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THEORY AND PRACTICE OF MULTINATIONAL POOLING

by

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1. Introduction

1.1 It may be said that the topic of this paper has hitherto received scant attention in the annals of our profession although, from another viewpoint, it can be seen to be no more than a fairly elementary application of risk theory. Certainly, when the author was commissioned by his office to develop the technical basis of multinational pooling, he found himself in what seemed to be uncharted territory. Relieved of the chore of ploughing through earlier work on the subject, he was able to indulge in the illusion of original thinking from first principles together with the delight of constructively playing with a newly-acquired small personal computer. (The personal note struck here should not mislead the reader into thinking that this was a solo project. The author was greatly assisted in the technical development by colleagues in his office and in the offices of other insurers belonging to the international network of which his office is a member.)

1.2 A full description of the commercial aspects of this topic will be found in an excellent paper, entitled "Multinational Experience Rating", presented to the Faculty of Actuaries Students' Society on 3 December 1979 by G.C. Archibald and J. Wallace. The author of the present paper hopes to rely on the former as compulsory prior reading so that, in a sense, he can pick up certain aspects of the subject where the previous authors left them. However, a brief sketch of the commercial background, bringing out the salient points, may be welcomed by some readers.

1.3 Multinational pooling is a technique used by insurers throughout the world to attract and hold the business of multinational companies. A network of cooperating insurers is put together, including as far as can be arranged a member in each of the main countries in Europe, America and the rest of the world where multinational companies operate their businesses. The network offers to its multinational customers the facility of combined information and accounting in respect of their employee benefit schemes throughout the world, underwriting by reference to the world-wide membership of these schemes, and the added attraction of an experience-rated international dividend.

The sole concern of this paper is with the experience-rated international dividend, the stop loss premium or risk charge which makes this possible and the apportionment of this charge, and the liability associated with it, among the network insurers.

1.4 The international dividend is the bottom line of a yearly International Profit and Loss Account, drawn up for each multinational customer of the network, putting together the results in separate National Profit and Loss Accounts (in their various currencies) prepared by the network insurers.
The following is a simplified layout for the National Profit & Loss Account:

**INCOME ITEMS**
- Reserves brought forward
- Experience-rated premiums
- Inward transfers of reserves
- Non-rated claims
- Investment income

**OUTGO ITEMS**
- Reserves carried forward
- Experience-rated claims
- Outward transfers of reserves
- Surrender values
- Non-rated risk premiums
- Local dividends
- Commissions
- Taxes
- Administrative expenses

PROVISIONAL PROFIT OR LOSS = INCOME \(\rightarrow\) OUTGO

1.5 Each contract type is represented by a separate column in the Account - there would be separate columns for death-in-service, disability, retirement, endowment, accident and medical benefits.

Note that the Account is not limited to pure risk benefits but includes substantive contracts as well. However, in the U.S.A. and U.K., and certain other countries whose employee benefit arrangements follow a similar pattern, substantive contracts - typically funded by deposit administration, managed fund or independent investment - are almost invariably excluded.

Reserves are included in the account not only in respect of substantive contracts but also to allow for annual premium costing, claims in payment, unexpired premiums, disability benefit deferred periods and deferred local dividends.

Transfers of reserves may take place between different insurers within the network, when employees of the multinational are relocated to other countries.

Some benefits may be excluded from experience-rating, e.g. the top slice of individual benefits which exceed the average by a factor of 4 or 5, and reserved for strictly non-profit treatment. The corresponding risk premiums and any relevant claims are explicitly removed from the Account.

1.6 Each National Profit & Loss Account produces a net profit or loss on the year, before deduction of the risk charge.

This is usually accumulated at the appropriate local rate of interest to the predetermined yearly settlement date and to it is added (algebraically) any profit or loss brought forward in the National Account from the previous annual settlement date, together with a year’s interest thereon.
The net overall profit or loss as at the yearly settlement date, before deduction of the risk charge, is then converted to a common currency, usually U.S. dollars, and carried to the International Profit and Loss Account. This latter Account deals with items which depend on the specific structure of the pooling arrangement, namely

- the stop loss premium or risk charge
- the reallocation of losses among the network insurers
- the writing-off of excess loss
- the operation of any contingency fund
- and the international dividend.

The precise meaning and method of determination of these items will become apparent as the paper develops.

1.7 The development of the paper is in accordance with the following sequence of subject headings:

1. Introduction.
2. The fundamental experience-rating structure: Full Stop Loss.
3. The concept of expected surplus.
4. Theme and Variations - the diversity of structure of Losses-carried-forward arrangements.
5. The interaction of structure and risk charge.
6. The interrelation of network insurers and the apportionment of risk and risk charge.
7. Miscellaneous problems.
8. Technical description of the computer simulation techniques and pascal programs used in arriving at the results presented in this paper.
9. Tables for approximate computation of the risk charge based on the poisson distribution.

1.8 It should be said at the outset that a purist will be disappointed by the lack of rigour in the mathematics to be found in this paper. The author's preference and degree of mathematical attainment incline him very much to a heuristic or geometrical approach, backed up by supporting evidence from computer simulation.
2. The fundamental experience-rating structure: Full Stop Loss.

2.1 The fundamental experience-rating structure, the basic theme on which all other structures are variations, is Full Stop Loss. In its simplest form this can be understood, and readily manipulated, in terms of a pure risk group assurance with uniform sums at risk on all the lives assured. For large groups, and risks with a relatively low incidence, such as the risk of death or of becoming seriously disabled, it is well known that the risk is closely modelled by the poisson random variable.

2.2 As we are deliberately starting from first principles it should be recorded here that underlying the poisson model is the still more fundamental binomial model. If the rate of incidence over a specified period is q, and n independent lives assured are exposed during that period, the binomial model gives

\[ \frac{n!}{x!(n-x)!} q^x (1-q)^{n-x} \]

as the probability of x occurrences, so that the mean number of occurrences is nq and the variance is npq. The poisson model is an excellent approximation to the binomial for "large" n and "small" nq so that, writing nq = m, whence q = m/n,

\[ \frac{n!}{x!(n-x)!} \left( \frac{m}{n} \right)^x \left( 1 - \frac{m}{n} \right)^{n-x} \rightarrow \frac{m^x}{x!} e^{-m} \]

as n \( \to \infty \) while m/n remains finite.

It follows that for large, albeit finite, m/n a much higher exposure is required by the poisson approximation. This will be true of the risk involved in medical expenses insurance, for example.

2.3 In the highly rarefied realm of abstract theory where this paper has started out it can be assumed that

(i) the expected number of claims per annum is m,

(ii) the fixed sum at risk for each life assured is B,

and

(iii) the expected surplus under the contract, i.e. the excess of the premiums over expected claims and expenses, is 100S% of the expected claims,

i.e. expected surplus = SmB.

If a stop loss charge of 100R% of the expected claims is specified, and a dividend of the excess, if any, of the premiums over claims, expenses and stop loss charge is paid each year, then the expected value of this dividend is

\[ B \sum_{x=0}^{x=c} \left[ m(1+S-R) - x \right] p(x) \]

where c is the greatest integer less than or equal to \( m(1+S-R) \)

and \( p(x) = \frac{m^x}{x!} e^{-m} \)
2.4 Because of the elegant property of the poisson process that
\[ \sum_{x=0}^{\infty} p(x) = m \sum_{x=0}^{\infty} p(x-1) \]
the summation in 2.3 reduces neatly to
\[ mB \left[ (S-R) \sum_{x=0}^{\infty} p(x) + p(c) \right] \]

2.5 With even greater elegance, should S equal R, the expected value of the dividend reduces to
\[ mB \sum_{x=0}^{\infty} p(m) \]
Further, if we then invoke Stirling’s approximation (which is very good), viz.
\[ n! \approx \sqrt{2\pi n} \left( \frac{n}{e} \right)^n \]
we obtain
\[ B\sqrt{\frac{m}{2\pi}} \]
where \( B\sqrt{m} \) is the standard deviation of the claims distribution and \( (\sqrt{2\pi})^{-1} \) is, very nearly, 40%.

2.6 Thus, if
(i) it is intended to give away the whole of the expected surplus in the form of dividends and
(ii) the expected surplus is equal to the resultant stop loss premium,
the latter turns out to be (for this highly simplified abstract model) 40% of the standard deviation of the claims distribution.

2.7 Let us now begin to move from the realm of idealized abstract theory towards down-to-earth practical reality. The first complication which needs to be introduced is the heterogeneity of the sums at risk. This immediately puts into question the relevance of the poisson model, but it has been found that, in coping with the degrees of heterogeneity normally encountered in group assurances, the model gives an excellent approximation. The procedure followed is simply to fit the poisson model to the heterogeneous claims distribution by equating means and variances.

2.8 Suppose the sum at risk on the ith life assured is \( B_i \) and the rate of incidence of the risk for that life is \( q_i \). Then the variance of the claims distribution is
\[ \sum_{i} B_i^2 q_i \]
In developing the matching poisson distribution it is convenient to define \( B \) and \( H \), where
\[ \bar{B} = \frac{\sum_{i} B_i q_i}{\sum_{i} q_i} \]
i.e. \( \bar{B} \) is the mean claim (not necessarily equal to the mean sum at risk, unless there is no correlation between \( B_i \) and \( q_i \)), and \( H \) is given by
\[ \sum_{i} B_i^2 q_i = H^2 \bar{B}^2 \sum_{i} q_i \]
The distribution can then be represented by a poisson variable with uniform sum at risk $B'$ and expected number of claims $m'$, such that

$$B' m' = \sum_i B_i q_i = \bar{B} m$$

and

$$(B')^2 m' = H^2 \bar{B}^2 m$$

Solving for $B'$ and $m'$, we obtain

$$m' = \frac{m}{H^2}$$

and

$$B' = H^2 \bar{B}$$

2.9 As we shall see, the most convenient statistical parameter for determining the equivalent poisson-based distribution is the coefficient of variation, i.e. the standard deviation scaled by the mean.

Thus, the coefficient of variation of the poisson-based distribution derived from the heterogeneous distribution characterized by $m$, $\bar{B}$ and $H$ is

$$\frac{1}{\sqrt{m'}} = \frac{H}{\sqrt{m}}$$

while the standard deviation = expected claims x coefficient of variation

$$= B' m' \frac{H}{\sqrt{m}} = \bar{B} m \frac{H}{\sqrt{m}} = \bar{B} H \sqrt{m}$$

2.10 It has been tacitly assumed so far in this paper that the sum at risk is a lump sum payment. Let us now consider the situation which arises when the sum at risk is a contingent series of payments, rather than a lump sum or a series of payments certain. The insurance of long-term disability is typically of this character.

It would be convenient if we could allow for the additional variation of the sum at risk, deriving from the varying duration of a contingent income benefit, as another dimension of the heterogeneity considered in section 2.8.

Let us see where this idea takes us. At this stage we are still limiting ourselves to the consideration of the Full Stop Loss model.

Suppose that, during the insurance period in question, we have $n$ active lives with average rate of incidence $q$, average cost per new claim (= payout during the period plus reserve at the end of the period) $B$, coefficient of heterogeneity of the annual benefit $H$ and a further adjustment factor of $K$ to allow for the variation within the year of the cost per new claim.

Suppose further that there are $m$ claims in payment at the beginning of the period with expected average termination rate $t$, average release of reserve per claim terminating $C$ and coefficient of heterogeneity $H'$. 
First consider $K$:

suppose we expect 30\% of new claims arising in the insurance period to terminate before the year's end with an average payout of 7\% of $B$ and the other 70\% to persist with a payout plus end-of-year reserve of

$$\frac{B \left(1 - 0.07 \times 0.3\right)}{0.7} = 1.4B$$

(ignoring interest);

then

$$K = \frac{\sqrt{3 \times (0.07B)^2 + 0.7 \times (1.4B)^2}}{3 \times (0.07B) + 0.7 \times (1.4B)} = 1.18$$

which argues for taking $K = 1.2$.

Then a reasonable estimate of the standard deviation of the net expected strain, i.e. the expected strain from new claims minus the expected release from terminations, is given by

$$\sqrt{\frac{nqH^2K^2B^2 + mt(H')^2C^2}{nqH^2B^2}}$$

and it can be shown that the extra dimension of heterogeneity can be accommodated by applying the multiplier

$$\sqrt{\frac{nqH^2K^2B^2 + mt(H')^2C^2}{nqH^2B^2}}$$

to the coefficient of variation for the incidence of new claims and taking the expected claims throughout as $nqB$.

Putting some realistic figures into this expression we can see how it might work in a typical case:

Let $m/n = .015$

$HK = 1.2 \times 1.2 = H'$ (say)

$C = 2B$

$q = 0.3\%$

$t = 6\%$

Then the adjustment factor for the coefficient of variation is

$$1.2\sqrt{0.003 + 0.015 \times 0.06 \times 2^2} = 1.18$$
The disability risk would be covered alongside more straightforward risk contracts in a multinational pooling arrangement. It is convenient to note here the formula for calculating the overall combined coefficient of variation, when the expected claims (expected new claims for LTD contracts) and coefficient of variation for the i th contract of the j th insurer are $E_{ij}$ and $V_{ij}$ respectively, viz.

$$V = \sqrt{\sum_{j} \sum_{i} \frac{E_{ij}^2 V_{ij}}{E_{ij}}}$$

The arithmetic in this section is very rough and approximate, giving no more than a broad idea of the order of magnitude involved. Some work already done with computer simulations suggests that the use of the poisson model for the disability risk errs on the side of the insurers to the extent that the adjustment factor found by the method of this section could be reduced by some 10%, e.g. from 1.78 to .9 x 1.78 = 1.6. More computer work needs to be done on disability insurance, though it is perhaps salutary to recall that we have in general much less statistical knowledge of the disability risk than of the mortality risk. It is also the case that the expected surplus under disability insurance would normally be enhanced by interest surplus on claim reserves and that this will go some way towards offsetting the effect of the greater stochastic variation.

This formula does, of course, assume that all the contracts are mutually independent. Clearly this is untrue of a combination of life and disability insurance, though the overlap between disability terminations and life claims may be ignored for practical purposes.

Similarly, the basic poisson model assumes independence of the lives assured. The only serious breach of this condition is the risk of multiple claims, which may give rise to concern in the case of a multinational which runs its own fleet of executive aircraft.

2.11 We can tabulate the function $A(U,V)$, where

$$A(U,V) = (U-1) \sum_{x=0}^{\infty} p(x) + p(c)$$

where $c$ is the greatest integer less than or equal to $\frac{U}{V^2}$, $V = \frac{H}{\sqrt{m}}$ and

$$p(x) = \frac{1}{x!} V^{-x} e^{-V}$$

for all relevant values of $U$ and $V$.

The stop loss premium for any claims distribution for which the poisson-based distribution is a good approximation is given by the equation

$$S = A(1+S-R,V)$$

It is then a simple matter to derive a tabulation of the explicit function

$$R = \phi(S,V)$$
The special case discussed in 2.5 is given by

\[ S = R = A \left( 1, \sqrt{\frac{V}{2\pi}} \right) \]

Clearly, if \( S > \sqrt{\frac{V}{2\pi}} \) or \( S < \sqrt{\frac{V}{2\pi}} \) then \( R < \sqrt{\frac{V}{2\pi}} \) or \( R > \sqrt{\frac{V}{2\pi}} \).

The functions \( A \) and \( \phi \) are tabulated in section 9 of this paper. As we shall see, they can be extended to give a good working approximation to the risk charge in those more complex experience-rating structures which involve the carrying-forward of losses and/or profits.

2.12 It is worth noting here the following useful properties of these functions:

\[ K \phi \left( \frac{S}{K}, \frac{V}{K} \right) = \phi (S, V) \]

and \( \frac{S}{K} = A \left( 1 + \frac{S-R}{K}, \frac{V}{K} \right) \; \rightarrow \; S = A \left( 1 + \frac{S-R}{K}, \frac{V}{K} \right) \)

2.13 It is now time to descend even further towards the murky depths of down-to-earth practical reality as we set out to determine what practical meaning can be attached to the crucial concept of "expected surplus".
3. **The concept of expected surplus**

3.1 We have so far defined *expected surplus* simplistically as the excess of the gross premium over the expected claims and expenses. This definition suffices if we are considering a one-year term assurance with premiums and claims payable continuously throughout the year. Even in this case, however, there is a further small element deriving from the persistence of the surplus throughout the year and the interest (say, half a year's worth) earned thereon. Most networks do not complicate their accounting by specific inclusion of this element in the Profit and Loss Accounts but it is a simple matter to allow for it in the determination of the risk charge. Thus the implicit equation for the risk charge may be

\[ S (1 + \frac{1}{2} i) = A (1 + S - R, V) \]

instead of

\[ S = A (1 + S - R, V) \]

3.2 It is convenient to relate all the other premium components to expected claims, expressing them as a percentage of this quantity. This seems to imply that the expected claims are a known fixed quantity at the very heart of the premium structure. Of course, this is far from the truth of the matter. In practice we simply do not know what claims to expect under any given multinational pooling arrangement.

3.3 The quantity "expected claims" has to be estimated on the basis of the available information. This comprises:

(i) the most recent claims experience of each insurer in respect of the whole of its group assurance business for the risk in question

(ii) the wider, national experience (e.g. in the U.K. that described in the publications of the Office of Population Censuses and Surveys) of the variation of the risk with respect to social, geographical and occupational factors, supplemented as far as it goes by the insurer's own differential claims experience

and

(iii) the claims experience to date of the given multinational policyholder.

Item (i) may be used to construct a standard table or to modify an existing standard table. If the former technique is used the raw data would be subjected to a graduation process while the latter technique has the advantage that the graduated table already exists and requires only adjustment by reference to an appropriately smoothed set of factors.

A rational procedure for deriving the expected claims for a given case would begin with an estimate based on the standard table and proceed by applying a factor, judged to be appropriate to the case, based on item (ii). The result at this stage may be referred to as the a priori expected claims.

3.4 Proceeding independently from item (iii) a second estimate of expected claims may be derived on the assumption that the future will mirror the past. We may then calculate what may be termed the a posteriori expected claims as a weighted mean of these two estimates.

The next section of this paper develops a possible theoretical basis for the rational determination of the weights to be used in the calculation of the a posteriori expected claims. The weight applied to the estimate based on the assumed simple duplication of the case's own past experience may be referred to as the credibility factor. For simplicity, it is assumed that there has been no significant change in the constitution of the risk or in the sum at risk or in the assured group of lives; should this not be the case a considerable degree of improvisation may be needed in constructing an estimate of the expected claims and, as always, when in doubt the actuary will err on the side of the insurers.
3.5 The theory begins with the actual claims experience to date, i.e. item (iii) in paragraph 3.3. Corresponding to this actual experience there was an expectation based on the standard table derived from item (i) in paragraph 3.3, as modified by item (ii). (If the actual claims experience goes back some years, a purist would argue for some backwards extrapolation of the secular trend in mortality, in order to ensure a comparison of like with like.)

The theoretical development starts with the consideration of numbers of claims rather than amounts. This is inevitable while we take the poisson to be the fundamental distribution but, as in 2.8, it will be found possible to accommodate the heterogeneity of the sums at risk, with a reasonable degree of approximation, by extending the poisson model.

3.6 We suppose the a priori expected number of claims (m, say) to be the central value - let us say, the arithmetic mean - of a distribution, ranging from "light" to "heavy" underlying claims potential, which may be termed the a priori distribution, with probability density function \( p_1(x) \).

Then

\[
E(x) = \int_0^\infty x \cdot p_1(x) \, dx = m
\]

and

\[
\sigma^2(x) = \int_0^\infty (x - m)^2 \cdot p_1(x) \, dx = K^2 \cdot m^2, \text{ say}
\]

The probability of obtaining \( d \) claims from a particular case whose a priori expected claims, defined by its position in the a priori distribution, number \( x \) is \( p_a(d|x) \).

In any given case we know only

(i) the actual claims \( d \)

and

(ii) the assumed a priori distribution.

If we had a large number of cases randomly selected from the population represented by the a priori distribution and we picked out all those cases which experienced \( d \) claims during a specified period we would expect the incidence of cases with a priori expected value \( x \) in our sample to be given by

\[
p_a(d|x) = \frac{\int_0^\infty p_a(d|x) \cdot p_1(x) \, dx}{\int_0^\infty p_a(d|x) \cdot p_1(x) \, dx}
\]

(Bayes' Theorem).

\( p_a(x|d) \) is then the probability density function of what we may call the a posteriori distribution and is the key to the theoretical determination of the a posteriori expected claims in respect of a "random" case with \( d \) actual claims.
3.7 We cannot proceed further (as is well known in Bayesian Theory) without postulating the shape of the a priori distribution, which means both assigning to it a mathematical function and quantifying the parameters of that function. It will come as no surprise that the function chosen by the author was the probability density function of the normal distribution, namely

$$p(x) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-m}{s} \right)^2}$$

The parameter $s$ is taken to be equal to $m/6$, so that the effective range of the a priori distribution is from $m/2$ to $3m/2$, with very low probabilities associated with the extremities of this range.

3.8 There are now at least two possible ways of proceeding and the one chosen may depend upon the relative intractability of the associated mathematics. We wish to determine the expected value of the a posteriori distribution, given the assumed normality of $p_1$, and poisson for $p_2$. As it stands, the author found this to be mathematically intractable. To remedy this we can either shift our focus of interest from the mean to the mode of the a posteriori distribution or we can substitute another, more mathematically tractable, distribution for the normal. The latter course was suggested by one of the author’s colleagues, Mike Bolton, who put forward the gamma function as giving a good approximation to the normal distribution over the ranges of values of $m$ and $s$ which are likely to concern us in dealing with multinationals. We shall consider both approaches.

3.9 We have

$$p_2(x|d) \propto \frac{x^d}{d!} e^{-x} \cdot \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-m}{s} \right)^2}$$

i.e. $\propto x^d e^{-\left[x + \frac{1}{2} \left( \frac{x-m}{s} \right)^2\right]}$

To determine the mode we are interested in the turning values of this function, as $x$ changes, and in fact in the largest of the turning values within the range $(d,m)$.

Differentiating, and eliminating non-zero factors, we obtain

$$x^2 - x (m-s^2) - s^2 d = 0$$

whence $x$ may be determined from

$$x = \frac{1}{2} \left[ (m-s^2) \pm \sqrt{(m-s^2)^2 + 4s^2 d} \right]$$

and $x \in (d,m)$

Thus, suppose $m = 10$, $s = 10/6$ and $d = 7$.

Then $x = \frac{1}{2} [7.222 \pm \sqrt{52.160 + 77.778}] = 9.3$.

This gives the mode of the a posteriori distribution. As we shall see, in this case the mean derived from the gamma approximation is also 9.3.
3.10 It can readily be seen that, for very small $s$, as $s \to 0$, $x = m$ as it should. Moreover, for very large $s$, as $s \to \infty$

$$\frac{s^2}{m} \rightarrow \infty$$

$$x^2 = (m - s^2) - s^2d \rightarrow x^2 + s^2x - s^2d = 0$$

i.e.,

$$d = \infty \left(1 + \frac{x}{s^2}\right)$$

but, since $x$ and $m$ are commensurate,

$$\frac{x}{s^2} \rightarrow 0 \text{ as } \frac{s^2}{m} \rightarrow \infty$$

$$d \rightarrow \frac{d}{1 + \frac{x}{s^2}}$$

as it should.

3.11 We next explore the gamma approximation.

We write

$$p(x) = \frac{1}{s \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-m}{s})^2} = \frac{1}{(a-1)!} \left(\frac{b}{a}\right)^x e^{-bx}$$

where

$$E(x) = m = \int_0^\infty x p(x) \, dx = \frac{1}{(a-1)!} \int_0^\infty \frac{b}{a} x^{a-1} e^{-bx} \, dx = \frac{a}{b}$$

and

$$E(x-m)^2 = \frac{s^2}{b^2} = E(x^2) - \left[E(x)\right]^2$$

$$= \left[\frac{1}{(a-1)!} \int_0^\infty \frac{b}{a} x^{a+1} e^{-bx} \, dx\right] - \left[\frac{a}{b}\right]^2$$

$$= \frac{(a+1)a}{b^2} - \frac{a^2}{b^2} = \frac{a}{b^2}$$

whence

$$a = \left(\frac{m}{s}\right)^2$$

and

$$b = \frac{m}{s^2}$$

Thus if, as before, $s = m/6$, we have

$$a = 36$$

$$b = \frac{36}{m}$$

Thence:

$$p_s(x \mid d) \propto \frac{\frac{x}{d!} e^{-x} \cdot \frac{1}{(a-1)!} \left(\frac{b}{a}\right)^x e^{-bx}}{\int_0^\infty \frac{x}{d!} e^{-x} \cdot \frac{1}{(a-1)!} \left(\frac{b}{a}\right)^x e^{-bx} \, dx}$$

$$\propto \frac{\left(\frac{b}{a}\right)^x e^{-bx}}{\left(\frac{b}{a}\right)^{x-d} e^{-(x-d)b}}$$

$$\propto \left(\frac{b}{a}\right)^x e^{((a-1)b - d)x}$$
Now the denominator =

\[
\int_0^\infty \frac{(a+d-1)!}{d! (a-1)!} \cdot \left(\frac{1}{1+b}\right)^d \cdot \left[\frac{1}{(a+d-1)!} (1+b)^{a+d-1} e^{-(1+b)x}\right] dx
\]

\[
= \frac{(a+d-1)!}{d! (a-1)!} \cdot \left(\frac{1}{1+b}\right)^d \cdot \left(\frac{1}{1+b}\right)^d
\]

since the expression in square brackets is a gamma function which sums to unity over the range \((0, \infty)\).

\[
\therefore p_\theta(x|d) = \frac{1}{(a+d-1)!} (1+b)^{a+d-1} e^{-(1+b)x}
\]

which is immediately recognisable as our friendly gamma function, where \((a+d)\) has replaced \(a\) and \((1+b)\) has replaced \(b\).

Whence:

\[
E(x|d) = \frac{a+d}{1+b} = d \cdot \left(\frac{1}{1+\frac{m}{s^2}}\right) + m \cdot \left(\frac{\frac{m}{s^2}}{1+\frac{m}{s^2}}\right)
\]

and

\[
\text{var}(x|d) = \frac{a+d}{(1+b)^2} = E(x|d) \cdot \left(\frac{1}{1+\frac{m}{s^2}}\right).
\]

Thus the credibility factor obtained from this theoretical development is which can be expressed more imaginatively as

\[
\frac{1}{1 + \left(\frac{V_m}{V_p}\right)^2}
\]

where

\(V_p\) is the coefficient of variation of the a priori distribution

and

\(V_m\) is the coefficient of variation of the poisson distribution with mean equal to the mean of the a priori distribution.

Clearly, this is a more elegant formulation than that of section 3.9 because it leads to an explicit formula for the credibility factor instead of leaving it implicit within the solution of a quadratic equation. As in section 3.10, if \(s \to 0\) then also \(V_p \to 0\), whence \(V_m/V_p \to \infty\) and the credibility weight tends to zero. If \(s^2/m \to \infty\) then \(m/s^2 \to 0\) and the credibility weight tends to unity.

In the example studied in section 3.9 the credibility was low - only 22%. Suppose now that \(m = 100\), \(s = 100/6\) and \(d = 70\).

As before \(V_p = 1/6\) but now \(V_m = 1/10\) and

\[
\frac{1}{1 + \left(\frac{V_m}{V_p}\right)^2}
\]

gives 73\(\frac{1}{2}\)% credibility.
The a posteriori modal value is
\[ \frac{1}{\alpha} \left[ (100 - 277.8) + \sqrt{(171.8)^2 + 280 \times 277.8} \right] = 76.5 \]

The a posteriori mean from the gamma approximation is
\[ (13\frac{1}{2}\% \text{ of } 100) + (26\frac{1}{2}\% \text{ of } 100) = 78. \]

Clearly, to repeat a familiar criticism of the a priori approach, the result depends on the shape and the spread assumed for the a priori distribution. A lower value of \( s \) pushes us back towards the a priori mean. Thus, for \( \mathbb{V}_p = 1/10 \) and \( \mathbb{V}_m/\mathbb{V}_p = 1 \), we have only 50\% credibility and an a posteriori mean of 85. The a posteriori mode is then
\[ \frac{1}{\alpha} \left[ (100 - 100) + \sqrt{0 + 280 \times 100} \right] = 83.7 \]

Taking one more example, suppose that \( m = 100, \mathbb{V}_p = 1/10 \) and \( d = 130 \) or 100. The a posteriori means are 115 and 100, using gamma, and the a posteriori modes are
\[ \frac{1}{\alpha} \left[ (m - s^2) + \sqrt{(m-s^2)^2 + 4s^2m} \right] \]

The latter result follows more generally, as it should, by putting \( d = m \) in the quadratic, so that
\[ \frac{1}{\alpha} \left[ (m - s^2) + \sqrt{(m-s^2)^2 + 4s^2m} \right] = 1 \]

3.13
This is as far as we shall take the theory of “expected claims” in this paper. It is, at best, a shaky foundation on which to base a necessary judgment, without which there can be no estimation of expected surplus. John Lockyer, in his excellent paper on “Group Life Assurance” presented to the Society on 30th March, 1982, mentioned that it is common practice in the U.S. Group Life assurance market to regard as few as 10,000 life-years of experience as enough to justify a credibility factor of 100\%. It is interesting to apply the theory of this paper to deduce the coefficient of variation of the a priori distribution which would indicate a credibility factor of, say, 90\% for 10,000 life-years’ experience with an a priori mean of, say, 40 claims. Allowing for a coefficient of heterogeneity (the \( H \) in section 2.8 of this paper) of 1.15, the a priori coefficient of variation is given by
\[ \frac{1}{\alpha} \left( \frac{\mathbb{V}_m}{\mathbb{V}_p} \right)^2 = 0.9 \]
\[ \therefore \mathbb{V}_p = (0.1)^2 \mathbb{V}_m = \frac{3 \times 1.15}{\sqrt{40}} = 0.55 \]
i.e. the a priori distribution would range, taking only 2 standard deviations on either side of the mean, from zero up to 200\% of the mean, except that the gamma distribution is now no longer approximately normal.

In the author’s view this is a reductio ad absurdum and implies that the philosophical basis for such a high credibility factor must be quite different from that discussed in this paper. Indeed there appears to be a denial of the fundamental assumptions of random selection from the a priori distribution (see section 3.6) and of the essential randomness of the claims themselves in favour of some mysterious alchemy which guarantees that history will repeat itself.
Of course, if the a priori distribution were uniform and rectangular, with equal incidence for all possible values of expected claims, rather than normally peaked as we have pictured it, we would have a radically different perspective. On this basis

\[ p_a(x|d) = \frac{x^d e^{-x}}{\int_0^\infty \frac{x^d}{d!} e^{-x} dx} = \frac{x^d}{d!} e^{-x} \]

which is a gamma distribution with \( b = 1 \) and \( a = d + 1 \). Whence

\[ E(x|d) = d + 1 = \text{var}(x|d) \]

while the mode of the distribution is \( d \).

As already indicated by the treatment of the example in this section, it is suggested that the increased variation of the claims experience introduced by the heterogeneity of the sums at risk be brought into the determination of the credibility factor by a suitable adjustment to \( V_m \), increasing the pure poisson factor by 10% to 30% depending on the heterogeneity of the sums at risk.

The following sequences give a useful indication of the heterogeneity implied by various values of \( H \):

<table>
<thead>
<tr>
<th>( H )</th>
<th>1.22</th>
<th>1.26</th>
<th>1.28</th>
<th>1.12</th>
<th>1.19</th>
<th>1.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>90% of lives assured for 1, 10% for 4</td>
<td>90% average 1 with ( H = 1.1 ), 10% for 4</td>
<td>90% of lives assured for 1, 10% for 4 but with double mortality of the other 90%</td>
<td>90% of lives assured for 1, 10% for 3</td>
<td>90% average 1 with ( H = 1.1 ), 10% for 3 but with double mortality of the other 90%</td>
<td>90% of lives assured for 1, 10% for 2</td>
<td></td>
</tr>
</tbody>
</table>

3.14 It was seen in section 3.11 that the variance of the a posteriori distribution is the product of the expected value and the credibility factor. For 100% credibility the distribution is effectively poisson with coefficient of variation \( d^{-\frac{1}{2}} \); for 50% credibility it is much more compact than poisson with coefficient of variation

\[ \frac{1}{a} \left( \frac{m+d}{a} \right)^{-\frac{1}{2}} \]

The a posteriori distribution is not symmetrical but, for practical purposes, its skewness may be disregarded. For the values of \( (a + d) \) with which we are likely to be concerned, the a posteriori distribution is approximately normal with variance equal to the product of the mean and the credibility factor. Further, it is convenient and not unduly inaccurate to represent the full distribution by two points situated at the “centres of gravity” of the two halves of the distribution on either side of the mean, i.e. at \( E(x|d) \times (1 \pm 0.7 V) \) where \( V \) is the coefficient of variation of the a posteriori distribution. (At this stage, to avoid possible confusion, we must recall that the theoretical development has been entirely in terms of numbers of claims, with adjustment for the heterogeneity of the sums at risk. In translating from numbers to amounts the dimensionality of the variance involves the square of the mean claim, so that if \( E(x|d) \) gives the expected number of claims and the mean claim amount is \( B \) we have

\[ \text{var}(x|d) = K \cdot B \cdot E(x|d) \]

in terms of claim amounts, where \( K \) is the credibility factor and the allowance for heterogeneity is already contained in \( K \).
3.15 It may now be perceived that the development in chapter 2 of this paper was simplistic in assuming that there is a unique quantity which may be determined for each pooling arrangement and called the expected surplus, derived inter alia from consideration of the expected claims. We can preserve the concept of overall expected surplus in relation to a notional “large number” of such arrangements sharing the same a posteriori expected claims, but we should now make due allowance for the placement of these arrangements within the a posteriori distribution. Thus, in place of the equation

\[ S = A \left( 1 + S - R \cdot V \right) \]

we should consider

\[ S = \frac{\sum E_i \cdot A \left( 1 + S_i - R \cdot V_i \right)}{\sum E_i} \]

where the summation is over the a posteriori distribution and

\[ S = \frac{\sum S_i \cdot E_i}{\sum E_i} \]

In practice it is sufficient to consider the 2 point representation of the a posteriori distribution discussed in section 3.14. One of the 2 points may involve negative expected surplus—clearly, it is possible for a subset of the a posteriori distribution to have expected claims in excess of the net premiums.

3.16 In considering the stop loss premium for a particular year’s pooling we cannot assume that the a priori distribution is static in time. It is generally believed, and on good historical evidence, that there is a secular trend towards lighter mortality, upon which there may be superimposed a cyclical variation. A similar process has not yet been demonstrated in relation to the incidence and duration of disablement, but the claims experience for this risk too is subject to a priori variation correlated with the trade cycle or other economic parameters.

In regard to mortality it may be reasonable to assume that the secular trend is either cancelled by or reinforced by cyclical variation so that by ignoring it we may leave a small margin in favour of the insurers. In regard to disability there is probably little we can do apart from trying to arrange that the risk is not underestimated.

3.17 We have so far considered expected surplus as a constant premium margin equal to the excess of the gross premium over expected claims and expenses. In the case of substantive contracts like endowment assurances and deferred annuities there may also be a major contribution to expected surplus from the excess of earned interest over the interest rate allowed for in the premium rates and valuation reserves. Unlike the premium margin (though even there we have the effect of the secular trend in mortality), this interest surplus is not constant but increases with duration, as the reserve builds up, though in the case of a large stable group in a long-standing arrangement it is conceivable that the group interest surplus may have reached a steady state.

In considering the risk charge for a Full Stop Loss arrangement it is appropriate to take account only of the expected surplus for the current insurance period, usually a year. When we come to consider Losses-carried-forward arrangements we shall have to allow for the pattern of emergence of interest surplus.

Note that deferred annuities will generally make a positive contribution to expected surplus, both by way of a premium margin and by way of interest surplus, while their expected claims are normally negative. Thus, whereas strains are produced under life assurance contracts when deaths are more than average, releases are produced in these circumstances under deferred annuities. For the purposes of calculating the risk charge or stop loss premium we simply reverse the sign of the expected claims and proceed as if we are dealing with term assurance. This is because the contribution to the variance of the overall claims distribution is the same, and because of the relation given in section 2.12.
A feature of many networks is the waiver of local discontinuance charges or “penalties” on discontinuance of the contract(s) of one of the subsidiary companies with its local network insurer. This concession is usually conditional upon a minimum period of inclusion in the network’s pooling arrangement prior to discontinuance. The local charge or penalty is automatically released into the surplus emerging under the pooling arrangement by way of the difference between the technical reserves brought forward in respect of the discontinuing contract and the surrender-value paid out locally: because of the way in which the Profit and Loss Accounts (see section 1.4) operate, this contribution to surplus could be withheld only by a suitable increase in the explicit changes levied against the account or, in the case of a “penalty” justified in terms of market-value depreciation, by an explicit reduction in the investment yield for the final year or an explicit depreciation charge.

The contribution to expected surplus from this source may be quite substantial in the case of endowment assurance contracts, particularly in Germany and Switzerland, and can make a very considerable difference to the risk charge appropriate to a losses-carried-forward arrangement. Again, the size of this contribution will depend very much upon the timing of discontinuance.

If there is a large front-end expense which is amortized by a suitable loading in the annual premiums, a Full Stop Loss arrangement would normally charge for this expense on a level amortization basis. Under a Losses-carried-forward arrangement there are two alternatives:

(i) to charge the full expenses in the first year, carrying forward any resultant loss, and releasing the corresponding premium loadings into surplus in subsequent years,

or

(ii) to spread the expense charge on a level amortization basis.

The author believes that method (i) is more realistic, though in either case the calculation of the risk charge requires some assumptions about the persistency of the business.

Before leaving the discussion of expected surplus we must touch on the knotty subject of local distributions of surplus, i.e. dividends paid under the individual network insurers’ own contracts. These are a deduction from the local surplus, shown in the National Profit and Loss Account. There is no great problem if they are assessed on the insurer’s wider national experience, but they do present a problem if they are experience-rated by reference to the policyholder’s own experience. Clearly, the expected surplus available to the International Profit and Loss Account is reduced by the expected value of the local dividend (which may be assessed by the techniques discussed in this paper). Moreover, not only is the expected surplus reduced, but the variance of this residual surplus, which is the same as the variance of the claims and local dividends (inversely correlated variables) combined, is also greatly diminished. It is possible to estimate the coefficient of variation of the sum of these negatively correlated variables, in the case of any given local experience-related dividend formula, by applying the principles enunciated in this paper. In general, it may be sufficient to assume that the variance of the claims and dividends combined is about half that of the claims alone.
4. **Theme and Variations**  
the diversity of structure of Losses-carried-forward arrangements

4.1 The risk charge for a one-year Full Stop Loss arrangement tends to be very high. Using the formula in 2.11 it can be seen that, with 10% expected surplus, a stop loss charge of as much as 10% is required when

\[
\frac{V}{\sqrt{m}} = 40\% \text{ of } V = 10\% \quad \text{i.e. } V = \frac{1}{4}
\]

If \( H = 1.15 \) then \( V = \frac{1.15}{\sqrt{m}} \), so that

\[
m = \left(\frac{1.15}{\frac{1}{4}}\right)^4 = 21.16
\]

If \( m = nq \) and \( q = \frac{1}{3}\% \) we have \( n = 6350 \).

In general, if an arrangement covers fewer than, say, 5000 lives the risk charge for a one-year Full Stop Loss is likely to make it unattractive, though a high level of expected surplus will make a considerable difference to the perspective here.

4.2 Full Stop Loss can be seen as one end of a whole spectrum of Partial Stop Loss arrangements. By stopping only part of any loss, or only the “top slice” of any outsize loss, and carrying the balance forward to be offset against future profits, a dramatic reduction in the risk charge can be achieved.

For Full Stop Loss the risk charge is simply the cost of writing off all losses at the end of the year, i.e.

\[
R = \frac{1}{E} \int_0^\infty \left[ x - (1 + S - R)E \right] p(x) \, dx
\]

while being identically defined by

\[
S = \frac{1}{E} \int_0^\infty \left[ (1 + S - R)E - x \right] p(x) \, dx
\]

since the sums of the LHS and RHS are clearly the same.

For Partial Stop Loss, or *Losses-carried-forward*, the risk charge is the combined cost of writing off a defined segment of loss at the end of each year and writing off the balance brought forward in the event of discontinuance of the arrangement. We have here a new dimension of risk such that the elegant simplicity of the *poisson* model for Full Stop Loss seems to explode into mind-boggling complexity.

4.3 A way forward in the theoretical development is to leave aside, for the time being, the complication of *selective* discontinuance. We may suppose that there is a known rate of discontinuance, from which may be deduced the expected lifetime of a losses-carried-forward arrangement. If we make the further simplification of assuming that both losses and profits are carried forward throughout the lifetime of the arrangement, we have a simple extension of our *poisson* theory which may serve as an imaginative starting-point for the consideration of losses-carried-forward.
Thus, if

$T = \text{the expected term of the arrangement and the dividend, if any, is paid at the end of } T \text{ years, with no intermediate dividends,}$

the losses-carried-forward arrangement is identical with a $T$-year Full Stop Loss arrangement and the risk charge is clearly given by

$$S = A \left( 1 + S - R, V T^{-\frac{1}{2}} \right)$$

where $V$ is the coefficient of variation of one year's claims. (It is further assumed here that losses and profits are carried forward at a rate of interest which precisely balances with the rate of inflation of premiums, claims and expenses).

It is then evident that we have defined a spectrum of values of $R$ whose extremes are given implicitly by the equations

(i) $S = A \left( 1 + S - R, V \right)$

and (ii) $S = A \left( 1 + S - R, V T^{-\frac{5}{8}} \right)$

By introducing such factors as intermediate dividends and a write-off limit to determine the incidence of partial stop loss we are defining points between the upper and lower bounds of this spectrum.

The author decided early in his investigations that this was likely to be as far as pure mathematics would take him, while being ready to acknowledge that this was more a consequence of his own limited mathematical attainments than of the ultimate potential of pure mathematics. The points in the spectrum may conveniently be specified by the quantity $\epsilon$ in

$$A \left( 1 + S - R, V T^{-\epsilon} \right)$$

where $\epsilon$ ranges between 0 and $\frac{1}{2}$, and it is imaginatively helpful to quantify $\epsilon$ for each variation on the basic theme of losses-carried-forward. However no way of estimating the $\epsilon$'s has been found other than the method of Monte Carlo simulation by electronic computer. Much of the remainder of the theoretical development of this paper therefore necessarily turns out to be a discussion of the design of a simple computer program, written in pascal, and analysis of some of the results obtained by running this program for a variety of cases as it happened, on a 48K Sinclair SPECTRUM, using HISOF'Ts excellent implementation of pascal for this computer.
4.4 Let us define the automatic write-off limit (AWOL) as that level, in terms of a specified multiple of the standard deviation of the claims distribution, above which losses are written off and below which they are carried forward to be offset against future profits. In the case of Full Stop Loss the AWOL is zero. It may reasonably be supposed, in the case of a losses-carried-forward (LCF) arrangement, that there is an increasing risk of selective discontinuance at the higher levels of AWOL: if the loss-carried-forward becomes so great that the policyholder sees little hope of a reversal of fortune he may well be tempted to shed the loss by the simple act of discontinuing the arrangement. Note that discontinuing the network's pooling arrangement need not in any way affect the operation of the insurance contracts with the separate network insurers. However it is more likely that the multinational policyholder would seek to renegotiate the pooling arrangement to bring in an AWOL, with retrospective effect, and it is partly in anticipation of such a prospect that a network would incorporate an AWOL into the initial design of a pooling arrangement, with due allowance for this feature in the assessment of the risk charge. Thus, the AWOL is not merely a design feature aimed at enticing the prospective network customer: it exists primarily to protect the network from the fool's paradise of too low a risk charge. Clearly, the lower the AWOL, the lower the risk of selective discontinuance but the higher the risk charge up to the level of Full Stop Loss in the limit—there is a balance to be struck, at a level of AWOL which is readily tolerable by an ongoing arrangement. Even so, it is sensible to incorporate in our computer program for assessing the LCF risk charge a discontinuance rate which is sensitive to the relative magnitude of the loss being carried forward: it may then be possible to adopt representative values of $T$ and $e$ in $\phi(s \sqrt{T^{-s}})$ to reflect our simulation findings for LCF structures with different AWOL's. We shall pursue this in the next chapter.

4.5 The AWOL may operate in two different ways. Most commonly, it operates on the net loss-carried-forward, after putting together the current year's results and any loss or profit brought forward from the previous year. Alternatively, with some advantage in terms of marketing publicity, it may operate first and independently on the current year's results, even though a loss in the current year may be reduced below the AWOL by offsetting against it a profit brought forward. The latter form of AWOL is more powerful and more expensive in terms of risk charge. Our simulation should investigate this differential.

4.6 We have already mentioned the possibility of carrying profit forward instead of releasing it all as current international dividend: this is the strategy of the contingency fund.

There are at least 6 different types of contingency fund. These fall neatly into 2 groups of three, which may be categorized as follows:-

**PERMANENT**

Type 1: **static**; no profit is released as international dividend until the aggregate brought forward, together with the current year's profit, exceeds the *contingency fund limit* (CFL); only the excess is released, the balance (= the contingency fund) being accumulated at interest and available to be offset against subsequent losses; the contingency fund is paid out on discontinuance of the pooling arrangement.

Type 2: **slow build-up**: here the CFL is dynamic, usually building up in arithmetic sequence (20%, 40%, 60% etc) to its ultimate level over the first 5 years.
Type 3:  
**dynamic;** here the contingency fund, including the whole of the current years’ results, is assessed at the end of each year and a specified percentage of the aggregate is released as international dividend; if the balance is still greater than the CFL the excess would also be released as dividend.

Each of these types of **permanent contingency fund** (PCF) may be operated in conjunction with an **automatic profit release** (APR) in respect of the current year’s results. Here a specified percentage of any current profit is released as dividend and only the balance is then available to consider as part of the ongoing contingency fund.

**TEMPORARY**

Type 4:  
**proportionate cyclical;** here the contingency fund operates over recurring n - year cycles, carrying forward \( \frac{n-t}{n} \) times the profit for the first year, \( \frac{n-t}{n} \) times for the t th. year and releasing the balance to date at the end of the n th. year; a new cycle then begins in the (n + 1)th year, and so on. In the event of discontinuance within a cycle the network may retain the balance in the contingency fund, though the effect of this would normally be to confine discontinuances to the end-point of a cycle.

Type 5:  
**offset cyclical;** here the dividend is determined as \( x\% \) of the aggregate profit or loss to date during the cycle less the dividends already paid during the cycle, both being accumulated at interest, and \( x \) is an increasing function of duration since commencement of the cycle, leading to \( x = 100 \) for the last year; the contingency fund is then simply the net undistributed profit to date.

Type 6:  
**deferred cyclical;** here we simply withhold all dividends until the end of the cycle, when all accumulated profit to date is released.

Type 6 is clearly the most powerful form of temporary contingency fund, resulting in a particularly low risk charge, but it presents marketing difficulties, similar to those associated with a static contingency fund, arising out of the natural impatience of the policyholder.

In cases where a temporary contingency fund (TCF) operates it would be normal for write-offs to be confined to the end-point of each cycle.
5. The interaction of structure and risk charge

5.1 It was primarily with the aim of studying the interaction of pooling structure and risk charge that the first suite of computer programs was written. It is not easy to report the results of this study in a lively, interesting way because of the vast array of permutations and combinations of the variable factors. To some extent the paper will have fulfilled its purpose by discussing the theoretical background and the application of computer simulation techniques to the determination of the risk charge and its apportionment. However, some attempt will be made to present an overview of the results and the presentational framework which has been chosen is that of Question and Answer and the drawing of comparisons.

It is important to bear in mind throughout that the results of any simulation contain random errors, which tend to vitiate comparisons. Every simulation has been repeated one thousandfold, but even so the coefficient of variation of the risk charge may be in the order of 5%. Some indication of the variability of the risk charge may be obtained by measuring the variability of the losses over the 1000 simulations and dividing this by \sqrt{1000}, but a better indication is given by repeating the thousandfold simulation several times (say, 5) with different pseudorandom number seeds. Thus, the results of repeating case 4, from the final table of section 5.2 (viz, (3,10,3) with (50,5)), five times are as follows:

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Eff. term.</th>
<th>Σ</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.104</td>
<td>.269</td>
<td>19.6%</td>
</tr>
<tr>
<td>2</td>
<td>9.775</td>
<td>.278</td>
<td>18.3%</td>
</tr>
<tr>
<td>3</td>
<td>9.046</td>
<td>.267</td>
<td>20.0%</td>
</tr>
<tr>
<td>4</td>
<td>9.649</td>
<td>.277</td>
<td>18.6%</td>
</tr>
<tr>
<td>5</td>
<td>9.609</td>
<td>.266</td>
<td>19.2%</td>
</tr>
</tbody>
</table>

Average: 19.14%
Estd. coeff. of variation: 3.6%

(This may be compared with the coefficient of variation of the standard deviation, which is approximately 1/\sqrt{2n} = 2%).

5.2 What is the effect of selective discontinuance?

As was said in chapter 4 an important function of the automatic write-off limit (AWOL) is to control selective discontinuance. It is therefore interesting to define a loss-dependent rate of discontinuance and to study the behaviour of the risk charge, the effective term and our epsilon index as we vary the awol.

Consider two alternative bases for the discontinuance function:

(1) \[ 5 + \frac{x}{s} \left( \frac{l}{s} \right)^3 \] % per annum

where \( l \) is the loss due to be carried forward
\( s \) is the standard deviation
\( x \) is \( \frac{1}{2} \) (i.e. disfac = 5)
and (2)

\[
5 + x \left( \frac{l+p}{s} \right)^3 \%
\]

per annum

where \( p \) is the expected surplus minus the risk charge (and will be negative if the expected surplus is low), summed over the next 3 years.

Basis (2) measures the propensity to selective discontinuance by comparing the loss minus 3 year's worth of "p", i.e. the margin available after risk charge (which may be negative, and has then to be added to the loss), with the standard deviation. The value of "x" taken here is 0.3, so that the extra probability of discontinuance if the extinction of the loss in 3 years would require claims at the level of 2 3-year standard deviations less than expected is

\[
\left[ 0.3 \times (2 \times 3^{\frac{3}{2}})^3 \right] \% = 1 \text{ in } 8
\]

It is as well to remind the reader at this stage that these functions are in no way empirically based. They are highly theoretical and serve only to indicate the possible consequences of selective discontinuance.

We can anticipate the effective term when

\[
\text{disconst} = 50 \text{ and disfac} = 0
\]

by summing

\[
\sum_{t=0}^{50} t (0.95)^{t-1} + 50 (0.95)^{50} = \bar{a}_{50}^{5.263\%} = 18.462
\]

Moreover, as we are relying on thousand-fold simulations, we are also interested in the standard deviation as a measure of our random error. We have:

\[
\text{Variance} = \frac{2 \bar{a}_{50}^{5.263\%} - 1 - 99 (0.95)^{50}}{0.05} - \left( \bar{a}_{50}^{5.263\%} \right)^2
\]

\[
= 225.28
\]

The standard deviation of the average from 1000 trials is therefore \( \sqrt{\frac{225.28}{1000}} = 0.47 \).

The following results explore 8 different permutations for each of 2 bases for the discontinuance function. The order of appearance is as follows:

(Expected claims, sur\%, AWOL)

| 1, 2 | 3, 10, 2 |  | 9, 10 | 9, 10, 2 |
| 3, 4 | 3, 10, 3 |  | 11, 12 | (9, 10, 3) |
| 5, 6 | 3, 30, 2 |  | 13, 14 | (9, 30, 2) |
| 7, 8 | 3, 30, 3 |  | 15, 16 | (9, 30, 3) |
Common factors are as follows:

Heterogeneity coefficient = 1.143
Interest = 10\% \text{ p.a.}
Inflation = 7\% \text{ p.a.}

Maximum term = 50
No. of simulations = 1000
No discontinuance release
No initial expenses
No interest surplus
No contingency fund.

Disfac is 5 for basis (1) and 3 for basis (2). Basis (1) is the first of each pair; basis (2) the second.

<table>
<thead>
<tr>
<th>Case</th>
<th>Exp.</th>
<th>Sur%</th>
<th>AWOL</th>
<th>Eff. Term</th>
<th>$\epsilon$</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>10</td>
<td>2</td>
<td>15.227</td>
<td>.260</td>
<td>16.0</td>
</tr>
<tr>
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<td>10</td>
<td>2</td>
<td>15.303</td>
<td>.263</td>
<td>15.9</td>
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<td>3</td>
<td>10</td>
<td>3</td>
<td>11.588</td>
<td>.282</td>
<td>16.7</td>
</tr>
<tr>
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<td>3</td>
<td>10</td>
<td>3</td>
<td>12.559</td>
<td>.288</td>
<td>15.5</td>
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<tr>
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<td>3</td>
<td>30</td>
<td>2</td>
<td>16.896</td>
<td>.230</td>
<td>5.1</td>
</tr>
<tr>
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<td>3</td>
<td>30</td>
<td>2</td>
<td>18.648</td>
<td>.237</td>
<td>4.6</td>
</tr>
<tr>
<td>7</td>
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<td>30</td>
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<td>.251</td>
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<td>30</td>
<td>3</td>
<td>18.303</td>
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<tr>
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<td>9</td>
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<td>2</td>
<td>15.733</td>
<td>.246</td>
<td>5.7</td>
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<td>10</td>
<td>2</td>
<td>17.562</td>
<td>.247</td>
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</tr>
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<td>10</td>
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<td>.274</td>
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</tr>
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<td>12</td>
<td>9</td>
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<td>16.515</td>
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<td>2</td>
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</tr>
<tr>
<td>14</td>
<td>9</td>
<td>30</td>
<td>2</td>
<td>18.193</td>
<td>.228</td>
<td>0.7</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
<td>30</td>
<td>3</td>
<td>17.633</td>
<td>.243</td>
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</tr>
<tr>
<td>16</td>
<td>9</td>
<td>30</td>
<td>3</td>
<td>18.193</td>
<td>.258</td>
<td>0.5</td>
</tr>
</tbody>
</table>

It is instructive to compare these results with those obtained by taking disfac = 0 and disconst at the level needed to reproduce the same effective terms. Let us consider both

\[(t = 50, \text{disconst} = 64) \text{ and } (t = \frac{6.87}{50} \text{ } \div 15, \text{disconst} = 0)\]

for \((3,10,2)\)

and

\[(t = 50, \text{disconst} = 83) \text{ and } (t = \frac{9}{50} \text{ } \div 12, \text{disconst} = 0)\]

for \((3,10,3)\)
We obtain:

$$ (3, 10, 2) \cdot$$

$$(t = 50, \text{disconst} = 64): \text{Eff. Term} = 15.2, \epsilon = 0.271, \text{RC} = 15.5$$

$$(t = 15, \text{disconst} = 0): \text{Eff. Term} = 15.0, \epsilon = 0.266, \text{RC} = 16.0$$

$$(3, 10, 3) \cdot$$

$$(t = 50, \text{disconst} = 83): \text{Eff. Term} = 11.9, \epsilon = 0.313, \text{RC} = 14.5$$

$$(t = 12, \text{disconst} = 0): \text{Eff. Term} = 12.0, \epsilon = 0.297, \text{RC} = 15.4$$.

Instinctively, the author prefers the Basis (2) formulation of the discontinuance function but is also inclined to think that $x = 0.3$ is too low and should be increased to, say, $x = 0.5$.

Reworking the above examples with this value of disfac, we obtain:

<table>
<thead>
<tr>
<th>Case</th>
<th>Exp.</th>
<th>Sur%</th>
<th>AWOL</th>
<th>Eff. Term</th>
<th>$\epsilon$</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>10</td>
<td>2</td>
<td>13.381</td>
<td>.261</td>
<td>17.2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>10</td>
<td>3</td>
<td>9.104</td>
<td>.269</td>
<td>19.6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>30</td>
<td>2</td>
<td>18.324</td>
<td>.239</td>
<td>4.6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>30</td>
<td>3</td>
<td>18.012</td>
<td>.279</td>
<td>3.3</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>10</td>
<td>2</td>
<td>17.125</td>
<td>.249</td>
<td>5.4</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>10</td>
<td>3</td>
<td>14.459</td>
<td>.269</td>
<td>5.3</td>
</tr>
<tr>
<td>14</td>
<td>9</td>
<td>30</td>
<td>2</td>
<td>18.054</td>
<td>.230</td>
<td>0.7</td>
</tr>
<tr>
<td>16</td>
<td>9</td>
<td>30</td>
<td>3</td>
<td>18.054</td>
<td>.262</td>
<td>0.5</td>
</tr>
</tbody>
</table>

5.3 What is the effect of ignoring interest/inflation?

The program religiously makes provision for the joint effects of interest and inflation on the accumulation of loss in a losses-carried-forward arrangement. Both of these parameters are stochastic and, though highly correlated, their inter-relationship is also stochastic but no attempt has been made to allow for the extra dimension of variation arising out of their stochastic character. But suppose we equate both with zero - what happens then to the risk charge? (It is assumed that both interest and inflation remain effective in the real situation).

For brevity, in the remainder of this paper, we shall use the vector (expected no. of claims, sur%, AWOL).

Compare, then, $(3, 10, 3)$, with heterogeneity index 1.143, for int = 10%, inf = 7% with the same for int = inf = 0, taking the discontinuance function as basis (2) with (50,5), i.e. disconst = 50 (i.e. 5%) and disfac = 5 (i.e. ½%).

We have:

<table>
<thead>
<tr>
<th>Int.</th>
<th>Inf.</th>
<th>Eff. Term.</th>
<th>$\epsilon$</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
<td>9.437</td>
<td>.271</td>
<td>19.1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>10.579</td>
<td>.289</td>
<td>16.9</td>
</tr>
</tbody>
</table>

5.4 Suppose we have a permanent contingency fund and an AWOL both at the level of 2 standard deviations. Compare the 3 types of permanent contingency fund, thus:-

Type 1 (static):
no dividend until the contingency fund slops over.
Type 2 (slow build-up):
target fund builds up over the first 5 years.

Type 3 (dynamic):
25% of the profit-carried-forward is released each year.

For (3, 10, 2), heterogeneity index 1.143, int = 10%, inf = 7% and the same discontinuance basis, we have:

<table>
<thead>
<tr>
<th>Type</th>
<th>Eff. Term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.234</td>
<td>.407</td>
<td>6.9</td>
</tr>
<tr>
<td>2</td>
<td>17.044</td>
<td>.387</td>
<td>7.8</td>
</tr>
<tr>
<td>3</td>
<td>16.237</td>
<td>.352</td>
<td>9.8</td>
</tr>
</tbody>
</table>

5.5 **What is the effect of a Type 1 AWOL (i.e. automatic write-off applies first to current losses)?**

Consider the effect on the case studied in section 5.4, with the three types of permanent contingency fund.

<table>
<thead>
<tr>
<th>Type</th>
<th>Eff. Term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.289</td>
<td>.397</td>
<td>7.2</td>
</tr>
<tr>
<td>2</td>
<td>16.994</td>
<td>.382</td>
<td>8.0</td>
</tr>
<tr>
<td>3</td>
<td>16.149</td>
<td>.349</td>
<td>10.0</td>
</tr>
</tbody>
</table>

5.6 Compare the effect of a 25% APR on the operation of the Type 1 and 2 contingency funds in the above example with Type 3 operating normally (i.e. releasing 25% of aggregate profit).

<table>
<thead>
<tr>
<th>Type</th>
<th>Eff. Term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.616</td>
<td>.321</td>
<td>11.9</td>
</tr>
<tr>
<td>2</td>
<td>15.492</td>
<td>.308</td>
<td>12.7</td>
</tr>
<tr>
<td>3</td>
<td>16.237</td>
<td>.352</td>
<td>9.8</td>
</tr>
</tbody>
</table>

5.7 **How does the dynamic contingency fund perform with a higher level of release?**

Again studying the same example:

<table>
<thead>
<tr>
<th>Release</th>
<th>Eff. Term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>16.237</td>
<td>.352</td>
<td>9.8</td>
</tr>
<tr>
<td>50%</td>
<td>15.447</td>
<td>.312</td>
<td>12.5</td>
</tr>
</tbody>
</table>

5.8 Compare the 3 temporary contingency fund bases for the following variations:

Expected no. of claims : 1 or 3
Heterogeneity : 1.143 (index = 2) or 1.458 (index = 3)
Surplus % : 10 or 30
This gives 8 permutations on each of the 3 bases.

We suppose further -

Type 4 = Proportionate cyclical:
3 year cycle for 1 expected claim; 2 year cycle for 3 expected claims.

Type 5 = Offset cyclical:
as above, distributing 50% for the first year of the cycle and 60% for the second year of a 3 year cycle, less the amount already distributed in the current cycle.

Type 6 = Deferred cyclical:
same cycles.

It is also assumed that discontinuance takes place only at the end of a cycle and is in accordance with the now-familiar discontinuance function, taking disconst = 50 times the number of years in the cycle and disfac = 5.

The AWOL is taken at 3 standard deviations, operating only at the end of each cycle.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Het</th>
<th>Sur%</th>
<th>Type</th>
<th>Eff. Term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>10</td>
<td>4</td>
<td>11.585</td>
<td>.327</td>
<td>33.8</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td>.326</td>
<td>33.8</td>
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<tr>
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<td></td>
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<td>.357</td>
<td>29.0</td>
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<td>30</td>
<td>4</td>
<td>16.765</td>
<td>.325</td>
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<td></td>
<td></td>
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<td>16.941</td>
<td>.324</td>
<td>10.9</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>17.553</td>
<td>.344</td>
<td>9.2</td>
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<td>11.312</td>
<td>.355</td>
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<td>11.841</td>
<td>.365</td>
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</tr>
<tr>
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<td>.314</td>
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<td></td>
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<td>10</td>
<td>4</td>
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<td>12.144</td>
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<td>.289</td>
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<td>18.108</td>
<td>.289</td>
<td>6.0</td>
</tr>
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<td></td>
<td>6</td>
<td>18.178</td>
<td>.302</td>
<td>5.5</td>
</tr>
</tbody>
</table>
5.9 What is the effect of a very low AWOL, say one standard deviation only?

Clearly, if AWOL = 0 we have Full Stop Loss.

We can consider this for (3,10,1) and (3,30,1) comparing the following:

(1) No contingency fund.
(2) Permanent contingency funds also at the level of 1 standard deviation (with 25% release for type 3).
(3) Temporary contingency funds as in 5.8.

Heterogeneity index = 1.143.

<table>
<thead>
<tr>
<th>Sur%</th>
<th>Type</th>
<th>Eff. term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>16.150</td>
<td>.185</td>
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<td>.275</td>
<td>14.1</td>
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<td>4</td>
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<td>16.6</td>
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<td>.154</td>
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<td>.261</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18.583</td>
<td>.246</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>18.583</td>
<td>.246</td>
<td>4.2</td>
</tr>
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<td>4</td>
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<td>6.0</td>
</tr>
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<td>6</td>
<td>17.986</td>
<td>.220</td>
<td>5.4</td>
</tr>
</tbody>
</table>

5.10 What is the effect of too low a risk charge?

Let us put this in the form of asking the effect on the risk charge of a deliberate in-built subsidy of, say, 2% of the expected claims. This will then indicate to us the degree of gearing between the risk charge and such an in-built subsidy. At the same time it is interesting to consider the effect of a negative subsidy, i.e. an in-built profit obtained through a margin in the risk charge, rather than a margin in the expenses charge which would communicate itself directly to the expected surplus.

The variable used to carry this in-built "subsidy" in the program is named \( \text{delta} \).

Consider, then the example in section 5.3 with \( \text{delta} = -2\% \) or \( +2\% \). We have:

<table>
<thead>
<tr>
<th>DELTA</th>
<th>Eff. term</th>
<th>ε</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2%</td>
<td>11.536</td>
<td>.276</td>
<td>12.5</td>
</tr>
<tr>
<td>0</td>
<td>9.437</td>
<td>.271</td>
<td>19.1</td>
</tr>
<tr>
<td>-2%</td>
<td>7.398</td>
<td>.241</td>
<td>26.8</td>
</tr>
</tbody>
</table>
In this case the gearing from \textit{delta} to RC is more than threefold. Evidently this has been enhanced by the behaviour of the discontinuance basis, but it can be readily seen that there would be substantial gearing even without this enhancement. Thus, solving

\[ S + \Delta = A \left(1 + S - R, \sqrt{V}\right) \]
\[ .12 = A \left(1.1 - R, .660 \times .437^{-27}\right) \]
\[ = A \left(1.1 - R, .3592\right) \]

\begin{array}{ccc}
\sqrt{V} & .9 & 1.0 & .95 \\
.3 & .073 & .119 \\
.4 & .113 & .160 \\
.3592 & .0967 & .1433 & .12 \\
\end{array}

Hence \(1.1 - R = .95\)
so \(R = 15\%\)

Thus, even without the enhancement from the supposed diminution of selective discontinuance, a reduction of 4.1\% or so in the risk charge results from a subsidy of 2\%. There will undoubtedly be some further enhancement from improved persistency but the precise quantum is not really predictable.

Going the other way, a “negative subsidy” of 2\% produces

\[ 0.8 = A \left(1.1 - R, .3592\right) \]

\begin{array}{ccc}
\sqrt{V} & .8 & .9 & .855 \\
.3 & .042 & .073 \\
.4 & .072 & .113 \\
.3592 & .0598 & .0967 & .08 \\
\end{array}

Hence \(1.1 - R = .855\)
so \(R = 24.5\%\),

which is an increase of 5.4\%.

5.11 \textbf{What is the effect of a discontinuance release?}

Suppose the network includes an endowment assurance scheme under which a substantial release (i.e. the excess of actuarial reserves over local surrender value) is expected on discontinuance after 5 years. The amount of this expected release, as a multiple of expected claims, may build up over a number of years - let us say, for our example, from zero to 150\% of one year's expected claims over 20 years. Suppose that the endowment assurance represents a third of the total expected claims, so that the overall discontinuance release builds up to 50\% of total expected claims.

As we are concerned with endowment assurance here we must also consider the effect of interest surplus. The characteristic of this type of surplus which exercises us is that, unless we start off with a scheme which is already mature, this too will start low and build up over a number of years, say from zero to 45\% of the expected claims, or 15\% overall, over 20 years.
Let us take, this time, a case which has a total of 6 expected claims per annum, heterogeneity of 1.458, and expected non-variable surplus of 10%. Let AWOL = 2, int = 10%, inf = 7% and there is no contingency fund.

Compare the situation described above with a parallel "pure risk" case with level expected surplus per annum equal to the average surplus (including discontinuance release), spread with due allowance for interest and inflation, in the former case.

<table>
<thead>
<tr>
<th>Surplus</th>
<th>Average p.a.</th>
<th>Eff. term</th>
<th>$\epsilon$</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>18.9%</td>
<td>17.847</td>
<td>.245</td>
<td>7.0</td>
</tr>
<tr>
<td>Constant</td>
<td>19.0%</td>
<td>18.026</td>
<td>.235</td>
<td>7.3</td>
</tr>
</tbody>
</table>

**5.12 What is the effect of a front end expense?**

Suppose now we have an overall front-end expense of 50% of expected claims, notionally amortized over 14 years. Let us study this in relation to the constant surplus example of section 5.11, taking the average surplus for each of the two cases in our comparison as the amount brought out by the amortization procedure described above, in conjunction with the (50, 5) discontinuance function.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Surplus %</th>
<th>Eff. term</th>
<th>$\epsilon$</th>
<th>RC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front-ended</td>
<td>18.9</td>
<td>16.298</td>
<td>.241</td>
<td>7.6</td>
</tr>
<tr>
<td>Level</td>
<td>19.0</td>
<td>18.026</td>
<td>.235</td>
<td>7.3</td>
</tr>
</tbody>
</table>

It is clear that, within the parametric structure of these simulations, the incidence of expenses has little effect upon the risk charge.

**5.13 What is the effect on the risk charge if a further contract, which distributes all of its available surplus via a separate local experience-rating formula, is included in the pooling?**

This situation was mentioned in section 3.20. Consider the following somewhat idealized model:

<table>
<thead>
<tr>
<th>Exp</th>
<th>Sur %</th>
<th>H</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>10</td>
<td>1.143</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>20</td>
<td>1.143</td>
</tr>
</tbody>
</table>

Suppose the AWOL is one year's expected claims (i.e. AWOL = 6) and there is no contingency fund.

The assumed term to discontinuance is 10 (say : 10% p.a. for 50 years, since $\frac{10}{50} = 10$) and we simulate over 1000 decennia.

The risk charge for A alone is 15.7% of expected claims = 0.628 and that for B alone is 18.1% of expected claims = 0.362.
Putting A and B together but assuming that A operates local experience-rating, also with 10% p.a. discontinuances and a local AWOL of one year's expected claims (i.e. AWOL = 4), so that the only surplus expected to be available for the combined second-stage experience rating is B's surplus, we obtain a risk charge of 9% of overall expected claims = 0.54.

It was suggested in section 3.20 that the effect of including A in a combined pooling arrangement is similar to that of including an office with zero expected surplus and a claims distribution with half the variance of A's. Repeating the simulation with A having 2 expected claims instead of 4, and zero expected surplus, we obtain an overall risk charge of 0.54 (no cheating!).
6. The interrelation of network insurers and the apportionment of risk and risk charge.

6.1 Insurers who join together to form an international network have to agree upon a set of principles for the sharing of the risk and the risk charge. It is likely to be required that these principles should satisfy the criteria of practicability, consistency and equity. Four generalized systems may be defined:

(1) **ARM’S LENGTH**

It is entirely possible to operate a network on the principle that, while the international dividend should reflect the overall balance of profit and loss, there should be no transfers of money between insurers (other than for onward transmission to the policyholder or as required by bilateral reinsurance arrangements). It is not, however, clear how such a principle can be documented in the form of a mutual reinsurance agreement, which many national supervisory authorities appear to require as a necessary precondition for the approval of multinational pooling. Nevertheless, this may be the only feasible basis for dealing with an insurer in a blocked currency area.

Under the ARM’S LENGTH pooling system there is no sharing of losses, whether write-offs or losses to be carried forward, among the network insurers, who therefore carry forward in their national accounts any excesses of local profit over their share of the international dividend (determined as the proportion borne by the net overall profit to the sum of the profits of those insurers whose accounts are showing profits), i.e. amounts corresponding to local losses being carried forward by other network insurers (before write-offs) together with any contingency fund.

In a Full Stop Loss case, as also on discontinuance of a Losses-carried-forward arrangement, these excesses are retained by the local insurer, as also are any losses.

(2) **LIMITED MUTUALITY**

Under this system loss-making insurers are reimbursed each year by profit-making insurers, to the extent of the latters’ profits. Any profit in excess of what is needed for this purpose feeds into the international dividend, or contingency fund. If there is a loss to be carried forward, this will be carried by the loss-making insurers in proportion to the amounts of their losses prior to partial reimbursement. Similarly, any contingency fund is apportioned among the profit-making insurers in proportion to their profits.

Under both of these systems any automatic write-off is apportioned in proportion to local insurers’ losses in excess of their write-off levels.

(3) **TOTAL MUTUALITY**

Under this system, not only are loss-making insurers reimbursed to the extent of profit-making insurers’ profits but an overall loss is also met by a levy among all the insurers. Some kind of apportionment rule is needed, but unlike the first two systems, in this case there is an infinite choice of possible rules. In the case of systems (1) and (2) above, the rule for dealing with losses theoretically determines a unique apportionment of the risk charge, in order that each insurer’s share of the risk charge shall equate his expected share of the losses. In the case of system (3), so long as the same rule of apportionment is applied to the risk charge, **any** rule of apportionment can be applied to the overall loss, automatic write-offs and the contingency fund. However, the rule normally chosen is an apportionment in proportion to **size** - either expected claims or net premiums.
It is noteworthy that each of the above three systems involves a sharing out of risk and risk charge among the participating insurers, and implies an attribution of origin of shares of the international dividend to these insurers. The Limited Mutuality system implies a sharing of the dividend in proportion to local profits, while the Total Mutuality system implies a sharing in proportion to expected claims or whatever is chosen for the apportionment of the overall loss, though there is nothing in theory to prevent an entirely different apportionment of the dividend, e.g. in proportion to expected surplus. Unless the latter expedient is followed in relation to Total Mutuality, the only one of the three systems which gives rise to a flow of locally apportioned dividends whose expected value is the local expected surplus is the Arm’s Length system.

The remaining system avoids apportionment altogether in virtue of being.

(4) CENTRALIZED

Under this system the whole of the risk charge goes to one of the insurers, usually the leading network insurer, who then undertakes to reimburse all the other insurers’ losses. The whole of any loss (or profit) to be carried forward is held by the leading insurer.

6.2 The second of the two suites of computer programs was written for the purpose of studying the apportionment of the risk charge under the Arm’s Length (“HAWKS”) and Limited Mutuality (“DOVES”) systems. Unlike the first suite it assumes a fixed term to discontinuance, though it could easily be modified to incorporate a discontinuance function or, with further sophistication and less easily, a different discontinuance function for each insurer. It was felt to be sufficient for our purpose to assume a fixed term, equivalent to a fixed rate of discontinuance per annum.

In addition to determining the requisite apportionment for each defined arrangement, the program builds up a matrix from which it is hoped that a relatively simple basis for doing this may eventually be derived. No results have yet been obtained from this matrix.

6.3 A very approximate simple formulation for the Limited Mutuality basis of apportionment averages together, with roughly equal weights, the ratios derived from expected claims and independent local risk charges. Thus, it is easy to determine a set of proportions based on expected claims. If we then assess the independent risk charges which each of the insurers would require if he were operating the experience-rating arrangement on his own, outside the network, we can determine a further set of proportions based on these risk charges. A first approximation to the Limited Mutuality apportionment is then mid-way between these two sets of proportions.

If there is marked heterogeneity in the average sum at risk among the insurers, two more sets of proportions can be determined bringing in the average sum at risk as a further multiplier, and all four sets of proportions can then be averaged together.

This formulation has a certain intuitive appeal and, in practice, seems to achieve rough justice. Let us now see what happens in a few examples.

The coefficient of heterogeneity throughout is 1.143 and the expected term to discontinuance is 10 years. The automatic write-off limit is 2 years’ expected claims and there is no contingency fund.
An interesting, and sometimes awkward in practice, feature of Arm's Length pooling arises out of the fact that the apportioned risk charges are not necessarily non-negative. The Limited Mutuality principle results in an apportionment which lies within the range between one based on expected claims and one based on independent local risk charges - the range is divided internally. Lateral thinking, pursuing this geometrical analogy, suggests that an approximation for the Arm's Length apportionment is given by dividing this range externally, and such external division will sometimes produce negative values.

In practice, because negative risk charges are commercially unappealing (if only because they violate the Arm's Length principle, introducing mutuality by the back door, as it were), the viability of Arm's Length pooling depends upon there being a reasonable balance of size and expected surplus among the contracts of the pooling insurers. One would define this "balance" circularly by reference to the absence of negative values in the equitable apportionment of the risk charge.

An obvious example of Arm's Length pooling giving rise to a negative risk charge is that of section 5.13. Even in the combined pooling situation, insurer A still needs a risk charge equal to his local "first stage" risk charge because his expected residual surplus, after local experience-rating, is zero and he cannot allow any local surplus to get through into the international dividend: there is no mechanism in the Arm's Length pooling system to provide compensation for subsequent losses. Thus, in order that the overall risk charge can be 9% of overall expected claims in this case, i.e. 0.54, insurer B's risk charge must be (0.54 - 0.628) = -0.088 or -4.4% of B's expected claims. The apportionment is 116% to A and -16% to B. That obtained by a thousandfold simulation was 116.5 : -16.5.

The examples in section 6.4 will now be repeated with Arm's Length pooling, but using different sequences of claims (i.e. different random number seeds).
### ARM'S LENGTH

<table>
<thead>
<tr>
<th>Office</th>
<th>B</th>
<th>E</th>
<th>S</th>
<th>Simulated Claims</th>
<th>Risk charges (%)</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

The "estimates" are based on weights of 150% for the proportions involving risk charges and -50% for those involving expected claims. As will be seen the estimates are not very good, even allowing for random error - hence the matrix project mentioned in chapter 8: however, it may well be that the only satisfactory method is Monte Carlo simulation.

#### 6.7

There is no problem of apportionment of the risk charge if either of the Total Mutuality or Centralized systems is adopted. Under the latter system the leading insurer shoulders the entire risk and will be concerned to ensure that the other insurers have made reasonably consistent and conscientious assessments of their expected surplus and have not reserved for themselves excessive expense charges. Under the former system all of the insurers together, in proportion to the size of their respective stakes in a given arrangement, are concerned about the same issues. All networks require a close relationship of mutual trust and confidence between the participating insurers, though this is perhaps less crucial under the Arm's Length and Limited Mutuality systems.
7. Miscellaneous problems

7.1 This chapter covers a miscellany of topics which are considered to be relevant and interesting, but which have been left out of the mainstream of the paper in order to avoid sidetracking. They appear in random order.

7.2 Partial discontinuance

We have considered the risk charge up to now as the provision for writing-off losses in excess of the AWOL and on discontinuance of the pooling arrangement. Reality, as always, is more complex, and we must also consider the situation arising on partial discontinuance, when one or more subsidiary companies discontinue their arrangements with the local network insurers and a resultant truncated pooling arrangement remains in being for the remaining companies of the multinational.

Suppose the insurers of the withdrawing subsidiaries are carrying forward losses. If their share of the risk charge had been assessed on the basis that they would remain parties to the pooling arrangement until its ultimate discontinuance, it would be unfair to ask them to write off these losses prematurely without compensation. Moreover, if the pooling arrangement continues, albeit in truncated form, and surpluses begin to emerge, there is a natural expectation that, at least for a limited number of years, the losses carried forward from past years will be offset as a prior charge against these surpluses. However, clearly, it would not be reasonable to expect a truncated arrangement to bear the full burden of heavy losses carried forward from earlier years when the total exposure was more substantial. The principle of automatic write-off, whose prime purpose (as we saw in chapter four) is to contain selective discontinuance, requires the application of the automatic write-off limit (AWOL) to the reduced level of expected claims (or rather the standard deviation of the expected claims). The risk charge is reassessed for the truncated pooling arrangement, ignoring for this purpose the loss being carried forward: in the limit where the discontinuing assurances are relatively trivial, so that the total exposure remains virtually unaltered, this procedure would ceteris paribus reproduce the original risk charge, irrespective of the size of any loss-carried-forward, which of course it should. A rule is then needed whereby to apportion the loss-carried-forward between the discontinued and continuing insurers.

Under the Arm’s Length system of apportionment there is no question of the losses of the discontinued insurers ever being compensated by other insurers’ surpluses: any carryforward of these losses can only benefit the ongoing insurers by depressing the amount of their surpluses due to emerge as international dividend, and this is appropriate because it was on the understanding that this would be the case that they had accepted a lower risk charge. Alternatively, the framework of assumptions on which the risk charge and its apportionment were originally assessed could have taken account of differential insurer-specific rates of discontinuance - it is perfectly possible to complicate the Monte Carlo simulation in this way - though, in the author’s view, the mind begins to boggle at this degree of artificial complexity.

Under the Limited Mutuality system, if each insurer’s AWOL is determined in relation to his current expected claims or net premiums, the proportion of any overall write-off to be borne by the discontinued insurers will be related to the whole of their losses-brought-forward - their AWOL is zero since their net premiums are zero.

On this basis their losses-carried-forward would be written off fairly quickly, though there will still be a reasonable chance of part of them being met from other insurers’ surpluses.
Under the Centralized system, the leading network insurer shoulders all losses, but will be concerned not to write off too quickly any loss-carried-forward associated with a discontinued insurer.

Under the Total Mutuality system, the network insurers have to make up their minds as to the procedure whereby the losses-carried-forward of discontinued insurers are to be eroded by write-offs dictated by the overall AWOL. Clearly, it is desirable that some limit be set to the period during which such losses are permitted to continue as a potential charge against emerging profits - say, 3 or 5 years. Within that period it has to be decided whether, as a matter of course, these losses are to feature first or last in the pecking order among the insurers for the application of emerging surpluses. A possible rule to determine the apportionment between discontinued and continuing insurers of any residual overall loss to be carried forward would be as follows: the discontinued insurers carry forward whichever is the lesser of

(i) their previous loss-brought-forward, plus interest, reduced in proportion to any amount of the overall loss written off at the end of the year

and

(ii) the whole of the residual overall loss to be carried forward.

The continuing insurers then apportion among them the balance, if any, of the residual overall loss to be carried forward in the usual way, by reference to expected claims or net premiums.

A similar procedure would operate in relation to a contingency fund, i.e. profit to be carried forward.

It is possible, in theory, to complicate the algorithm on which the risk charge and its apportionment are assessed, using the computerized simulation techniques we have been studying, by building into the simulation the procedures which are to be followed on partial discontinuance. It is even possible to introduce differential insurer-specific discontinuance rates into the model. However, it was not thought worthwhile to pursue this degree of complexity because of the highly artificial character of the assumptions involved.

7.3 Inclusion of a new subsidiary company in a losses-carried-forward arrangement

The other side of the coin from the situation considered in section 7.2 is the expansion of a pooling arrangement, e.g. by bringing in a new subsidiary company and its insurer. The existing structure and its associated risk charge were framed on the assumption that any losses (and any contingency fund) would be carried forward, subject to the automatic write-off provisions.

The need to carry a loss (or contingency fund) forward is not altered by expansion of the arrangement, though this will be the occasion for a reassessment of the risk charge and possibly some reshaping of the structure.

The motivation for any reshaping of the arrangement is likely to be provided by the network’s multinational client if it is not volunteered by the insurers themselves. He may not be very keen to bring contracts which contain large profit margins, e.g. because the insurer concerned operates within a conservative tariff system, into an existing arrangement with a massive loss-carried-forward with little prospect of emerging surplus for some time to come.
While the insurers will argue that the loss must still be cleared, and will be cleared much more rapidly in the new situation, the client may be happier if the arrangement is redesigned - for example, it may be changed to incorporate an automatic profit release (the APR of chapter 4) or a dynamic contingency fund (type 3 of chapter 4), with an appropriate resetting of the AWOL. With the larger exposure and the new level of expected surplus it could turn out, after due assessment, that there is little change in the overall percentage risk charge for the modified arrangement. Clearly there is scope for renegotiation of the structure, provided that the insurers understand what they are doing.

If the relationship between the insurers is based on the Total Mutuality principle it is likely that all will agree to a rule whereby the new insurer (new, that is to say, to the given multinational arrangement, but probably not new to the network) participates from the outset in any losses-carried forward, or contingency fund, on the basis of the standard principle of apportionment, e.g. by reference to the yearly expected claims or net premiums. It is, however, possible to complicate the working of the network by introducing some principle for postponing the new insurer's participation in a loss-carried-forward, e.g. until the earlier of the expiry of 2 or 3 years, or the first year in which his own National Profit and Loss Account generates a loss. A word of warning here: these multinational pooling arrangements can be complicated enough already, without superimposing upon them additional levels of complexity so that the book-keeping, even if carried out on a computer, becomes virtually incomprehensible.

7.4 Negative expected surplus

Some insurance contracts in some countries run at an expected loss, though in tandem with other contracts which contain margins hopefully adequate to provide the necessary compensation. Long term disability cover is a notorious case in point, with expected losses offset against expected surplus under associated Group Life assurance or substantive contracts.

It is usual for the total expected surplus contributed by each insurer to a multinational pooling arrangement to be positive. Otherwise, from the client’s viewpoint, there is little point in including such an insurer. Equally, of course, the insurer with overall negative surplus must have some peculiar private motivation of his own in order to tolerate such a situation and it is unlikely that the other network insurers will wish to be involved.

However, situations may arise in which it is considered to be openly beneficial to all concerned to accommodate a set of contracts, and their insurer, which operate on nil or negative expected surplus. Provided that the overall expected surplus in a pooling arrangement is adequate, an appropriate risk charge may be assessed. The choice of apportionment systems may well then depend on the underlying motivation, but it seems likely that the Arm’s Length principle will be appropriate in determining the relationship between the loss-making insurer and the rest of the network. One’s first, and last, instinct may well be to avoid such situations altogether.

7.5 Attribution of profits to origins

A multinational company may wish to know how much each of its areas of operation is contributing to the international dividend. It will not know the various insurers’ assessments of their expected surplus; in general, it will only know the breakdown of the sources of the dividend shown each year in the International Profit and Loss Account.
The only one of the apportionment systems we have considered which produces a realistic relationship between apportioned dividend and expected surplus, such that the expected value of the former is equal to the latter, is the Arm’s Length system. The dominant principle of each system is necessarily the equitable apportionment of the risk charge, and this coincides with the equitable apportionment of the dividend only in the case of Arm’s Length apportionment. The desired result can also be achieved under the Total Mutuality system but then only by deliberately apportioning the dividend by reference to expected surplus, which would not normally be considered either practicable or prudent.

It is interesting to see what distortion in the apportionment of the risk charge among the insurers results, under the Limited Mutuality system, from homing in on equity for the dividend apportionment (in proportion to locally emerging profits) instead of equity between the insurers.

Let us take the second example considered in chapter 6. The equitable apportionment of the risk charge in that case was 24.267% of office 1’s expected claims and 5.830% of office 2’s. The expected surpluses were both 10% of expected claims but it turned out, averaging dividends over the 1000 simulations that the expected dividends were 13.258% and 6.846% respectively, though still 10.027% overall.

Further simulation showed that in order to get expected dividends of 10.094% and 10.005% respectively we needed risk charges of 43.606% and -13.294%, resulting in a totally unacceptable net loss for office 2 and a corresponding net gain for office 1.

### 7.6 Long Term Disability (LTD) and Stop Loss

Long Term Disability is, in a very real sense, a two-dimensional contract, giving rise to a strain when a reserve has to be set up in respect of a new claim and a release when the claim terminates. If the setting up of the reserve is separated from the release on termination by one or more yearly pooling assessment dates, a low level of AWOL under a Losses-carried-forward arrangement, and a fortiori the situation under a Full Stop Loss arrangement, may result in the writing off of much of any resultant loss at the first of these assessment dates. On subsequent termination the whole of the reserve is released into current surplus and there may be little or no loss brought forward, corresponding to the inception of the claim, to offset against this surplus. No anomaly arises if the first year reserves were fully covered by the net premiums, or if any excess was still below the write-off limit and was therefore carried forward. But consider what happens if there is a high volume of claims in a given year, giving rise to a strain which is partly written off at the end of the year. The next year starts off with heavy reserves in respect of the claims in course of payment. Suppose then, as may well be the case, all or nearly all of these claims terminate in the next year such that the total outlay on benefits may prove to have been commensurate with that provided for in the net premiums, albeit on different assumptions as to incidence and persistency. The heavy reserves which started the year will then fall into surplus and a massive dividend may result, out of all proportion to that justified by the relationship of actual to expected claims.

If LTD cover bulks large under any given pooling arrangement it may be considered imprudent to operate with too low a level of automatic write-off limit, a fortiori to allow Full Stop Loss. LTD premium rates are a balanced compromise between incidence and persistency, high persistency being combined with low incidence or vice versa: for most of us, they are a classic case of “making bricks without straw”. Too high a persistency assumption will result in reserves, on the premium basis, which are too high, and vice versa.
If reserves are too high, and result in write-offs, the network can be involved in the payment of excessive dividends. Only if the exposure is very large, and both the premium rates and the reserving bases for a given arrangement can be framed in relation to past claims experience, giving statistically significant measures of both incidence and persistency, or there is massive expected surplus, may it be considered prudent to offer Full Stop Loss in a case where LTD cover bulks large.

In any event, it can be seen that over-reserving is not necessarily prudent practice if a powerful AWOL is operating.

### 7.7 Over-estimating expected surplus

The risk charge is a function of the expected surplus and the coefficient of variation. Because it is essentially an amalgam of imponderables the actuary may well find himself losing sleep over the possibility that he has over-estimated expected surplus. Let us consider the consequence of under-estimating the expected claims by 100k%, so that the risk charge should have been

\[
\phi \left[ (1-k) S - k \sqrt{(1-k)} VT^{-\epsilon} \right] \times \frac{E}{1-k}
\]

instead of

\[
\phi \left( S, VT^{-\epsilon} \right) \times E
\]

What is the effect upon the expected value of the dividends of using the latter rather than the former?

As we have seen, if we assume that the probability of discontinuance is in part a function of the loss due to be carried forward, one consequence of using too low a risk charge may be greater persistency of the pooling arrangement. Let us ignore this (essentially unquantifiable) effect and assume that the persistency is unaffected, i.e. that \(T\) and \(\epsilon\) are unaltered.

Suppose \(S = .15, E = 6, V = .5\),

\[
k = \frac{.05}{1.15}, \quad T = 15, \quad \epsilon = 0.3
\]

so that we should have used

\[
\phi \left( 0.1, \sqrt{\frac{1.1}{1.15}} \times 0.5 \times 15^{-0.3} \right) \times 6 \times \frac{1.15}{1.1}
\]

\[
= \phi \left( 0.1, 0.217 \right) \times 6.27
\]

instead of

\[
\phi \left( 0.15, 0.5 \times 15^{-0.3} \right) \times 6 = \phi \left( 0.15, 0.222 \right) \times 6
\]
i.e. 7.63% of 6.27 = 0.48

instead of 5.00% of 6.00 = 0.30.

Because of lower persistency the risk charge should perhaps have been still greater, but even on this assessment it has been understated by 2.9% of the expected claims. But the real point here is: how much more dividend will now be given away than can be afforded?

The expected value per annum of the dividend is approximately

\[
A \left( 1.1 - \frac{0.30}{6.27} \cdot 0.217 \right) \times 6.27
\]

\[
= A(1.052, .217) \times 6.27 = .74,
\]

so that the use of a risk charge which is too low by at least 2.9% of the expected claims results, in this case, in dividends which are too great by 1.8% of the expected claims.
8. Technical description of the computer simulation techniques and PASCAL programs used in arriving at the results presented in this paper.

8.1 Equipment:

48K Sinclair Spectrum
"HISOFT" PASCAL compiler

8.2 Programs:

There are two suites of programs for

(1) computation of risk charge and associated parameters
(2) apportionment of risk charge between insurers.

Each suite comprises

(i) an input organization program
(ii) a simulation processing program
(iii) an output organization program.

The structuring described above was dictated by the limitations imposed by 48K of RAM. These limitations were not found to be unduly onerous.

8.3 The input program for suite (1) will accept up to 18 different "cases" at a time, each case being specified by the following data:

- e = expected number of claims p.a.
- h = index to heterogeneity coefficient (1 for 1, 2 for 1.143, 3 for 1.458)
- sur = expected surplus as % of e
- int = rate of interest for accumulation of losses/profits
- inf = rate of inflation for yearly increases in premiums and claims
- t = maximum term of pooling arrangement
- n = number of simulations (usually 1000)
- kind = structure type (1 to 3 for static, slow build-up and dynamic PCF, 4 to 6 for proportionate cyclic, offset cyclic and deferred cyclic TCF)
- atype = AWOL type (1 for application to current year's loss first; otherwise 0)
- K = percentage taken for automatic profit release (APR) or proportionate release of contingency fund (structure type 3)
- AWP = write-off limit (AWOL) as multiple of standard deviation of one year's claims
- TSR = no of years in TCF cycle
- PSR = limit of PCF as multiple of standard deviation
- RND = constant (any number between 1 and $2^{16}$-1) used to initialize pseudorandom number generator
Discontinuance parameters

disconst = fixed rate of discontinuance per thousand p.a.
disfac = coefficient (% of) of \( \left( \frac{1 + p}{s} \right)^3 \)

where \( I \) is the loss due to be carried forward
\( p \) is 3 years' worth of (expected surplus minus risk charge)
\( s \) is the standard deviation of one year's claims (the missing multiplier, viz. \( \sqrt[3]{3} \) is absorbed into the coefficient).

rel = maximum contribution to surplus arising on discontinuance (e.g. difference between full reserve and local surrender-value), as percentage of expected claims
relnil = initial period (no. of years) during which the discontinuance release is suppressed
relspd = initial period (no. of years) over which the potential discontinuance release is deemed to mature

Expense parameters

strn = initial expense strain in first year, as percentage of expected claims
strnspd = amortization period of initial expense strain

Surplus parameters

survar = percentage of sur taken to be variable (e.g. contribution from interest surplus)
surspd = initial period (no. of years) over which the variable component of surplus is deemed to mature

8.4

The output program for suite (1) provides the following information:

RC = risk charge, as percentage of average claims
Loss = average write-off, as percentage of average claims (which equates RC)
Sur = percentage surplus, as input
Gain = average dividend, as percentage of average claims (which equates Sur, except where survar > 0)
Index = computed value of \( e \), derived from
Gain = \( A (1 + Gain \cdot RC, VT) \)

where \( T \) is the mean lifetime to discontinuance.

Expected and Actual numbers of claims (averaged over \( t \) years)
Expected and Actual coefficients of variation (averaged over \( t \) years)
Effective term (= \( T \) used for Index)

Weighted = actual number of claims averaged over lifetime of pooling arrangement (i.e. up Dths to discontinuance, or \( t \) if not discontinued earlier)

In computing the effective term (\( T \)) use is made of the following approximate "identity".

\[
\sqrt{\sum \frac{x_t (1+i)^t}{E}} \approx \sqrt{\frac{\sum x_t (1+i)^t}{E}}
\]
so that the coefficient of variation over $T$ years is virtually independent of weighting by compound interest/inflation factors.

8.5

An attempt has been made to print out the full Pascal programs for insertion in this paper, using the Sinclair printer. These programs are not particularly elegant and are probably capable of considerable technical improvement. However, they do have the virtue that, for anyone who has a nodding acquaintance with one of the FORTRAN-type high level languages, they should be relatively easy to read and understand, given the definitions in 8.3 and 8.4 above.

The programs simply carry out a simulation of the claims experience, repeated $n$ times (where $n$ is 1000, usually), over the duration of the pooling arrangement, and compute the risk charge and index value ($e$) for the specified pooling structure, by iteration.

8.6

The pseudorandom number generator is as follows:

$$x := \frac{1}{\sqrt{E \sum_{t} x_{t}}} = \sqrt{\sum_{t} x_{t}}$$

8.7

The succession of equally likely probabilities ($r$) is mapped into a succession of simulated claims via the appropriate cumulative probability distribution. Thus, if

$$p(x) = \text{probability of less than } x \text{ claims}$$

and $r \in [p(x), p(x+1)]$ we have $x$ claims $[p(0) = 0]$

The program uses the poisson distribution via an expected number of claims.

An alternative technique would consider the timing of claims in respect of each individual life and associated sum at risk (duly replaced after each claim) over the total period of $t \times n$ years. Thus, the probability of $(x-1)$ claim-free years followed by a claim in year $x$ is

$$(1-q)^{x-1}q$$
and the probability that the first claim occurs within $x$ years is therefore

$$p(x) = q \sum_{t=1}^{t=x} (1-q)^{t-1} = 1 - (1-q)^x$$

Hence, if

$$r \in [p(x), p(x+1))$$

we have the first claim in the $(x+1)$th, year.

It follows that $x$ is given by

$$(1-r) \in [(1-q)^x, (1-q)^{x+1})$$

and, since all $r$ are equally likely, by

$$r \in [0,1) \in [(1-q)^{x+1}, (1-q)^x) ; \quad x \in \mathbb{N}_0$$

Taking logarithms, we have

$$\frac{\log_e(r)}{\log_e(1-q)} \in \left[ x, x+1 \right) ; \quad x \in \mathbb{N}_0$$

(taking care to change $r=0$ to $r=1$, if it occurs).

Thus, $x = \text{TRUNC} \left( \log (r)/\log (1-q) \right)$.

In this way, the timing of claims in respect of each "life" can be established by spacing them out over the $\text{txn}$ years, each successive claim occurring within the $(x+1)$th, year after the year of the previous one, for successive "throws" of $r$ mapping to $x$.

8.8

The technique described in 8.7 allows us to deal with individual lives covered for different sums at risk. It requires considerably more computing capacity than that available in a 48K machine and the author preferred to use instead the simpler technique based on the cumulative poisson distribution and 3 alternative distributions of the sums at risk, assumed to be independent of age. The sum at risk attributed to each claim is drawn randomly from the 3 alternative distributions, indexed 1,2 and 3, viz.

- $H=1$ uniform sums at risk
- $H= \sqrt{(14.375/11)} = 1.143 : \frac{1}{4},\frac{1}{4},\frac{1}{2},1,1,1,1,1,1,2,2.$
- $H= \sqrt{(23.375/11)} = 1.458 : \frac{1}{4},\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},1,1,1,2,4.$
E103 150 END;
E107 151 IF totxs > 0.0 THEN BEGIN
E109 152 xs := BOOL(-h[l] > amp) * (-h[l] - amp);
E114 153 FOR case := 1 TO z DO BEGIN
E121 155 l[2, case] := ll + l[2, case] +
E123 156 r[4, 2, case] := r[4, 2, case] - ll;
E126 157 END; END;
E129 158 FOR case := 1 TO z DO BEGIN
E131 159 ll := BOOL(-l[1, case] > a[case]) * (-l[1, case] - a[case]) / l[1, case];
E134 160 l[1, case] := ll + l[1, case] +
E137 161 r[4, 1, case] := r[4, 1, case] - ll;
E140 162 END; END;
E143 163 FOR case := 1 TO z DO BEGIN
E145 164 IF type := 1 TO 2 DO BEGIN
E147 165 ll := x[type, case];
E150 166 y := BOOL(l[0,0] + l[1,0] + l[2,0] + l[3,0]) * BOOL(l[0,0] + l[1,0] + l[2,0] + l[3,0]);
E153 167 r[4, type, case] := r[4, type, case] + y;
E156 168 END; END;
E159 169 END;
E162 170 END;
E164 171 index := 0;
E167 172 FOR case := 1 TO z DO BEGIN
E169 173 r[4, 2, 22] := r[4, 2, 22] + r[4, 2, case];
E172 174 FOR type := 1 TO 2 DO BEGIN
E175 176 r[4, type, case] := r[4, type, case] +
E178 177 IF (ABS(r[2, type, case] - r[4, type, case]) + loss) > 0.001 THEN
E181 178 index := index + 1;
E184 179 r[5, type, case] := r[5, type, case] + loss;
E189 181 END; END;
E192 182 BEGIN
E194 183 input;
E196 184 FOR case := 1 TO z DO BEGIN
E198 185 FOR type := 1 TO 2 DO BEGIN
E201 186 r[1, type, case] := 1.0 + s[case];
E203 187 r[2, type, case] := 0.0;
E205 188 r[3, type, case] := 1.0;
E208 189 END; END;
E210 190 count := 0;
E212 191 REPEAT
E214 192 count := count + 1;
E216 193 IF count > 1 THEN BEGIN
E218 194 FOR case := 1 TO z DO BEGIN
E221 195 FOR type := 1 TO 2 DO BEGIN
E223 196 r[1, type, case] := r[1, type, case];
E225 197 r[2, type, case] := r[2, type, case];
E227 198 r[3, type, case] := r[3, type, case];
E230 199 r[4, type, case] := r[4, type, case];
CC20 99 END;
CC22 100 l[(case):=clm;
CC50 101 END;
CC54 102 b[0,0; h[0,0;
CCAC 103 FOR C358;=i TO Z DO BEGIN
CCCD 104 FOR type.=i TO 2 DOCCE7 105 l[type, case]:=l[type, case]+p[type, case]-c l £ case 3 ; END;
CE03 177 IF[NOT (q=term) THEN BEGIN
CE17 1©S ll :=BOOL (ll, case) • v [cases) • v [case] +BOOL (NOT (ll, case)
CF9D 110 l[(1, case):=ll;
CDE2 111 END;
D03C 112 ELSE
D03F 113 div:=BOOL ((1, case) >0)
*([1, case];
DF03 114 r[(5,1, case):=r[(5,1, case
D188 115 IF (((case=1) AND (ind=1)) THEN ll:=[2,1]:=[2,1]-div;
D283 116 h[1]=h[1]+[2, case];
D289 117 h[2]=h[2]+BOOL (ll, case) 1<0,0)!*ll, case1;
D3EC 118 w[case]:=0,0; END;
D41C 119 adj: =0,0;
D429 120 IF hawks=1 THEN BEGIN
D43B 121 FOR case:=1 TO 2 DO
D459 122 w[case]:=BOOL (ll, case) 1=0,0)!*ll, case); END;
D522 123 IF (NOT (q=term)) THEN BEGIN
D52F 124 BEGIN
D53F 125 FOR case:=1 TO Z DO BEGIN
D55D 126 IF (ll, case) >0,0 THEN BEGIN
D570 127 IF (ll, case) >zr (case)
D59F 128 IF hawks=1 THEN BEGIN
D61C 129 h[1]=h[1]-ll;
D63C 130 w[case]:=w[case];
D65C 131 l[2, case]:=l[2, case] -ll;
D705 133 END;END;
D72C 134 IF h[1]>0,0 THEN BEGIN
D749 135 IF h[1]>psr THEN adj:=
D765 136 ELSE adj: =1,0; END
D781 138 IF h[2]>0,0 THEN BEGIN
D809 139 ELSE quot:=BOOL (h[1] >
D835 140 IF h[2]=0,0 THEN quot:=
D851 141 ELSE quot:x:=h[1]/h[2];
D871 142 IF hawks=1 THEN quot:x:=
D892 143 ELSE adj:=
D917 144 FOR case:=1 TO Z DO BEGIN
DA0B 145 r[(5,2, case):=r[(5,2, case)
DE2F 147 l[2, case]:=BOOL (h[1] <0,0) • h[1] • l[2, case] • quot • adj • BOOL (hawks =1) 1=1-quot);
DFAF 148 w[case]:=w[case] +ll;
DFF3 149 IF l[2, case] >0,0 THEN quot:=
DF03 149 IF l[2, case] =0,0 THEN l[2, case]:=w[case];
DF03 149 totxs:=totxs+BOOL (-l[2, case] (£ case) 1= case) 1= (l[2, case] +a[case]);
F282 201 IF denom=0 THEN r1:=(r1/r2)/2.0
F2C0 202 ELSE r1:=(r1*r4-r2*r3+1.0)/denom;
F335 203 r2,type,case]:==r1;
F39A 204 r[1,type,case]:=r2;
F5F5 205 r[3,type,case]:=r4;
F64C 206 END;END;END;
F6BC 207 simulation;
F77D 208 WRITELN;WRITELN('count = ',count,';index = ',index);
F4C9 209 UNTIL (index=2*Z);
F5FF 210 WRITELN;WRITELN('Enter zero to start microdrive.');READ
F46B 211 TOUT ('2:RESUFI',ADDR(r)
F51F 212 END.
F46B 213 simulation;
F4E9 214 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 215 UNTIL (index=2*Z);
F51F 216 END.
F46B 217 simulation;
F4E9 218 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 219 UNTIL (index=2*Z);
F51F 220 END.
F46B 221 simulation;
F4E9 222 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 223 UNTIL (index=2*Z);
F51F 224 END.
F46B 225 simulation;
F4E9 226 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 227 UNTIL (index=2*Z);
F51F 228 END.
F46B 229 simulation;
F4E9 230 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 231 UNTIL (index=2*Z);
F51F 232 END.
F46B 233 simulation;
F4E9 234 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 235 UNTIL (index=2*Z);
F51F 236 END.
F46B 237 simulation;
F4E9 238 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 239 UNTIL (index=2*Z);
F51F 240 END.
F46B 241 simulation;
F4E9 242 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 243 UNTIL (index=2*Z);
F51F 244 END.
F46B 245 simulation;
F4E9 246 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 247 UNTIL (index=2*Z);
F51F 248 END.
F46B 249 simulation;
F4E9 250 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 251 UNTIL (index=2*Z);
F51F 252 END.
F46B 253 simulation;
F4E9 254 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 255 UNTIL (index=2*Z);
F51F 256 END.
F46B 257 simulation;
F4E9 258 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 259 UNTIL (index=2*Z);
F51F 260 END.
F46B 261 simulation;
F4E9 262 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 263 UNTIL (index=2*Z);
F51F 264 END.
F46B 265 simulation;
F4E9 266 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 267 UNTIL (index=2*Z);
F51F 268 END.
F46B 269 simulation;
F4E9 270 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 271 UNTIL (index=2*Z);
F51F 272 END.
F46B 273 simulation;
F4E9 274 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 275 UNTIL (index=2*Z);
F51F 276 END.
F46B 277 simulation;
F4E9 278 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 279 UNTIL (index=2*Z);
F51F 280 END.
F46B 281 simulation;
F4E9 282 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 283 UNTIL (index=2*Z);
F51F 284 END.
F46B 285 simulation;
F4E9 286 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 287 UNTIL (index=2*Z);
F51F 288 END.
F46B 289 simulation;
F4E9 290 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 291 UNTIL (index=2*Z);
F51F 292 END.
F46B 293 simulation;
F4E9 294 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 295 UNTIL (index=2*Z);
F51F 296 END.
F46B 297 simulation;
F4E9 298 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 299 UNTIL (index=2*Z);
F51F 300 END.
F46B 301 simulation;
F4E9 302 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 303 UNTIL (index=2*Z);
F51F 304 END.
F46B 305 simulation;
F4E9 306 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 307 UNTIL (index=2*Z);
F51F 308 END.
F46B 309 simulation;
F4E9 310 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 311 UNTIL (index=2*Z);
F51F 312 END.
F46B 313 simulation;
F4E9 314 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 315 UNTIL (index=2*Z);
F51F 316 END.
F46B 317 simulation;
F4E9 318 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 319 UNTIL (index=2*Z);
F51F 320 END.
F46B 321 simulation;
F4E9 322 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 323 UNTIL (index=2*Z);
F51F 324 END.
F46B 325 simulation;
F4E9 326 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 327 UNTIL (index=2*Z);
F51F 328 END.
F46B 329 simulation;
F4E9 330 URITELN;URITELN('Enter zero to start microdrive.');READ
F4CC 331 UNTIL (index=2*Z);
F51F 332 END.
F46B 333 simulation;
F4E9 334 BEGIN IF x THEN BOOL:=
F51F 335 ELSE BOOL:=
F51F 336 END;
F46B 337 simulation;
F4E9 338 BEGIN IF x<y THEN MIN:
F51F 339 ELSE MIN:
F46B 340 END;
FUNCTION MAX(x, y: REAL)
BEGIN
  IF x > y THEN MAX := x
  ELSE MAX := y
END;

PROCEDURE input;
BEGIN
  TIN('DATFILE1', ADDR(data_tint)),
  TIN('DATFILE2', ADDR(data_tint)),
  TIN('DATFILE3', ADDR(data_tint)),
  TIN('DATFILE4', ADDR(data_tint)),
  TIN('DATFILE5', ADDR(data_tint)),
  TIN('DATFILE6', ADDR(data_tint)),
  TIN('DATFILE7', ADDR(data_tint)),
  TIN('DATFILE8', ADDR(data_tint)),
  TIN('DATFILE9', ADDR(data_tint)),
  TIN('DATFILE10', ADDR(data_tint)),
  TIN('DATFILE11', ADDR(data_tint)),
  TIN('DATFILE12', ADDR(data_tint)),
  randgen := data[1];
  loss := data[2];
  haw := data[3];
  awp := data[4];
  psr := data[5];
  term := data[6];
  sims := data[7];
  z := data[8];
  ind := data[9];
  z := z + 1;
  TIN('2: RESUFL', ADDR(value));
END;

PROCEDURE printout;
BEGIN
  WRITELN('RandO TO no. seed is', randgen:5:0);
  IF hawks = 1 THEN WRITELN('Allocation is hawks-type');
  ELSE WRITELN('Allocation is by DOVES 3');
  IF ind = 1 THEN BEGIN
    WRITELN('The first office distributes');
    WRITELN('all its surplus locally');
  END;
  IF loss > 0 THEN WRITE('Inbuilt LOSS is', loss * 100:5:2, ' % of exp. claims');
  WRITELN('Expected term is', term);
  WRITELN('No. of simulations is', sims);
  WRITELN('AWP is', awp:6:2);
  WRITELN('PSR is', psr:6:2);
  WRITELN('Off Ben Exp Act is');
  FOR J := 1 TO X DO
    WRITELN(j:2, ENTIER(b[i, j]), c[i, j]:6:2, c[i, j]:6:2);
  FOR J := 1 TO 31 DO
CE45  93 WRITE(‘ - ’); WRITELN;
CE53  94 WRITELN(’ALL’, c[1, zz] : 15 : 2, c[2, zz] : 12 : 2);
CEFC 95 FOR j := 1 TO 81 DO
CF10  96 WRITE(‘ - ’); WRITELN;
WRITE;
CF27  97 IF printervideo=0 THEN
READ(ind);
CF3F  98 WRITELN(’Office Surplus Reserve Reserve Reserve Ratio’);
CF6C  99 WRITELN(
CF98 100 sur := 0.0;
CF9B 101 FOR j := 1 TO z DO
CF6C 102 sur := sur + r[5, 2, j] * c[1, j] ;
DF07 103 sur := 100.0 * sur / c[1, zz] ;
D0DC 104 FOR j := 1 TO z DO BEGIN
D0FD 105 WRITE(j:3, ROUND(s[j] * 1.0:7, c[2, j] / c[1, j] : 6:2, ;
D27E 107 WRITELN;
D281 108 IF prntorvideo=0 THEN READ(ind) ;
D3E3 111 WRITE(’ ’); WRITELN;
D4F7 115 sur := 0.0;
D404 116 FOR j := 1 TO z DO
D422 117 sur := sur + r[j] ;
D45F 118 IF sur = 0 THEN WRITELN(’Stabilization reserves are collective.’);
D4B1 119 ELSE WRITELN(’Stabilization reserves are localized.’);
D507 121 IF printervideo=0 THEN READ(ind);
D51F 122 x := i * 2.0 - 0.5 * (i - 1) * (i - 2) ;
D57A 123 k := ENTIER(x) ;
D587 124 CASE 1 OF
D590 125 1: WRITELN(’Risk Charges’);
D5B7 126 2: WRITELN(’Losses’);
D5DB 128 3: WRITELN(’Dividends’);
D5F6 129 END;
D5FD 130 WRITELN;
D5FF 131 WRITE(’Office Alone Company’);
D61F 132 WRITELN;
D625 133 sum := 0.0;
D653 135 WRITELN(case:3, r[k, 1, case] * 100:9:3, r[k, 2, case] * 100:9:3);
D748 136 sum := sum + r[k, 2, case] * c[1, case] ;
DEFE 137 WRITELN;
D801 138 sum := sum * 100.0 / c[1, zz] ;
D85E 140 FOR j := 1 TO 21 DO
D873 143 WRITE(’ - ’); WRITELN;
D888 141 WRITELN(’ALL
D8B4 142 FOR j := 1 TO 21 DO
D904 145 WRITE(’ - ’); WRITELN;
D924 150 IF num = 1 THEN BEGIN


FOR i:=1 TO 13 DO
  FOR j:=1 TO 14 DO
    matrix[i,j] := 0.0;
END;
num := 0;
WRITELN; WRITELN('Print Then finish? ');
WHILE num=0 DO
  BEGIN
    READ (printvideo/num);
    IF printvideo=1 THEN
      WRITELN(CHR(16));
    END;
    WRITELN ('Is the matrix to be updated?');
    READ(num);
    FOR i:=1 TO 14 DO
      ss[i] := 0.0;
    FOR case:=1 TO Z DO
      BEGIN
      END;
    FOR case:=1 TO Z DO
      BEGIN
        vector[1] := c[1, case] / ss[1];
      END;
VECTOR(14) := r[2,2,case1,case]/ss[14];
FOR i := 1 TO 13 DO
  FOR j := 1 TO 14 DO
    MATRIX[i,j] := MATRIX[i,j] + VECTOR[i]*VECTOR[j];
  END;
END;
TOUT('2: MATRIX', ADDR(MATRIX), SIZE(MATRIX));
FOR i := 1 TO 13 DO BEGIN
  READ(num); PAGE;
  FOR j := 1 TO 14 DO
    WRITELN(MATRIX[i,j] : 8:2);
  END;
END.
End Address: F73C
9.1 **DIVIDEND VALUATION FUNCTION: A(U,V)**

<table>
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<th>(V)</th>
<th>(U = 0.5)</th>
<th>(U = 0.6)</th>
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### 9.2 RISK CHARGE FUNCTION: \( \varphi(S,V) \)

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In the practical situation, the coefficient of heterogeneity, \( H \), may be calculated to give effect to correlation with age by taking

\[
H^2 = \frac{\sum B_i q_i}{\left(\sum B_i q_i\right)^2}
\]

instead of, simply,

\[
H^2 = \frac{n \sum B^2}{\left(\sum B\right)^2}
\]

Since all of these calculations are primarily an aid to judgment, and fine precision is evidently out of place, a sensible compromise is achieved by pulling out the "sore thumbs" -i.e. lives associated with abnormally high sums at risk - and estimating \( H^2 \) from

\[
\frac{(m_i + m_a) \left( H^2 \bar{B}_i m_i + H_a^2 \bar{B}_a m_a \right)}{(\bar{B}_i m_i + \bar{B}_a m_a)^2}
\]

where the suffices relate to the two groups, i.e. "sore thumbs" and the rest.

The appropriate poisson distribution is then used, with

\[
\lambda = \frac{H}{\sqrt{m_i + m_a}}
\]

8.9 The second suite of programs was designed to explore the effect of different principles of apportionment on the allocation of the risk charge among the network insurers participating in a given pooling arrangement. In fact, of the three different principles discussed in chapter 6, only two occasion recourse to computer simulation. These two are the arm's-length and limited mutuality principles, referred to in the program somewhat quizically as "hawks" and "doves".

8.10 The input program will accept data for a network of up to 6 different insurers, each carrying one contract characterized by the following data items:

- \( b[i] \) = average benefit size for ith insurer
- \( e[i] \) = expected no. of claims
- \( s[i] \) = expected surplus, as percentage of \( e[i] \)
- \( v[i] \) = contingency fund limit for assessment of independent risk charge for ith insurer, given as an amount
- \( zr[i] \) = local contingency fund limit, defining a contingency fund which is reserved for the local insurer in computing the international dividend, given as an amount.
The common input items are

- **term** = expected term to discontinuance (usually taken as 10, 15 or 20); there is no provision for loss-dependent discontinuance rates
- **sims** = number of simulations (usually 1000)
- **het** = coefficient of heterogeneity, either 1 or \( \sqrt{\frac{14.375}{11}} \)
- **loss** = inbuilt loss (or profit) to the insurers, designed to be shared in proportion to expected claims, expressed as a percentage of expected claims (this is relevant when, for example, the risk charge is set with a deliberate margin, which may be negative in a highly competitive situation)
- **randgen** = pseudorandom number seed
- **hawks** = index for setting apportionment principle: 1 for arm’s-length, 0 for limited mutuality
- **awp** = AWOL here expressed as an amount
- **psr** = overall contingency fund limit, here expressed as an amount.

The program incorporates an option to treat the whole of insurer 1’s surplus as being distributed by local experience-rating.

**8.11** The output program for suite (2) provides the following information in respect of each insurer:

- Expected and average actual claims and standard deviation of claims.

- Independent local risk charge, using \( v[i] \) and apportionment of awp by reference to expected claims, expressed as a percentage of expected claims.

- Apportioned overall risk charge, expressed as a percentage of expected claims.

- Average loss, as a percentage of expected claims (which should equate the apportioned risk charge adjusted for any inbuilt loss).

- Average dividend, as a percentage of expected claims. (Although the average total dividend should equate the average expected surplus, adjusted for any inbuilt loss, this will not be the case for individual insurers, except on the arm’s length principle).

**8.12** The program simulates the claims experience of the network insurers and the working of the pooling and apportionment structures.

The automatic write-off is assumed to be a fixed multiple of expected claims, both overall and for individual insurers, the same multiple applying to all insurers irrespective of size. The apportionment of the overall write-off among the insurers is in proportion to the individual insurers’ excesses over their own write-off limits. Clearly, a different apportionment of the overall write-off, e.g. simply in proportion to individual insurers’ losses, rather than their excess losses, would lead to a different apportionment of the risk charge.
1. RISK CHARGE PROGRAMS.

Run

2 PROGRAM DATAIN;
B711 3 CONST
B711 4 last=16;
B711 6 VAR
B71A 6 i, case, repeat, fin, s
B71A 7 number : ARRAY [1..2] OF
B71A 8 INTEGER;
B71A 8 data : ARRAY [1..last] OF
B71A 9 ARRAY [1..23] OF INTEGER;
B71A 9 PROCEDURE headings;
B71A 10 BEGIN
B71A 11 WRITELN;
B71A 12 WRITE('Case Exp H Su
B71A 13 INT In T N ');
B71A 14 WRITE('Kind K AW
B71A 15 WRITE('Dis.paras** E
B71A 16 WRITE('Paras** Sur.Paras');
B71A 17 END;
B71A 18 PROCEDURE cases;
B71E 19 VAR i, case, repeat, chan
g, fin, stop : INTEGER;
B71E 20 BEGIN
B71E 21 headings;
B71E 22 WRITELN('X....X...X..xx..xx..xx.xxxx');
B721 23 WRITELN('....X.X...X..
x.x.xxx.xxx.xxx');
B721 24 WRITELN('x..x.xxx.x.x
x***xxx.xxx***.xxx');
B721 25 fin := 0;
B721 26 WHILE fin=0 DO BEGIN
B721 27 READ(case, repeat, fin);
B721 28 IF repeat = 0 THEN BEGIN
B721 29 WRITE('Case: 2');
B721 30 FOR i := 1 TO 7 DO
B721 31 READ(data[case, i]);
B721 32 WRITE('');
B721 33 FOR i := 8 TO 14 DO
B721 34 READ(data[case, i]);
B721 35 WRITE('');
B721 36 FOR i := 15 TO 23 DO
B721 37 READ(data[case, i]);
B721 38 END;
B721 39 ELSE BEGIN
B721 40 FOR i := 1 TO 23 DO
B721 41 data[case, i] := data[rep
B721 42 eat, i];
B721 43 stop := 0;
B721 44 WHILE stop=0 DO
B721 45 READLN(change, data[cas
B721 46 , change, stop]);
B721 47 WRITE('Case: 2');
B721 48 WRITE(data[case, 1] : 5);
B721 49 FOR i := 2 TO 5 DO
B721 50 WRITE(data[case, i] : 4);
B721 51 WRITE(data[case, 6] : 3);
B721 52 WRITE(data[case, 7] : 5);
B721 53 WRITE(data[case, 8] : 5);
B721 54 WRITE(data[case, 9] : 2);
B721 55 FOR i := 10 TO 13 DO
B721 56 WRITE(data[case, i] : 4);
B721 57 WRITE(data[case, 14] : 7);
B721 58 WRITE(data[case, 15] : 2);
B721 59 WRITE(data[case, 16] : 3);
B721 60 WRITE(data[case, 17] : 4);
B721 61 WRITE(data[case, 18] : 2);
B721 62 WRITE(data[case, 19] : 3);
B721 63 WRITE('**');
B721 64 WRITE(data[case, 20] : 4);
BFB4 62 WRITE(data[case,21]:3);
C007 63 WRITE(data[case,22]:2);
C04D 64 WRITE(data[case,23]:3);
C093 65 END;
C0A1 66 BEGIN
C0AA 67 cases;
C0AF 68 WRITELN;WRITELN;
C0B5 70 WRITE("Number of cases is ");
C0D3 71 READLN(num);
C0F6 72 FOR i:=1 TO 2 DO
C11F 73 number[i] :=num;
C138 74 WRITE(\"Start drive to receive data by entering zero.\";
C15A 75 READLN(fin);
C163 76 TOUT\(2:\\"DATAFI\\", ADDR(data), SIZE(data)));
C163 77 TOUT\(2:\\"NUMFIL\\", ADDR(number), SIZE(number))
C1A3 78 END.

Run Address: CIA5

2 PROGRAM RISK;
CAE9 3 CONST m9=75.0;
CAE9 4 d9=65537.0;
CAE9 5 e9=65536.0;
CAE9 6 discind=0;
CAE9 7 PI=3.14159;
CAE9 8 VAR denom, gain, loss, sf, epsilon, r1, l1, r, e, x1, mean, 
CAE9 9 sd, meanx, sx, prc, awp, psr, 
CAE2 11 sr, v, index, sur, k, randg, 
CAE9 12 en, int, info, dth, pf, disconst, disfa, 
CAE9 13 c, y, Project, 
CAE9 14 spread, tspread, tloss, t 
CAE9 15 gain, inc, dic, awp, psr, x, lint, lin 
CAE2 16 r, rel, relind, relspd, 
CAE2 17 strn, rls, strs, surva, 
CAE2 18 prs, psuv, survy, tsr, q, icf, dvs, w, 
CAE2 19 cv, v1, v2, z, dd, intx, sdx 
CAE2 20 x, infx, meanxx, manxx, pp, ps, psuv, s 
CAE2 21 sd 
CAE2 22 : REAL;
CAE2 23 i, j, a, q, lim, h, n, 
CAE2 24 kind, t, surc, awpc, psr, case, num, a 
CAE2 25 type, ben, 
CAE2 26 prntorvideo, fin, count, 
CAE2 27 kount, tot 
CAE2 28 : INTEGER;
CAE2 29 d : ARRAY[1..61] OF REAL;
CAE2 30 o : ARRAY[1..3,1, 
CAE2 31 .11] OF REAL;
CAE2 32 hadj : ARRAY[1..3] OF REAL;
CAE2 33 data : ARRAY[1..last] 
CAE2 34 OF ARRAY[1..last] OF INTEGER;
CAE2 35 number : ARRAY[1..2] OF 
CAE2 36 INTEGER;
CAE2 37 results : ARRAY[1..last] 
CAE2 38 OF ARRAY[1..11] OF REAL;
CAE2 39 FUNCTION BOOL(x: BOOLEAN) : REAL;
CAE2 40 BEGIN IF x THEN BOOL :=1.0;
CAE2 41 ELSE BOOL :=0.
CAE2 42 END;
CAE2 43 PROCEDURE rnd; 
CAE2 44 BEGIN 
CAE2 45 r :=(x1+1.0)*#9; 
CAE2 46 x1 :=r-1-(ENTIER(r/
CAE2 47 d9))*d9;
103 awpx := awp;
104 psrx := psr;
105 sdxx := dxx;
106 gain := 0.0;
107 loss := 0.0;
108 spread := 0.0;
109 mnxx := 0.0;
110 REPEAT
111 if (disconst = 0.0) AND (disfac = 0.0) THEN y := 1.0
113 ELSE BEGIN
114  rnd; y := r END;
116 q := qfl;
117 sdx := sdx*intx/ints;
118 i := 1;
119 IF tsr > 0.0 THEN BEGIN
120  qq := q/tsr;
121  IF qq = 0 THEN qq := 1;
122 END;
124 jcc := jcc*intx/int;
125 tot := tot + 1;
126 loss := loss + int;
127 gain := gain + int;
128 awpx := awpx + inf;
129 psrx := psrx*inf;
130 ps := psur*int/urspd;
131 i := i + 1;
132 WHILE x >= d(1) DO
133 BEGIN
134  i := i + 1;
135  ben := ENTIER(r*111) + 1;
136  dth := dth + 0.01
137  mnxx := (mnxx + int) + (dth + inf);
138  END;
139 END;
140 if (psur*q/psur) THEN pf := p + psur*q/psur;
142 ELSE pf := p + psur;
143 IF pf > 0.0 THEN BEGIN
144  pp := (pf - meanx)*int;
145  ps := psur*int/psur;
146  i := 0.0;
147 IF q = l THEN pf := pf - meanx;
148 IF (atype = 1) AND (-pf > awpx) THEN
149 BEGIN
150  loss := loss - awpx;
151  awpx := awpx;
152  END;
153 IF NOT (kind = 3) AND (k > 0.0) THEN BEGIN
154  gain := gain + k*pf;
155  pf := (1.0 - k)*pf;
156  END;
157  cf := cf + int;
158  cf := cf + int;
159  disc := disc + int;
160  i := 0.0;
161 BEGIN
162 IF ((kind < 4) OR (qq = 1)) THEN BEGIN
163  cf := cf + int;
164  cf := cf + int;
165  disc := disc;
166  i := 0.0;
167  IF (cf > awpx) THEN BEGIN
168  cf := cf - awpx;
169  loss := loss + wf;
170  cf := awpx;
171  END;
172  END;
173  END;
174  END;
175  END;
176  END;
177  END;
178  END;
179  END;
180  END;
181 BEGIN
182 IF ((kind < 4) OR (qq = 1)) THEN BEGIN
183  cf := cf + int;
184  cf := cf + int;
185  disc := disc;
186  i := 0.0;
187  IF (cf > awpx) THEN BEGIN
188  cf := cf - awpx;
189  loss := loss + wf;
190  cf := awpx;
191  END;
192  END;
193  END;
194  END;
195  END;
196  END;
197  END;
198  END;
199  END;
200  END;
201  END;
202  END;
203  END;
204  END;
205  END;
206  END;
207  END;
DB19 175 \( x := \text{BOOL}(x > d\text{vs}) \ast (x - d\text{vs}) \)

DB6E 176 \( d\text{vs} := \text{dvs} + x; \)

DB6S 177 \( \text{gain} := \text{gain} + x; \)

DBA2 178 \( cf := cf - x; \)

DC0 179 IF \( qq = 1 \) THEN BEGIN

DBE 180 \( d\text{vs} := 0.0; \)

DBEE 181 \( cf := \text{BOOL}(cf < 0.0) \ast cf \)

DC1 182 END END;

DC2B 183 IF \( cf < 0.0 \) THEN BEGIN

DC46 184 IF \( \text{disfac} > 0.0 \) THEN BEGIN

DC5E 185 \( \text{psum} := 0.0; \)

DC7B 186 FOR \( i := 1 \) TO 3 DO BEGIN

DC98 187 \( pp := pp + \text{BOOL}((i + q - 1) < \text{spd}) \ast pp; \)

DCF6 188 \( \text{psum} := \text{psum} + pp; \)

DD0F 189 END;

DD12 190 \( \text{psum} := \text{psum} + cf; \)

DD20 191 IF \( \text{psum} > 0.0 \) THEN \( \text{psum} := 0.0; \)

DD6F 192 \( \text{disc} := \text{disc} - \text{disfac} \ast \text{psum} \)

DDAC 193 IF \( \text{disc} > 1.0 \) THEN \( \text{disc} := 1.0; \)

DDDE 194 END;

DDDE 196 ELSE BEGIN

DDFF 197 \( x := 0.0; \)

DE0 198 IF \( \text{kind} = 4 \) THEN \( x := qq; \)

DEB 199 IF \( \text{kind} = 6 \) THEN \( x := \text{BOOL}(qq < 1); \)

DE65 200 \( \text{gain} := \text{gain} + x \ast cf; \)

DE7B 201 \( cf := cf \ast (1 - x); \)

DE81 202 IF \( \text{kind} = 3 \) THEN BEGIN

DE96 203 \( \text{gain} := \text{gain} + cf \ast cf; \)

DECC 204 \( cf := (1 - k) \ast cf \)

DEF4 205 END;

DF05 206 IF \( \text{kind} = 4 \) THEN BEGIN

DF19 207 \( \text{sr} := \text{psr} \ast x; \)

DF27 208 IF \( (\text{kind} = 2) \) AND \( (q < 5) \)

DF4C 209 THEN

DF7B 210 IF \( cf > \text{sr} \) THEN BEGIN

DF97 211 \( \text{gain} := \text{gain} + cf - \text{sr}; \)

DFC1 212 \( cf := \text{sr} \)

DFC3 213 END END;

DFCF 214 END END;

DFE1 215 IF \( \text{disfactor} = 0 \) THEN

DFE 216 IF \( ((\text{kind} = 3) \) AND \( (qq < 1) \)

E00E 217 THEN \( \text{disc} := 0.0; \)

E01E 218 UNTIL \((y < \text{disc}) \) OR \((q = t) \)

E05C 219 IF \( ((\text{rel} > 0) \) AND \((q < \text{relmin}))) \)

E07 220 THEN \( cf := \text{cf} - \text{rel} \ast \text{meanxx} \ast \text{in}(\text{BOOL}(q < \text{relspd}) \ast q \ast \text{relspd}) + \text{BOOL}(q > \text{relspd}) \ast q \ast \text{relspd}); \)

E17B 221 LOSS := LOSS - wf;

E1BE 222 IF \( cf < 0.0 \) THEN

E1AC 223 LOSS := LOSS - cf;

E1BC 224 ELSE \( \text{gain} := \text{gain} + cf; \)

E1D8 225 \( \text{tspread} := \text{tspread} + \text{spread} \ast \text{intx}; \)

E20D 226 \( \text{meanxx} := \text{meanxx} + \text{meanxx} \ast \text{intx}; \)

E233 227 \( t\text{gain} := t\text{gain} + \text{gain} \ast \text{intx} \)

E259 228 \( t\text{loss} := t\text{loss} + \text{loss} \ast \text{intx} \)

E27F 229 END;

E283 230 \( \text{meanxx} := \text{meanxx} / \text{tspread} \)

E29D 231 \( \text{loss} := \text{tloss} / \text{tspread} \)

E2C3 232 \( \text{gain} := \text{tgain} / \text{tspread} \)

E2DC 233 END;

E2EF 234 BEGIN

E2FF 235 PAGE;
E2FD 236 FOR i := 1 TO 11 DO BEG
E31A 237 o[i, 1] := 1.0;
E31D 238 o[i, 2] := 0.25 + BOOL(i > 2) + 0.25 + BOOL(i > 7) + 0.5 + BOOL(i > 11) + 2.0;
E428 239 o[i, 3] := 1.0 + 2.0;
E517 240 END;
EBAE 242 TIN('2: DATAF', ADDR(id ta));
EBD0 243 TIN('2: NUMFIL', ADDR(id number));
EBE6 244 WRITELN; WRITELN('NOW COMPUTING'); WRITELN;
EBE7 245 num := number[1];
EBE8 246 FOR case := 1 TO num DO BEGIN
E717 247 e := data[case, 1];
E71D 248 h := data[case, 2];
E78A 249 src := data[case, 3];
E7F1 250 int := 1.0 + data[case, 4]/100.0;
E7F4 251 lnf := LNI(int);
E85E 252 inf := 1.0 + data[case, 5]/100.0;
E8F4 253 infx := inf*inf*inf;
E90A 254 inf := LNI(inf);
E916 255 t := data[case, 6];
E956 256 project := 1.0;
EB6A 257 FOR i := 1 TO t DO BEGIN
EB6F 258 project := project*int,
EBEF 259 h := data[case, 7];
EC0E 260 kind := data[case, 8];
EC6E 261 atype := data[case, 9];
ED5A 262 k := data[case, 10]/0.0;
EB9A 263 awpc := data[case, 11];
EBE7 264 lim := TRUNC(BOOL(e > 0 AND (e < 25)) +4*BOOL(e > 25) +100); EFA8 265 poi;
EBAD 266 tsr := data[case, 12];
EB81 267 psrc := data[case, 13];
EB2E 268 randgen := data[case, 14];
E772 269 disconst := data[case, 1]/1000.0;
EBC1 270 disfac := data[case, 1]/1000.0;
ED18 271 rel := data[case, 17];
ECDF 272 relint := data[case, 18];
EC33 273 relspd := data[case, 19];
EC77 274 strn := data[case, 20]/100.0;
ED36 275 strnspd := data[case, 21];
ED7A 276 survar := data[case, 22];
EDC9 277 surspd := data[case, 23];
E80D 278 IF (inf = int) THEN v :=
E840 279 ELSE BEGIN v := int/inf
E869 280 v := (1-EXP((LNI-LNI) *(strnspd-1))/(v-1)) END;
EC6F 281 IF (NOT(v > 0)) THEN rls := strn/v;
E2F8 282 ELSE rls := 0;
E31C 283 xl := randgen;
E329 284 claims;
E32F 285 sdx := sdx*sdx*sdx/sdx/i
INFX;
EF5D 186 cv =ndx/meanx;
EF77 100 cv =par*ndx/inf;
EF9C 160 arw =arw*ndx/inf;
EF10 200 r1 =1.0+sur;
EF10 200 l1 =1.0;
EF47 101 epsilon =1.0E-3;
EF10 200 loss =l1;
F022 200 rc =1;
F004 294 results [case,7] :=TRUNC
(mean*1000.0+0.5)/1000;
F004 294 results [case,8] :=TRUNC
(cv*1000.0+0.5)/1000;
F004 298 meanx :=meanx;
F016 298 count =0;
F104 298 p =1;
F111 293 REPEAT
F111 300 count :=count+1;
F116 301 xi :=ranagen;
F122 302 MOUNT
F122 304 UNTIL (ABS(rc+delta-loss) (epsilon)
AND (ABS(meanx-meanx));
F122 306 index :=0.0;
F122 306 results [case,1] :=TRUNC
(rc*1000.0+0.5)/10;
F229 307 results [case,2] :=TRUNC
(loss*1000.0+0.5)/10;
F229 308 results [case,3] :=TRUNC
(sur*1000.0+0.5)/10;
F30F 309 results [case,4] :=TRUNC
(gain*1000.0+0.5)/10;
F30F 310 results [case,5] :=TRUNC
(index*1000.0+0.5)/1000;
F3EF 311 results [case,6] :=TRUNC
(mean*1000.0+0.5)/1000;
F45F 312 results [case,7] :=TRUNC
((sd/mean)*1000.0+0.5)/1000;
F45F 313 results [case,10] :=p;
F45F 314 results [case,11] :=TRUNC
(mean*x*1000.0+0.5)/1000;
F594 315 WRITELN("Case , case ,");
F594 315 done .' ,count, simulations .)
F5E5 315 ENDO;
F5E5 317 WRITELN('Start drive t o receive results by entering zero.');
F526 318 READLN(fin);
F526 319 TOUT(2:RESUL', ADDR(results), SIZE(results));
F54F 320 END.
End Fiddress.
SC76 10 PROGRAM inandout;
SC76 19 EI=3.14159;
SC76 90 VAR cv, v1, v2, x, y, z, index, epsilon, gain, rc, r, dd:
REAL;
BCEF 7 1, num, case, fin, printo
C:video, J, kount, lim: INTEGER;
BCEF 8 data : ARRAY[1..last] OF ARRAY[1..2] OF INTEGER;
BCEF 9 number : ARRAY[1..2] OF REAL;
BCEF 10 results : ARRAY[1..last]
OF ARRAY[1..11] OF REAL;
BCEF 11 PROCEDURE headings;
BCEF 12 BEGIN
BCEF 13 WRITELN;
BCEF 14 WRITELN('Case Exp H Su
(In Int T N')
BCEF 15 WRITELN;
BCEF 16 WRITELN('Kind K AW
08R PSR AND');
BCEF 17 WRITELN;
BCEF 18 WRITELN('Dis. paras* Exp
,Paras**Sur, Paras');
BCEF 19 WRITELN;
BCEF 20 ENDO;
BD20 20 END;
SD20 20 END;
PROCEDURE printout;
BEGIN
  headings;
  WRITE; WRITELN;
  FOR case:=1 TO num DO
    BEGIN
      WRITE(case:2);
      WRITE(data[case,1]:5);
      FOR i:=2 TO 5 DO
        WRITE(data[case,i]:12);
      END;
      WRITELN;
      FOR i:=10 TO 13 DO
        WRITE(data[case,i]:14);
      END;
      WRITELN;
      IF (case=TRUNC(case/3) AND (printorvideo=0) AND (case<5)) THEN READ(x);
      WRITE('Case RC Loss Surf Gain Index');
      WRITELN;
      BEGIN
        FOR case:=1 TO num DO
          BEGIN
            FOR i:=1 TO 2 DO
              WRITE(results[case,i]:5);
            FOR i:=3 TO 4 DO
              WRITE(results[case,i]:7);
            END;
            WRITELN;
          END;
        END;
      END;
    END;
  END;
END;
C63B 79 IF ((case=TRUNC(case/3) *3) AND (printorvideo=0) AND (case <num)) THEN READ (x) C690 80 END END; C69A 81 PROCEDURE variance; C69D 82 BEGIN C705 83 COUNT = 0; C718 84 REPEAT C728 85 COUNT := COUNT + 1; C738 86 IF COUNT = 1 THEN V2 := CV C748 87 IF COUNT = 2 THEN V2 := CV/SORT (p); C757 88 IF COUNT > 2 THEN BEGIN C767 89 IF ABS(x1-x) < Epsilon/100.0 THEN V2 := CV/2 C777 90 ELSE V2 := V1 + (V - V1) * (x1 - gain) / (x1 - x); C787 91 END; C797 92 V1 := V; C7A7 93 X1 := X; C7B7 94 V := V2; C7C7 95 dd := 1/SORT (v); C7D7 96 r := ENTIER ((1.0 + gain - rc) * dd); C7E7 97 IF r > 0.0 THEN x := EXP(r - dd - (1.0/star)) + r * LN(dd) - LN(r); C7F7 98 END IF COUNT > 2 THEN BEGIN C807 99 J := ENTIER (r); C817 100 FOR i := 1 TO COUNT DO C827 101 y := y - fi * dd; C837 102 END END END; C847 103 x := x + y; C857 104 END; C867 105 BEGIN C877 106 TIN('2:DATA', 'ADDR(daf t a)) ; C887 107 '2: NUMFIL', 'ADDR (num b er) ; C897 108 '2: RESUFI', 'ADDR (re sults) ; C8A7 109 num := number (11) ; C8B7 110 FOR case := 1 TO num DO C8C7 111 BEGIN C8D7 112 epsilon := 1.0E-3; C8E7 113 VARCASE := results [case, 9] ; C8F7 114 p := results [case, 10] ; CBD7 115 gain := results [case, 4] / 100.0; CBB7 116 rc := results [case, 11] / 100.0; CBC7 117 variance := C8C7 118 V := M/VCV; C8C8 119 index := LN(V) / LN(p); CCC7 120 writeln (‘Case , , , done (kount , simulations for U .) ’); CDA7 121 results [case, 5] := TRUNC (index * 1000.0 + .5) / 1000; CDB7 122 END; CDB7 123 FIN := 0; CDB7 124 WHILE FIN = 0 DO BEGIN CDD7 125 READLN (printorvideo, fin ) ; CDE7 126 IF printorvideo = 1 THEN CDF7 127 writeln (CHR(16)); CDD7 128 printout; CDE7 129 IF printorvideo = 1 THEN CDF7 130 writeln (CHR(16)); CDD7 131 END; CDE7 132 END; End Address: CDF2
2. APPORTIONMENT PROGRAMS.

```c
PROGRAM INPUT;
CONST
m=75.0;
d=65537.0;
e=65536.0;
VAR
data:ARRAY[1..5] OF REAL;
datint:ARRAY[1..5] OF INTEGER;
a:ARRAY[1..6] OF REAL;
lim:ARRAY[1..6] OF INTEGER;
v:ARRAY[1..6] OF REAL;
c:ARRAY[1..7] OF REAL;
b:ARRAY[1..6] OF REAL;
e:ARRAY[1..6] OF REAL;
o:ARRAY[1..11] OF REAL;
m:ARRAY[1..2] OF ARRAY[1..7] OF REAL;
z:ARRAY[1..6] OF REAL;
r, x1, randgen, loss, amp, lens, het, x, y, id, mean, sd, clm, clmx, claim, adu;
I, quot, sum, x, xs, totx, r2, r3, r4:

FUNCTION BOOL(x:BOOLEAN):REAL;
BEGIN IF X THEN BOOL:=1.0; ELSE BOOL:=0.0; END;
FUNCTION MIN(x,y:REAL):REAL;
BEGIN IF X<y THEN MIN:=X; ELSE MIN:=Y; END;
FUNCTION MAX(x,y:REAL):REAL;
BEGIN IF x>y THEN MAX:=X; ELSE MAX:=Y; END;
FUNCTION REAL;
BEGIN IF x THEN THEN BOOL:=1.0; ELSE BOOL:=0.0; END;
FUNCTION int(x,y:REAL);
BEGIN IF x<y THEN MIN:=X; ELSE MIN:=Y; END;
FUNCTION MAX(x,y:REAL);
BEGIN IF x>y THEN MAX:=X; ELSE MAX:=Y; END;
FUNCTION rnd;
BEGIN
rl:= (xl + 1.0) * d; X1:= x - 1.0 - d * ENTER(r);
ENDIF
BEGIN
WRITELN;
WRITE('How many offices? ') ;
READ(Z) ;
Z:=Z + 1.0;
WRITELN;
```

`
Does the first office distribute all its surplus by local exp-ratings?

Hawks? Enter built-in random number seed is, Read(random);

Enter expected term to discontinuance.

Enter number of simulations.

Enter PSR.

Now for the office-specific data.

Office Ben Exp Sur Reserve.

Coll Local.

END.
PROCEDURE statistics;
BEGIN
FOR i := 1 TO 2 DO BEGIN
  FOR case := 1 TO zz DO BEGIN
    cri := s(J,B;
    sl := randgen;
    FOR K := 1 TO sims DO BEGIN
      FOR q := 1 TO term DO BEGIN
        clm := 0;
        FOR case := 1 TO 2 DO
          BEGIN
            rnd := ENTIER (rl*li:i + 3:
            clm := 0 + clm;
            x := rl;
            i := 1;
            WHILE x >= d [case,i] DO
              BEGIN
                i := i + 1;
                IF hh = 1 THEN ben := 1 ELSE BEGIN
                  rnd := ENTIER (rl*li:i + 3:
                  ben := ENTIER (rl*li:i + 3:
                  clm := 0 + clm;
              END;
            c [l,case] := c[l,case]+c[2,case];
          END;
          cE2, case := cE2, case+c[l,case];
        clm := clm+x;
      END;
    END;
    FOR case := 1 TO zz DO
      BEGIN
        mean := c [1, case]/term/sims;
        ben := c [1, case] * mean;
        adj := cE2, case - ben;
      END;
    dataset;
    FOR case := 1 TO zz DO
      BEGIN
        dd := e [case];
        lim [case] := TRUNC (15*10 *
        BOOL (dd > 4) + BOOL (dd > 9) + BOOL (dd > 16) + BOOL (dd > 25));
      END;
    Poisson;
  statistics;
  data [1] := randgen;
  psr := datint [1];
  hawks := datint [2];
  sims := datint [3];
  term := datint [4];
  ind := datint [5];
  writeln('Now start tape to receive data and enter zero');
  read (num);
  Tout ('DATFILE1', ADDR (data), SIZE (data));
  Tout ('DATFILE2', ADDR (datint), SIZE (datint));
D5E1 177 TOUT ('DATFILE3', ADDR (b), SIZE (b));
D5E1 178 TOUT ('DATFILE4', ADDR (s), SIZE (s));
D5E1 179 TOUT ('DATFILE5', ADDR (v), SIZE (v));
D6A1 180 TOUT ('DATFILE6', ADDR (z), SIZE (z));
D6A1 181 TOUT ('DATFILE7', ADDR (o), SIZE (o));
D6A1 182 TOUT ('DATFILE8', ADDR (e), SIZE (e));
D6A1 183 TOUT ('DATFILE9', ADDR (m), SIZE (m));
D6C1 184 TOUT ('DATFILE10', ADDR (c), SIZE (c));
D6B1 185 TOUT ('DATFILE11', ADDR (d), SIZE (d));
D7A1 186 TOUT ('DATFILE12', ADDR (a), SIZE (a));

WRITELN; WRITELN ('Now enter the simulation program.');

RUN?C3A0 2 PROGRAM Doves;
C3A0 3 CONST m9=75.0;
C3A0 4 d9=65537.0;
C3A0 5 e9=65535.0;
C3A0 6 VAR
C3A0 7 data:ARRAY [1..5] OF REAL;
C3A0 8 datint:ARRAY [1..5] OF INTEGER;
C3A0 9 v: ARRAY [1..6] OF REAL;
C3A0 10 c: ARRAY [1..7] OF REAL;
C3A0 11 a: ARRAY [1..63] OF REAL;
C3A0 12 b: ARRAY [1..6] OF REAL;
C3A0 13 e: ARRAY [1..6] OF REAL;
C3A0 14 s: ARRAY [1..6] OF REAL;
C3A0 15 o: ARRAY [1..6] OF REAL;
C3A0 16 d: ARRAY [1..6] OF ARRAY [1..5] OF REAL;
C3A0 17 m: ARRAY [1..2] OF ARRAY [1..7] OF REAL;
C3A0 18 c: ARRAY [1..2] OF ARRAY [1..6] OF REAL;
C3A0 19 p: ARRAY [1..2] OF ARRAY [1..6] OF REAL;
C3A0 20 r: ARRAY [1..5] OF ARRAY [1..7] OF REAL;
C3A0 21 zr: ARRAY [1..6] OF REAL;
C3A0 22 l: ARRAY [1..2] OF ARRAY [1..6] OF REAL;
C3A0 23 w: ARRAY [1..6] OF REAL;
C3A0 24 h: ARRAY [1..2] OF REAL;
C3A0 25 r1, x1, randgen, loss, awp, P, S, het, x, y, dd, mean, sd, elm, clmx, claim, adj;
C3A0 26 ll, quot, quotx, xs, total, r1, r3, r4, denom, div
C3A0 27
C3A0 28 i, j, hawks, hh, terms, sim, k, q, case, len, type, index, count, n
C3A0 29 um, z, zz, ind
C3A0 30
C3A0 31
C3A0 32
C3A0 33 FUNCTION BOOL (x: BOOLEAN)
C3A0 34 BEGIN IF x THEN BOOL := 1.0;
C3A0 35 ELSE BOOL := 0.0;
C3A0 36 END;

C3E8 37 END;
PROCEDURE rnd;
BEGIN
1 := (x + 1.0) * 89,
1 := x / 89;
END;
PROCEDURE input;
BEGIN
TIN ('DATFILE1', ADDR (a1));
TIN ('DATFILE2', ADDR (a2));
TIN ('DATFILE3', ADDR (a3));
TIN ('DATFILE4', ADDR (a4));
TIN ('DATFILE5', ADDR (a5));
TIN ('DATFILE6', ADDR (a6));
TIN ('DATFILE7', ADDR (a7));
TIN ('DATFILE8', ADDR (a8));
END;
PROCEDURE simulation;
BEGIN
FOR CASE := 1 TO 2 DO BEGIN
FOR TYPE := 1 TO 2 DO BEGIN
FOR CASE := 1 TO 2 DO BEGIN
T [TYPE, CASE] := 0.0;
END;
END;
END;
FOR K := 1 TO SIMS DO BEGIN
FOR CASE := 1 TO 2 DO BEGIN
RND; X := RND;
END;
END;
END;
FOR CASE := 1 TO 2 DO BEGIN
IF HE = 1 THEN BEGIN
BEN := ENTIER(RND * 11) + 1;
ELSE BEGIN
BEN := RND;
END;
END;
END;
FOR CASE := 1 TO 2 DO BEGIN
CASE := 1;
END;
END;
FOR CASE := 1 TO 2 DO 
