The Actuarial Profession
making financial sense of the future

GIRO Convention

23-26 September 2008
Hilton Sorrento Palace

A Practical Study of Economic Scenario Generators For General Insurers

Gareth Haslip
Benfield Group

Agenda

- Introduction to economic scenario generators
- Building blocks of an ESG
- Time horizons
- Interest rate modelling
  - Real world versus risk neutral
  - Calibration techniques
- Equity return modelling

The Actuarial Profession
making financial sense of the future

What is an “ESG”?

A stochastic model of key economic variables, e.g. equity return, inflation, interest rates, FOREX etc

- Two key purposes: risk management or pricing
- That is: either “real world” or “risk neutral”
- For general insurance we are primarily concerned with real world ESG’s (risk neutral is used in life insurance to price contingent liabilities)

The Actuarial Profession
making financial sense of the future

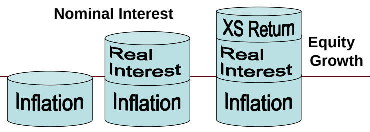
What do general insurers need in an ESG?

Primarily interested in risk management

- Need to project how the assets and liabilities of an insurer will evolve time
- Asset value driven by equity return, interest rates, inflation
- Liabilities are impacted by claims inflation and interest rates (discounting)

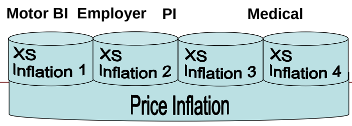
Building Blocks of an ESG

- Can model inflation, nominal interest rates, equity returns as distinct random processes
- … and introduce correlations between them to reflect the co-dependencies
- Or build the model up in terms of key economic building blocks:
  inflation, real interest, excess equity return



Building Blocks of an ESG

- The second approach is normally favoured and can also be applied to model claims inflation
- Claims inflation = Price inflation + XS Inflation
- Excess claims inflation can be modelled using variety of processes (e.g. Jump processes for court claims inflation)



Some ESG's in the Public Domain

- The Wilkie model
- "A Stochastic Asset Model & Calibration For Long-Term Financial Planning Purposes" by John Hibbert, Philip Mowbray & Craig Turnbull (June 2001)
- "Modeling of Economic Series Coordinated with Interest Rate Scenarios" Sponsored by the CAS and the SOA (July 2004)

Understanding ESG's

- Many companies purchase ESG's from external providers
- How does this fit into regulatory models?
  - Article 119 Statistical Quality Test - Should be able to justify the assumptions underlying the internal model to the supervisory authorities
  - UK ICA – The firm remains responsible for the reliability of the underlying assumptions - this responsibility cannot be passed on to a third party
- It is important that actuaries understand the models and calibration process
  - Not sufficient to delegate this process entirely to the provider

Time Horizons

- How the ESG is calibrated depends on the use
- 1 year model ➔ Short term focus
- Calibration should reflect current market conditions, not long term averages
  - How can the market value of assets change over the next year?
- This is significant issue for the equity model
  - Need a forward looking measure of volatility
- Less significant for interest rates and inflation

Modelling Interest Rates (and inflation)

“Short rate” models

- Popular choice for ESG’s, idea is to develop a model for the real world evolution of the instantaneous interest rate ...
- ... and use this to compute the full term structure of interest rates under the risk-neutral measure
- Correlation with other economic variables is provided through Brownian motion ‘shocks’

The Actuarial Profession
making financial sense of the future

A Selection of Short Rate Models

- One factor models have the following form:

$$dr_t = \mu(r_t)dt + \sigma(r_t)dW_t$$

- Vasicek: $\mu(r_t) = \alpha(\mu - r_t),\ \sigma(r_t) = \sigma$
- Cox-Ingersoll-Ross: $\mu(r_t) = \alpha(\mu - r_t),\ \sigma(r_t) = \sigma\sqrt{r_t}$
- Black-Karasinski: $\mu(r_t) = \alpha(\ln(\mu) - \ln(r_t)),\ \sigma(r_t) = \sigma$

The Actuarial Profession
making financial sense of the future

Using a Short Rate Model

- Recall that we are interested in modelling the real world evolution of interest rates ...
- ... but we also need the risk neutral model for deriving the yield curve
- This is a common point of confusion in their use
- Moving from real world to the risk neutral world is achieved via the “market price of risk”
  - This represents a “term premium” that investors demand for holding money for longer periods

The Actuarial Profession
making financial sense of the future

The Market Price of Risk

Example: Vasicek

- Real world model: $dr_t = \alpha(\mu - r_t)dt + \sigma dW_t$
- If we assume market price of risk is: $\lambda(t) = \lambda$
- Then risk neutral model is: (via Girsanov)

$$dr_t = \alpha(\mu - r_t)dt + \sigma\left(\lambda(t)dt + d\tilde{W}_t\right)$$

$$\Leftrightarrow dr_t = \alpha\left(\mu + \tfrac{\lambda\sigma}{\alpha} - r_t\right)dt + \sigma d\tilde{W}_t$$

Vasicek – Real World vs. Risk Neutral

- Real world: $dr_t = \alpha(\mu - r_t)dt + \sigma dW_t$
- Risk neutral: $dr_t = \alpha\left(\mu + \tfrac{\lambda\sigma}{\alpha} - r_t\right)dt + \sigma d\tilde{W}_t$
- Model differs only by the long term mean reversion rate
- How do we estimate the parameters?

Calibration Methods

1) Proxy Approach - Method

- The short rate cannot be directly observed in the market, but short term interest rates should be similar – e.g. 1 month interest rate
- The **real world** model for the short rate is fitted to the historical proxy time series data
- Use maximum likelihood estimation or method of moments

Calibration Methods

1) Proxy Approach - Difficulties

- Historical data for very short dated treasury stock limited
  - Usually have to look at 3 month rate as proxy
- We do not make use of historical information about the yield curve at other durations
  - In reality the term structure provides information about the expected path of the short rate
- Only provides real world parameters
  - The market price of risk is not recoverable

Calibration Methods

2) Cross-Sectional Approach – Method

- Historical data is rich – full yield curve is available at every date
- Cross-sectional method uses all data
  - It assumes n spot rates are observed without error (to 'back-out' the unobservable short rate)
  - and remaining spot rates are observed with error
  - Parameters found using maximum likelihood estimation along all historical dates and across all spot rates

Calibration Methods

2) Cross-Sectional Approach – Properties

- Complicated to implement and runs slowly
- Maximum likelihood problem 'ill-posed'
  - Many local maxima
  - This is due to the model implied short rate changing for each combination of parameters
- Provides both real world parameters and 'average' market price of risk
- Optimal in the sense that it incorporates all available information

Calibration Methods

3) Swaption Implied - Method

- Rather than use historical data, we can use observed market prices to calibrate risk neutral model
- Parameters are chosen to minimise the sum of square difference between modelled and observed swaption prices
- Provides calibration suitable for pricing liabilities contingent on interest rates – e.g. guarantees

Calibration Methods

3) Swaption Implied - Difficulties

- The method is only strictly necessary if you need to price interest rate derivatives
- For risk management we need real world model
  - It's possible to move back to real world using an assumption for the market price of risk...
  - ...but volatility assumption is unlikely to be valid
  - Swaption implied volatility will not be consistent with realised volatility
  - Although arguably a forward looking measure of interest rate volatility is more appropriate for 1 year models...
  - ...can consider stochastic volatility approaches (complicated for interest rate modelling)

Example of Proxy Method

- Data set: Daily US Treasury Bond Yields
- Proxy: 3 month spot rate
- Model: 1 Factor Black-Karasinski
- Real world: $d\ln r_t = \alpha(\ln\mu - \ln r_t)dt + \sigma dW_t$
- Risk neutral: $d\ln r_t = \alpha(\ln\tilde{\mu} - \ln r_t)dt + \sigma d\tilde{W}_t$

$$\tilde{\mu} = \mu\exp\left(\tfrac{\sigma\lambda}{\varepsilon}\right)$$

Example of Proxy Method

- Historical MLE estimate provides real world model parameters:

$$\hat{\alpha} = 0.14115,\ \ \hat{\mu} = 0.026338,\ \ \hat{\sigma} = 0.35224$$

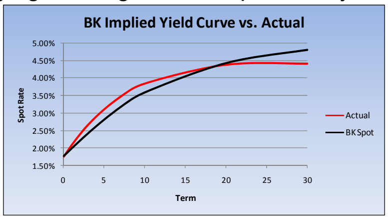
- How to find λ?
- The risk neutral version of the model should be consistent with the current yield curve
- Therefore we should use λ to fit the model to the current yield curve

Example of Proxy Method

- We therefore need to calculate spot rates under the Black-Karasinski model for parameters $\hat{\alpha}, \hat{\mu}, \hat{\sigma}$ and some value for λ
- There is no closed form for spot rates under BK
- They are computed by numerical methods
  - Trees, finite differences, Monte Carlo
- We apply a trinomial tree (see Brigo & Mercurio)
- λ is selected to minimise the sum of square error between actual and modelled spot rates

Example of Proxy Method

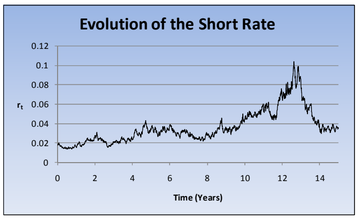
- Varying λ changes the shape of the yield curve



- Optimal λ = -0.09749 provides a good fit
- Note that through time varying parameters an exact fit is also possible

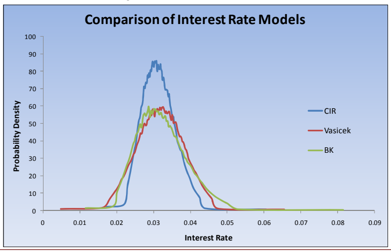
Example of Proxy Method

- Output of a single scenario of the real world evolution of the short rate:



Comparison of Interest Rate Models

- Probability density for the 3 models calibrated to the data set (5 year spot rate)



Comparison of Interest Rate Models

- Black-Karasinski has heavy right tail (due to log-normally distributed short rate)
  - Not representative of historical interest rate
- Cox-Ingersoll-Ross is underweight in the tails
- Vasicek provides closest match to historical distribution of interest rates
  - Issue with negative interest rates can be ignored for real interest rate models ...
  - ... or simply truncated at zero if modelling nominal interest rate

Comparison of Interest Rate Models

- Useful to look at 0.5th and 99.5th percentiles for % change in spot rates over 1 year

Interest Rate Model	0.5th Percentile of 5 Year Spot Rate	99.5th Percentile of 5 Year Spot Rate
Cox-Ingersoll-Ross	-31%	49%
Vasicek	-55%	61%
Black-Karasinski	-42%	81%
Solvency II	**-40%**	**56%**

- Again it can be seen that BK overstates right hand tail and CIR underweight in tails: Vasicek best choice

Short Rate Models - How Many Factors?

- Common criticism of one factor models is they induce perfect correlation between spot rates of different durations
- Below is a table of realised correlations on spot rate from US Treasury bonds

Correlation	1 year	2 year	3 year	5 year	7 year	10 year	20 year	30 year
1 year	1.00	0.99	0.97	0.91	0.87	0.81	0.68	0.64
2 year	0.99	1.00	0.99	0.96	0.93	0.88	0.78	0.77
3 year	0.97	0.99	1.00	0.99	0.96	0.92	0.83	0.84
5 year	0.91	0.96	0.99	1.00	0.99	0.97	0.91	0.92
7 year	0.87	0.93	0.96	0.99	1.00	0.99	0.95	0.94
10 year	0.81	0.88	0.92	0.97	0.99	1.00	0.98	0.98
20 year	0.68	0.78	0.83	0.91	0.95	0.98	1.00	0.99
30 year	0.64	0.77	0.84	0.92	0.94	0.98	0.99	1.00

Short Rate Models - How Many Factors?

- Clear that spot rates beyond 5 year duration are all perfectly correlated
- Slightly less correlation between < 5 year and longer term rates
- Not necessarily a problem for non-life insurers who
  - Do not have complicated fixed income portfolios
  - Are not valuing embedded interest rate options (life insurers need more factors to match the swaption implied volatility surface)
- If desirable to de-correlate spot rates move to multi-factor models
  - N.B. calibration becomes more involved

Excess Equity Return Models

What is required from the equity model?

- Only interested in real world evolution of excess equity return
  - Unless equity derivatives held in asset portfolio
- It should capture the heavy tails observed in historical equity return
- It must be suitable for short term projections
  - i.e. volatility should reflect current market conditions

Excess Equity Return Models

There are many different models described in the literature for modelling equity return

- Exponential Brownian motion
- Regime switching models (e.g. Hardy)
- Exponential jump diffusion (e.g. Merton, Kou)
- Stochastic volatility (e.g. Heston)

Which one is appropriate?

Exponential Brownian Motion

This is the model underlying the Black-Scholes equation

$$dS_t = S_t\left(\mu dt + \sigma dW_t\right)$$

- Easy to parameterise to historical data, but this provides historical average volatility
  - What time period is appropriate for measuring historical volatility?
- Difficult to achieve appropriate calibration that captures short term volatility

Regime Switching Model

- The economy is assumed to have two states:
  - Stable state with low variance and steady returns
  - Volatile state with returns highly positive or negative
- Equity risk premium is assumed to follow a log-normal distribution in both states (but with different mean and variance)
- Switching between states follows a continuous two state Markov process
- Easy to calibrate and provides good fit to historical data
  - Although calibrated parameters change if discretisation step is altered

Exponential Jump Diffusion

Extension to exponential Brownian motion to include jumps

$$dS_t = S_{t^-}\left(\mu dt + \sigma dW_t + \left(e^J - 1\right)dN_t\right)$$

- dN(t) indicates if a jump has occurred in [t, t+dt]
- J is the size of the jump
  - Merton → Normal distribution
  - Kou → Asymmetric double exponential distribution
- Works very well for risk neutral modelling (Exotics)
- Poor fit for real world – jump frequency is found to be approx. 40 per annum using MLE methods

Heston Model (Stochastic Volatility)

Extension to exponential Brownian motion to allow for time varying volatility

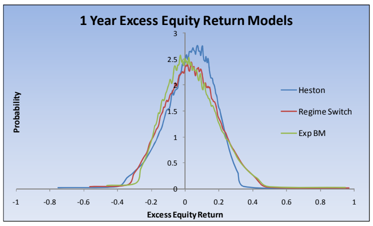
$$dS_t = S_t\left(\mu dt + \sqrt{Y_t}\, dW_t^{(1)}\right)$$

$$dY_t = \kappa\left(\gamma - Y_t\right)dt + \sigma\sqrt{Y_t}\, dW_t^{(2)}$$

- Y(t) represents the variance of stock prices
- Model provides fit to current market conditions
  - Good choice for Solvency II models capturing 1 year equity risk

Excess Equity Models Fitted to S&P 500

- Regime Switching and Exp BM similar in distribution
  - They both have higher upside semi-variance that historically observed



- Heston model captures current market conditions and provides better fit to historical data

Excess Equity Models Fitted to S&P 500

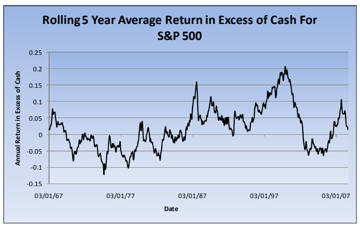
- Useful to compare 0.5th percentile for excess equity return under each calibrated model

Equity Model	0.5th Percentile Over 1 Year
Exponential Brownian Motion	-32%
Regime Switching	-38%
Heston Model	-42%
Solvency II (Global)	**-32%**

- Significant difference in 1 year change in market value when using **current** market conditions
- Are Solvency II assumptions appropriate?

Calibration Issues

- Expected excess equity return is historically much more stable than nominal equity return but still difficult to choose appropriate future value – what's an appropriate time window?



Calibration Issues

- Similar issue for choosing volatility assumption
  - What's an appropriate historical time horizon to measure realised volatility?
- Can resolve this using Heston model by using forward looking volatility measure
  - E.g. VIX index for S&P 500
  - Implied volatility for other markets
  - Question of appropriate adjustment – implied volatility normally over-estimates realised volatility

Conclusion

- Many different models for interest rates and equity return
  - Each have respective pros/cons, but some clear winners
  - Complicated underlying financial theory
- Vital that actuaries understand the implication of model choice and calibration methodology
- ESG's for one year models should reflect current market conditions, not historical conditions