would like to use stochastic methods either frequently or sometimes (80% in total). Few people use them frequently at present (6%), but it is interesting to note that out of those who use them "sometimes", most would like to use them more. The most common combination of replies to questions 1 and 4 were from the following groups:

- Those who don't currently use stochastic methods (or do so only occasionally), but would ideally like to use them "sometimes" (37%); and

- Those who currently use them "sometimes", but would ideally like to use them "frequently" (17%).

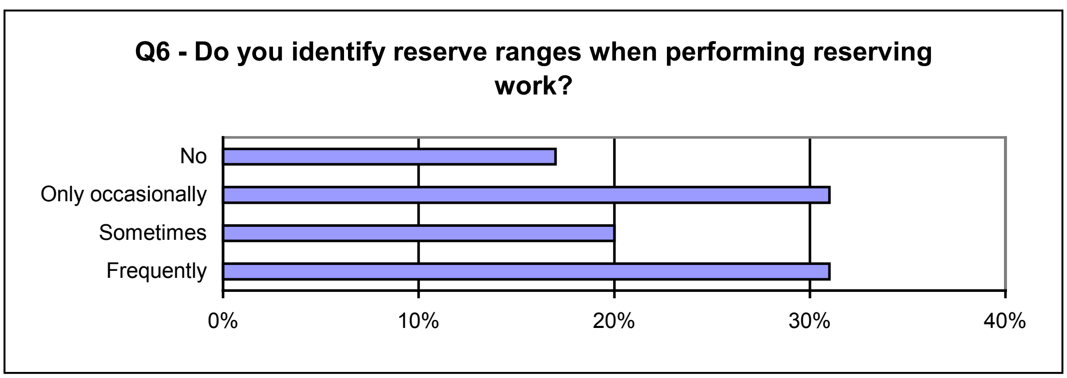
The main obstacles cited by those who don't use stochastic methods as often as they would like were that the methods are time-consuming (50%) and that they need to learn more about them (50%). Several other problems were noted: budget constraints, lack of data, lack of suitable software, issues with the general approach (such as the reliance on assumptions) and difficulties in convincing others to accept these methods. Some replies noted that external factors such as regulatory requirements and fair value accounting are likely to lead to increased use of stochastic methods.

For the most part, stochastic methods are currently used to identify reserve ranges (indicated by 80% of those who have used them in practice) rather than to estimate reserve requirements (30%). 50% use them for DFA/Asset-Liability Modelling and 30% use them for other purposes – the most common of these being reinsurance modelling and issues relating to solvency or capital requirements.

Question 3 asked which stochastic methods have been used and whether they were found to be useful in practice. Encouragingly, most people found the methods they had used to be useful. By far the most commonly used methods were Bootstrap (60%) and Mack (50%).

4.2.2.2 Reserve Ranges

Most actuaries responding to the survey identify reserve ranges when performing reserving work (about 80%), although only 30% do so frequently (see chart). Those working for consultancies tend to do so more often than those working for insurers.



A variety of types of range are produced and some actuaries produce more than one type of range. The most common types of range identified were: Range of reserve estimates that I would consider reasonable/acceptable (50%), Range of likely outcomes (30%) and Range of possible outcomes excluding very unlikely outcomes (30%). A smaller proportion of people said that they identified ranges with specified associated probabilities (20%), although a few noted that they would like to be in a position to attach probabilities to their ranges.

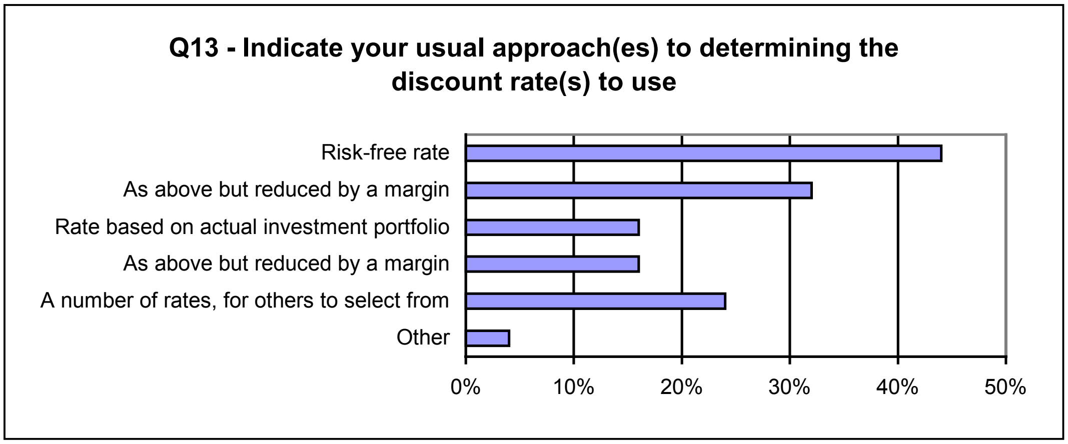
The principal methods used to identify reserve ranges are: varying projection assumptions (70%) and judgement (70%). Stochastic methods are regularly used to identify reserve ranges by 30% of people responding to the survey and a rules-based framework is used by about 20%. Consultants more often used stochastic methods than those employed by insurers, but otherwise there were no clear differences in methods.

Question 9 asked what people would like to change about their current approach to reserve ranges and question 10 asked about the obstacles preventing them from doing so. The responses varied, but a clear theme was that the actuaries responding would like to be able to be more scientific in their approach to identifying ranges. In particular, a number of people commented that they would like to reduce the amount of subjectivity involved and several said that they would like to use stochastic methods. Constraints included time and data issues, plus a lack of methods accurately reflecting the risks (for example allowing for correlations and reflecting how reserves are selected in practice). A few people said that, for them, it was not worthwhile spending more time on reserve ranges or that it was not currently an important enough issue to do so – although some felt that this would change.

4.2.2.3 Risk Margins and Discounting

When asked how they would generally describe their undiscounted reserve estimate, most people said they would describe it as a mean (50%) or as a mean plus a margin (25%), with the remainder evenly split between median and median plus a margin. Those working for insurers were more likely to describe it as including a margin.

In answer to question 12, most people said that they did not produce discounted estimates or did so only occasionally (35% and 30% respectively). Of those who produce discounted estimates, the most popular approach to determining discount rates was to use a risk-free rate, with or without a margin (see chart).



Explicit allowance for risk margins is not particularly common, with only 10% of those responding indicating that they frequently incorporate an explicit risk margin. 15% do so sometimes, and 20% occasionally, but 55% do not use explicit risk margins. One person noted that it can be difficult to convince others to accept explicit margins.

Those who do use explicit risk margins generally do so by adding a margin to the reserve estimate (80%) rather than by deducting a margin from the discount rate (20%). The approach to selecting the margin is usually subjective. Few people specified an approach to calculating a risk margin. Of those that did, the most popular approaches were stochastic methods and varying the original reserving assumptions.

The final two questions asked how actuaries would ideally like to approach risk margins and discounting in their reserving work, if this differs from their current approach, and the main reasons for any differences. Fewer comments were made here than in the equivalent sections on the use of stochastic reserving methods and approaches to reserve ranges, perhaps indicating that people are more satisfied with their current approach in this area or perhaps just that this area feels further away from the work of most actuaries.

The most common response was a desire to reduce the subjectivity inherent in the approach currently used. However, one person noted that they preferred to use a subjective risk margin and commented that a more scientific approach lends an air of accuracy to the assessment that is not justified given the number of subjective assumptions involved.

5. Stochastic Methods in Practice

5.1 Range value selection in practice is often done by:

- Using percentages of reserves or of outstanding claims and IBNR, by origin year. The percentages would be judgementally selected based on previous experience and a view as to the degree of variability in the possible outcome of the forecast. This would give a range of results which would possibly be presented as : “ The ranges are provided as a guide to uncertainty only. The ranges selected are intended to be a range of possible outcomes for which it is believed possible but unlikely that losses would fall outside the range.”

- By varying the parameters selected. For example, with the Bornhuetter-Ferguson chain ladder method, upper and lower values for both the development factors and initial expected loss ratios would be selected to determine upper and lower values for the ultimate values and hence for the reserves.

- By using a stochastic method which would provide a statistical range. The paper “Stochastic Claims Reserving in General Insurance” by England and Verrall gives a review of some of these stochastic models.

A stochastic method might be thought to be a preferable method for estimating a range of results. However, as intimated above, the method used in practice to select the reserve required for statutory purposes may not be the mid-point of the values found using a stochastic method. This may be because the estimated reserve has implicit or explicit margins but even if these are taken into account, there still may be problems. One particular example in practice is when the reserve is obtained by using smoothed or benchmark development factors and a Bornhuetter-Ferguson methodology with initial expected loss ratios based on rate changes from a particular base year. There is no straight-forward stochastic method that can be used to give a range around the reserves derived using this methodology.

One suggestion that might be worth looking into is to use a Monte Carlo simulation approach. For the underlying BF method, one could work out a distribution for development factors around the selected values and hence simulate the possible outcomes. Also, one could simulate around the trend line for the a priori ultimate loss ratios. The England-Verrall paper suggests a Bayesian approach to accommodate the BF method. Questions that arise are:

- How to allow for correlations, firstly between development factors and secondly between chain-ladder ultimates and initial expected ultimates particularly where the initial expected loss ratios are trended from chain-ladder estimates for earlier origin years.

- If the mean values of the chain-ladder and initial expected values are very similar, does this give a wider range than if the chain-ladder and initial expected values are completely different (in which case the range of possible results might be considered to be rather narrow)?

- The BF method is usually considered to be a credibility approach between two methods, the chain-ladder and the naïve ultimate loss ratio method and hence statistically superior to the two underlying methods (see section 6.2.7). The Mack paper suggests that the Bayesian model is a credibility approach between two methods, namely chain-ladder and BF and hence that using this model would give results that are statistically superior to BF (but only if the naïve loss ratio is in itself a credible estimate). Which method is "superior"?

  Note that in Benktander's "Approach" he derives the standard BF formula for IBNR by suggesting the credibility factors that should be given to the a priori and incurred development IBNR estimates. He then selects IBNR "modified by experience" as BF IBNR x BF ultimate / a priori ultimate, with no more justification than "It is thus natural to put" followed by this formula. The Benktander method is in fact an iterative credibility calculation that effectively uses the result of a normal BF credibility calculation as the prior estimate for a second credibility calculation. Mack (2000) has since shown that the so-called Benktander method is usually "superior" (by which he means it has a lower mean-square error) to either the Chain Ladder or BF methods.

Conversely, Gluck shows that the BF ultimate is a weighted average of the development based ultimate and the initial expected ultimate and that the weights are optimal with certain constraints.

- Some other approaches in the England-Verrall paper also need further examination.

Other ways to adjust the ranges from the results of a stochastic method to match the value that has been selected for a particular valuation (which may be put forward as a best estimate or mid-point value, which could be supposed to be a mean or median value), are:

- Just use the range round the mid-value. For example if the method gives a mean and a 90% value, take mean of distribution as selected reserve amount and 90% value as reserve amount +(90% value from stochastic method – mean value from stochastic method). This is arguable using Bayesian theory as the range should presumably not be more than the range found if more information is used to obtain the actual valuation reserve.

- Pro-rata adjustment: take 90% values as 90% value from stochastic method x reserve amount / mean from stochastic method

5.2 Stochastic Reserving – A Case Study (1)

5.2.1 Introduction

The results of the survey relating to stochastic reserving suggest that many actuaries would like to use stochastic reserving techniques more extensively but feel they do not know enough about these methods to use them with confidence. This feeling is shared by some members of the working party. Therefore, it was decided that it would be useful for one member of the working party with no experience of stochastic reserving to attempt it for the first time and report on some of the issues encountered.

The actuary who carried out the work:

- had no experience of stochastic reserving;

- does not have a particularly strong statistical background;

- had not recently taken any of the relevant actuarial exams.

Because of this, stochastic reserving veterans are likely to learn little from the following section.

5.2.2 Approach

There is a fairly daunting volume of literature on the subject so it was decided that a good place to start was volume 2 of the Claims Reserving Manual. Most of the papers in this volume turned out to be to be both fairly accessible and reasonably interesting. In addition to the papers in the claims reserving manual, the recent paper by England and Verrall (2002), which provides a fairly comprehensive summary of the range of approaches, was also useful.

It became clear after reading the England and Verrall paper that the level of mathematics required for several of the methods exceeded the current level of the actuary carrying out the work. A fair amount of progress can be made, but sooner or later, actuaries who are not comfortable with techniques such as Generalised Linear Modelling are likely to have to become so in order to be confident using several of the methods.

It was mainly for this reason that the Mack (1994) method was chosen for this first attempt, since the mathematics required is reasonably accessible (although some of the appendices are fairly hard work). Other reasons were:

- it is easy to implement in a spreadsheet – there is no need for specialist software;
- its basis is the chain ladder method, which is the most common method used to derive central reserve estimates (at least for earlier origin years);
- it can cope with development factors of less than one, so can be used with incurred projections;
- it can be extended to incorporate tail factors, and factor selections using weights other than the volume of claims (as described in Mack 1999).

An Excel spreadsheet will be attached to the institute website to accompany this paper. The spreadsheet enables the user to derive reserve ranges using the Mack (1994) method (that is before the extension for tail factors described in Mack 1999). While programming a spreadsheet such as this is a relatively simple task, it is hoped that this spreadsheet will make it easier for stochastic reserving novices to experiment with the method using their own data.

One of the most useful aspects of the Mack 1994 paper is the discussion of the assumptions underlying the chain ladder method and the description of methods to test the validity of those assumptions for the current data set. (These tests are implemented in the Mack spreadsheet which accompanies this paper.) If the data does not adequately satisfy the chain ladder assumptions then not only will the central reserve estimates be likely to be invalid, but so will any ranges calculated using the model.

Four different classes were reserved using the model. They were chosen to represent a range of different tails and development characteristics. Each of these had previously been reserved using a combination of the Chain Ladder and Bornhuetter-Ferguson techniques – although the reserves for several of the origin years had been adjusted to allow for known factors not reflected in the triangles.

5.2.3 Problems Encountered

Several problems were encountered while performing the analyses. The paragraphs below set out these problems and suggest potential approaches to overcome them. It is likely that experienced stochastic reserving practitioners are familiar with these problems and have their own approaches for dealing with them. Papers which deal with these issues, written by experienced stochastic reserving practitioners, would therefore be very welcome.

5.2.3.1 Treatment of large claims

These were not a big problem for the classes chosen for this exercise. One class had a particularly large claim in it, which was fully settled, so could just be removed from the analysis and added back (to the central estimate and the upper and lower confidence limits) at the end. Large claims will be more of a problem in cases where there is more uncertainty associated with them (for example: they are not fully settled, they take longer to emerge in the data, there is a greater probability of new large claims emerging in respect of past origin periods). In these situations, just extracting large claims from the triangles and adding selected large-claim ultimates back at the end will be likely to understate the uncertainty. One possible way around this would be to assume a distribution for the total of large claims and combine this in some way with the distribution for the remaining reserve. If large claims can be considered independent of the residual claims just adding variances and assuming a lognormal distribution for the overall reserve may suffice. Depending on the significance of the large claims and the distribution assumed, simulation may be necessary.

5.2.3.2 Tail factors

The Mack method requires the actuary to estimate the standard error of any tail factor selected. Even where a tail factor is not required the method requires the user to make an assumption about the standard error of the last factor (it cannot estimate this as there is only one observed value). The ranges derived for the early origin years are sensitive to these assumptions. In his 1999 paper Mack gives some advice on a possible approach for selecting the standard error of tail factors. Since the selection of tail factors themselves is a difficult exercise it seems unlikely that estimating the standard error will be any easier. It could be argued, though, that since estimates of these factors are fairly subjective it is not worth spending long periods of time agonising over the estimates of standard errors.

5.2.3.3 Selected central estimates that are not pure chain ladder results

This was a common problem. Where this was the case consideration was given to why the selected estimates were different from the pure chain ladder estimates. In many cases this was because less weight had been given to factors of less than one in the top right hand corner of the triangle (in order to add a small margin for prudence). Providing the difference between the estimates was relatively small it would appear that any of the following approaches could be justified:

- Use the upper and lower confidence limits as suggested by the pure chain ladder method (recognising that the *central* estimate has a small margin for prudence).
- Shift the range upwards by the same amount as the difference between the selected best estimate and the pure chain ladder estimate (to allow a degree of prudence in the confidence limits as well as the central estimate).
- Similar to the above only adjusting the limits so the ratios between the central estimates and each limit remain constant.

Alternatively the weights given to factors in the stochastic model could be adjusted so that the central estimates match fairly closely. If this is done by excluding factors less than one in later years, the range derived from the model will reduce to reflect the fact that the remaining factors are less volatile.

The approaches were less satisfactory when the difference between the pure chain ladder estimate and the selected best estimate was more marked and no easy and satisfactory solution was found for this exercise. In cases where the difference is caused by some IBN(E)R in relation to a specific issue / issues (this could be positive or negative) then it may be possible to follow an approach similar to the one suggested for large claims (that is assuming a distribution for this component of the IBNR). Where the difference is caused by the fact that the chain ladder result isn't considered trustworthy for some reason, then it seems reasonable to question whether the chain ladder method should be used to derive the range.

5.2.3.4 Recent origin years

Since the selected reserves for recent years were based, at least partially, on the Bornhuetter-Ferguson result, these years presented more of a problem. As mentioned above, the use of the chain ladder model to derive ranges in this case would appear to be highly questionable. The regression and residuals plots for the first development year (see Mack 1994) tended to show that the chain ladder model could not be trusted for the most recent origin year. Once again no easily implemented and satisfactory solution was found in the time available. The Bayesian Models described by England and Verrall (2002) may help to solve this problem as they would appear to be more consistent with a central estimate derived using the Bornhuetter-Ferguson method.

Using different methods to derive ranges for different origin years raises the question of how to combine them to estimate a range for the overall reserve. Even where we can consider the individual origin year reserves to be independent (which is, after all, an implicit assumption of the chain ladder method) reserve estimates will not be if we have used the same estimates of development factors to derive them (see Mack 1994). A move toward a Bayesian approach for all origin years would appear to solve this problem.

5.2.3.5 The size of the range estimates

In general the ranges derived were larger than those that the actuary would previously have considered reasonable. This is possibly owing to a tendency on behalf of the actuary concerned to underestimate the volatility of the results, but it is also likely to be, at least to a degree, owing to the fact that the actuary could take into account information which was not reflected in the triangles. This external information not only affects the central estimate, but may also reduce the degree of uncertainty around this estimate. If a Bayesian approach is used it may be possible to incorporate some of these factors in prior estimates.

5.2.3.6 Combining estimates over several classes

While underwriters are likely to be interested in the ranges of reserves for a particular class, management are more likely to be interested in ranges over several classes.

Combining the result for classes will be simple enough if we can assume independence (providing we are happy to assume a specific distribution for the overall reserve). In many cases this is unlikely to be a valid assumption (especially where there are recent origin years with unexpired exposures). It should be possible to use a simulation approach to do this allowing for correlations between classes. Care may need to be taken where there is unexpired exposure leading to tail dependencies, which may not be adequately captured by specifying correlations in a package such as @Risk. However the probability of these events may be small enough to be outside the confidence limits that would normally be under consideration.

Most actuaries are happy to believe that the widths of their carefully selected ranges are proportional to the standard deviations of the results. Therefore, a simple method for estimating ranges for a combination of classes is as follows. If we define H_1 and H_2 to be the difference between central and higher estimates for classes 1 and 2, we can estimate the difference between central and higher estimates of the classes combined as

$$H_{1+2} = \sqrt{H_1^2 + 2\rho H_1 H_2 + H_2^2}$$

where ρ is between -1 and 1. It is of course necessary to select a value of ρ to complete the estimate and to do this, we note that it is difficult to think of two ordinary classes of business that would be negatively correlated which means that we can set ρ to zero to define our narrowest possible range. Similarly, the widest possible range is derived by setting ρ to one which can only approach being true for very similar classes of business.

5.2.3.7 Consistency of Gross and Net estimates

This was found to be a big problem. The central estimates for net ultimates had previously been derived by explicitly netting down the gross IBNR. However, for this exercise the net chain ladder was used to derive net range estimates. Where the mid-points of the two methods did not agree the range was just shifted by the difference between the two central estimates. This approach did not feel very satisfactory especially for the net data used for this exercise which, owing to historic adjustments (for example: corrections of errors, re-allocations of RI accruals between classes, re-allocation between origin years), resulted in net development factors that were believed to be unrealistically volatile.

One approach that could be used in practice (but was too time consuming to implement this time) was to deduce a curve of recovery percentages (RI/Gross) against the gross IBNR for each origin year (interpolating between a small number of points may be sufficient). Then the various outcomes of the gross IBNR could be simulated, and the percentage recovery read from the curve accordingly. If necessary some additional variation could be simulated around the selected percentage. The net IBNR could then be derived from the gross IBNR and the recovery percentage for each trial. The distribution of net outcomes could be built up over the entire simulation. This method would require knowledge of the RI program for each year and assumptions about what might drive changes to the gross IBNR and what the corresponding net effect would be. It would clearly be fairly approximate but may be a practical method to use particularly for an in-house actuary who carries out regular reserving exercises on the same business.

5.2.4 Summary

- Looking at stochastic reserving for the first time is likely to be intimidating for many actuaries. In this situation, Volume 2 of the Claims Reserving Manual is a reasonable place to start as most of the papers in it are fairly accessible.

- There may now be reason to add some more papers to this volume (for example: Mack 1999 – in which he extends his analysis to included tail factors, and England and Verrall 2002 – which is a fairly comprehensive review of some different approaches, although the original papers on which England and Verrall is based would likely provide more thorough analyses).

- Using Mack's method it is reasonably easy to make a start and estimate confidence intervals for reserves in a fairly short period of time.

- Sooner or later, however, it is likely that many actuaries will have to revise their statistics (for example by using GLM) before they become comfortable in using some of the available methods to estimate ranges.

- Stochastic reserving takes time (although this no doubt improves with experience). Given the relatively small number of actuaries that use these methods at present, there appears to be little demand from employers for actuaries to sacrifice other activities in order to spend time producing range estimates that are theoretically more justifiable than the ad-hoc estimates that are often produced at the moment. If, as many people suspect, the importance of more theoretically sound range estimates is working its way up the agenda, then this situation may well change. This may have implications for actuarial training both before and after qualification. Alternatively, maybe it is just for individual actuaries to train themselves. The actuary who carried out this exercise initially found it quite intimidating, and wasn't able to solve all the problems that he encountered, but did find it much more interesting that a standard deterministic reserving exercise.

- There are many papers which cover theoretical aspects of stochastic claims reserving, but there appears to be limited coverage of the practical issues that are faced, such as

  - Treatment of large claims

  - Consistency of gross and net projections

  - Estimating ranges where considerable judgement has been used to adjust, say, chain ladder estimates

Research and examples in this area from experienced stochastic reserving practitioners would be very helpful for actuaries attempting to use these methods in practical situations for the first time.

5.3 Stochastic Reserving – A Case Study (2)

5.3.1 **Introduction**

There is a large body of material on Stochastic Reserving, much of it impractical to use. Those methods which are practical to use tend to be divorced from the practical methodology used to arrive at a "mean" value for the reserves. Also, the ranges produced tend to be too large as all the information available has not been used to reduce the ranges in line with Bayesian theory (refer to the Sanders and Leifer paper). This case study is intended to provide practical guidance on how a method can be constructed to give ranges of results around the mean value obtained from a practical reserving methodology. Note that the Sanders and Leifer paper gives some guidance on this approach.

5.3.2 Underlying Practical Deterministic Reserving Methodology

The data used here is the triangle of data, with ten origin and development years, used by Thomas Mack in his paper "Measuring the variability of Chain Ladder Reserve Estimates". We have invented ultimate premiums by origin year. We have assumed for this exercise that the triangle data are incurred claims as these are what are used in practice. Hence, the difference between ultimate claims and the incurred claims (the last diagonal of the triangle) are IBNR claims.

The practical methodology assumed, and around which we will build the stochastic model, is:

- Projection of claims using standard chain ladder methodology with hand-smoothed development factors presumed based on observed averages and whatever other information is available (see section 3.1).

- Selection of a tail factor to proceed from the 10-year development horizon of the triangle to ultimate, presumed based on curve-fitting, graphical methodology, benchmarks, et cetera

- Selection of initial expected loss ratios, in this example for the latest four origin years, presumed based on knowledge of the account (see section 3.7).

- Selection of ultimate claims based on the Bornhuetter-Ferguson ("BF") calculation for the most recent four origin years and the chain ladder method for the first six origin years.

5.3.3 Practical Stochastic Reserving Methodology

The steps used were as follows, using @Risk as the stochastic platform.

5.3.3.1 Development Factors

The basic assumption was made that for chain ladder factors for most development years the selected values, $1+r_j$ are the mean values of the distribution and that the standard errors are proportional to r_j. For later development years, there is still variation even though r_j are close to zero. For these years, we have assumed that the standard deviation is constant. Note that this is a practical solution as the standard duration should reduce by development year, eventually becoming zero. One way to overcome this problem would be to derive benchmarks based on data for comparable lines of business with more years of data.

We examined various statistics, including straight standard deviation, skewness and kurtosis and standard error based on replacing the statistical mean by the selected value in the standard deviation formula. These were calculated for absolute value, and for values based on proportionate values, that is $r_{i,j}/r_j$ for single development years and for ranges of development years. The values selected were:

- For development years 8-9 and 9-10, a standard error of 0.018 based on the standard error for years 7-8 up to 9-10.

- For development years 2-3 to 7-8, a proportionate standard error of 0.8 (standard error for years 2-3 up to 6-7 is 0.826 and for years 2-3 up to 7-8 is 0.789), that is standard error for selected factors of $1+r_j$ is $0.8 r_j$.

- For development years 1-2, there was observed to be fairly high negative correlation between the observed development factors and the amounts, or rate-adjusted first year loss ratios (see below). We based the standard error on the deviation from the regression line as 10.65, which is lower than the 12.34 standard deviation 1 or 13.52 standard error) on the basis that the additional information used has reduced the uncertainty, in line with Bayesian theory.

- For the tail, we assumed a further three years of development and selected the standard deviation as √3 times the year 9-10 value.

Because of the high skewness for development years 1-2 to 3-4 we assumed log-normal distributions for the development factors. We assumed normal distributions for all other years.

5.3.3.2 Initial Expected Loss Ratios

For the initial expected loss ratios, we assumed that we know about rate changes and could adjust historic loss ratios for these. It is assumed that these rate changes allow for the effects of inflation, as appropriate. It is likely that this sort of approach had been used to derive the deterministic IELRs.

Based on the rate-adjusted ultimate loss ratios from the deterministic chain-ladder projection, standard deviations from the mean were calculated for origin years 1–6 up to 1–10. A value was selected of 14.5%. Initial expected loss ratios (IELRs) were generated for origin years 7–10 as normally distributed with the deterministic values as means and standard deviations of 14.5%.

5.3.3.3 Variable BF Simulation

The first attempt at simulation was made with the credibility varying for each iteration, that is, the formula for ultimate for origin years 7 to 10 was:

$$(1-c).\chi P + c.E$$

with χ = initial expected loss ratio

P = Premium

E = Ultimate from Chain-Ladder

A = Actual incurred claims

c = A/E (with minimum of 0, maximum of 1)

This did not work as the formula (ignoring the restrictions on the credibility factor, c) reduces to:

IBNR = $\left(1 - A/E\right)\chi P$

with E and χ both derived stochastically and giving an unstable result.

5.3.3.4 Fixed BF Simulations

The same formula was used,

that is $\left(1 - c\right)\chi P + c.E$ but with c=A/E from the deterministic chain-ladder method, that is fixed for each iteration.

This is inherently a straight credibility formula between the two choices of ultimate, the initial expected estimate and the chain ladder estimate, with both estimates varying stochastically.

5.3.4 Results of Simulation (10,000 iterations)

RESULTS OF SIMULATION (10,000 ITERATIONS)
IBNR:

		Chain Ladder	IELR	Bornhuetter-Ferguson
Deterministic:		58,263	66,410	62,954
Stochastic:				
	mean	58,284	66,410	62,960
	Standard deviation	76,388	8,104	10,526
Percentiles:	5%	28,846	53,027	50,024
	10%	30,348	56,107	52,852
	50%	45,753	66,356	62,220
	90%	83,301	76,773	72,883
	95%	120,171	79,768	76,844
	Minimum	13,652	37,358	32,057
	Maximum	4,393,941	97,160	503,252
Differences from Deterministic:	5%	(31,416)	(13,383)	(12,930)
	10%	(27,914)	(10,302)	(10,102)
	90%	25,038	10,363	9,929
	95%	61,908	13,358	13,890

5.3.5 Observations

5.3.5.1 The practical approach via the BF method gives much lower ranges of results than the CL method. This is in line with Bayesian theory, that is the use of more information reduces the range.

5.3.5.2 The standard deviation could be misleading (for both the CL method and BF method) because the results are highly skewed by particular outliers; note the maximum value of 4,393,841 for the CL method which is a complete outlier. Consideration could be given to imposing some restrictions on ultimates that are considered to be impossible.

5.3.5.3 The mean values from the stochastic distributions are in all three cases very close to the deterministic values. This backs up the assumption that the value produced by the practical deterministic approach is the mean value, rather than the median (see Section 4.2.2.3), although the mean has been increased significantly by outliers.

5.3.5.4 Even though the standard deviation could be misleading, the ranges should be considered as useable, with, for example, the result being quoted as an IBNR of 62,954 with a probability of 5% that it would not exceed 76,844 and of 5% that it would not be below 50,024 (subject to standard caveats).

5.3.5.5 There is reasonably high negative correlation between RTR factors for years j to $j+1$ and the amount for year j, for j from 1 to 5 (varying from -0.504 for $j = 2$ to -0.776 for $j = 5$). This has not been allowed for in the stochastic approach used, apart from development year 1-2, and was not mentioned by Mack. It is possible that results would be different if this had been allowed for, but probably not significantly.

5.3.6

Practical Stochastic Reserving: Test

Development Triangle of Incurred

Origin	1	2	3	4	5	6	7	8	9	10
1	5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834
2	106	4,285	5,396	10,666	13,782	15,599	15,496	16,169	16,704	
3	3,410	8,992	13,873	16,141	18,735	22,214	22,863	23,466		
4	5,655	11,555	15,766	21,266	23,425	26,083	27,067			
5	1,092	9,565	15,836	22,169	25,955	26,180				
6	1,513	6,445	11,702	12,935	15,852					
7	557	4,020	10,946	12,314						
8	1,351	6,947	13,112							
9	3,133	5,395								
10	2,063									

RTR Factors

	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10
1	1.650	1.319	1.082	1.147	1.195	1.113	1.033	1.003	1.009
2	40.425	1.259	1.977	1.292	1.132	0.993	1.043	1.033	
3	2.637	1.543	1.163	1.161	1.186	1.029	1.026		
4	2.043	1.364	1.349	1.102	1.113	1.038			
5	8.759	1.656	1.400	1.171	1.009				
6	4.260	1.816	1.105	1.226					
7	7.217	2.723	1.125						
8	5.142	1.887							
9	1.722								

Averages	*1 - 2*	*2 - 3*	*3 - 4*	*4 - 5*	*5 - 6*	*6 - 7*	*7 - 8*	*8 - 9*	*9 - 10*	
WAD	2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009	
SAD	8.206	1.696	1.315	1.183	1.127	1.043	1.034	1.018	1.009	
WAD 5 yr	4.234	1.748	1.245	1.175	1.113	1.042	1.033	1.017	1.009	
SAD 5 yr	5.420	1.889	1.229	1.190	1.127	1.043	1.034	1.018	1.009	
WAD 3 yr	3.246	2.054	1.232	1.157	1.093	1.024	1.033	1.017	1.009	
SAD 3 yr	4.694	2.142	1.210	1.166	1.103	1.020	1.034	1.018	1.009	
SAD xHL	4.540	1.597	1.229	1.176	1.144	1.033	1.033			
Trend	9.525									
										Tail
Selected	**3.000**	**1.800**	**1.250**	**1.175**	**1.120**	**1.040**	**1.033**	**1.020**	**1.010**	**1.010**
Cumulative	9.930	3.310	1.839	1.471	1.252	1.118	1.075	1.041	1.020	1.010
% of Ultimate	10.1%	30.2%	54.4%	68.0%	79.9%	89.5%	93.0%	96.1%	98.0%	99.0%
Incremental	2.000	0.800	0.250	0.175	0.120	0.040	0.033	0.020	0.010	0.010

Projected Triangle

Origin	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Ult
1	5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834	19,022
2	106	4,285	5,396	10,666	13,782	15,599	15,496	16,169	16,704	16,871	17,040
3	3,410	8,992	13,873	16,141	18,735	22,214	22,863	23,466	23,935	24,175	24,416
4	5,655	11,555	15,766	21,266	23,425	26,083	27,067	27,960	28,519	28,805	29,093
5	1,092	9,565	15,836	22,169	25,955	26,180	27,227	28,126	28,688	28,975	29,265
6	1,513	6,445	11,702	12,935	15,852	17,754	18,464	19,074	19,455	19,650	19,846
7	557	4,020	10,946	12,314	14,469	16,205	16,853	17,410	17,758	17,935	18,115
8	1,351	6,947	13,112	16,390	19,258	21,569	22,432	23,172	23,636	23,872	24,111
9	3,133	5,395	9,711	12,139	14,263	15,975	16,614	17,162	17,505	17,680	17,857
10	2,063	6,189	11,140	13,925	16,362	18,326	19,059	19,688	20,081	20,282	20,485

<---------lognormal----------> <---------------------------------------normal-->

Chain Ladder Projection | **Premium Data**

Origin	Actual	Ult%	CL Ult	IBNR	Premium	CL LR%	IELR%
1	18,834	99.0%	19,022	188	23,257	81.8%	
2	16,704	98.0%	17,040	336	19,015	89.6%	
3	23,466	96.1%	24,416	950	22,421	108.9%	
4	27,067	93.0%	29,093	2,026	22,810	127.5%	
5	26,180	89.5%	29,265	3,085	20,327	144.0%	
6	15,852	79.9%	19,846	3,994	23,789	83.4%	
7	12,314	68.0%	18,115	5,801	24,526	73.9%	**95%**
8	13,112	54.4%	24,111	10,999	23,118	104.3%	**95%**
9	5,395	30.2%	17,857	12,462	24,423	73.1%	**85%**
10	2,063	10.1%	20,485	18,422	30,257	67.7%	**75%**
Total				58,263			

Ratios:	*1 - 2*	*2 - 3*	*3 - 4*	*4 - 5*	*5 - 6*	*6 - 7*	*7 - 8*	*8 - 9*	*9 - 10*
1. Based on Observed Values									
Standard Deviation	12.340	0.474	0.317	0.067	0.075	0.050	0.009	0.021	
Skewness	2.770	1.636	1.896	0.786	-1.139	1.105	0.565		
Kurtosis	7.947	3.130	3.781	0.482	1.227	2.075			
Standard Error	13.520	0.487	0.325	0.067	0.075	0.050	0.009	0.022	
S.E. to End	5.981	0.642	0.216	0.116	0.088	0.034	0.018	0.017	
Selected S.E.							**0.018**		
2. Based on Proportional Values									
S.D./Incr Sel	1.713	0.681	1.008	0.365	0.588	1.160	0.250	1.186	
Standard Error	6.760	0.609	1.299	0.385	0.626	1.260	0.265	1.077	
S.E. from Yr 1	6.760	4.797	4.055	3.615	3.346	3.184	3.066	2.999	2.964
S.E. from Yr 2		0.609	0.953	0.820	0.775	0.826	0.789	0.787	0.776
S.E. from Yr 3			1.299	0.951	0.855	0.905	0.850	0.843	0.828
Selected						**0.800**			
Value from Select	1.600	0.640	0.200	0.140	0.096	0.032	0.026	0.016	0.008

3. Based on Correlation for Year 1

	Straight Amount	Rate Adjusted LR	Selected S.E.
Correlation Year 1 to 2 Ratio v Yr 1 Amount or LR	-0.583	-0.593	
Standard deviation from regression line	10.721	10.624	**10.650**

	<-Correl->	<------------Pro-rata to Selected Incremental Ratios------------>						<---Constant---->		<--Tail-->
Selected S.E.	**10.650**	**0.640**	**0.200**	**0.140**	**0.096**	**0.032**	**0.026**	**0.018**	**0.018**	**0.031**

Rate Adjustment Calculations

			<--------Rate Adjusted-------->					
Origin	Rate Change	Rate Adjust Factor	Premium	CL Loss Ratio	1st Year Loss Ratio	Stochastic IELR	Stochastic IE Losses	Initial Expected IBNR
1		1.190	27,667	68.8%	18.1%		19,022	188
2	**7.50%**	1.107	21,042	81.0%	0.5%		17,040	336
3	**-10.00%**	1.230	27,569	88.6%	12.4%		24,416	950
4	**-10.00%**	1.366	31,163	93.4%	18.1%		29,093	2,026
5	**0.00%**	1.366	27,771	105.4%	3.9%		29,265	3,085
6	**20.00%**	1.139	27,084	73.3%	5.6%		19,846	3,994
7	**-10.00%**	1.265	31,025	58.4%	1.8%	95.0%	23,299	10,985
8	**0.00%**	1.265	29,245	82.4%	4.6%	95.0%	21,963	8,851
9	**15.00%**	1.100	26,865	66.5%	11.7%	85.0%	20,760	15,365
10	**10.00%**	1.000	30,257	67.7%	6.8%	75.0%	22,693	20,630
Total								66,410

Observations on Rate Adjusted CL Loss Ratios

	1 to 6	1 to 7	1 to 8	1 to 9	1 to 10
Correlation v Origin Year	39.74%	-14.37%	-10.07%	-26.49%	-35.62%
Standard deviation from trend-line	13.88%				
Mean	85.05%	81.24%	81.39%	79.73%	78.53%
Standard Deviation	13.53%	15.94%	14.77%	14.68%	14.35%
Skewness	36.18%	10.60%	6.83%	32.76%	53.62%
Selected Standard Error	**14.50%**				

Fixed Crediblity BF Simulation

Origin	Actual Claims	CL Ultimate	Initial Expected Ultimate	CL Credibility Fixed	BF Ultimate	BF IBNR
1	18,834	19,022	19,022	100.0%	19,022	188
2	16,704	17,040	17,040	100.0%	17,040	336
3	23,466	24,416	24,416	100.0%	24,416	950
4	27,067	29,093	29,093	100.0%	29,093	2,026
5	26,180	29,265	29,265	100.0%	29,265	3,085
6	15,852	19,846	19,846	100.0%	19,846	3,994
7	12,314	18,115	23,299	68.0%	19,775	7,461
8	13,112	24,111	21,963	54.4%	23,131	10,019
9	5,395	17,857	20,760	30.2%	19,883	14,488
10	2,063	20,485	22,693	10.1%	22,470	20,407
Total						62,954

Summary of Results for IBNR Claims

	IBNR
Chain Ladder	58,263
IELR	66,410
BF Fixed Credibility	62,954

5.4 Bayesian Model

One other practical approach suggested in the England and Verral paper is construction of a Bayesian model supplementing a Markov Chain Monte Carlo (MCMC) approach, possibly using WinBUGS (Note: BUGS is an acronym for Bayesian inference Using Gibbs Sampling) software. This approach is also considered in the paper by Scollnik, "Actuarial Modelling with MCMC and BUGS". The Bayesian statistical method treats all unknown parameters appearing in a statistical model as random variables and derives the distribution conditional upon known information. It may be argued that the Bayesian paradigm is the most natural and convenient one to adopt for the implementation and analysis of range models arising in actuarial science, insurance and risk management.

Until recently fully Bayesian analyses of these statistical models had been computationally infeasible. This changed in the early 1990's following the rediscovery in the statistical literature of computer-intensive MCMC simulation methods like the Metropolis-Hastings algorithm and the Gibbs sampler. Also, specialised software for implementing MCMC analyses is now available, the foremost being BUGS and WinBUGS software packages. WinBUGS is available free of charge via the internet from www.mrc-bsu.cam.ac.uk/bugs.

It is recommended that further work be done implementing WinBUGS in a practical reserving context and establishing if it could be of use for stochastic reserving. It may be that a more rigorous approach to Bayesian modelling may be more appropriate than the empirical methodology suggested in section 5.3.

6. Generalised Cape Cod Method

6.1 Introduction

Definitions

6.1.1. "Exposure base" is defined as a measure that is directly correlated with the quantity being estimated and is known or accurately estimated in advance. Premium (ideally after adjusting for rate changes and inflation) and vehicle-years are well known examples of exposure bases that are often used for reserving purposes.

6.1.2. A "leading indicator" is defined as a measure that is directly correlated with the quantity being estimated but is not known in advance. When performing projections for reserves, leading indicators may be used as exposure bases, although the ultimate value of the leading indicator also needs to be estimated. The reported number of claims is a well-known example of a leading indicator that is also often used as an exposure base for reserving purposes.

6.1.3. "A priori" here refers to any estimate of the amount being projected that is based on an exposure measure. Multiplying the earned premium by an expected loss ratio is an example of an exposure-based estimate.

6.1.4. For generality we will refer to the amount to be estimated as "losses" and the exposure base as "exposures".

Blending and Bornhuetter-Ferguson

6.1.5 Common actuarial procedures involve projections of the traditional loss development triangle in two directions:

- the development direction
- the trend direction.

6.1.6 The development direction refers to the emergence of information for a single year of origin, such as the development of cumulative paid claims in respect of a particular origin year. The trend direction refers to the expected changes in the ratio of a projected amount to an exposure base, such as an increasing trend in the claim frequency from one origin year to the next.

6.1.7 Stochastic reserving methods are increasingly used to reflect both the development and the trend directions simultaneously. However, given the still-low level of understanding of the complexities and the time and difficulty in implementing these methods, other methods that reflect development and trend directions in projections will still be in high demand.

6.1.8 Stanard found significantly higher prediction errors when using the loss development (chain ladder) method and concluded that this method is clearly inferior to the methods that give weight to expected losses. Murphy, Patrik and Mack all give reasons why a method which blends development and trending projections is preferred. The Bornhuetter-Ferguson method is the most commonly used approach for blending development and trend projections.

6.2 Traditional Cape Cod Method

Introduction

6.2.1 Both Stanard and Bühlmann described the Cape Cod method. Although Stanard originally presented the method assuming that the exposure is constant for each year of origin, Gluck allowed for varying levels of exposure.

The Method

6.2.2 Losses and/or exposures are adjusted for trend so that the adjusted loss ratios are expected to be equal for all years. The expected loss ratio calculated using the data from all available years is then used to calculate a priori expected losses in the Bornhuetter-Ferguson procedure. The Cape Cod method is, therefore, an application of the Bornhuetter-Ferguson method with the a priori estimates being determined from a specified, trend-based calculation.

6.2.3 The expected loss ratio for all origin years can be calculated as the weighted average of the trended developed ultimate loss ratio for each year of origin as follows:

where

$$\hat{E}(LR) = \frac{\sum_i \left(LTD_i \times TF_{ij} \times \frac{DF_i}{E_i} \right) \times \left(\frac{E_i}{DF_i} \right)}{\sum_i \frac{E_i}{DF_i}}$$

- $\hat{E}(LR)$ is the expected loss ratio
- LTD_i is the current evaluation of losses for each year of origin i
- TF_{ij} is the trend factor from year of origin i to year of origin j
- DF_i is the development factor to ultimate for each year of origin i
- E_i is a measurement of the relative exposure for each year of origin i .

6.2.4 The weights used in the calculation of the expected loss ratio are:

- proportional to exposure
- inversely proportional to the development to ultimate.

This means that larger weight is given to those years of origin with greater exposure and those years that are more mature.

6.2.5 Equation 6.2.3 can be simplified to the ratio of reported losses to the amount of exposure expected to relate to the reported losses for all years combined. This means that the Cape Cod method could be seen as applying this expected loss ratio to the amount of unreported exposure in order to estimate the ultimate level of losses.

6.2.6 The ultimate losses can then be estimated by applying the Bornhuetter-Ferguson method to blend the development projection with the expected losses as follows:

$$\hat{ULT}_i = \left(\frac{1}{DF_i}\right) \times LTD_i \times TF_{ij} \times DF_i + \left(1 - \frac{1}{DF_i}\right) \times E_i \times \hat{E}(LR)$$

where

- $\hat{E}(LR)$ is the expected loss ratio
- LTD_i is the current evaluation of losses for each year of origin i
- TF_{ij} is the trend factor from year of origin i to year of origin j
- DF_i is the development factor to ultimate for each year of origin i
- E_i is a measurement of the relative exposure for each year of origin i
- $\hat{ULT}_i$ is the estimated ultimate losses for year of origin i.

6.2.7 Gluck demonstrated that the Bornhuetter-Ferguson weights are optimal, that is they produce the minimum variance of the prediction error, subject to certain constraints.

6.2.8 Section 6.5 shows an example of the Cape Cod method.

6.3 Generalised Cape Cod method

6.3.1 Gluck developed a modification to the Cape Cod method that allows the a priori trended loss ratio to vary for each year of origin. More importantly, however, it takes into account the relationship between the variance and trending, which if not considered could cause excessive weight to be given to years that are out of date.

6.3.2 The variance related to trending is taken into account by introducing an exponential decay factor. Equation 6.2.3 then becomes:

$$\hat{E}(LR) = \frac{\sum_i \left(LTD_i \times TF_{ij} \times \frac{DF_i}{E_i} \right) \times \left(\frac{E_i}{DF_i} \right) xD^{|j-i|}}{\sum_i \frac{E_i}{DF_i} xD^{|j-i|}}$$

where

- $\hat{E}(LR)$ is the expected loss ratio for year of origin j
- LTD_i is the current evaluation of losses for year of origin I
- TF_{ij} is the trend factor from year of origin i to year of origin j
- DF_i is the development factor to ultimate for year of origin I
- E_i is a measurement of the relative exposure for year of origin I
- D is the exponential decay factor, assuming a value between 0 and 1, inclusive.

6.3.3 The expected loss ratio for a particular year of origin can now be seen to be a weighted average of the trended developed ultimate loss ratio for each year of origin where the weights are

- proportional to exposure
- inversely proportional to the development to ultimate
- inversely proportional to the length of the trending period.

6.3.4 The ultimate losses can then be estimated by applying the Bornhuetter-Ferguson method using equation 6.2.6. Section 6.6 shows an example of the Generalised Cape Cod method including decay.

6.3.5 The traditional loss development and Cape Cod methods can be viewed as special cases of the Generalised Cape Cod method since when D=0 the Generalised Cape Cod method returns the loss development method result and when D=1 it returns the traditional Cape Cod method result.

6.3.6 Among the constraints required for the Bornhuetter-Ferguson weights to be optimal is that for a given year of origin the variance of the development-based estimate of ultimate losses is proportional to the development factor. Although this assumption is often adequate, it is sometimes the cause of the Bornhuetter-Ferguson and Cape Cod methods being unusable or of limited effectiveness, for example where the development factors are less than 1 or approach 1 faster than the uncertainty surrounding the development projection is eliminated. Gluck demonstrated that alternative weights could be used that are inversely proportional to the development-based and the a priori projections to overcome this problem.

6.3.7 Although determination of the alternative variance factors might be based on the actual data triangles, it usually requires sufficiently detailed data making it more practical to use a reference pattern. Gluck suggested the possibility of using the paid claims development pattern when projecting incurred claims.

6.3.8 Including an alternative variance pattern, equation 6.3.2 becomes:

$$\hat{E}(LR) = \frac{\sum_i \left(LTD_i \times TF_{ij} \times \frac{DF_i}{E_i} \right) \times \left(\frac{E_i}{VF_i} \right) x D^{|j-i|}}{\sum_i \frac{E_i}{VF_i} x D^{|j-i|}}$$

where

- $\hat{E}(LR)$ is the expected loss ratio for year of origin j
- LTD_i is the current evaluation of losses for year of origin i
- TF_{ij} is the trend factor from year of origin i to year of origin j
- DF_i is the development factor to ultimate for year of origin i
- VF_i is the variance factor for year of origin i
- E_i is a measurement of the relative exposure for year of origin i
- D is the exponential decay factor, assuming a value between 0 and 1, inclusive.

6.3.9 The ultimate estimated losses can be determined by applying the Bornhuetter-Ferguson equation to blend the development-based projection and the expected loss ratio as follows:

$$U\hat{L}T_i = \left(\frac{1}{VF_i}\right) \times LTD_i \times TF_{ij} \times DF_i + \left(1 - \frac{1}{VF_i}\right) \times E_i \times \hat{E}(LR)$$

where

- $\hat{E}(LR)$ is the expected loss ratio for year of origin j
- LTD_i is the current evaluation of losses for year of origin i
- TF_{ij} is the trend factor from year of origin i to year of origin j
- DF_i is the development factor to ultimate for year of origin i
- VF_i is the variance factor for year of origin i
- E_i is a measurement of the relative exposure for year of origin i
- $U\hat{L}T_i$ is the estimated ultimate losses for year of origin i.

6.3.10 An example showing the use of alternative variances with the Generalised Cape Cod method is shown in section 6.7.

6.4 Practical Considerations

6.4.1 As with many other methods, there are some situations when it is prudent not to use the Generalised Cape Cod method, for example high excess layers with no claims reported.

6.4.2 Struzzieri and Hussian suggested a number of reasons, however, why the Generalised Cape Cod method could be the preferred method to estimate ultimate losses:

- information from all origin years is used when estimating the a priori loss ratio for a particular year
- more weight is given to surrounding years, which is preferable since:
  - insurance is subject to underwriting cycles
  - pricing and underwriting changes are usually implemented gradually
  - imprecision of using one year to make projections
- less weight is given to immature years
- less weight is given to low-volume years
- the a priori loss ratio estimate is optimal under certain constraints (as determined by Gluck)
- internal or external changes can be systematically reflected through exposure base adjustments
- generalised case of loss development and traditional Cape Cod methods.

The inclusion of the exponential decay factor is particularly useful since it allows the actuary to vary the weight given to the loss development projections in the blending algorithm. In many situations the Bornhuetter-Ferguson method gives excessive weight to the loss development projections, which can be overcome with the use of the Generalised Cape Cod method.

6.5 *Example of traditional Cape Cod method*

The following data are available for origin years 1992 to 2001, inclusive:

Cumulative incurred claim amounts

	1	2	3	4	5	6	7	8	9	10
1992	1,792,798	1,909,210	1,922,547	1,928,577	1,902,838	1,883,939	1,887,271	1,880,739	1,882,227	1,887,796
1993	1,871,855	1,985,941	1,990,172	1,974,750	1,967,028	1,958,514	1,942,655	1,943,831	1,944,657	
1994	2,001,820	2,107,168	2,104,298	2,125,908	2,115,309	2,090,456	2,080,043	2,077,725		
1995	2,138,686	2,290,017	2,305,967	2,294,540	2,274,294	2,264,598	2,259,433			
1996	2,381,205	2,575,848	2,579,848	2,595,063	2,563,442	2,555,219				
1997	2,530,821	2,733,345	2,738,086	2,745,514	2,753,200					
1998	2,756,916	3,020,781	3,047,314	3,037,936						
1999	3,046,380	3,244,523	3,240,900							
2000	3,306,631	3,562,098								
2001	3,455,161									

Cumulative paid claim amounts

	1	2	3	4	5	6	7	8	9	10
1992	1,041,391	1,432,010	1,541,077	1,638,158	1,715,111	1,771,338	1,810,952	1,939,088	1,855,220	1,867,005
1993	1,043,712	1,439,455	1,556,512	1,665,090	1,750,317	1,815,040	1,858,740	1,893,833	1,912,635	
1994	1,073,184	1,494,802	1,641,730	1,775,523	1,881,630	1,947,711	1,997,867	2,028,776		
1995	1,143.520	1,633,759	1,789,949	1,927,919	2,037,936	2,117,859	2,172,802			
1996	1,297,185	1,848,145	2,027,654	2,200,467	2,340,330	2,420,412				
1997	1,410,213	1,993,082	2,198,251	2,383,230	2,533,690					
1998	1,576,499	2,243,986	2,468,201	2,656,864						
1999	1,90,849	2,379,622	2,634,131							
2000	1,771,627	2,556,548								
2001	1,755,208									

Example of Generalised Cape Cod method including decay

Development factors based on last 4 years data

	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9	tail
Incurred	1.077	1.004	1.000	0.994	0.994	0.997	0.999	1.001	1.003	1.020
Paid	1.424	1.102	1.081	1.061	1.036	1.025	1.016	1.010	1.006	1.050

Cape Cod method of estimating the ultimate losses (Bornhuetter-Ferguson method)

	Exposure (1)	Paid LTD (2)	Incd LTD (3)	Trend Factor (4)	Trended Paid (5)	Paid CDF (6)	E/DF (7)	LTD*DF/E (9)	E(LR) (10)	E(ULT) (11)	ULT (12)
1992	10,038	1,867,005	1,887,796	1.5513	2,896,337	1.0500	9,560	303	180	1,809,053	1,953,150
1993	20,709	1,912,635	1,944,657	1.4775	1,825,833	1.0567	10,135	279	189	2,026,627	2,021,324
1994	11,315	2,028,776	2,077,725	1.4071	2,854,691	1.0668	10,606	269	199	2,248,270	2,169,643
1995	11,899	2,172,802	2,259,433	1.3401	2,911,763	1.0844	10,973	265	209	2,482,485	2,365,966
1996	12,616	2,420,412	2,555,219	1.2763	3,089,127	1.1111	11,355	272	219	2,763,762	2,696,711
1997	12,497	2,533,690	2,753,200	1.2155	3,079,716	1.1514	10,854	284	230	2,874,591	2,911,707
1998	12,704	2,656,864	3,037,936	1.1576	3,075,652	1.2218	10,398	296	242	3,068,273	3,213,822
1999	13,186	2,634,131	3,240,900	1.1025	2,904,129	1.3203	9,987	291	254	3,344,068	3,445,473
2000	14,097	2,566,548	3,562,098	1.0500	2,694,875	1.4550	9,689	278	266	3,753,748	3,740,429
2001	14,302	1,755,208	3,455,161	1.0000	1,755,208	2.0719	6,903	254	280	3,998,625	3,823,857
					28,087,332		100,457				28,342,082

Notes
(4) Trend Factor to 2001 calculated at 5% pa which can also reflect other adjustments to losses
(5) Trended paid claims = (2) x (4)
(7) Reported exposure = (1) / (6)
(8) Expected ultimate loss ratio = (total col. 5) / (total col. 7)
(9) Reported loss ratio = (5) / (7)
(10) Detrended expected ultimate loss ratio = (8) / (4)
(11) Expected ultimate losses = (10) x (1)
(12) Estimated ultimate losses = (2) + [1 – 1/(6)] x (11)

6.6 Example of Generalised Cape Cod method including decay

	Exposure (1)	Paid LTD (2)	Incd LTD (3)	Trend factor (4)	Trended Paid (5)	Paid CDF (6)	E/DF (7)	LTD*DF /E (8)	Decay (9)	*D/DF (10)	Iterated E(LR (12)	E(LR) (13)	E(ULT) (14)	ULT (15)
1992	10,038	1,867,005	1,887,796	1.5513	2,896,337	1.050	9,560	303	0.075	718	282	182	1,826,417	1,953,977
1993	10,709	1,912,635	1,944,657	1.4775	2,825,933	1.057	10,135	279	0.100	1,015	280	189	2,027,844	2,021,389
1994	11,315	2,208,776	2,077,725	1.4071	2,854,691	1.067	10,606	269	0.133	1,416	278	198	2,235,212	2,168,825
1995	11,899	2,172,802	2,259,533	1.3401	2,911,763	1.084	10,973	265	0.178	1,953	277	207	2,463,538	2,364,492
1996	12,616	2,420,412	2,555,219	1.2763	3,089,127	1.111	11,355	272	0.237	2,695	278	218	2,751,775	2,695,513
1997	12,497	2,533,690	2,753,200	1.2155	3,079,716	1.151	10,854	284	0.316	3,434	280	230	2,879,259	2,912,321
1998	12,704	2,656,864	3,037,936	1.1576	3,075,652	1.222	10,398	296	0.422	4,387	281	243	3,087,678	3,217,345
1999	13,186	2,634,131	3,240,900	1.1025	2,904,129	1.320	9,987	291	0.563	5,618	281	255	3,361,540	3,449,712
2000	14,096	2,556,548	3,562,098	1.0500	2,694,875	1.455	9,689	278	0.750	7,266	279	266	3,750,672	3,739,467
2001	14,302	1,755,208	3,455,161	1.0000	1,755,208	2.072	6,903	254	1.000	6,903	277	277	3,964,741	3,806,327
								2,791		35,403				28,329,368

Notes

(4) Trend Factor to 2001 calculated at 5% pa which can also reflect other adjustments to losses
(5) Trended paid claims = (2) x (4)
(7) Reported exposure = (1) / (6)
(8) Reported loss ratio = (5) / (7)
(9) Decay factor = 0.75 abs (2001 – I)

The table shows the decay factor for the 2001 origin year, but will vary depending on which origin year is being considered. An iterative process is followed whereby the decay factor is calculated for each origin year and the results of (11) are recorded in column (12)

(10) Weight assigned to indicated ultimate loss ratio = (7) x (9)
(11) Expected loss ratio for origin year under consideration, in this case 2001, = [total of (8) x (10)] / total col. 10)
(12) Iterated expected loss ratio = results of (11) for each origin year
(13) Detrended expected ultimate loss ratio = (12) / (4)
(14) Expected ultimate losses = (13) x (1)
(15) Estimated ultimate losses = (2) + [1 – 1/(6)] x (14)

6.7 Example of Generalised Cape Cod method including alternative variance factors

	Exposure (1)	Paid LTD (2)	Incd LTD (3)	Trend factor (4)	Trended Incurred (5)	Incurred CDF (6)	E/DF (7)	LTD*DF/E (8)	Decay (9)	VF (10)	E*D/VF (11)	Iterated E(LR) (13)	E(LR) (14)	E(ULT) (15)	ULT (16)
1992	10,038	1,867,005	1,887,796	1.5513	2,928,592	1.0200	9,841	298	1.0000	1.0500	9,559.544	276	178	1,782,867	1,918,758
1993	10,709	1,912,635	1,944,657	1.4775	2,873,144	1.0230	10,468	274	0.7500	1.0567	7,601.188	273	185	1,977,864	1,988,799
1994	11,315	2,028,776	2,077,725	1.4071	2,923,568	1.0236	11,053	264	0.5625	1.0668	5,965.770	271	192	2,176,410	2,129,942
1995	11,899	2,172,802	2,259,433	1.3401	3,027,857	1.0223	11,639	260	0.4219	1.0844	4,629.112	269	201	2,392,810	2,316,292
1996	12,616	2,420,412	2,555,219	1.2763	3,261,179	1.0188	12,383	263	0.3164	1.1111	3,592.689	270	211	2,663,995	2,609,335
1997	12,497	2,533,690	2,753,200	1.2155	3,346,532	1.0129	12,337	271	0.2373	1.1514	2,575.606	270	222	2,778,477	2,787,477
1998	12,704	2,656,864	3,037,936	1.1576	3,516,791	1.0073	12,612	279	0.1780	1.2218	1,850.580	271	234	2,972,972	3,044,196
1999	13,186	2,634.131	3,240,900	1.1025	3,573,092	1.0074	13,089	273	0.1335	1.3203	1,333.113	271	246	3,237,403	3,258,299
2000	14,097	2,566,548	3,562,098	1.0500	3,740,203	1.0119	13,931	268	0.1001	1.4550	969.947	270	257	3,625,991	3,611,404
2001	14,302	1,755,208	3,455,161	1.0000	3,455,161	1.0902	13,119	263	0.0751	2.0719	518.292	270	270	3,854,362	3,812,066
								2,715			38,595.842				27476,467

Notes

(4) Trend Factor to 2001 calculated at 5% pa which can also reflect other adjustments to losses

(5) Trended incurred claims = (2) x (4)

(7) Reported exposure = (1) / (6)

(8) Reported loss ratio = (5) / (7)

(9) Decay factor = 0.75 abs (1992 – *i*)
The table shows the decay factor for the 1992 origin year, but will vary depending on which origin is being considered. An iterative process is followed whereby the decay factor is calculated for each origin year and the results of (11) are recorded in column 12.

(10) Alternative variance factor, in this case the paid claim development factors are used

(11) Weight assigned to indicated ultimate loss ratio = (7) x (9)

(12) Expected loss ratio for origin year under consideration, in this case 1992, = [total of (8) x (10)] (total col. 10)

(13) Iterated expected loss ratio = results of (12) for each origin year

(14) Detrended expected ultimate loss ratio = (13) / (4)

(15) Expected ultimate losses = (14) x (1)

(16) Estimated ultimate losses = (2) +[1 – 1/(10)] x (15)

7. Standard Linear Actuarial Reserving Methods

7.1 Introduction

In this section, the models we discuss are based on a triangle of cumulative data $C_{i,j}$ where i represents the origin period and j is the development period since the start of the origin. Our aim is to produce a model capable of estimating each of the $C_{i,j}$s where j>1. In order to estimate the parameters in our model we have, at time t, a triangle of data $C_{i,j}$ for all $i + j \leq t+1$.

We illustrate the ideas here using the following incurred loss data triangle.

	Devt 1	Devt 2	Devt 3	Devt 4	Devt 5	Devt 6	Devt 7	Devt 8	Devt 9
1993	24	378,203	1,138,355	1,657,239	1,742,723	2,005,301	2,130,884	2,205,691	2,234,437
1994	3,901	279,487	1,263,809	1,950,588	2,106,054	2,376,390	2,513,937	2,601,627	
1995	38,863	316,975	1,498,175	2,293,172	2,601,832	2,920,168	2,992,439		
1996	28,848	800,798	1,778,722	2,623,833	2,981,742	3,354,018			
1997	37,227	645,216	1,774,319	2,487,082	3,291,414				
1998	29,421	831,025	1,931,249	3,159,232					
1999	2,451	460,993	2,104,527						
2000	32,807	679,917							
2001	40,381								

These data are taken from genuine Lloyd's US casualty data grouped by underwriting year and in a disguised form. Using our notation we have, for example, $C_{1993,2}$ = 378,203.

To construct our model, we first assume that $C_{i,j+1}$ is a function of $C_{i,j}$.

The second assumption is that this relationship is linear and so most common actuarial models are of the form:

$C_{i,j+1} = a_{i,j} + C_{i,j}\, R_{i,j} + \varepsilon_{i,j}$,

where $a_{i,j}$ and $R_{i,j}$ are "intercept" and "slope" parameters and $\varepsilon_{i,j}$ is a random error term.

The models examined here all have the simplifying assumptions that $a_{i,j}$ and $R_{i,j}$ are independent of i, that is

$a_{i,j} = a_j$ and $R_{i,j} = R_j$ for all i.

This assumption should not be blindly accepted but it is usually sufficient to proceed on the basis that this assumption holds and, at the end of the analysis, to form a judgement as to whether or not this assumption holds. We will discuss methods for making this assessment in section 7.3.

An alternative, and entirely analogous, notation is preferred by some practitioners. This is based on incremental developments which we shall denote using lower case letters.

The incremental developments are defined as

$$c_{i,j+1} = C_{i,j+1} - C_{i,j},$$

and the associated incremental slope parameters are

$$r_{i,j} = R_{i,j+1} - 1$$

$$= c_{i,j} / C_{i,j}.$$

7.2 Slope only Models (aka Chain Ladder or Link Ratio Models)

7.2.1 Theory

The models in this section are of the form

$$C_{i,j+1} = C_{i,j} R_j + \varepsilon_{i,j}.$$

In full, these models can all be described as multiple regression models and solved using standard mathematical techniques. This has the advantage that the variance of the parameters, r_j, can be found and the significance of the results tested.

In practice, it is often informative for the actuary to follow a more pragmatic route. In the next sub-section, we describe the steps that are commonly followed and, by following this procedure, further insight into the analysis can be gained. This insight is considered useful when applying judgement to the final selection. There is of course no reason why the pragmatic model described below can not be followed by a more rigorous regression based analysis.

It is worth noting that the resulting R_j may be referred to as development factors; link ratios; cumulative link ratios (as opposed to the incremental r_j); age-to-age factors; report-to-report factors or by a variety of similar terms. In this section, we will use the expression "age-to-age" factors.

7.2.2 Practice

7.2.2.1 We can apply the multiplier-only methods to the derivation of a "pattern". The resulting "pattern" of age-to-age factors is used as the basis for several reserving methods such as the Chain Ladder, Bornhuetter-Ferguson and (Generalised) Cape Cod methods.

In the models described here, our aim is to calculate estimators for each of the R_j based on the triangular data. Having done this, the simplest reserve estimation method, known as the Chain Ladder method, is to multiply the latest developed amount for each origin by each R_j that represents a future development. Some further notation is useful. Firstly, the "age-to-ultimate" factor is defined as:

$$S_j = \prod_{\text{all } k \geq j} R_k$$

and, as the name suggests, is the expected ratio of the developed amount at ultimate to the developed amount at time j. Many practitioners refer to S_j as the cumulative link ratio when the cumulative (R_j) or incremental nature (r_j) of the model is not in question. The reciprocal of this,

$$D_j = \frac{1}{S_j}$$

is then the proportion of the ultimate amount that is expected at time j. For example, the fitted model may say that incurred losses are expected to be 10% of ultimate after 1 year, 20% after 2 years and so on. A complete set of these development percentages or the underlying (cumulative) link ratios is generally referred to as a "pattern".

As a first step, we calculate the set of observed cumulative link ratios $C_{i,j+1} / C_{i,j}$. Using our example data, we get

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
1993	16,080.04	3.010	1.456	1.052	1.151	1.063	1.035	1.013
1994	71.647	4.522	1.543	1.080	1.128	1.058	1.035	
1995	8.156	4.726	1.531	1.135	1.122	1.025		
1996	27.759	2.221	1.475	1.136	1.125			
1997	17.332	2.750	1.402	1.323				
1998	28.246	2.324	1.636					
1999	188.064	4.565						
2000	20.725							

It is often useful to examine any patterns arising here. For example we would wish to see whether, for each development period, the ratios are increasing or decreasing over time as this may suggest a speeding up or slowing down of the claims process (this and other effects that should be investigated are discussed in Section 2). If this were the case then our assumption that $R_{i,j} = R_j$ for all i cannot hold true and an alternative model should be used. One such two-dimensional model is described by Sherman. If we believe that the simple models are appropriate, or if we wish to fit a simple model and then assess its ability to describe the data, then we next need to assign values to each R_j. There are several methods for doing this. Some of the more common, simple methods are to look at the simple average of the development ratios (SAD) shown above, the (loss) weighted average (WAD) or the geometric average (GAD).

Murphy proposes that each of the different averaging methods can be considered "Best Linear Unbiased Estimators" of the model under different assumptions for the error term $\varepsilon_{i,j}$. It is unlikely that all data triangles are best described by any one assumption over the error term, but the most commonly used estimators are the weighted averages. From a practical viewpoint, WAD estimates are preferred as they give most weight to the origin years that have the most data and as such are least likely to be heavily distorted by random fluctuations (although the cells with the most data are most likely to be those distorted by catastrophes or large losses). This is consistent with the generally accepted wisdom that weighted averages are often the most appropriate method for estimating parameters to be used when predicting future events.

A common extension to these simple average methods is to look at averages (WAD, SAD or GAD) for the last *n* years only in order to better reflect recent experience. Common sense tells us that this should be done if there are reasons to think that development factors will have changed over time but that in the absence of such changes we should use the full triangle.

Some calculated averages for our example data are shown below.

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
WAD	25.312	3.095	1.510	1.155	1.130	1.046	1.035	1.013
SAD	2,055.25	3.446	1.507	1.145	1.132	1.048	1.035	1.013
GAD	42.953	2.525	1.313	1.076	1.056	1.016	1.008	1.001
WAD last 8	25.312	3.095	1.510	1.155	1.130	1.046	1.035	1.013
WAD last 6	22.020	3.104	1.510	1.155	1.130	1.046	1.035	1.013
WAD last 4	25.682	2.772	1.513	1.174	1.130	1.046	1.035	1.013
WAD last 2	32.359	3.124	1.524	1.227	1.124	1.040	1.035	1.013

The WAD averages for the last 2, 4, 6 and 8 periods are not materially different to the full WAD figures. This is especially true when we bear in mind that for the restricted periods we have far less data and so the estimators are more prone to being distorted by randomness within the data. Therefore we select the WAD figures as our preliminary estimates.

Simple models would stop at this stage. We have a set of age-to-age factors for each of developments 1 through 8 and as such could construct a pattern and go on to make reserve estimate calculations. However, there are a number of potential problems here that can be resolved by extending the analysis through curve fitting.

Firstly, the averaging processes used so far may not have sufficiently removed the randomness from the data. It is sometimes desirable to graduate the selected age-to-age factors to ensure that we do not project future randomness. This can be especially true for classes with few claims and a volatile development history. The smoothing process can also be useful for the latest developments in a triangle which have little or no reliable data and the age-to-age factors calculated above, probably based on only one or two data items, are clearly questionable.

The second important case when curve fitting is desirable is when the data does not appear to be fully run off within the triangle (the earliest origin period has not finished developing) as in our example. In these cases we can use a curve fitted to our model in order to calculate age-to-age factors beyond the most developed point.

Our example here uses simple techniques that can be easily programmed into a spreadsheet package. The curves that we consider are as follows:

- Exponential Curve – the incremental age-to-age factors are modelled by $r_j = \exp(a + b\,t)$. Taking natural logs, gives $\ln r_j = a + b\,t$. Therefore, we can estimate a and b by regressing $\ln r_j$ against t.

- Weibull Curve – the cumulative age-to-age factors are modelled by $R_j = 1/(1 - \exp(-a\,t^b))$. This can be manipulated into a form suitable for regression as follows:

  $$1 - 1/R_j = \exp(-a\,t^b)$$

  $$\ln (1 - 1/R_j) = -a\,t^b$$

  $$\ln(-\ln (1 - 1/R_j)) = \ln a + \mathrm{b} \ln t.$$

  Therefore, we can estimate $\ln a$ and b by regressing $\ln(-\ln (1 - 1/R_j))$ against $(\ln t)$.

- Power Curve: the cumulative age-to-age factors are modelled by $R_j = a^{(b\text{^}t)}$. where b^t means b^{t}. Taking natural logs twice, gives

  $\ln (\ln R_j) = \ln (\ln a) + (\ln b)\,t$. Therefore, we can estimate $\ln (\ln a)$ and $\ln b$ by regressing $\ln (\ln R_j)$ against t.

- Sherman Curve: also known as the Inverse Power curve: the incremental age-to-age factors are modelled by $r_j = a\,(t + c)^b$. Since this is a three-parameter curve, we cannot use linear regression to fit the parameters. However, for a given value of c, we can regress ln r_j against ln $(t + c)$ to estimate ln a and b. Using such a routine, we then need to find the value of c that minimises the standard error in order to fully fit the curve.

Note that in Excel "Solver" can be used to derive the parameters, by minimising weighted sums of square differences between actual and fitted values. This can be done with "actual" values being the selected averages or using the original link ratios. Lower weights would probably be given to earlier development periods which have much larger development factors.

The functions described above usually fit the loss development factors well over part of the development history but often not the entire history. Of the functions we describe, the inverse power curve will generally be seen to fit over a wider range simply because it has three parameters instead of two. Typically, the two-parameter curves are less able to describe the large development factors commonly seen during the first few development periods. A practical solution to this problem is to use the average-based age-to-age factors, as calculated above, for the early development periods and fit the curves only to the factors that can reasonably be described by a formula.

For our example data, we have:

7.2.2.2 Selection of initial age-to-age factors:

This is based on simple averaging techniques or after applying some judgement. We mentioned above that the WAD averages seem appropriate.

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
Initial selection	25.312	3.095	1.510	1.155	1.130	1.046	1.035	1.013

7.2.2.3 Fitting each of the curve formulae to the selected factors:

The results for each curve are shown below.

Exponential Curve

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
X*= *t	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
***Y*= ln r_t**	3.191	0.739	-0.673	-1.861	-2.042	-3.081	-3.353	-4.340
Fitted	1.930	0.971	0.011	-0.948	-1.907	-2.867	-3.826	-4.785

a	17.9820
b	-0.9593

Weibull Curve

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
X=ln t	0.000	0.693	1.099	1.386	1.609	1.792	1.946	2.079
Y=ln(-ln(1-1/rt))	-3.211	-0.941	0.082	0.696	0.772	1.140	1.220	1.471
Fitted	-2.714	-1.214	-0.337	0.285	0.768	1.162	1.495	1.784

a	17.3706
b	-0.1054

Power Curve

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
X*= *t	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
***Y*= ln(ln R_t)**	1.173	0.122	-0.886	-1.935	-2.104	-3.103	-3.370	-4.347
Fitted	5.252	5.725	6.198	6.671	7.143	7.616	8.089	8.562

a	119.0244
b	0.4728

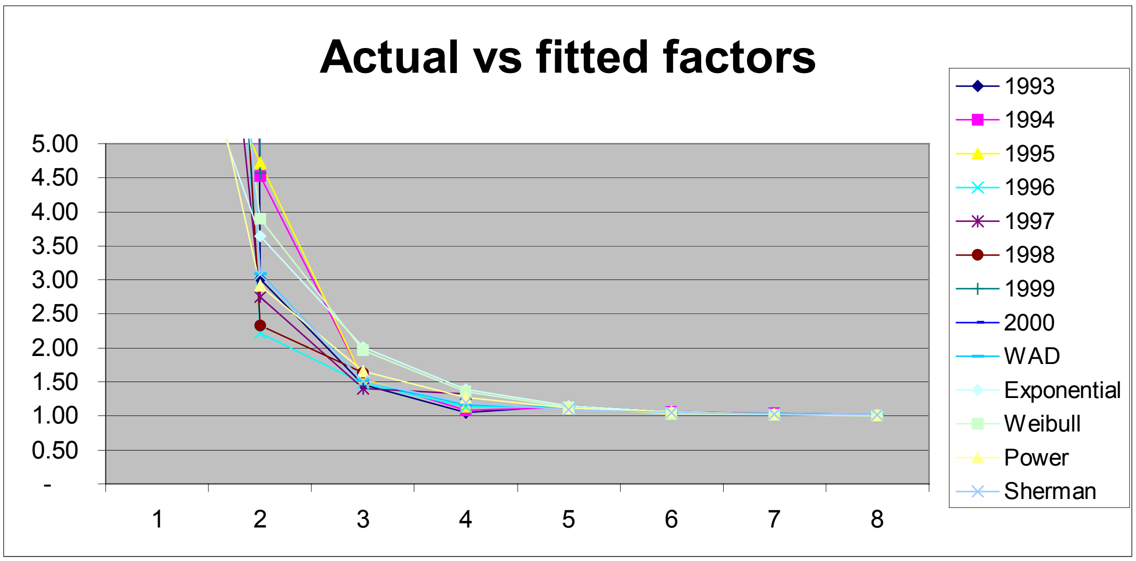
Sherman curve

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8
***X*=ln (*t*+*c*)**	-0.111	0.639	1.063	1.360	1.588	1.774	1.931	2.066
***Y*=ln(*t*)**	3.191	0.739	-0.673	-1.861	-2.042	-3.081	-3.353	-4.340
Fitted	3.225	0.731	-0.677	-1.663	-2.423	-3.041	-3.561	-4.011

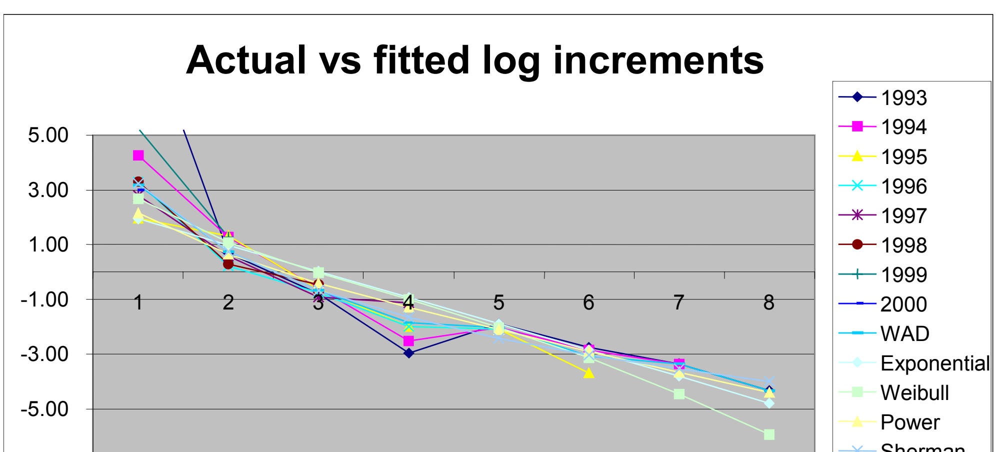
a	17.3706
B	-3.3231
C	-0.1054

7.2.2.4 Assessing the goodness of fit:

We have now produced several sets of estimators and we must now assess how well they describe the data. There are several methods of doing this before we calculate reserve estimates. Firstly, we could simply plot the historic age-to-age factors against the estimators, as shown below.



From this plot, we can see that the fitted averages look reasonable but the scales involved prevent us from using this plot to decide which curve is best. To get around this problem, we plot a similar graph but of the log-incremental factors ($\ln r_j$) against development period, as below.



This plot has the added advantage that the Exponential Curve appears as a straight line, the other curves being slightly curved. From this plot we can see that most curves would over-estimate the development from four to five years (R_4), the Sherman appearing to be better here. At the extreme right, we see that the Sherman and Power curves fit closest to the data but the Exponential and Weibull may underestimate the age-to-age factors.

A more sophisticated method that that gives far more information on the goodness of fit is to examine the residuals. Residuals are taken to be the difference between actual and expected values but the diagnostics are improved by looking at standardising the residuals. This is achieved by dividing each residual by an estimate of the standard error. There are several alternatives for the estimate of standard error, σ, as discussed by Venter but of these we opt for the most common:

$$s^2 = \sum_{\text{all data}} \frac{(\text{actual} - \text{expected})^2}{\text{data points} - \text{parameters}}$$

The standardised residuals for the Sherman curve fitted above are shown below by origin year, development period, calendar year (this being origin year plus development period).

Year	Devt	Calendar	Previous	Fitted ratio	Expected	Actual	Error squared	Residual
1993	1	1994	24	26.15	592	378,179	1.4257E+11	1.22
1993	2	1995	378,203	3.08	785,797	760,153	6.5760E+08	-0.08
1993	3	1996	1,138,355	1.51	578,322	518,883	3.5330E+09	-0.19
1993	4	1997	1,657,239	1.19	314,058	85,485	5.2246E+10	-0.74
1993	5	1998	1,742,723	1.09	154,532	262,578	1.1674E+10	0.35
1993	6	1999	2,005,301	1.05	95,867	125,582	8.8302E+08	0.10
1993	7	2000	2,130,884	1.03	60,519	74,807	2.0416E+08	0.05
1993	8	2001	2,205,691	1.02	39,940	28,745	1.2532E+08	-0.04
1994	1	1995	3,901	26.15	98,113	275,587	3.1497E+10	0.57
1994	2	1996	279,487	3.08	580,695	984,322	1.6291E+11	1.30
1994	3	1997	1,263,809	1.51	642,057	686,778	2.0000E+09	0.14
1994	4	1998	1,950,588	1.19	369,650	155,467	4.5874E+10	-0.69
1994	5	1999	2,106,054	1.09	186,749	270,336	6.9867E+09	0.27
1994	6	2000	2,376,390	1.05	113,607	137,547	5.7313E+08	0.08
1994	7	2001	2,513,937	1.03	71,398	87,689	2.6540E+08	0.05
1995	1	1996	38,863	26.15	977,449	278,112	4.8907E+11	-2.25
1995	2	1997	316,975	3.08	658,583	1,181,200	2.7313E+11	1.68
1995	3	1998	1,498,175	1.51	761,122	794,998	1.1475E+09	0.11
1995	4	1999	2,293,172	1.19	434,572	308,659	1.5854E+10	-0.41
1995	5	2000	2,601,832	1.09	230,711	318,336	7.6782E+09	0.28
1995	6	2001	2,920,168	1.05	139,604	72,271	4.5337E+09	-0.22
1996	1	1997	28,848	26.15	725,564	771,950	2.1516E+09	0.15
1996	2	1998	800,798	3.08	1,663,829	977,923	4.7047E+11	-2.21
1996	3	1999	1,778,722	1.51	903,650	845,111	3.4267E+09	-0.19

1996	4	2000	2,623,833	1.19	497,234	357,909	1.9412E+10	-	0.45
1996	5	2001	2,981,742	1.09	264,399	372,275	1.1637E+10		0.35
1997	1	1998	37,227	26.15	936,300	607,989	1.0779E+11	-	1.06
1997	2	1999	645,216	3.08	1,340,573	1,129,103	4.4720E+10	-	0.68
1997	3	2000	1,774,319	1.51	901,413	712,763	3.5589E+10	-	0.61
1997	4	2001	2,487,082	1.19	471,319	804,332	1.1090E+11		1.07
1998	1	1999	29,421	26.15	739,967	801,604	3.7991E+09		0.20
1998	2	2000	831,025	3.08	1,726,632	1,100,224	3.9239E+11	-	2.02
1998	3	2001	1,931,249	1.51	981,139	1,227,983	6.0932E+10		0.80
1999	1	2000	2,451	26.15	61,652	458,542	1.5752E+11		1.28
1999	2	2001	460,993	3.08	957,812	1,643,534	4.7021E+11		2.21
2000	1	2001	32,807	26.15	825,131	647,110	3.1692E+10	-	0.57

The sum of the squared error terms is 3,176,053,663,209 which, when divided by the number of data items less the number of fitted parameters (36-3) and then square rooted gives an error value of 310,232. The standardised residuals in the final column of this table have been calculated as the actual value (column 7) less the expected value (column 6), all divided by the error.

The error term is useful in assessing the goodness of fit without having to make judgements regarding the graphical output: it should be minimised. There are two other useful statistics than can also be used. Based on the assumption that the errors are randomly distributed, the proportion of standardised residuals that are positive can be calculated and should be roughly 50%. Secondly, by further assuming that the errors are roughly normally distributed with mean zero and variance one, the proportion of standardised errors lying outside the range (-2,2) should be around 5%. The process can easily be repeated for each of the curves. The results for our example are summarised below.

	Expon	Power	Weibull	Sherman
Error	384,877	332,880	530,313	310,232
+ve residuals	69%	58%	44%	56%
Exceptional	2.8%	2.8%	5.6%	11.1%

From these statistics, the Weibull curve looks to be the poorest fit based on the error value. The Exponential also appears poor owing to a high error value and a high positive residual count. These facts agree with our judgmental assessment of the log incremental plot.

Of the other two curves, it is again harder to pick a clear winner. The Sherman curve has the lower error, is closer to a 50-50 split of positive residuals but performs badly on the exceptional residual count. The Power curve is worse than the Sherman on the first two statistics, but not massively so, but is far superior based on the third statistic. For the third statistic, we note that based on the assumption that the standardised residuals have a Binomial(36,0.5) distribution then a 90% confidence interval level would be the range 13-22 (i.e. 36%-61%); therefore the Sherman's 11.1% is significant and we need to further investigate this problem with the Sherman curve.

This may seem surprising since the Sherman curve was by far the best looking fit to the log increments, had the lowest standard error and a good positive/negative residual split. This demonstrates that simply looking at plots of fitted factors against actual or even the superior log incremental plots is not enough to assess goodness of fit- many problems can go unnoticed.

The choice of fitted curve is an important one as can be seen when we look at the implied tail factors from each curve. Since we believe that the data are not fully run off within the triangle, we must extrapolate a tail factor. This is simply done using the fitted curves and, in the table below, we show the fitted cumulative link ratios at each development within the triangle and the extrapolated tail factors based on the assumption that the class stops developing after 17 years (that is development factors for ratios 9-16 have been estimated).

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8	Tail
Expon	3.929	2.321	1.596	1.269	1.121	1.055	1.025	1.011	1.009
Weibull	15.591	3.893	1.960	1.360	1.131	1.043	1.012	1.003	1.001
Power	9.581	2.911	1.657	1.270	1.120	1.055	1.026	1.012	1.011
Sherman	26.151	3.078	1.508	1.190	1.089	1.048	1.028	1.018	1.043

From this table it is clear that the Sherman curve will add around 3% extra to the ultimate values for all years compared to the Power curve.

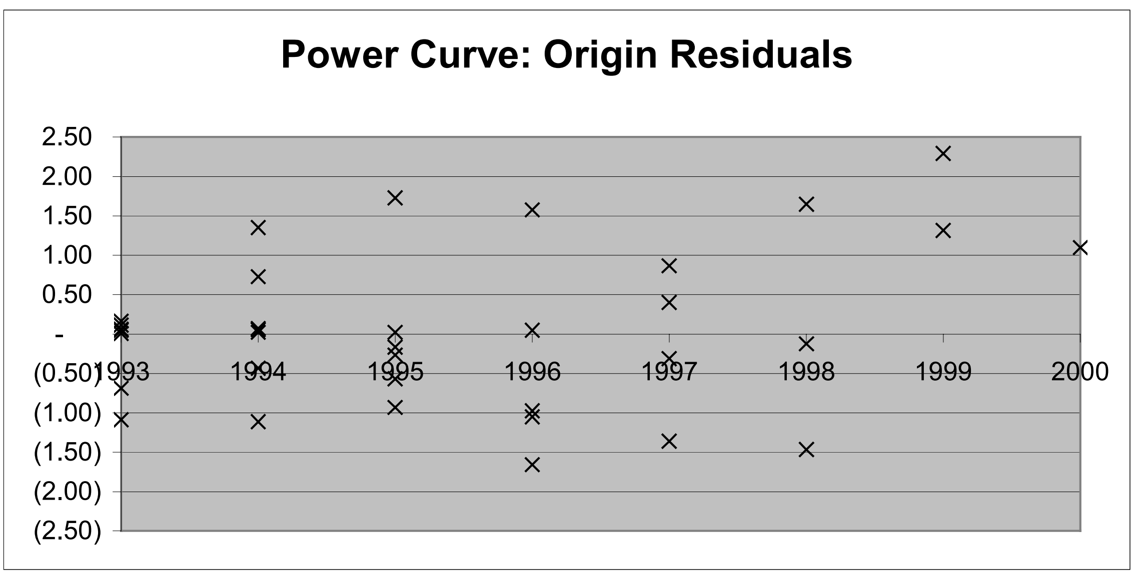
7.2.2.5 The next step is to plot the standardised residuals against

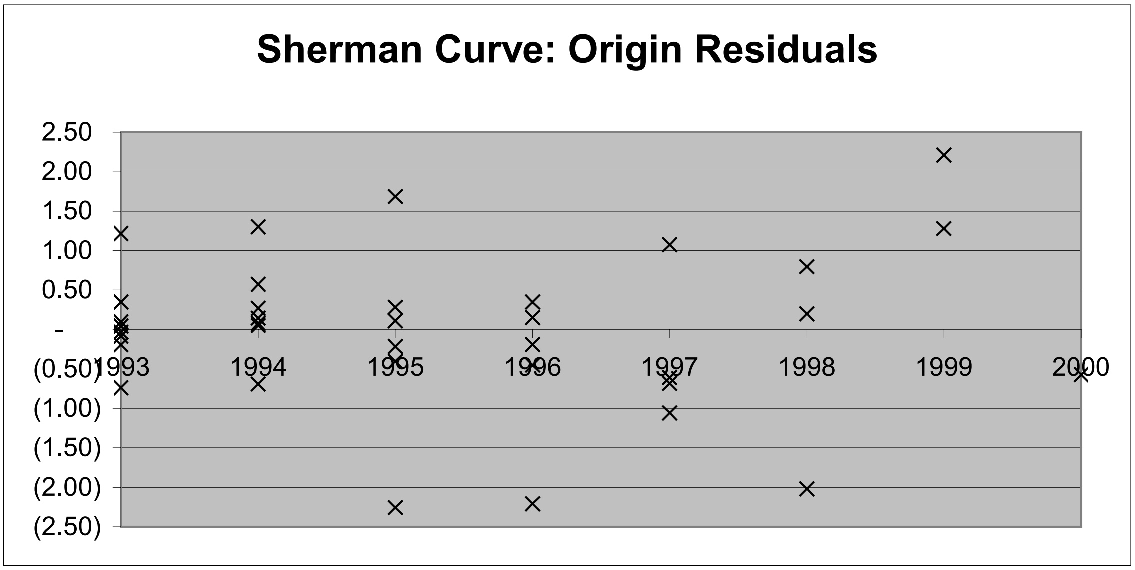
- Development period.
- Origin period.
- Calendar period.
- Fitted value (not actual value since the residuals are automatically correlated to the data).

If our model is correct then the standardised residuals should be randomly spread around zero and independent of the factor they are plotted against. A common problem is non-constant variance ("Heteroscedasticity") which typically shows up as a cone-shaped structure in the residual plot.

Each of the standardised residual plots for the fitted Power and Sherman curves in our example are given below.

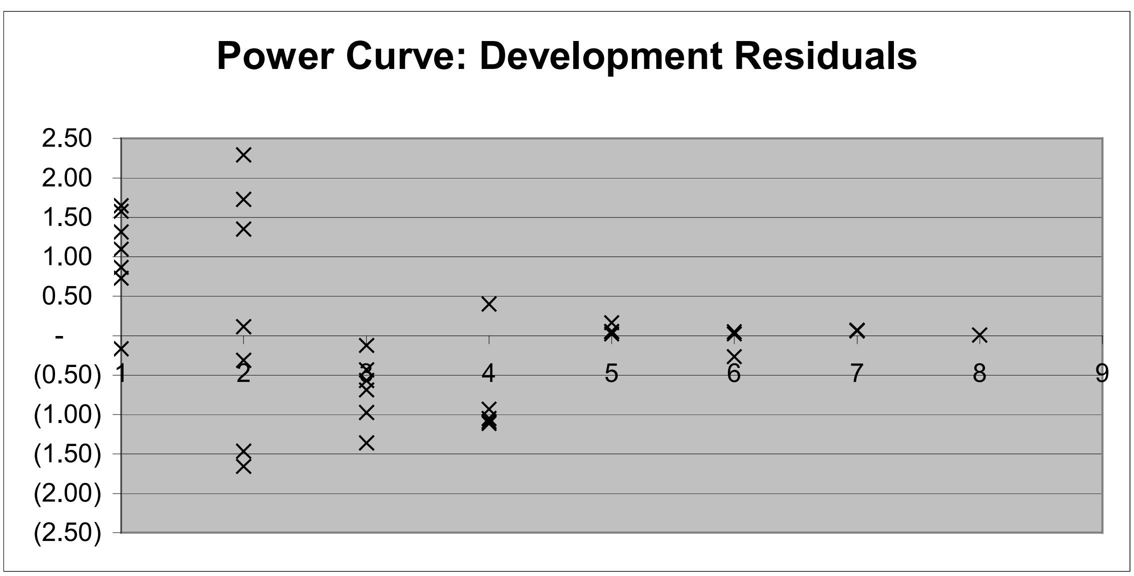
Residual 1. **Plot standardised residuals against origin period**

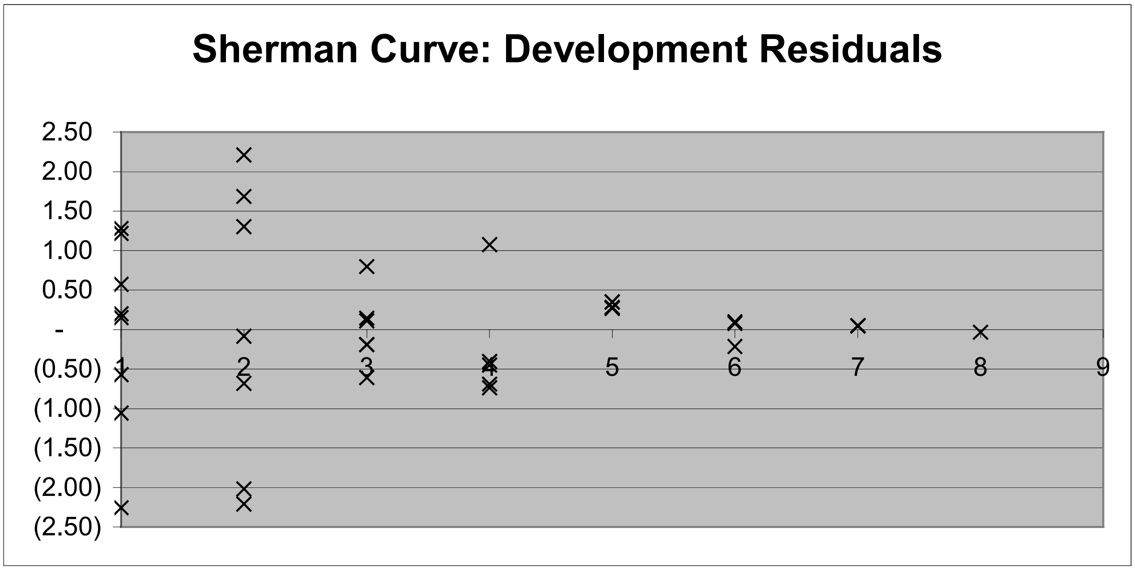




In these plots, each element on the x-axis refers to a different origin period (underwriting year). The residuals appear to be reasonably randomly spread and broadly comparable. However, we note that for the Power curve the 1999 and 2000 year residuals are all positive. Statistically, the sample is too small to draw any conclusions, with 25% chance that all three residuals have the same sign, but we may wish to bear in mind that this seems odd.

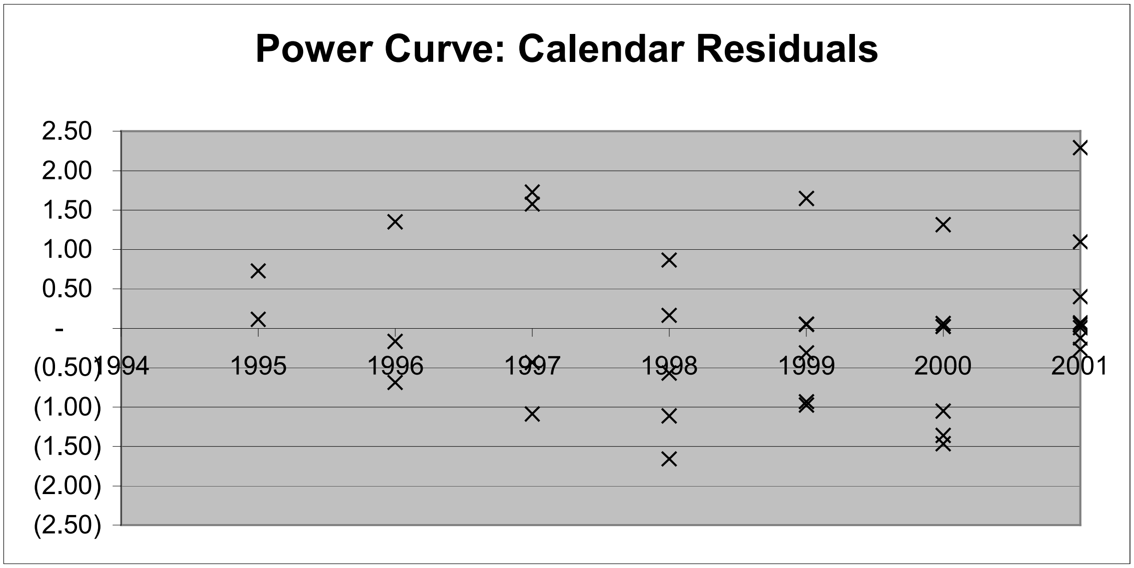
Residual 2. Plot standardised residuals against development period

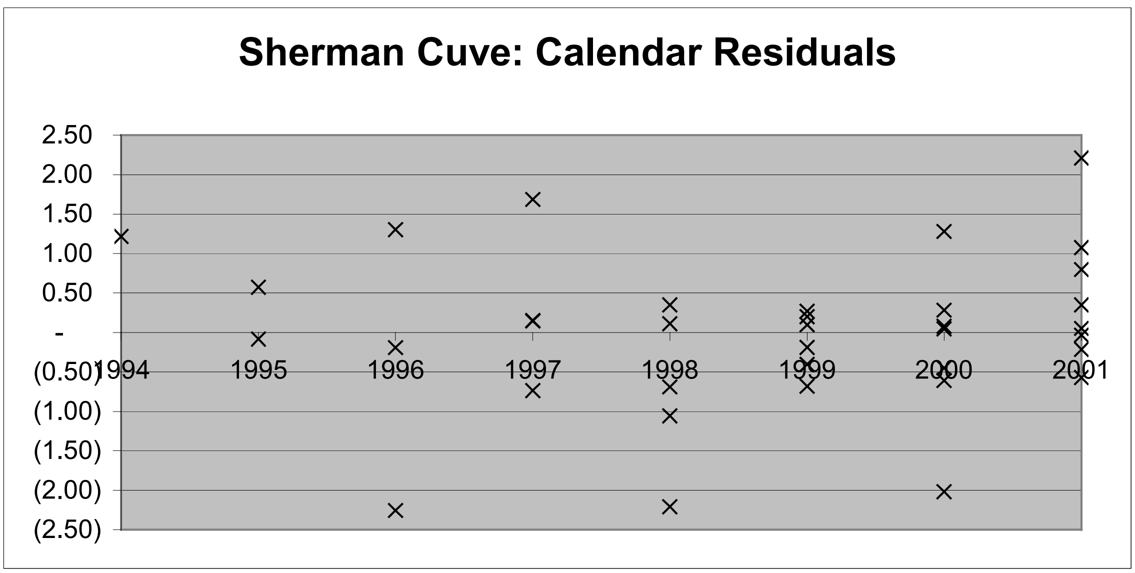




Each element on the x-axis here relates to an individual slope parameter, R_j, and is in some senses comparable to the log incremental plots discussed above. This plot shows a more marked difference between the two curves than the origin year plot. In comparison to the Sherman curve, the Power curve typically underestimates R_1 (since actual less expected and therefore the residuals are predominantly positive) and overestimates R_3 and R_4 (since the residuals are typically negative). Residual plots at each other development are broadly comparable. In some senses the Sherman curve is now looking better although the number of exceptional items is still far higher than for the Power curve.

Residual 3. Plot standardised residuals against calendar period

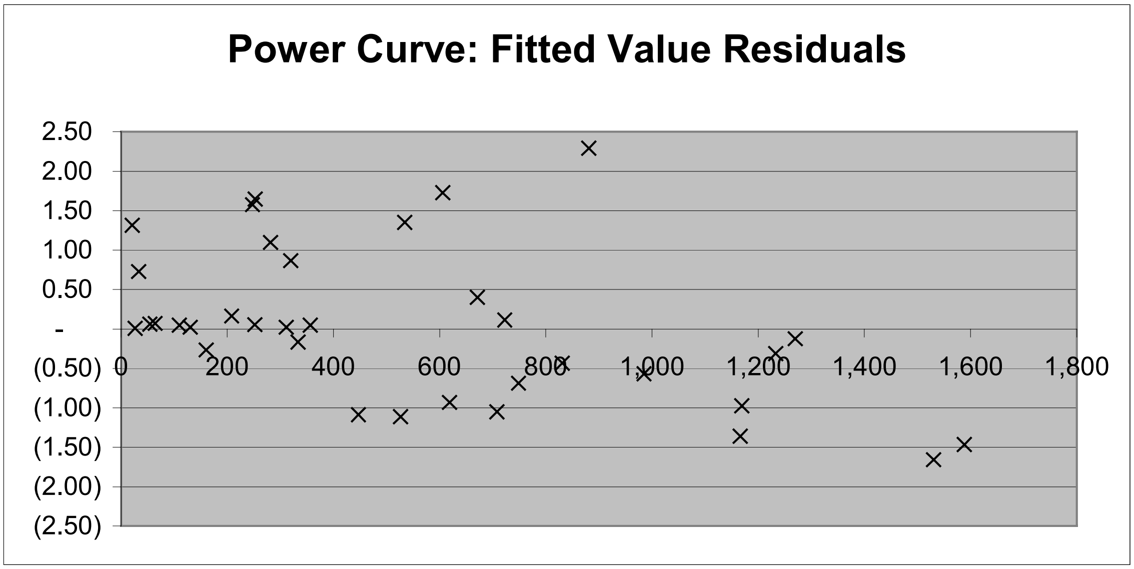


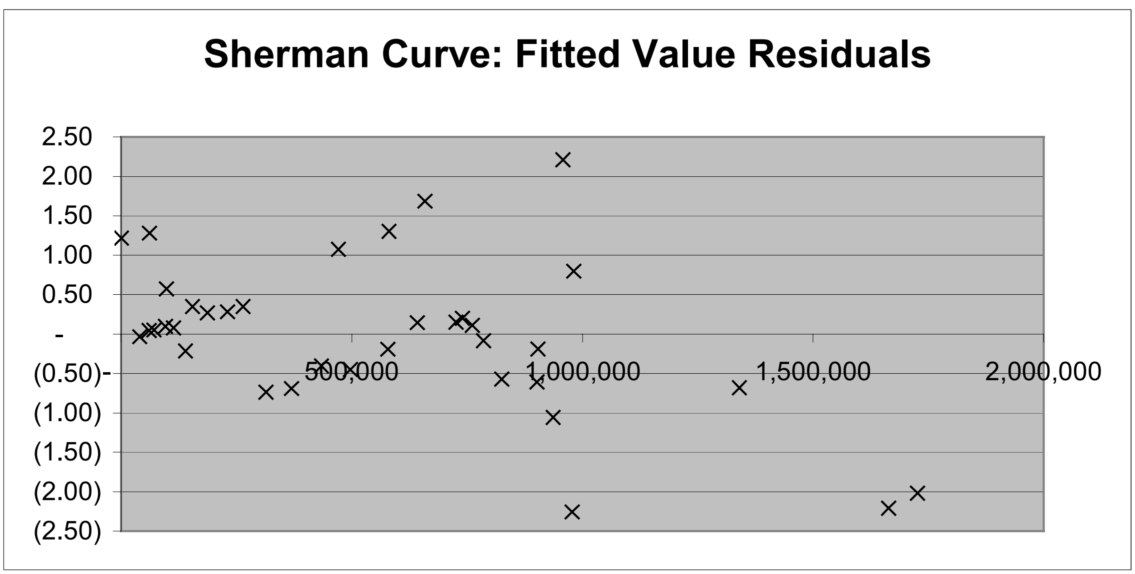


Each element on the x-axis relates to a period in time. For example, the residuals appearing for 2001 are the residuals from each origin period arising during the 2001 calendar year.

The residuals appear reasonably randomly spread but we note that both fitted curves appear to generally underestimate the 2001 development for each origin period. In fact, on further investigation we find that for the Sherman curve, three of the eight residuals are negative and for the Power curve, two of eight are negative. Neither of these observations is statistically significant but we again see some evidence that the Sherman Curve better describes the data.

Residual 4. **Plot standardised residuals against fitted values**





In this plot we show the residuals against fitted values ($C_{i,j}\ r_j$). The plots suggest that the largest values of $C_{i,j}\ r_j$, those in excess of 1,000,000, tend to be over-estimated since the residuals (actual less expected) are negative. However, we note that only 3 of the 36 observations lie in this range.

7.2.2.6 Modifications:

Our current models do not seem conclusive: there is strong evidence that the Sherman Curve best describes the data but there is a question mark over the number of exceptional residuals.

We have already noted that the fitted curves are rarely appropriate over the entire range of developments and usually are poorest for the earliest factors. From our development residual plots we note that the Sherman Curve has one exceptional item for development one and three for development two with none after that. Based on our feeling that it is usually the earliest developments that are most difficult to fit with a curve, we start by removing the first development rather than the one that currently appears worst. As we will see, this step is sufficient.

Following the same routine as before, but using the WAD average for the first development, our fitting statistics and fitted parameters are now as follows:

	Expon	Power	Weibull	Sherman
Error	388,853	353,450	357,537	299,741
+ve residuals	67%	64%	67%	56%
Exceptional	2.8%	2.8%	5.6%	5.6%
A	6.1019	33.7053	0.1560	48.6821
B	-0.7792	0.4976	1.6452	-3.7526
C				0.3277

When calculating the error values, we now have three parameters for the Exponential, Power and Weibull Curves (the WAD average plus two curve fit parameters) and four for the Sherman Curve (WAD plus the three curve parameters).

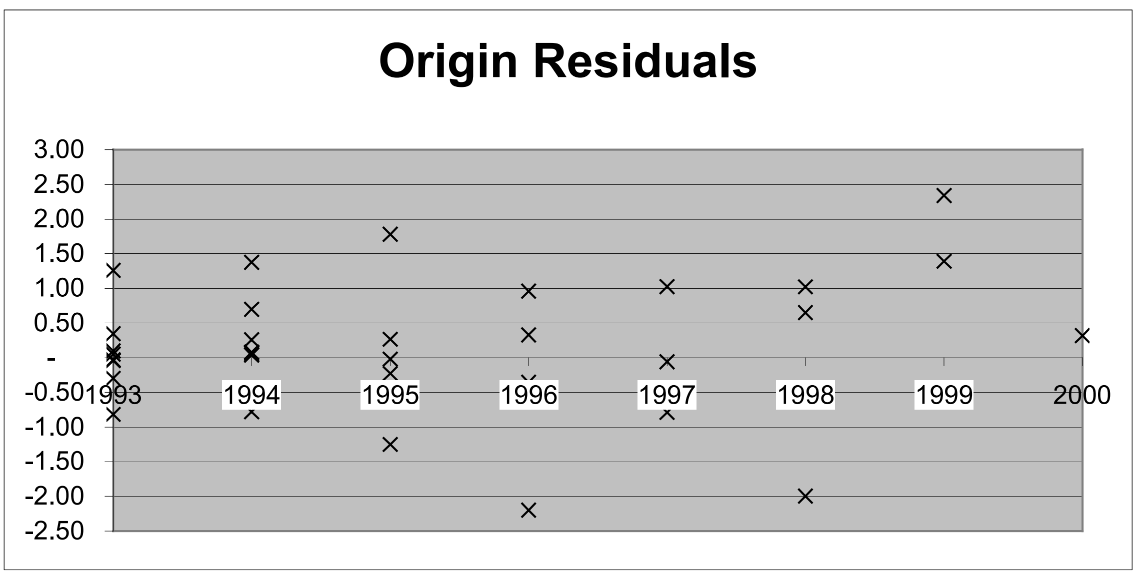
Compared to our previously fitted curves, we note that the Exponential is debatably no better or worse; the Power Curve has in fact deteriorated (that is the cost of introducing an extra parameter has not been outweighed by a sufficient improvement in fit); the Weibull

Curve has reduced its error term but become more biased; and the Sherman Curve has improved both its error term and exceptional residual proportion.

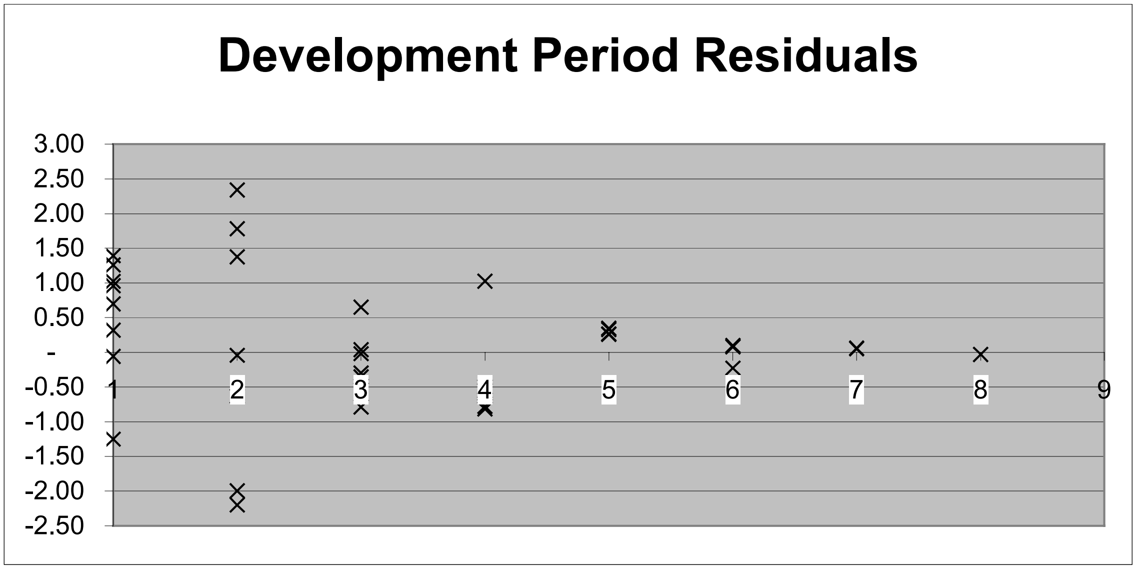
The revised Sherman curve is now the clear favourite. However, we should check whether or not this model is indeed better or worse than our initially selected WAD averages. The fitting statistics for the 8-parameter (R_1 to R_8) WAD model are shown below:

	WAD
Error	331,829
+ve residuals	44%
Exceptional	8.3%

The WAD average model was never going to be usable without creating a tail factor but we do need to check that our fitted curve model is superior to the simple model for developments observed within the triangle. Once again, we find that the revised Sherman Curve appears to be the most appropriate model. It only remains to check the residual plots for any problems.



The origin period residual plot is broadly similar to earlier, although with fewer residuals outside the (-2,2) range. Similar comments apply to the calendar period and fitted value residual plots.



The development residual plot is improved especially for R_1 and R_2 where the residuals are more evenly spread.

Having discovered no problems within the residual plots, we now have a clear winner: the revised Sherman Curve, as given below.

	Ratio 1	Ratio 2	Ratio 3	Ratio 4	Ratio 5	Ratio 6	Ratio 7	Ratio 8	Tail
Age-to-age	25.312	3.044	1.535	1.199	1.091	1.048	1.028	1.017	1.037
Age-to Ultimate	175.776	6.944	2.281	1.487	1.240	1.136	1.084	1.055	1.037
Pattern	1%	14%	44%	67%	81%	88%	92%	95%	96%

7.2.3 Reserve estimates

The Chain Ladder estimate of the ultimate developed amount, U_{CL},in respect of origin year I based on data as at development J is then

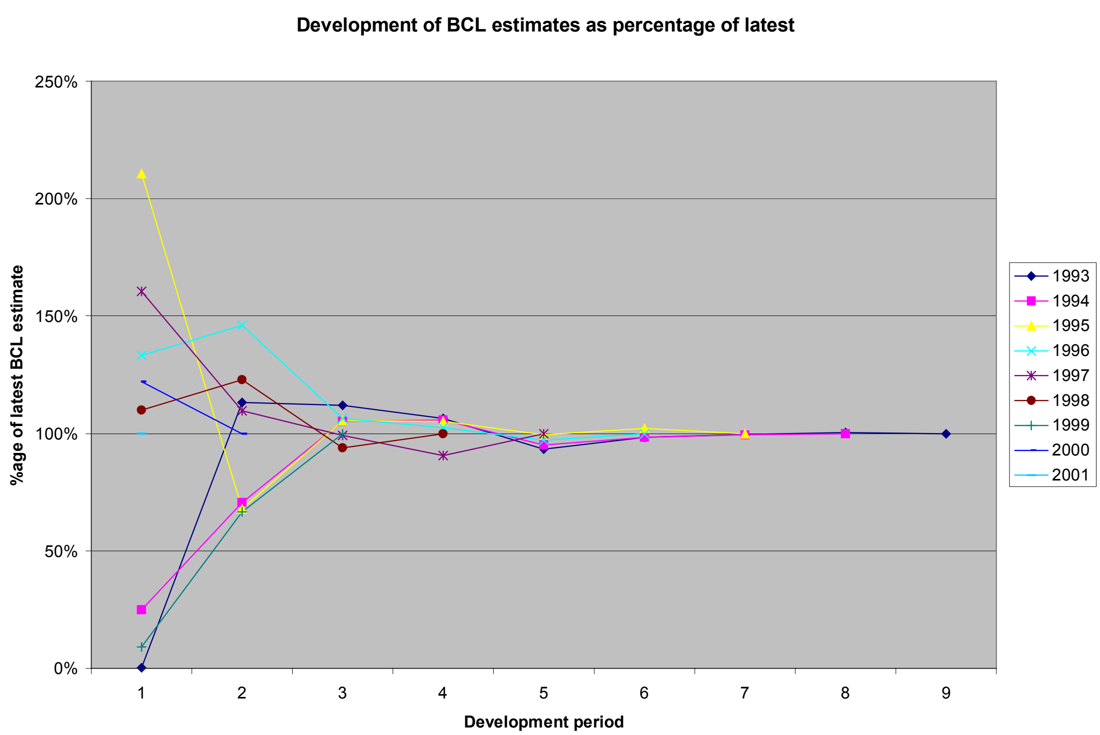
$$\begin{aligned} U_{\mathrm{CL}} &= C_{I,J}\, S_J \\ &= C_{I,J} / D_J, \end{aligned}$$

where S_J is the Age-to-Ultimate factor and D_J is the pattern, as shown above. The Chain Ladder logic is simple and easily communicated: if the pattern says that half the losses should have been incurred by now and $1M have been incurred then the ultimate losses will be $2M.

Before applying the Chain Ladder method, it is often useful to check the stability of the projections. In the table below we show Chain Ladder ultimate loss estimates using the derived pattern but based on incurred losses at the latest and 3 preceding developments.

	BCL @0	-BCL @1	-BCL @2	-BCL @3
1993	2,316,925	2,326,231	2,309,447	2,277,514
1994	2,743,804	2,724,600	2,698,977	2,610,580
1995	3,243,198	3,316,571	3,225,125	3,409,372
1996	3,809,314	3,696,047	3,900,982	4,058,114
1997	4,079,904	3,697,667	4,048,068	4,480,667
1998	4,696,985	4,406,101	5,771,010	5,171,446
1999	4,801,431	3,201,343	430,872	
2000	4,721,647	5,766,638		
2001	7,097,940			

As an extension to this, we can examine the development of Chain Ladder estimates graphically by plotting the ratio of ultimate losses calculated at each development against the latest Chain Ladder estimate, as below:



From this we see that the Chain Ladder estimates are volatile and up to 100% wrong at development one and 50% wrong at development 2. From Development 3 onwards the estimates are relatively stable. This pattern is typical in that the estimates based on immature origin years can be volatile. Note that we are comparing with the latest estimate, which may well be wrong.

To overcome this, we turn to one of the exposure-based methods referred to in Section 6. In terms of the theory described here, these models are "Intercept only" models, although in practice the more pragmatic Bornhuetter-Ferguson or (Generalised) Cape Cod approaches are used in preference to generalised linear modelling techniques.

In this example we will use BF estimates based on the previous origin period's ULR adjusted for estimates of claims inflation and premium rate adequacy going into the next origin period. The table below shows the BF estimates, again calculated at the latest and preceding 3 development periods.

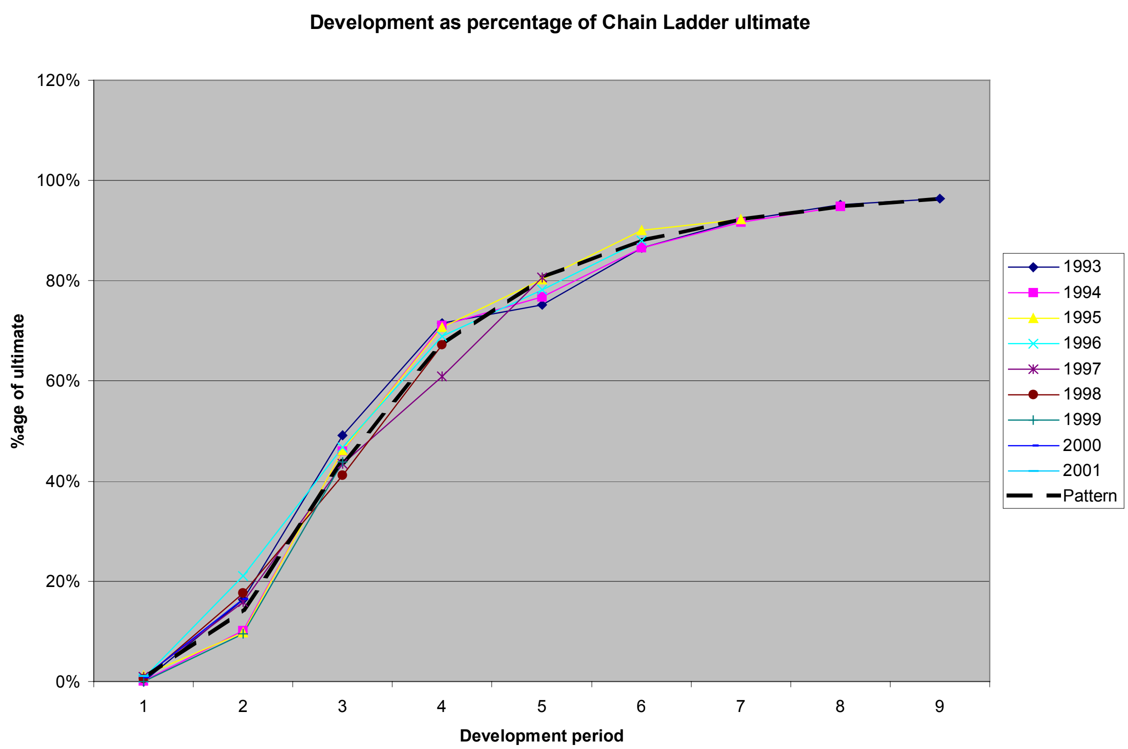
	Premiums	Chain Ladder ULR	Inflation	Rates	BF prior	BF @0	-BF @1	-BF @2	-BF @3
1993	2,659,299	87%				2,234,437	2,205,691	2,130,884	2,005,301
1994	3,181,861	86%	6%	5%	88%	2,746,644	2,730,322	2,710,886	2,646,919
1995	3,261,283	99%	6%	0%	91%	3,222,928	3,276,466	3,177,951	3,269,136
1996	3,507,538	109%	6%	-3%	108%	3,807,266	3,714,626	3,865,360	3,908,741
1997	3,395,122	120%	6%	-5%	121%	4,086,526	3,834,026	4,085,196	4,166,944
1998	3,776,338	124%	6%	-10%	142%	4,909,064	4,933,339	5,406,147	5,343,782
1999	4,067,302	118%	6%	0%	132%	5,116,530	5,051,222	5,334,361	
2000	3,874,269	122%	6%	20%	104%	4,138,135	4,049,798		
2001	3,273,228	217%	6%	20%	108%	3,544,079			

Therefore, out final selected ultimate loss estimates are the BF for 2000 & 2001 and the BCL estimates for all other years, as shown below.

Premiums	Incurred	Estimated	ILR	ULR

		losses	ultimate		
1993	2,659,299	2,234,437	2,316,925	84%	87%
1994	3,181,861	2,601,627	2,743,804	82%	86%
1995	3,261,283	2,992,439	3,243,198	92%	99%
1996	3,507,538	3,354,018	3,809,314	96%	109%
1997	3,395,122	3,291,414	4,079,904	97%	120%
1998	3,776,338	3,159,232	4,696,985	84%	124%
1999	4,067,302	2,104,527	4,801,431	52%	118%
2000	3,874,269	679,917	4,138,135	18%	107%
2001	3,273,228	40,381	3,544,079	1%	108%

One final check that is often carried out is to plot the development of incurred losses as a percentage of the selected ultimate, as shown below.



This plot serves several purposes. Firstly it is a simple goodness of fit check as the pattern should be a reasonable approximation to the development of each year (it should be noted that this “goodness of fit” assessment is far less accurate than the checks already applied). Secondly, the historic spread of incurred loss developments around the selected pattern indicates the volatility of the projection results and hence the inherent uncertainty in any estimates (for example at development 2, the historic losses have varied between 10% and

20% of ultimate losses which suggests considerable uncertainty). Thirdly, the plot can be used to assess the reasonableness of the selected figure. In our example, we have used the BF estimate for 2000 and, according to the plot above, this looks reasonable since the implied IBNR is very similar to that of 1997 and 1998 after the second development period.

7.2.4 Slope and intercept models

So far, we have discussed "Slope only" models in detail and mentioned that the Bornhuetter-Ferguson and (Generalised) Cape Cod methods can be described as "Intercept only" models. The remaining option within this framework is of course to look at models using both Slope and Intercept terms:

$$C_{i,j+1} = a_{i,j} + C_{i,j}\, R_{i,j} + \varepsilon_{i,j},$$

Models of this form can be fitted using standard generalised linear modelling techniques and have been described in some detail by Christofides in the Institute's Claims Reserving Manual.

In practice, this method can be useful especially for the earliest developments where Slope or Intercept only models do not give a good description of previous incremental loss developments. A full model of this kind has the considerable advantage that the user can calculate which development periods should have slope parameters and which should have intercept parameters, or both. However, in order to apply this model well, it is necessary to normalise each origin period so that they represent a consistent level of exposure and this information may not be readily available.

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