The latest issues surrounding catastrophe modelling

Peter Taylor

Janus, Cerberus, Hydra
Where Business meets Solvency II

29 March 2011
“Understanding the models, particularly their limitations and sensitivity to assumptions, is the new task we face. Many of the banking and financial institution problems and failures of the past decade can be directly tied to model failure or overly optimistic judgements in the setting of assumptions or the parameterization of a model.”

Tad Montross, 2010, Chairman and CEO of GenRe in “Model Mania”

“What insurance needs, as does banking, is for seriously experienced business people to look at the firm’s business plan and challenge whether it has the competence to execute it well and the risk controls to alert it in time if something goes wrong.

What we are getting instead is micromanagement of the worst sort – the FSA, through its models, trying to tell the industry how to manage itself. Board meetings in future will be all about compliance, not about trying to make a profit.”

Anthony Hilton, London Evening Standard 17th September 2010
Themes – Myths and Monsters

- **Janus** – Transitions
- **Cerberus** – Lessons from the Financial Crisis
- **Hydra** – Solvency II and Business

Transitions
Janus

- Baby Boomers
  - Control
  - Model
  - Quantify
- EP Curves
  - Primary Uncertainty
  - Secondary Uncertainty
  - Multiple Models
- VaR – the Measure of Risk

Transition Bibliography

- Useless Arithmetic - Orin Pilkey & Linda Pilkey-Jones
- The Science of Prediction - David Orrell
- The Origin of Wealth - Eric Beinhocker
- The Failure of Risk Management - Douglas Hubbard

EP Curves
Secondary Uncertainty

Source: Edouard von Herberstein, Master’s Thesis, University of Colorado at Boulder, April 2004

EP Curve with Secondary Uncertainty

Source: AIR
Applying Secondary Uncertainty

Multiple Models
Model Comparison – Different Models
The Financial Crisis
Cerberus

- Finance - what went wrong?
- The Hidden dimensions of Risk
- Recognising Uncertainty

What went wrong?
Turner Review Conclusions

MODELLING PROBLEMS

- Short observation periods
- Non-normal distributions
- Systemic versus idiosyncratic risk (Correlated behaviour)
- Non-independence of future events (The past not a guide to the future)

Basel II and the idiot brother of Insurance

- **Framing errors**: over-reliance on data sets, often artificial, that did not include a sufficient range of outcomes
- **Model risk**: over-simplistic model and risk distribution assumptions
- **Parameter risk**: insufficient consideration of error due to lack of calibration of models
- **Behavioural risk**: anyone pointing out model failings would have been shouted-down - it would have stopped a lot of lucrative business and things were “different this time”
The Hidden Dimensions of Risk

Dimensions of Risk Decisions

- Harm
- Chance
- Time

plus

- Reward
- Judgement

2D Projections

Risk

- Harm
- Chance
- Time

plus

- Measures
The Dutch Group Risk Criteria

Source: Probabilistic Risk Analysis, Bedford and Cooke, CUP, 2001

Tackling Model Risk
Tackling Model Risk – Round 1

- On the Quantitative Definition of Risk
- Stress Tests
- Evidence
- Model Comparison
- Independent Estimates

yielding a

- Revised EP Curve

On the Quantitative Definition of Risk

Source: Kaplan and Garrick, Risk Analysis Vol 1 No 1 1981
Scenario Tests

Evidence

Table 1: Number of Atlantic Hurricanes:

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>LH</th>
<th>AH</th>
<th>EQC</th>
<th>CAT</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>5</td>
<td>5</td>
<td>5.6</td>
<td>0.6</td>
<td>2.7</td>
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<tr>
<td>2007</td>
<td>6</td>
<td>6</td>
<td>6.0</td>
<td>0.0</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>5</td>
<td>5</td>
<td>5.0</td>
<td>0.1</td>
<td>0.4</td>
<td></td>
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<tr>
<td>2009</td>
<td>5</td>
<td>5</td>
<td>5.0</td>
<td>0.1</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>5.6</td>
<td>2.7</td>
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</tr>
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</table>

Table 2: Number of U.S. Landfalling Hurricanes:

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>LH</th>
<th>AH</th>
<th>EQC</th>
<th>CAT</th>
<th>RMS</th>
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<tbody>
<tr>
<td>2006</td>
<td>1</td>
<td>1</td>
<td>2.4</td>
<td>1.8</td>
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<tr>
<td>2007</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>2.9</td>
<td>2.4</td>
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<tr>
<td>2008</td>
<td>2</td>
<td>2</td>
<td>2.0</td>
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<tr>
<td>2009</td>
<td>2</td>
<td>2</td>
<td>2.0</td>
<td>2.9</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>2.9</td>
<td>2.4</td>
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</table>

Table 3: U.S. Insured Losses from Hurricane ($Millions):

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>LH</th>
<th>AH</th>
<th>EQC</th>
<th>CAT</th>
<th>RMS</th>
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<tr>
<td>2006</td>
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<td>20</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
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<tr>
<td>2007</td>
<td>20</td>
<td>20</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
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<tr>
<td>2008</td>
<td>20</td>
<td>20</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
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<tr>
<td>2009</td>
<td>20</td>
<td>20</td>
<td>16.0</td>
<td>16.0</td>
<td>16.0</td>
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<tr>
<td>Total</td>
<td>80</td>
<td>80</td>
<td>64.0</td>
<td>64.0</td>
<td>64.0</td>
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</tr>
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</table>

Source: 2006 - 2010 Loss Model “Near-Term” Predictions
Karen Clark & Company. “Near Term Hurricane Models Performance Update” January 2010
More Evidence

![Raw Losses, 1926-1995](image1)

![Normalized Losses, 1926-1995](image2)

Figure 1: Raw loss data shows a rapid increase in hurricane losses in recent decades. However, the normalized record shows that losses in recent years have been near the long term average, and that normalized losses from 1975-1990 were significantly worse than from 1986-1995.

Source: Evaluation of Catastrophe Models Using a Normalized Historical Record 1999
Roger A. Pielke, Jr., Christopher W. Landsea, Rade T. Musulin, and Mary Downton

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More Evidence

### Hurricane Isabel (2003)

<table>
<thead>
<tr>
<th>Company</th>
<th>Modelled</th>
<th>Actual</th>
<th>Multiple</th>
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<tbody>
<tr>
<td>A</td>
<td>0.47</td>
<td>3.30</td>
<td>6.9</td>
</tr>
<tr>
<td>B</td>
<td>1.44</td>
<td>7.30</td>
<td>5.1</td>
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<tr>
<td>C</td>
<td>2.36</td>
<td>15.00</td>
<td>6.4</td>
</tr>
<tr>
<td>D</td>
<td>0.31</td>
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<td>E</td>
<td>0.11</td>
<td>0.30</td>
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<tr>
<td>F</td>
<td>1.20</td>
<td>2.30</td>
<td>1.9</td>
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<tr>
<td>G</td>
<td>2.54</td>
<td>10.00</td>
<td>3.9</td>
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<td>H</td>
<td>3.32</td>
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<tr>
<td>I</td>
<td>1.55</td>
<td>5.70</td>
<td>3.7</td>
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</table>
More Evidence

**Katrina Loss Estimate Development**

<table>
<thead>
<tr>
<th>Event Type</th>
<th>RMS Industry</th>
<th>AIR Industry</th>
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<tbody>
<tr>
<td>Pre-Event Est (no flood)</td>
<td>$10-25bn (30/08)</td>
<td>$12-26bn (29/08)</td>
</tr>
<tr>
<td>August Close (no flood)</td>
<td>$20-35bn (09/09)</td>
<td>$18-25bn (30/08)</td>
</tr>
<tr>
<td>Lloyd's Pick (inc flood)</td>
<td>$40-60bn (13/09)</td>
<td>$42-61bn (27/09)</td>
</tr>
<tr>
<td>Sept Close</td>
<td>$40-60bn (27/09)</td>
<td>$42-61bn (27/09)</td>
</tr>
<tr>
<td>Oct 9th</td>
<td>$40-60bn (27/09)</td>
<td>$42-61bn (27/09)</td>
</tr>
</tbody>
</table>

Actual insurance industry loss (Swiss Re figure) $66bn

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**Model Comparison**

**2008 Industry Florida Hurricane**

<table>
<thead>
<tr>
<th>Annual Aggregate Loss Costs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td>RMS 7.0</td>
<td>$2,609</td>
<td>$2,483</td>
<td>$24</td>
<td>$140,716</td>
</tr>
<tr>
<td>RMS 6.0</td>
<td>$1,724</td>
<td>$1,637</td>
<td>$18</td>
<td>$85,798</td>
</tr>
<tr>
<td>AIR 9.5</td>
<td>$2,510</td>
<td>$2,785</td>
<td>$32</td>
<td>$134,544</td>
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<tr>
<td>EQECAT 3.1</td>
<td>$3,030</td>
<td>$3,888</td>
<td>$2</td>
<td>$176,318</td>
</tr>
</tbody>
</table>

Source: A Comparison of Hurricane Loss Models, Journal of Insurance Issues, Cole, Macpherson, McCullough
### More Model Comparison

**1999-2001 Survey of 180 cat layers**

*Expected Loss variability between models*

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td><strong>Gulf (Texas – W Florida):</strong></td>
<td>50%</td>
</tr>
<tr>
<td><strong>Nationwide or Worldwide:</strong></td>
<td>30-40% (mostly due to Florida)</td>
</tr>
<tr>
<td><strong>NE Risks:</strong></td>
<td>Factor of 3</td>
</tr>
<tr>
<td><strong>New Madrid:</strong></td>
<td>Factor of 10</td>
</tr>
<tr>
<td><strong>FF Cal Quake:</strong></td>
<td>Thought to be &gt; factor of 10</td>
</tr>
</tbody>
</table>

*Florida convergence considered possibly due to Florida Commission on Hurricane Loss Projection Methodology*


### Your own Model Comparison

![Graph comparing model predictions for hurricane losses](image)
Independent Estimates

**Northern California Quake**

Source: Mistry and McSharry (2010), Working paper, Smith School of Enterprise and the Environment, University of Oxford
[www.smithschool.ox.ac.uk/crf](http://www.smithschool.ox.ac.uk/crf)

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**Effective EP Curves**

**Model EP Curve(s)**

**Multi-Model mixed**

**Effective EP Curve**
“Solvency II is the gift of financial services to the physical sciences”

*Lenny Smith*, Royal Society Conference 2010, "Handling uncertainty in science"
Lloyd’s SII Guidance - External Models and Data

A managing agent’s use of external models and data sets should be:

- appropriate to the nature and complexity of the risks incorporated in its risk strategy, business objectives and modelling methodologies;
- appropriate to the availability of internal data;
- and should be suitable for use in its internal model.

Its (Internal Model) documentation explains how it meets the requirements of the six internal model tests, namely the use test, the statistical quality standards test, the calibration standards test, the profit and loss attribution test, the validation standards test, and the documentation standards test. It must also identify whether the use of external models or data sets introduces any deficiencies into its internal model or data, and document how it has dealt with any such deficiencies.

A managing agent must recognise and document the risks arising from the use of external data sets and models. If the risks are material and quantifiable they should be taken into account in the SCR calculation. This should be accompanied by an explanation of how it has managed or mitigated those risks, and how it has reflected any material and quantifiable residual risks in its SCR calculation.
Data

Lloyd’s SII Guidance – Data Quality Standards

“A managing agent must establish a data policy, setting out its requirements on data quality and data update. This policy is subject to agreement with Lloyd's, and any major changes to it require prior approval from Lloyd's.

Accuracy, completeness and appropriateness must be demonstrated against these criteria:

• data used is free from material mistakes, errors and omissions (accuracy);
• data is to a large degree consistent in time such that the model output refers to a well-defined point in time (accuracy);
• it has at its disposal comprehensive data for all business lines under consideration and, where possible, all relevant model variables (completeness);
• no relevant data available is excluded from consideration without justification (completeness);
• the granularity of data is sufficient to allow for adequate actuarial and statistical techniques to be used (appropriateness);
• data used is relevant to its business and the portfolio of risks being analysed (appropriateness);
• data used for prediction exercises is a good guide to the future (appropriateness).”
Data Quality

CRITERIA:
- Timely?
- Accurate?
- Complete?
- Sensitivity Tests?

Source: Ernst & Young 2008, survey with a group of leading reinsurers

Sensitivity Tests

Source: Conor McMenamin, private communication
Internal Model

Solvency II – Internal Model

Source: Lloyd's
**Internal Model**

"An insurer must demonstrate that its internal model is widely used in and plays an important role in its system of governance."

- **Article 120 – Use Test**
- **Article 121 – Statistical Quality Standards**
- **Article 122 – Calibration Standards**
- **Article 123 – Profit & Loss Attribution**
- **Article 124 – Validation Standards**
- **Article 125 – Documentation Standards**

"Where practicable, insurers shall derive the SCR directly from the probability distribution forecast generated by the internal model, using:"

"Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period."

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**Lloyd's SII Guidance - Statistical Quality Standards**

"A managing agent must:"

- Identify all assumptions inherent to the internal model.
- Be able, at any time, to explain and justify in detail those assumptions to Lloyd's, taking account of all the following factors:
  - their significance;
  - how they limit the model, whether in terms of application or performance;
  - their implications for model risk, i.e. deviations between the model and reality;
  - possible alternative assumptions and their implications.
- Assess the materiality of assumptions chosen and possible alternatives. This requires a qualitative assessment. In line with the proportionality principle and where practicable and reasonable, an agent must conduct a quantitative assessment in addition.
- Document all internal model assumptions, their justifications and the corresponding procedure."
Tackling Model Risk – Round 2

• Model Risk
  – Fold it back into pdf
  or
  – Apply Adjustment Factors
  or
  – Add as Operational Risk

Adjustment Factors

<table>
<thead>
<tr>
<th>Category</th>
<th>Factor</th>
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<tbody>
<tr>
<td>Data</td>
<td>Insured Values</td>
</tr>
<tr>
<td></td>
<td>Interest Characteristics</td>
</tr>
<tr>
<td></td>
<td>Out-of-date Schedules</td>
</tr>
<tr>
<td></td>
<td>Geo-coding</td>
</tr>
<tr>
<td>Modelled inadequacies</td>
<td>Base Model Risk</td>
</tr>
<tr>
<td></td>
<td>BI</td>
</tr>
<tr>
<td></td>
<td>Demand Surge</td>
</tr>
<tr>
<td></td>
<td>Flood</td>
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<td></td>
<td>Storm Surge</td>
</tr>
<tr>
<td></td>
<td>Fire Following</td>
</tr>
<tr>
<td></td>
<td>Anti-selection</td>
</tr>
<tr>
<td>Non-modelled inadequacies</td>
<td>Tsunamis</td>
</tr>
<tr>
<td></td>
<td>Contingent BI</td>
</tr>
<tr>
<td></td>
<td>Loss Adjustment Expenses</td>
</tr>
</tbody>
</table>

ADJUSTING:
• Insured Values?
• Estimated Exposures?
• Damage Factors?
• Calculated Losses?
Operational Risk

- A frequency/severity pair?

- Frequency plus a severity histogram?

Still has to end up as a pdf for Kernel

Risk Appetite
Realistic Disaster Scenarios?
Once in 200 years VaR?

er ... what was it we were in business for?

Profitable Return on Capital
Around once every 25 years

Dimensions of Risk Decisions

Decision
Judgement!
Risk-Reward
The Risk-Reward View

- Solvency II Regulators assess risk at a once in 200 year risk of ruin (VaR) for 1 year's capital
  *whereas*
- Businesses run on a shorter time horizon such as once in 25 years risk of a certain level of loss (TVaR) and longer capital period such as 5-10 years
  *and*
- Make decisions on risk appetite of profit against loss
  *so*
- Regulators and Businesses are measuring two different risks in different ways
Different Return Periods, Different Risk Measures

- 1 in 100 Year VaR = $13m
- 1 in 25 Year VaR = $7m
- 1 in 200 Year VaR = $15m

Mean Profit = $8m

Probing the Risk Curve for Appetite

• High inflation environment (multi-year)
• High trade combined ratio + high growth environment
• Large catastrophe
• Severe equity market decline
• Large catastrophe
• Severe operational / continuity event

Examples of risk events:

<table>
<thead>
<tr>
<th>Severity</th>
<th>Capital Volatility</th>
<th>Model Economic</th>
<th>Rating Agency</th>
<th>Economic Rating Agency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Volatility</td>
<td>Static</td>
<td>Loss Severity</td>
<td>Capital Required for Ratings Targets</td>
<td></td>
</tr>
<tr>
<td>Tolerance</td>
<td>Horizon</td>
<td>Multi-year</td>
<td>Multi-year</td>
<td>Single-year</td>
</tr>
<tr>
<td>≥ X%</td>
<td>≤ Y%</td>
<td>≤ Z%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warning Level</td>
<td>Appetite Violation</td>
<td>Within Tolerance</td>
<td>Applicable Scenario</td>
<td></td>
</tr>
</tbody>
</table>

Source: ERM
Risk Appetite

- Set “loss measure” for Risk
- Reward relative to this Risk
- Portfolio Benefits
  - Diversification
  - Cash-flow
- Subject to regulatory capital constraint

Risk Appetite - Losses

**Harm/Chance**: What timescale for embarrassment?
**Time**: How long before failure is judged?
**Measure**: What is being protected?

<table>
<thead>
<tr>
<th>Factor</th>
<th>Factor</th>
<th>Regulator</th>
<th>Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harm/Chance</td>
<td>Loss Horizon</td>
<td>Risk of Ruin Once in 200 years</td>
<td>Risk of volatility embarrassment – 25 years (say)</td>
</tr>
<tr>
<td>Time</td>
<td>Capital Period</td>
<td>1 year</td>
<td>5 years (say)</td>
</tr>
<tr>
<td>Measure</td>
<td>Risk Measure</td>
<td>How much capital to stay within Loss Horizon?</td>
<td>How much would lose on average if Loss Horizon exceeded?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VaR</td>
<td>TVaR</td>
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</table>
Risk-Reward charts

Correlations
Correlations

- Typically:
  - Correlation matrix
  - Gaussian copula
  - (for cat) common event set

- But what of long-tail fat-tail correlations?

<table>
<thead>
<tr>
<th></th>
<th>LoB 1</th>
<th>LoB 2</th>
<th>LoB 3</th>
<th>Risk 2</th>
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<td></td>
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<td>Risk 2</td>
<td>0.1</td>
<td>0.1</td>
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</table>

Percentile Correlation Method
Percentile Correlation Matrices

1. Independent

<table>
<thead>
<tr>
<th>Percentile</th>
<th>25%</th>
<th>25%</th>
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<td>51-75%</td>
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<tr>
<td>26-50%</td>
<td>0%</td>
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2. Full Four by Four Correlation

<table>
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<tr>
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<th>0%</th>
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<tr>
<td>76-100%</td>
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<td>0%</td>
<td>0%</td>
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<td>51-75%</td>
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<td>0%</td>
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<td>0%</td>
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<tr>
<td>26-50%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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</table>

3. Low-end Correlation

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<thead>
<tr>
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<th>0%</th>
<th>0%</th>
<th>0%</th>
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<tr>
<td>26-50%</td>
<td>25%</td>
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</table>

4. High-end Correlation

<table>
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<tr>
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<tbody>
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</tr>
<tr>
<td>51-75%</td>
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<td>0%</td>
</tr>
<tr>
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<td>0%</td>
</tr>
</tbody>
</table>

5. Fuzzy Low-end

<table>
<thead>
<tr>
<th>Percentile</th>
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</thead>
<tbody>
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<tr>
<td>51-75%</td>
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<td>0%</td>
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<td>0%</td>
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</tbody>
</table>

6. Fuzzy High-end

<table>
<thead>
<tr>
<th>Percentile</th>
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<th>0%</th>
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</thead>
<tbody>
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<td>0%</td>
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</tr>
<tr>
<td>51-75%</td>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Independent

- Correlation Matrix
- Mean: 48.2, SD: 17.2, Var: 76.0, VaR: 97.0
- Diversification Benefit: 11.3

Marginal pdfs

Total pdf
Correlation Matrix

<table>
<thead>
<tr>
<th>Mean</th>
<th>SD</th>
<th>Var 25</th>
<th>Var 50</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>48.3</td>
<td>16.9</td>
<td>75.0</td>
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<tr>
<td>B</td>
<td>37.7</td>
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<tr>
<td>Total</td>
<td>86.1</td>
<td>37.9</td>
<td>155.0</td>
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Diversification Benefit: 1.9 5.0 17.0

High-end

<table>
<thead>
<tr>
<th>Mean</th>
<th>SD</th>
<th>Var 25</th>
<th>Var 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>48.1</td>
<td>17.4</td>
<td>76.0</td>
</tr>
<tr>
<td>B</td>
<td>37.7</td>
<td>21.0</td>
<td>80.0</td>
</tr>
<tr>
<td>Total</td>
<td>85.8</td>
<td>38.4</td>
<td>158.0</td>
</tr>
</tbody>
</table>

Diversification Benefit: 3.6 7.0 20.0
Capital Allocation

Classes of Business with same 99.5% capital requirement and mean profit
When is a Loss a Profit?

Profit for Cost of VaR Capital 20% pa

Representing Return Variability
RI Credit Risk

The usual
• Credit Ratings
• Diversification with multiple reinsurers
• Correlations between reinsurers and
• Market Risk Correlations to Catastrophe!
Where Solvency II Meets Business

Regulators are obsessed with the downside for extremities about which we know little

whereas

Business is about making profit at levels of exposure and over timescales about which we know quite a lot

Contact

peter.taylor@philosophy.ox.ac.uk

peter.taylor@conductor.com
Loss Probability Curve and VaR

The illustration is the Value at Risk for 99% (1 in 100 years)

Area under curve to right of $20m is 1%

Annual Probability Density \( p(x) \)

Area under curve to right of $20m is 1%

\[
\text{Loss} \begin{array}{c}
\text{Area under curve to right of } \$20m \text{ is } 1\
\end{array}
\]

\[
\int_{\text{Loss}}^{\infty} \left( 1 - \alpha \right) p(x) \, dx = 1 - \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} xp(x) \, dx
\]

EP Curve and TVaR

\[
\text{ES} = \text{renormalised TVaR} \quad \text{ES}(\alpha) = \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} xp(x) \, dx
\]

TVaR = average excess of VaR for percentile \( \alpha \):

\[
\text{TVaR}(\alpha) = \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} xp(x) \, dx
\]

The EP curve is:

\[
\text{EP}(y) = \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} p(x) \, dx = 1 - \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} p(x) \, dx
\]

Hence:

\[
\text{TVaR}(\alpha) = \text{VaR}(\alpha) \left( 1 - \alpha \right) + \frac{1}{1 - \alpha} \int_{\text{VaR(\alpha)}}^{\infty} \text{EP}(x) \, dx
\]

Up to VaR point:

\[
\text{VaR}(\alpha) \left( 1 - \alpha \right)
\]

Area under curve:

\[
\int_{\text{VaR(\alpha)}}^{\infty} \text{EP}(x) \, dx
\]

1%