Pricing excess of loss treaty with loss sensitive features: an exposure rating approach

by

Ana J. Mata
Brian A. Fannin
Mark A. Verheyen

The problem

- Given:
  - The expected loss cost for the treaty.
  - The characteristics of the portfolio of policies: mixture of lines of business, limits and deductibles.
  - The reinsurance layer: \( m \times s \times l \)

- Estimate an aggregate loss distribution (frequency and severity) that includes all these characteristics.

The components of reinsurance pricing (I)

- Expected loss cost
  - Experience methods (burning cost, development triangles, etc)
  - Exposure methods (benchmark curve from industry or risk specific)
  - Mixed methods (combination of experience and exposure methods)

- We do not discuss the methods for estimating the loss cost.
The components of reinsurance pricing (II)

- **Premium:**
  - Fixed rate
  - Increase or decrease with losses incurred (loss sensitive)
- **Other costs:** expenses, commissions
  - Fixed % or $ amount
  - Loss dependent
- **Profit margin:** fixed load or through modelling of cash flows.

Loss sensitive features

<table>
<thead>
<tr>
<th>Feature</th>
<th>Variable component</th>
<th>Deduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margin plus Swing rating</td>
<td>Premium</td>
<td>Premium adjusts with losses incurred times a load, subject to a minimum and a maximum.</td>
</tr>
<tr>
<td>Profit commission</td>
<td>Commissions/Expenses</td>
<td>Profit is shared with ceding after a load for reinsurer’s expenses.</td>
</tr>
<tr>
<td>Loss Corridor</td>
<td>Losses</td>
<td>Cedant retains part of losses attaching at pre-determined value of LR.</td>
</tr>
<tr>
<td>Reinstatement</td>
<td>Premium and Loss</td>
<td>Limits the number of total losses. Extra premium is received to reinstate the layer.</td>
</tr>
<tr>
<td>Annual Aggregate Deductible (AAD)</td>
<td>Losses</td>
<td>Cedant retains the first D losses in aggregate.</td>
</tr>
</tbody>
</table>

The need of an aggregate model

- If $S$ represents the aggregate losses to the layer, then Loss Cost $= E[S] = E[X][E[N]]$.
- When premium and expenses vary with losses they become random variables (functions of the aggregate losses $S$).
- In general Jensen’s inequality holds:
  $$f(E[S]) \neq E[f(S)]$$
The need of an aggregate model

- We need to estimate the expected value of premiums and commissions when they are variable.
- Therefore we need an aggregate loss distribution for $S$ such that

\[ \text{Loss Cost} = \mathbb{E}[S] \]

Method 1: Parametric distribution

- Fit a parametric distribution (lognormal, gamma, etc.) using the method of moments.
- \( \mathbb{E}[S] \) given by the loss cost.
- \( \text{Var}(S) \) estimated assuming a Poisson or Negative Binomial distribution for frequency.
- Estimate the parameters.

Method 1: Parametric distribution

- Very easy to implement and understand.
- It ignores the probability of having zero losses to the layer (not realistic for some lines of business).
- Does not separate frequency and severity distributions.
- Does not account for mixtures of policy limits and deductibles. (E.g. $1m$ policy limit with no deductible or with $10m$ deductible).
Method 2: benchmark severity distribution

- Select an appropriate severity distribution for the line of business. Industry benchmark (ISO) or account specific. Calculate $E[X]$.
- Choose a frequency distribution (Poisson or Negative Binomial). Estimate the parameters.
- For Poisson: $\lambda = E[N] = \frac{\text{Loss Cost}}{E[X]}$

Method 2: benchmark severity distribution

- Compute aggregate losses (Panjer recursion, Fourier Transforms, etc.) See Appendix A.
- Improvement over Method 1: allows for probability of zero and at layer limit.
- When different policy limits are covered the severity might be overestimated since not every claim might reach the full layer limit.

Method 3: Exposure based severity curve

- Objective: estimate a “blended” severity distribution that:
  - Takes into account all combinations of policy limits and deductibles written by cedant.
  - Allows for multiple lines of business.
- How?: Using the exposure rating method.
- Given this severity, the frequency distribution is estimated as in Method 2.
Review of the exposure method

- Estimates the proportion of the risk ceded to the reinsurance layer.
- Basic ingredients:
  - Ground-up loss ratio
  - Ground-up severity distribution (benchmark or risk specific)
  - Limits profile: policy limits, deductibles, % of premium for each combination.

The exposure method for a $4 \times \$1m layer

The formula

- \( X = \text{Ground-up loss severity} \)
- \( PL = \text{Policy Limit} \);
- \( d = \text{deductible}; \)
- \( \text{Layer: } l \times m \)

\[
\text{Loss Cost} = (X \wedge \text{PL}) \cdot (X \wedge d) \\
\times \frac{E[X \wedge \min(PL + d, l + m + d)] - E[X \wedge \min(PL + d, l + m + d)]}{E[X \wedge PL + d] - E[X \wedge d]}
\]

Where

\[
E[X \wedge a] = E[\min(X, a)]
\]
Estimating frequency with the exposure method

- If we use the exposure method in a layer $S \times m$, it can be shown that the result is the expected frequency in excess of $m$.

- Given frequency at various attachments the distribution function can be estimated. All math is explained in the paper.

- This is the key result in developing our “blended severity”.

The basic recipe (by line of business)

- Split the layer $l \times m$ in sub-layers of size $h$ (small enough to keep resolution but not too small to save computing time).
  
  \[
  h \times m \\
  h \times m+h \\
  h \times m+2h \\
  \vdots \quad \vdots \quad \vdots \\
  h \times l+m-h
  \]

The basic recipe (cont'd)

- For each sub-layer estimate the expected frequency using the exposure method.

- Given frequency at each sub-layer, estimate the severity distribution (by line of business)

- With the distribution function estimate the severity density function (by line of business).
The basic recipe (cont’d)

- Mix all the density functions by LOB weighted by expected frequency to the layer. (Assumes independence between lines)
- All the mathematical details are explained in the paper.
- Result: a “blended” severity curve that takes into account all the policy limit combinations and mixture of lines of business.

With the “blended severity” calculate $E[X]$ and then

$$E[X] = \frac{\text{Loss Cost}}{E[X]}$$

- Fit a frequency distribution (Poisson or Negative Binomial).
- Compute aggregate losses (Panjer recursion, Fourier Transforms, etc.). Estimate the expected value of all loss sensitive features.

Worked example: professional liability $500k \times $500k

<table>
<thead>
<tr>
<th></th>
<th>Lawyers</th>
<th>E&amp;O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductible</td>
<td>$10,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Limit</td>
<td>$750,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Premium</td>
<td>$1,000,000</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>FGU LR</td>
<td>65% 65%</td>
<td>75% 75%</td>
</tr>
</tbody>
</table>

Worked example: professional liability $500k \times $500k

<table>
<thead>
<tr>
<th></th>
<th>Lawyers</th>
<th>E&amp;O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductible</td>
<td>$10,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Limit</td>
<td>$750,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Premium</td>
<td>$1,000,000</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>FGU LR</td>
<td>65% 65%</td>
<td>75% 75%</td>
</tr>
</tbody>
</table>
Assumptions and computation

- Using the expected implied frequency and a variance multiplier of 2 (see Appendix B) we fitted a Negative Binomial distribution.

- Using the severity and frequency distribution we computed the aggregate distribution using Panjer’s recursive algorithms.

<table>
<thead>
<tr>
<th>Expected Loss Cost</th>
<th>$750,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method 2: benchmark severity</td>
<td>Severity</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
</tr>
<tr>
<td>Method 3: exposure severity</td>
<td>Severity</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
</tr>
</tbody>
</table>
Calculating the expected value of the treaty features

- For each output of the aggregate losses (0, 1000, 2000,...,100000) defined by the sub-layers calculate the value of the premium, profit commission, etc.
- With the corresponding probability function calculate the expected value of the feature.

\[ P(Y = y) = P(S = s) \]
Treaty features

- Subject premium: $7.2m
- Margin plus rated: 7% minimum, 12.5% provisional and 18% maximum. Loss load 107.5%.
- Profit commission: 15% after 20% for reinsurer’s expenses.
- Brokerage: 10% on provisional.

Expected results $500k vs $500k

<table>
<thead>
<tr>
<th>Method 1: lognormal</th>
<th>Method 2: benchmark curve</th>
<th>Method 3: exposure method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>% Prem.</td>
<td>Amount</td>
</tr>
<tr>
<td>Prov. Prem.</td>
<td>900,000</td>
<td>900,000</td>
</tr>
<tr>
<td>Margin plus</td>
<td>222,739</td>
<td>167,989</td>
</tr>
<tr>
<td>Tot. prem.</td>
<td>1,122,739</td>
<td>1,068,041</td>
</tr>
<tr>
<td>Losses</td>
<td>750,000</td>
<td>750,000</td>
</tr>
<tr>
<td>PC</td>
<td>25,659</td>
<td>32,930</td>
</tr>
<tr>
<td>Brokerage</td>
<td>90,000</td>
<td>90,000</td>
</tr>
<tr>
<td>Marg. CR</td>
<td>865,659</td>
<td>872,930</td>
</tr>
</tbody>
</table>

Comments

- Key difference is the probability of zero losses. The parametric curves do not allow for this.
- If probability of zero losses is high, expected premium is lower and PC is higher.
- Practical relevance for high layers (or CAT layer) that have low frequency.
Communicating the results to underwriters: no need to understand the mathematical details (severity, frequency, Panjer’s recursion, etc.) but rather to communicate the relevance of the model in pricing and profitability.

Practical considerations
- How to choose the size of the sub-layers?
- How to include expenses ALAE?
- When does it fail?: theoretically it always works but:
  - For high frequency layers the resulting aggregate distribution is approximately Normal (CLT).
  - The lognormal might be more reasonable in this case: we need skewness and thicker tail.

Further aspects to consider
- How to allow for correlations and dependencies between lines of business ceding to the same treaty?
- How to use this technique to assess profitability for multi-layer treaties? (Strong dependence between layers)