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making financial sense of the future

Olivier-Smith Model

Mortality and Longevity Seminar
28 June 2006

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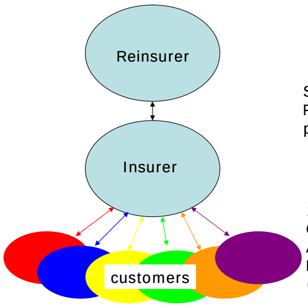
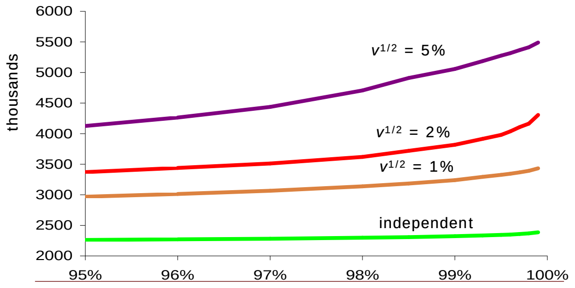
Reinsurance Contract



Stop loss reinsurance contract
Pays annuities once aggregate payout exceeds £2.5m

100 policyholders
One-off premium aged 65
Annual pension until death
Expected payments £2.12m
(under my own assumptions)

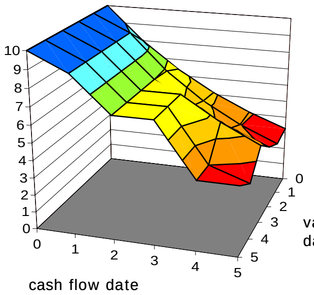
Example Olivier-Smith Model Output



Olivier-Smith Model

- Start with initial mortality curve, including best estimate of future improvements
- Revise each survival probability, p, as time passes according to the following formula:
  - $p_{new} = p_{old}$ ^ deterioration factor
  - p_{new} is the probability measured at the current valuation date, p_{old} is the survival probability measured at the previous valuation date
- Deterioration factor is stochastic, following a Gamma distribution with mean 1, variance v
- Applied inductively for multiple time horizons
- This model produces bias, which we will later correct

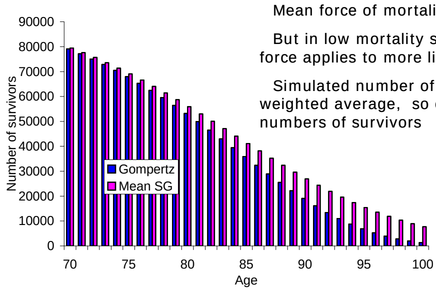
Olivier-Smith Model: Sample Output



Initial valuation based on expected table

Assumptions gradually replaced by reality

Stochastic Bias



Mean force of mortality is correct

But in low mortality scenarios, force applies to more lives

Simulated number of lives is a weighted average, so overstates numbers of survivors

Olivier-Smith Model

Mathematical Specification

- Notation:
  - Take a homogeneous cohort
  - L(t) = number of survivors at time t
  - L(s,t) = $\mathbf{E}_s$\{L(t)\}
  - Conditional on information at time *s*

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Olivier-Smith Model

$$L(s,s) \sim Bin\left[L(s-1,s-1), \frac{L(s-1,s)}{L(s-1,s-1)}\right]$$ binomial model

$$\frac{L(s,t)}{L(s,t-1)} = \left(\frac{L(s-1,t)}{L(s-1,t-1)}\right)^{\beta(s,t)*G(s)}$$ gamma deterioration factor

$$G(s) \sim \Gamma(v^{-1}, v^{-1})$$ gamma mean 1, variance v

$$\beta(s,t) = v^{-1}\ln\left(\frac{L(s-1,t-1)}{L(s-1,t)}\right)^{-1}\left\{\left(\frac{L(s-1,s)}{L(s-1,t)}\right)^{v} - \left(\frac{L(s-1,s)}{L(s-1,t-1)}\right)^{v}\right\}$$

Bias correction so that L(s,t) is a martingale in *s*

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Olivier-Smith Model

Multiple cohorts

- Split population by age, sex, smoking status and all other required underwriting factors.
- Project separately for each cohort, using initial best estimate for each cohort separately.
- BUT use the same deterioration factor in a given calendar year for all cohorts
- Mortality improves or deteriorates for everyone at once.
- Multi-factor extensions using principal components analysis

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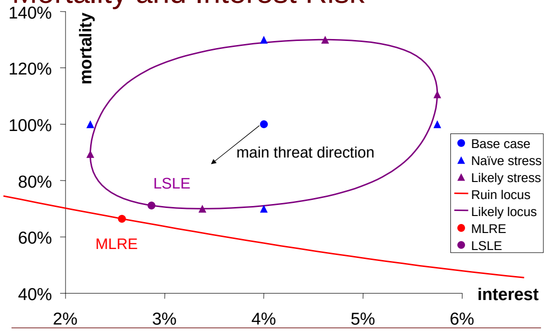
Mortality and Interest Risk



Conclusion

Problem	Olivier-Smith Model
I can't use my mean mortality assumptions.	✓ Initial mortality curves are used as input.
I need to project future balance sheets.	✓ Models the experience cash flows as well as estimates of future mortality rates.
I don't get the same mean number of survivors.	✓ Bias correction factor introduced to ensure cash flows are unbiased.

Conclusion

Problem	Olivier-Smith Model
I need to model multiple cohorts.	✓ Different assumptions can easily be used for different cohorts, by using different initial mortality tables for different cohorts as required.
I need to look at all my risks together.	✓ Perform smarter stress tests to determine which combinations of interest rates and longevity (and other risks) have the most significant impact.

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