A Short Introduction to Extreme Value Theory

Paddy Paddam

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Email: paddy.s.paddam@uk.pwcglobal.com Tel: +44 (0)20 7804 0830

Contents

- Introduction and Context
- Theory
- Short example and application issues
- Comments and discussion

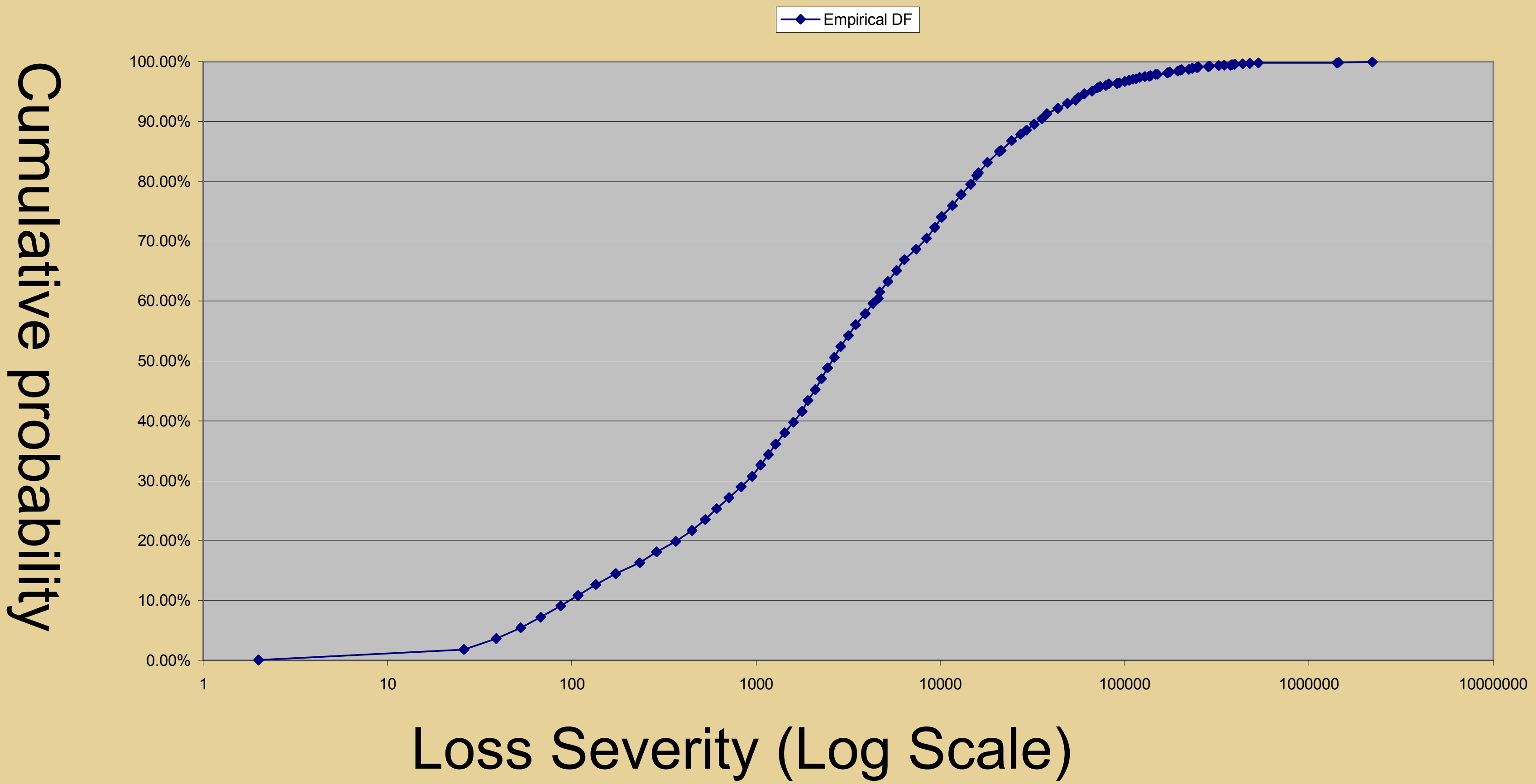
Loss severity

- The distribution of the size of a loss
- Common problem:
  - Pricing
  - Reinsurance pricing and modelling
  - Catastrophe models
  - Securitisation
  - Capital Management/DFA
  - Operational Risk Management

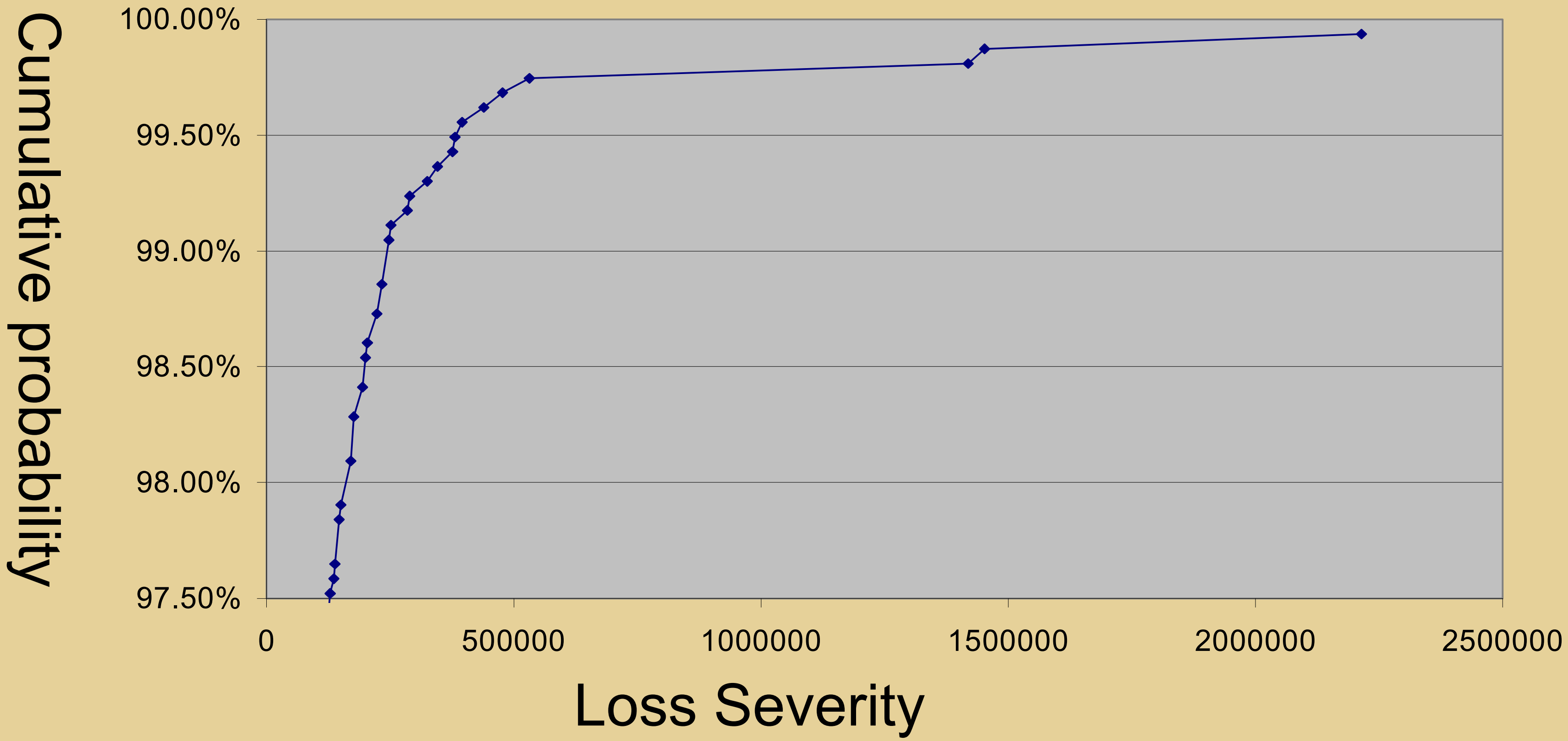
Loss severity - Central Limit Theorem

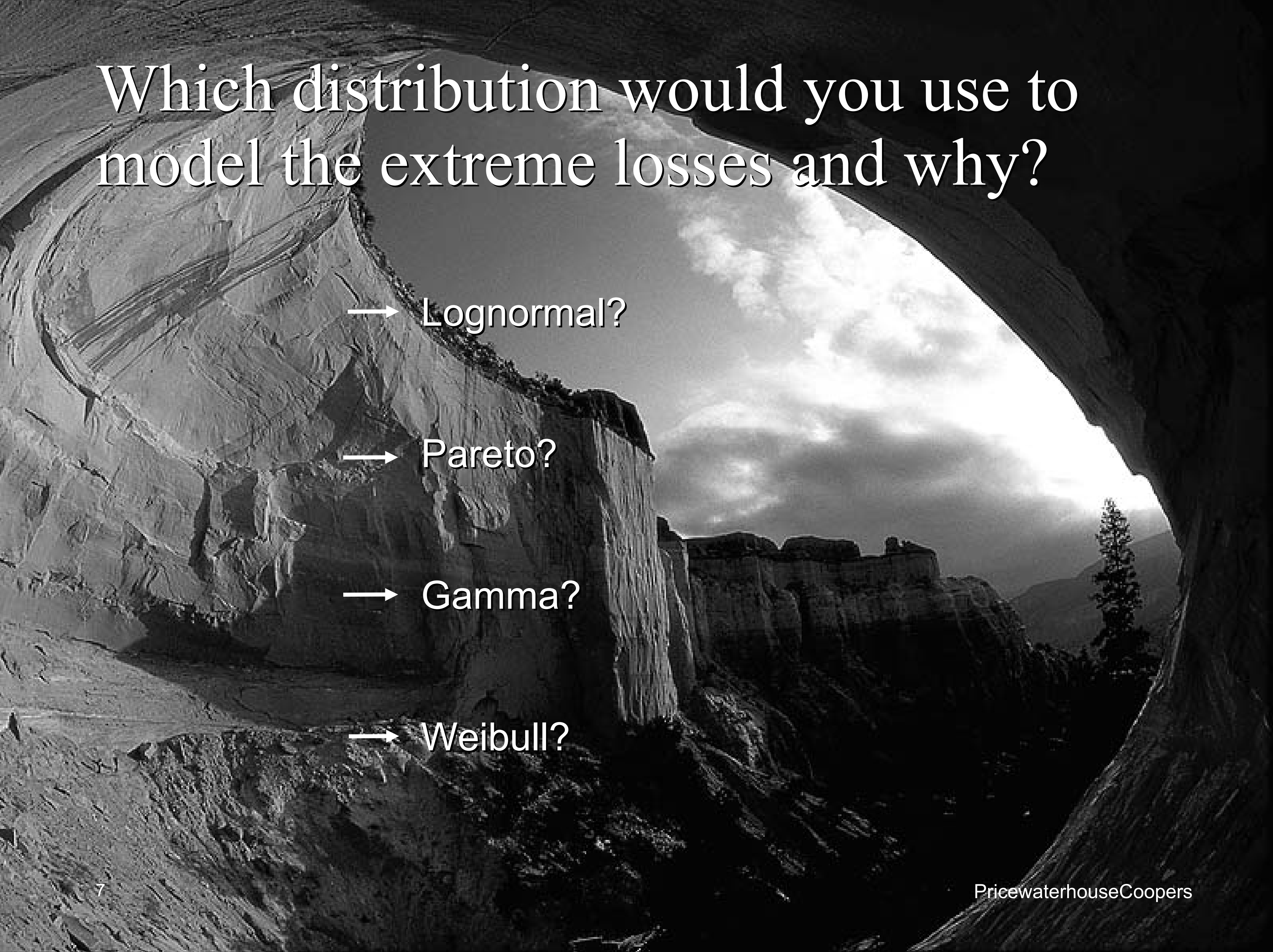
- Use the normal distribution for modelling sample means
- Common problem:
  - Pricing - Property damage claims on motor book
  - Reinsurance pricing and modelling - low layers
  - Securitisation - whole account portfolios
  - Capital Management - attritional losses

Context - The problem



The problem - The Tail





Which distribution would you use to model the extreme losses and why?

- Lognormal?
- Pareto?
- Gamma?
- Weibull?

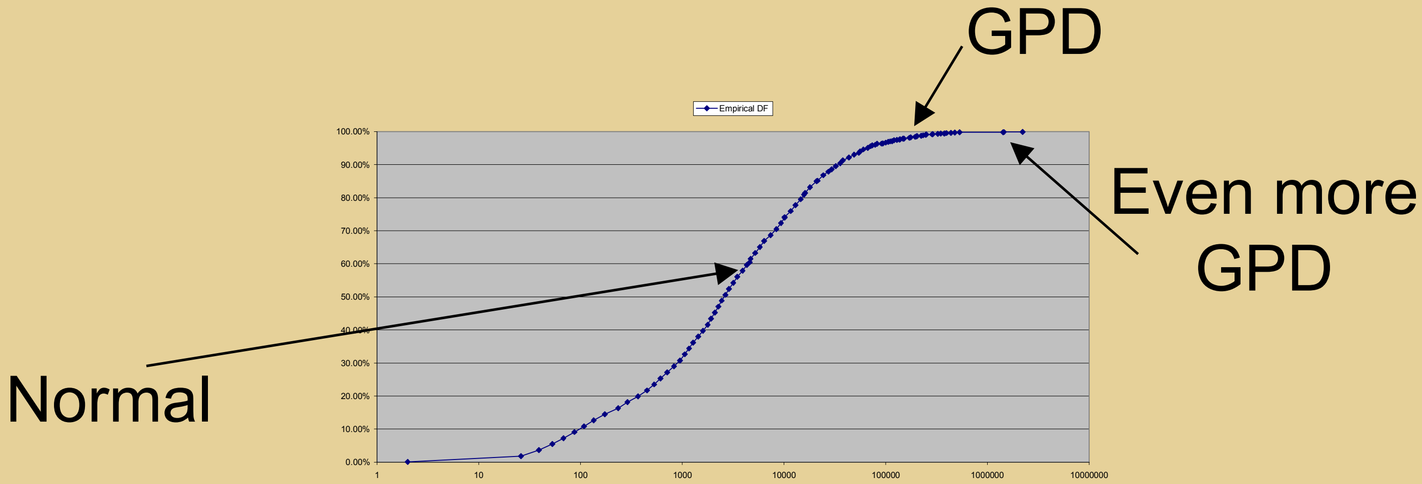
What is Extreme Value Theory?

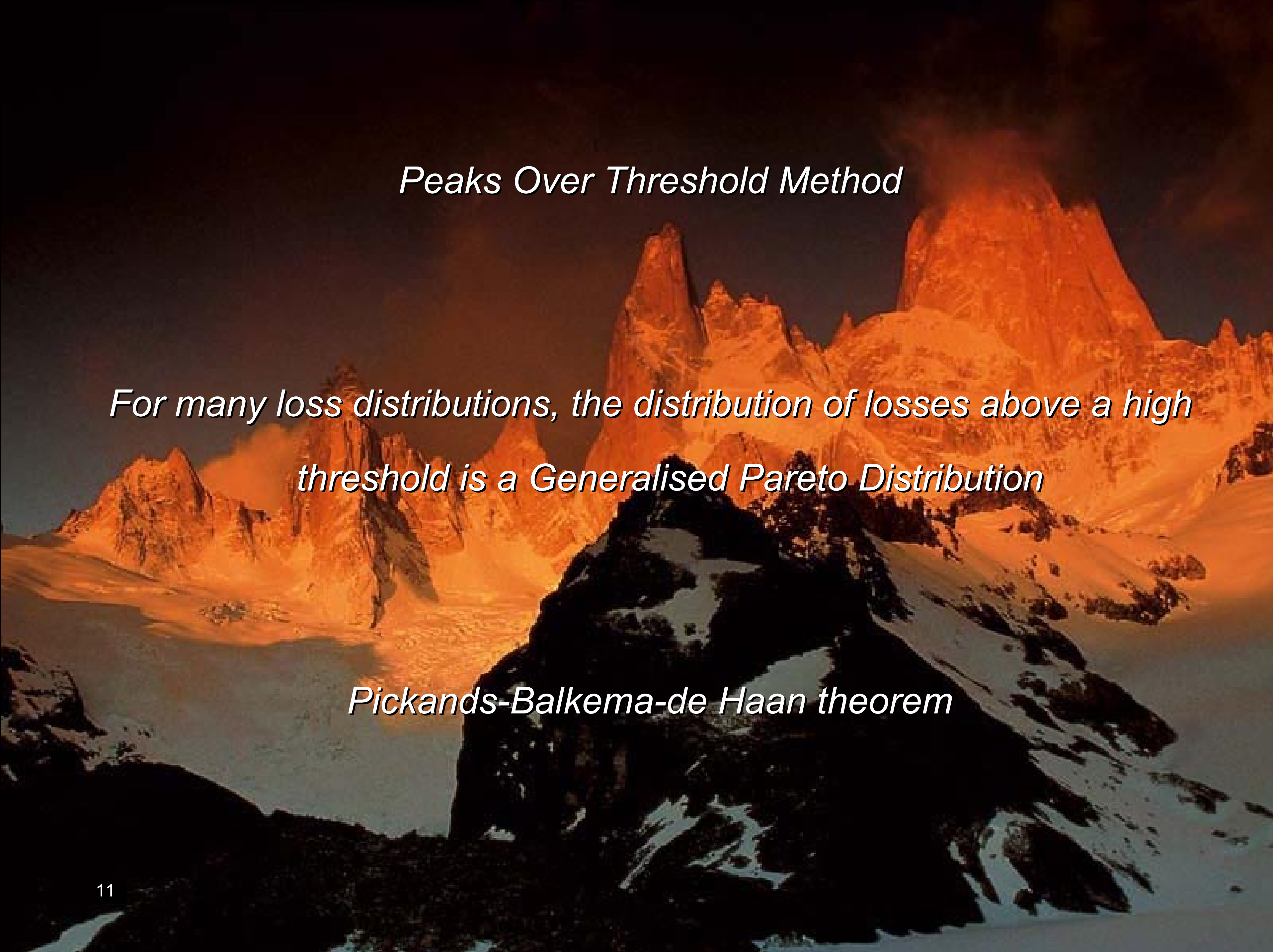
- Statistical Theory of Extreme Events
- Fisher-Tippet Theorem
  - For many loss distributions, the distribution of the maximum value of a sample is a generalised extreme value distribution.
- Generalised extreme value distributions are
  - Heavy tailed => Frechet
  - Medium tailed => Gumbel
  - Short tailed => Weibull


Useful result in Hydrology and Climatology
Not so useful in Insurance?

PBH Theorem

- Pickands-Balkema-de Haan theorem:
  - For many loss distributions, the distribution of losses above a high threshold is a Generalised Pareto Distribution.





Peaks Over Threshold Method

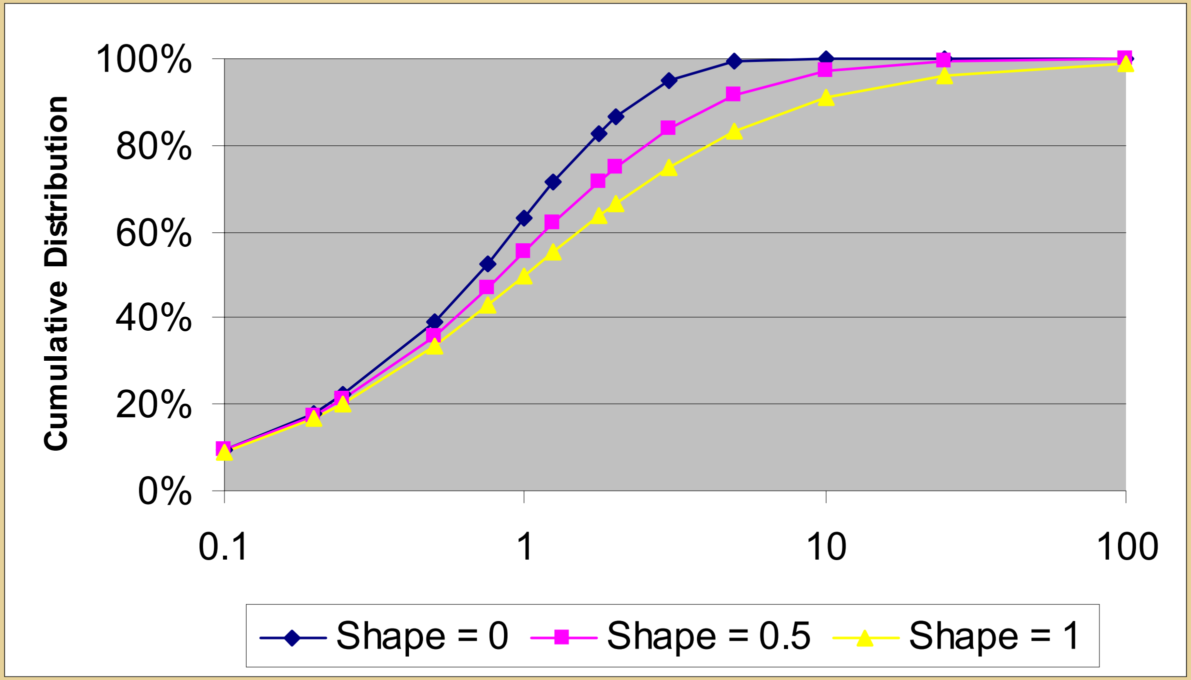
For many loss distributions, the distribution of losses above a high threshold is a Generalised Pareto Distribution

Pickands-Balkema-de Haan theorem

Generalised Pareto Distribution

- P(X<x|X>u) = 1 - [1+g(x-m)/s]^(-1/g) for g <> 0
  1 - exp[-(x-m)/s] for g = 0

- Parameters:
  - m = location
  - s = spread
  - g = shape
  - u = threshold



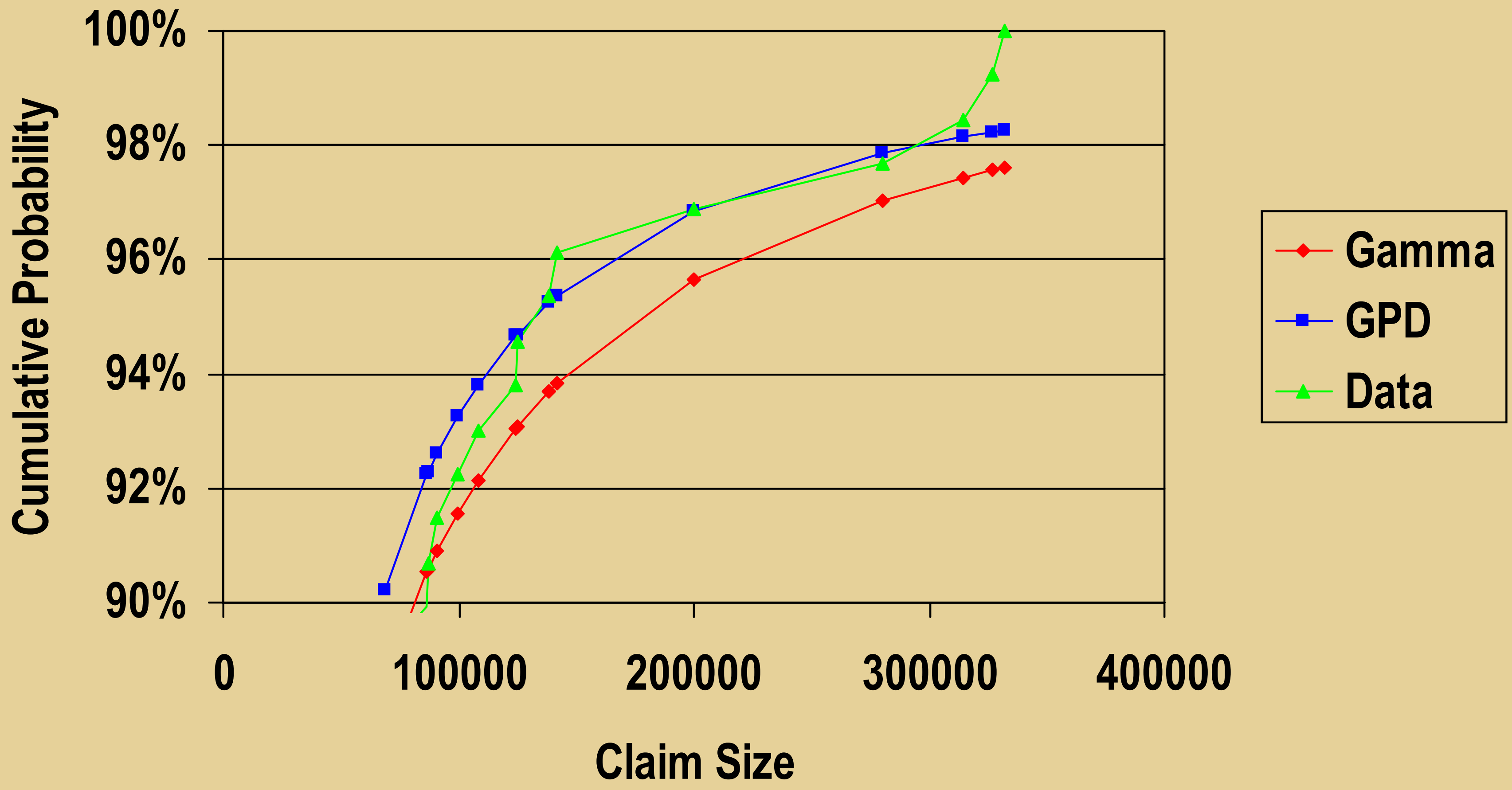
Fitting EVT distributions

- Maximum likelihood methods
- Probability weighted moments
- Variety of methods and software

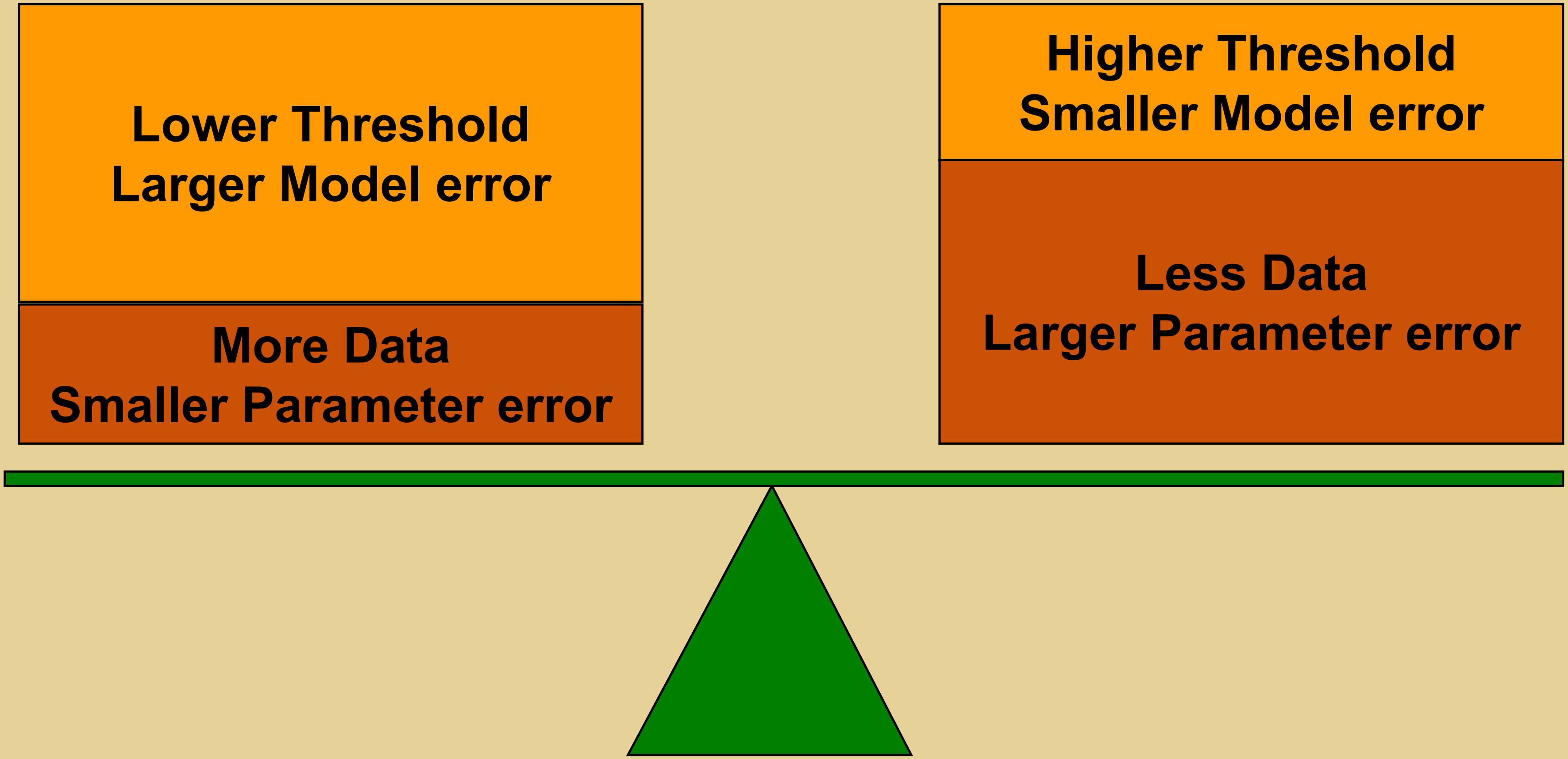
Applying EVT - Example Reinsurance modelling

- Considered a portfolio of 11 classes including Liability and Property accounts
- Performed a standard stochastic claims frequency and severity analysis
- In addition attempted to fit a GPD to the claims severity
- In our exercise, for 9 out of the 11 classes, the GPD was about as good or better than a standard loss distribution in modelling the extreme tail values of the loss severity distributions.

Fit to Claims Severity of a Non-Marine Liability Class



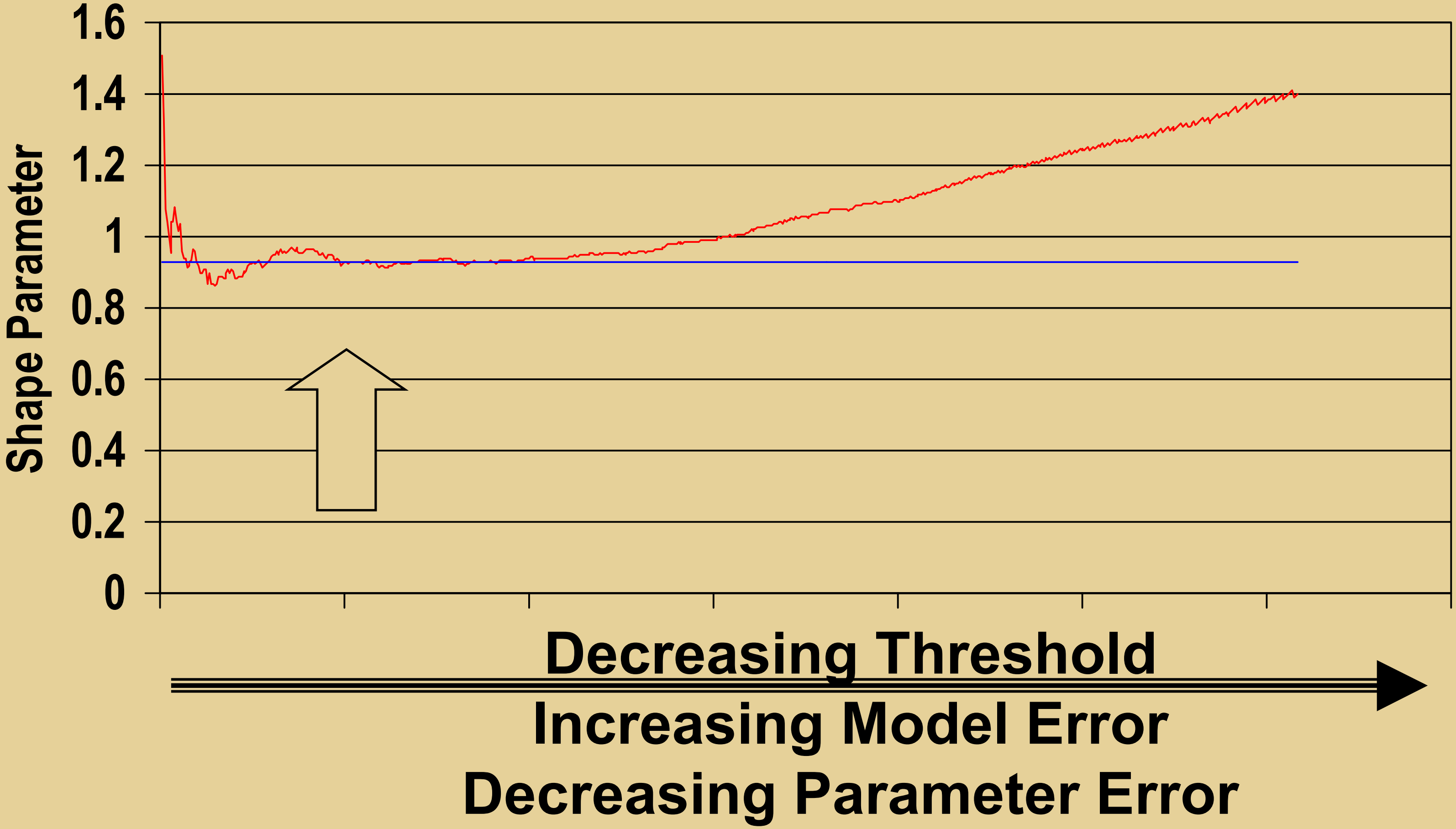
Choosing the Threshold
A Balancing Act



One approach
- Focus on shape parameter

- Shape parameter is the most important
- Higher shape parameter => Thicker tailed => More losses
- Compare the fitted shape parameter with the change in threshold (or equivalently the number of exceedances).
- Identify a stable plateaux
- Threshold < Excess attachment point

Fitted GPD Shape Parameter Compared to chosen threshold



Simple Conclusion

- Fit a GPD to the tails of claims severity distributions because it reduces Model error
- However Parameter error & Random fluctuations risks remain

Discussion

- Applications?
- Concerns?
- Questions?

References and Further Reading

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