LLOYD’S

Capital Allocation: Challenges and Options
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Lloyd’s Market Risk Unit with Imperial College, London

- MRU is a centre of expertise for the Market, with 3 actuaries, 3 actuarial students, 5 technical experts and 2 general analysts
- The Capital Modelling and Systemic Risk teams generate opportunities and requirements for research
- Imperial College, through the Centre for Quantitative Finance, provides tuition on financial mathematics and has seconded a PhD student, Andreas Tsanakas, to work in the MRU for three years
- Although it is expected that the resulting thesis will relate to the MRU’s activities, there have also been opportunities to incorporate research within current development plans for RBC
Lloyd’s Chain of Security, Funds at Lloyd’s and RBC

<table>
<thead>
<tr>
<th>Premiums Trust Funds</th>
<th>Premiums Trust Funds</th>
<th>£10,635</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funds at Lloyd’s</td>
<td>Funds at Lloyd’s</td>
<td>£7,324</td>
</tr>
<tr>
<td>FAL = Min (MinFAL, RBC)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Personal Wealth</td>
<td></td>
<td>£323*</td>
</tr>
<tr>
<td>Central Fund</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

LLOYDS

RBC - Inputs and Outputs

| Risk Assessment | µ, σ, ρ |
| Risk Measure    | |
| Prudential Calibration | ELC = 0.0464p |
| Syndicate Business Plans | Syndicate RBC |
| Member Participations | Member RBC |

ELC = 0.0464p

Syndicate RBC

Member RBC

Gamma Distribution and ELC

Gamma pdf:

\[ p(x; \alpha, \beta) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \]

RBC equalsises

\[ \int [x - (100 \times RBC)] p(x) dx \]

to the Expected Loss Cost per unit of net premium/reserve

\[ \beta \Gamma(\alpha) \]

Risk Profile
Properties of RBC

- **Risk Based** ⇒ differential capital requirements that reflect the risk posed by each member
- **Equity** ⇒ each member poses the same ELC to Central Fund for each £1 of net premium or net reserve
- **Diversification** ⇒ recognises benefits from business mix, spread across managing agents and years of account
- **Capital Efficiency** ⇒ sub-optimal as diversification within the Central Fund is not reflected in the risk measure

Rule-based Allocation

- 80:20 solution would suggest that a risk-based approach should focus on larger and more complex entities
- For the remainder, we should seek to achieve a broad reflection of comparative risk, based on some general rules
- Criteria for fixed capital may include:
  - no concentration >20%
  - limited exposure to high risk syndicates

Coherent Risk Measures

- Risk is defined as the amount of capital required to cover for future liabilities
- A risk measure is a real valued function, defined on the set, \( G \), of all random variables representing risks (losses)
- Coherent risk measures satisfy the four properties:
  - Monotonicity: \( X, Y \in G, X(\omega) \leq Y(\omega) \Rightarrow p(X) \leq p(Y) \)
  - Positive Homogeneity: \( \lambda \geq 0, X \in G \Rightarrow p(\lambda X) = \lambda \cdot p(X) \)
  - Subadditivity: \( X, Y \in G \Rightarrow p(X + Y) \leq p(X) + p(Y) \)
  - Translation invariance: \( X \in G, a \in \mathbb{R} \Rightarrow p(X + a) = p(X) + a \)
Expected Shortfall

- Expected shortfall:
  \[ E[X \mid X > \text{VaR}_\alpha(X)] \]
- A generalisation of Value at Risk
- “How bad is bad?”
- It is a coherent risk measure and satisfies the properties listed previously
- It is additive under comonotonicity

Cooperative Games

- Economies of scale: allocating savings from cooperation
- Stability of the grand coalition:
  - individual rationality
  - collective rationality
- In our case costs correspond to risk capital
  - The cost function corresponds to a risk measure
- The Shapley value:
  \[ \phi_i = \sum_{S \subseteq N \setminus \{i\}} \left( \frac{|S|}{n} \right) \left( \frac{1}{2^n} \right) \left( \rho(N-S) - \rho(S) \right) \]

Non-atomic Cooperative games

- Players are (divisible) portfolios: non-atomic games
- The Aumann-Shapley value:
  \[ \phi_i = \int \frac{\partial \rho(A)}{\partial a_i} dy = \frac{\partial \rho(A)}{\partial a_i} \]
- Coherent risk measure & AS \Rightarrow Coherent allocation
- For expected shortfall AS is:
  \[ E \left[ X_i \left| \sum_{i=1}^n X_i > \text{VaR}_\alpha \left( \sum_{i=1}^n X_i \right) \right. \right] \]
- A measure of systemic risk
Application to Lloyd’s

- Lloyd’s both accepts excess risk from members and regulates the market.
- The two distinct roles suggest different approaches to capital allocation:
  - “Reinsurer”: Determine aggregate risk to Central Fund and allocate excess risk to members according to AS - Risk capital is determined indirectly, as a retention.
  - “Regulator”: Determine aggregate risk capital and allocate capital directly according to AS.

Equations

- “Reinsurer”:
  \[ R_x = E \left( X_x - K_x \right) + \sum_i (X_i - K_i) > VaR_x \left[ \sum_i (X_i - K_i) \right] \]

- “Regulator”:
  \[ K_m = E \left[ X_m \right] + \sum_i X_i > VaR_x \left[ \sum_i X_i \right] \]

\( X_m \): Claims for member’s portfolio; \( K_m \): Total capital for member \( m \);
\( R_x \): Risk contribution for member \( m \) (proportional to capacity).

Capital efficiency

- The capital at Lloyd’s is only partially mutualised.
- There are several possible allocation methodologies.
- Each methodology might result in a different amount of required risk capital.
- Aggregate capital is not fixed!
- We need to investigate which the most capital efficient methodology is.
Dependence Structures

- Modelling dependent risks
- Copulas de-couple marginal behaviour from the dependence structure:
\[ P(X \leq x, Y \leq y) = C(P(X \leq x), P(Y \leq y)) \leq F_{x,y}(x, y) = C(F_x(x), F_y(y)) \]
- Can model both asymptotically dependent and independent risks
- How does capital efficiency of different methodologies relate to the dependence structure between risks?

Key References


Specific RDS - Florida Windstorm

- Saffir-Simpson Category
  - CAT 1 (<74 mph)
  - CAT 2 (74-95 mph)
  - CAT 3 (96-109 mph)
  - CAT 4 (110-155 mph)
  - CAT 5 (>155 mph)
Summary

- Lloyd’s RBC
- Coherent Risk Measures
- Cooperative Games
- Capital Efficiency
- Dependence Structures
- Systemic Risk