Mortality and Smoking Prevalence
A Mortality Model for Multiple Populations
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Mortality rates

UK (black), CA (blue) – age 70
observed death rates

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Smoking prevalence

Objectives

- Develop a consistent model for the mortality experience of multiple populations that can be used for scenario generation
- Explain differences between mortality rates in different populations
- Identify common factors influencing mortality in a number of populations
Building a model – Available Data

$D_i(x, t)$: Number of deaths,
$D_i^N(x, t)$, $D_i^S(x, t)$ for non-smokers, smokers (not observed)
$D_i(x, t) = D_i^N(x, t) + D_i^S(x, t)$

$E_i(x, t)$: Exposure-to-risk

$m_i(x, t)$: $= D_i(x, t)/E_i(x, t)$,
$m_i^N(x, t)$, $m_i^S(x, t)$, death rates

$s_i(x, t)$: Smoking prevalence, in $[0, 1]$,
the number of smokers is $s_i(x, t)E_i(x, t)$

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Building a model

Total number of deaths:

$D_i(x, t) = D_i^N(x, t) + D_i^S(x, t)$

$= m_i^N(x, t)[1 - s_i(x, t)] E_i(x, t) + m_i^S(x, t)s_i(x, t)E_i(x, t)$

Death rates:

$m_i(x, t) = \frac{D_i(x, t)}{E_i(x, t)} = m_i^N(x, t) + [m_i^S(x, t) - m_i^N(x, t)] s_i(x, t)$
Building a model

Modelling assumptions:

- Smoking prevalence has the same effect on mortality rates in all observed countries.
- Total mortality in country $i$ is the weighted average of non-smokers' and smokers' mortality adjusted by a "country effect"

$$m_i(x, t) = [m_N(x, t) + \left[m_S(x, t) - m_N(x, t)\right] s_i(x, t)] \exp(\Gamma_i(x, t))$$

Linear Regression
Linear Regression

Correlation between smoking and mortality
Adjusted Maximum Likelihood Estimation

Model:

\[ m_i(x, t) = \left[ m^N(x, t) + [m^S(x, t) - m^N(x, t)]s_i(x, t) \right] \exp(\Gamma_i(x, t)) \]

Assume country effects are normally distributed

\[ \Gamma_i(x, t) \sim N(0, \sigma^2_i(x, t)) \]

We assume that annual changes in the log mortality rates \( \mu^S \) and \( \mu^N \) for a given cohort are positively correlated.

Log death rates at age 65 in the UK

![Graph showing log death rates at age 65 for smokers and non-smokers from 1960 to 2000.](image)
Log death rates at age 65 in the Canada

Average country effect for different ages

\[ m_i(x, t) = \left[ m^N(x, t) + [m^S(x, t) - m^N(x, t)] s_i(x, t) \right] \exp(\Gamma_i(x, t)) \]

Average absolute value of the country effect for all ten countries as a function of age. The average is taken over all years for which smoking prevalence data are available.

Smoking explains mortality at old ages well compared to young ages
Possible Applications

Scenario generation for future mortality using stochastic models for non-smokers' and smokers' mortality rates, and assumptions (or models) for smoking prevalence

- Risk management of pension funds
- Assessment of basis risk
- Quantifying hedge ratios for longevity hedges as the mortality experiences of two populations can be modelled simultaneously
- ...

Conclusions

- The empirical results indicate that there is a significant downward trend in non-smokers' mortality rates while the rates for smokers seem to fluctuate around a constant level.
- Model fits the observed mortality rates rather well
- Impact of country effect is lowest for high ages
- However, we have also considered implied cessation rates, and they indicate that we underestimate high age non-smokers' mortality
- The methodology can be applied to other factors
Papers

- Mortality and smoking prevalence: An empirical investigation in ten developed countries, 2013, British Actuarial Journal / FirstView Article, pp 1-15
  - A Common Age Effect Model for the Mortality of Multiple Populations
  - Modelling England & Wales Mortality with Smoking Prevalence as a Covariate