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Projecting Realistic Balance Sheets Workshop C11

Or how to make global warming a certainty?

Participation models

- Interdependence of assets & liabilities
  - Leads to Monte-Carlo valuation
  - Path dependent cash-flows influenced by
    - Accounting
    - Management actions
- Understand these dependencies and you understand the cash-flows

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UK - model

- Guarantee + call option

  ↓

  Terminal bonus
- Asset share + Put option

  ↓

  Guarantee = Floor

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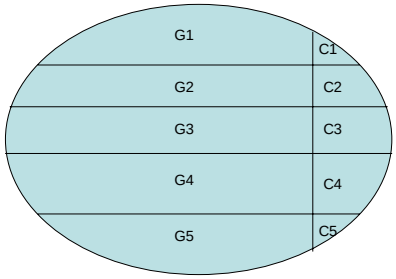
Pooling WP principle and hedge



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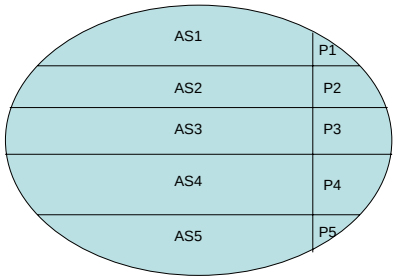
Pooling WP principle and hedge



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Illustrative model

- Demonstrate cohort effect
- Effectiveness of hedge
- Endgame = Lower Capital

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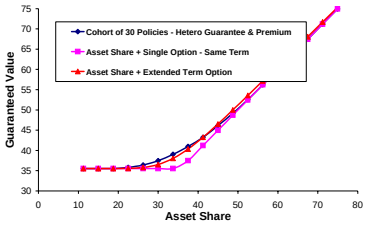
Cohort Effect – Simple Example



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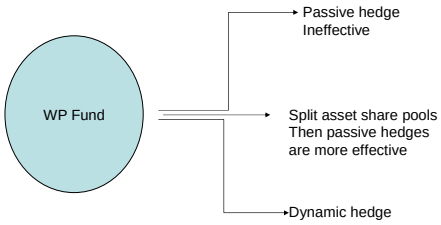
Three approaches to hedging



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Problems with the static hedge

- WP Pooling principal undermines effectiveness
  - Multiple strikes
  - WP Pool return applied to asset share not cohort return
- Underlying asset share mixture of bonds and equity
- Premium on long options (market less transparent)
- Out of the moneyness- Smile and Skew
- Option on the TRI – CRI more easily available
- Regular premiums
- Tax
- Other promises

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Problems with splitting fund by cohort

- At variance with one of the tenets of WP business
- Demutualisation and other contractual issues
  - Note sometimes equity investment specifically mentioned
- FSA (wears two hats)

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Problems with dynamic hedges

- CPPI approach
  - Like the options an ill fitting shoe
- DIY
  - Delta hedging / Rho hedging
- The gap
- Frictional costs
- Frequency of resetting the hedge

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This is why we need projected RBS

- Path dependency and circular arguments
- Closed form proxy RBS

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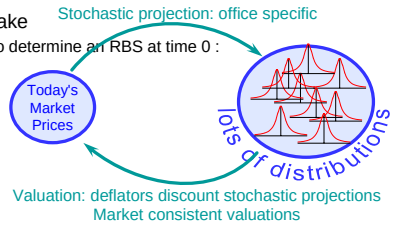
Simulation Fan Method

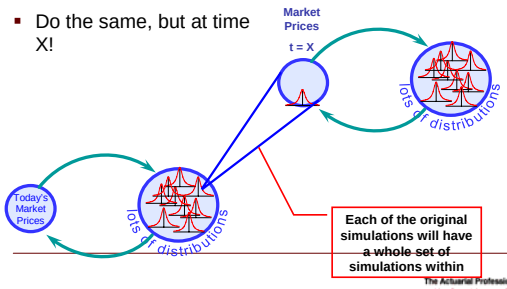
- Re-take
  - To determine an RBS at time 0 :



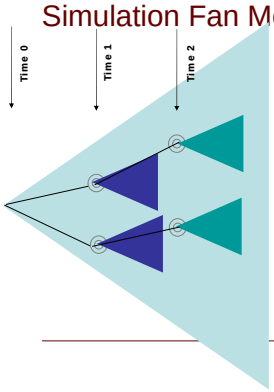
- Hence

Simulation Fan Method

- Do the same, but at time X!



Simulation Fan Method



- So we end up with fans within fan
- Hence the reason it is called the « Simulation Fan » or « Fan within a Fan » method

Normalisation techniques

- Re-use the same set of simulations
- Re-calibrate by
  - Changing ZCB prices
  - Changing Future ZCB prices
  - Changing deflators or stochastic discount mechanism
  - Inflation
  - Equity total return indices etc.
- Note – You are stuck with the same correlations and this eventually can lead to problems with auto calibration by proxy.

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Closed forms

- Black-Scholes (Flexibility)
- Hull-White
- Others
- For the sake of simplicity we concentrate on Black-Scholes

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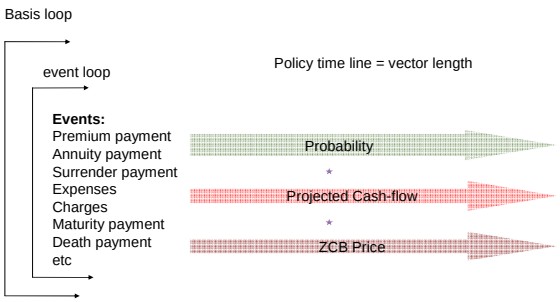
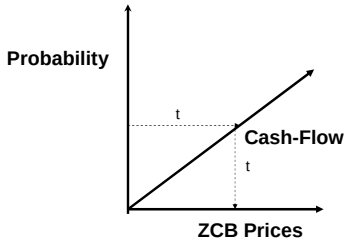
Generic Reserving algorithm



A generic reserving algorithm



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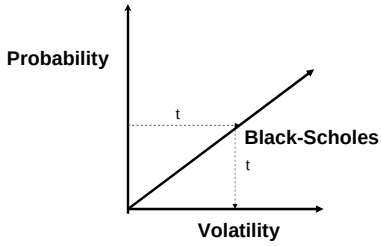
A generic reserving algorithm



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Delta Hedging Principle

$$\Delta p = \frac{\partial p}{\partial S}\Delta S$$

Change in derivative price

Delta:

The rate of change of the derivative price with respect to the underlying asset price.

Change in underlying asset price

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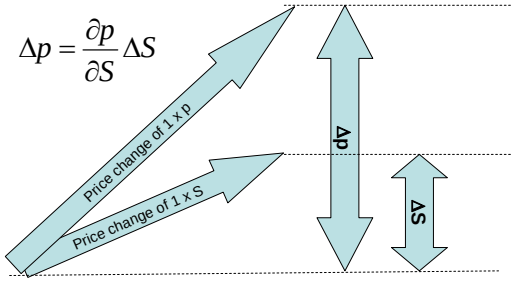
Delta Hedging Principle

$$\Delta p = \frac{\partial p}{\partial S}\Delta S$$



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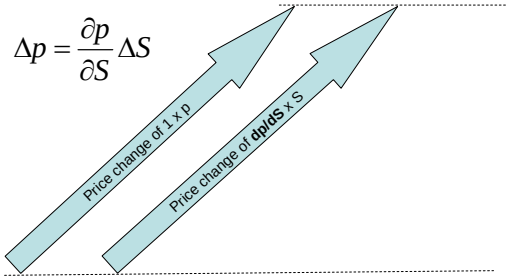

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Delta Hedging Issues

$$\Delta p = \frac{\partial p}{\partial S}\Delta S$$

- Linear relationship
  - but the delta is dependent on underlying asset price
    - creating a non-linear relationship.
- Breaks down for large movements in the underlying asset price.
  - Could happen if:
    - Hedge not reset for a long period
    - A large sudden move in the underlying asset price
    - Management actions?
- Transaction costs
- Stochastic interest rates

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Delta Hedging Life Application

$$\Delta p = \frac{\partial p}{\partial S}\Delta S$$

- **The derivative (p) can be equated to**
  - **E.g. the realistic value of liabilities**
- **The underlying asset price (S) can be related to the fund value**
  - **e.g. the asset share**

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Delta Hedging – Transaction Cost vs. Frequency of Hedge Reset

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Delta Hedging – Stochastic Interest Rates Issue

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Rho Hedging Principle

$$\Delta p = \frac{\partial p}{\partial r}\Delta r$$

Change in derivative price

Rho:

The rate of change of the derivative price with respect to the risk free rate.

Change in underlying asset price

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Rho Hedging Issues

$$\Delta p = \frac{\partial p}{\partial r}\Delta r$$

- **Linear relationship**
  - **but rho is dependent on the risk free rate**
    - **creating a non-linear relationship.**
- **Exposure to r typically comes from assets non-linear in r e.g. bonds and swaps.**
- **Swap market is generally the most liquid but this introduces a basis risk**
  - **RBS calculated on gilts+ not relative to swaps. Swap spread will introduce basis risk.**
- **Transaction costs**

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Rho Hedging In Practice

$$\Delta B = \frac{\partial B}{\partial r}\Delta r = -\frac{T}{B}\Delta r$$

$$B = \exp(-rT)$$

- **Linear relationship**
  - **but rho is dependent on the risk free rate**
    - **creating a non-linear relationship.**
- **Exposure to r typically comes from assets non-linear in r e.g. bonds and swaps.**
- **Swap market is generally the most liquid but this introduces a basis risk**
  - **RBS calculated on gilts+ not relative to swaps. Swap spread will introduce basis risk.**
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Combining Delta + Rho Hedges

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GAO's – a variation on Black's formula

Value of GAO=K * ZCB(n) * probability(n) *

$$\{N((\ln(a^g / a^i) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2$$

$$+ a^i / a^g \quad *N(\ln(a^i / a^g) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2\ ) - 1\}$$

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European business

- Annual guarantee
- Profit sharing – earned rate
- Guaranteed surrender values!!!!

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The Napoleonic Code – The relics of the tariff system and the impact of the Euro

- Most territories operated a tariff system now not permitted under the EU Life directives.
- Consequence was no competition on price or design within a territory and uniformity of guarantees. In different territories this led to guarantees that only started to fall as the Euro consolidated.
  - Spain 6%
  - Belgium 4.75%
  - Portugal 4%
  - Italy 4%
  - Holland 4%
  - Germany 4%
  - Switzerland 3% *

* Note – Pensions business used to have the guarantee set by the regulator. Bizarrely on one occasion the regulator increased the guaranteed rate when interest rates fell.

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Accounting matters

- Impact of realisation of gains - earned rate.
- Reducing volatility in earned rate traditionally done by accounting mechanisms.
- Mismatching and imprudent distribution of gains following falls in interest rates leave Companies exposed.
- Profit sharing mechanisms are particularly nasty asymmetrical derivatives.

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Simple design plus participation type (1)

- Policyholder guaranteed 3% per annum
- Policyholder receives 80% of investment (surplus = excess over guaranteed rate)
  - If we earn 4% then Policyholder gets 3.8%
  - 3% + .8*(4 – 3)
  - Shareholder profit 0.2% = 4% - 3.8% in that year

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Simple design plus participation type (1)

- Policyholder guaranteed 3% per annum
- Policyholder receives 80% of investment (surplus = excess over guaranteed rate)
  - If we earn 2% then Policyholder gets 3.0%
  - 3% + Max{.8*(2 – 3), 0}
  - Shareholder profit -1% = 2% - 3% in that year

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Simple design plus participation type (2)

- Policyholder guaranteed 3% per annum
- Policyholder receives 100% of investment (surplus = excess over guaranteed rate + management charge)
  - If we earn 4% then Policyholder gets 3.5%
  - 3% + .8*(4 – 0.5%)
  - Shareholder profit 50bps in that year
- Discourages risk taking as the extra volatility transfers value from the shareholder to the policyholders.
- Modification is to make the shareholder margin dynamic. Risk / Reward

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Simple design plus participation type (2)

- Policyholder guaranteed 3% per annum
- Policyholder receives 100% of investment surplus (surplus = excess over guaranteed rate and management charge)
  - If we earn 2% then Policyholder gets 3.0%
  - 3% + Max{(2 – 3 - 0.5), 0}
  - Shareholder profit -1% = 2% - 3% in that year

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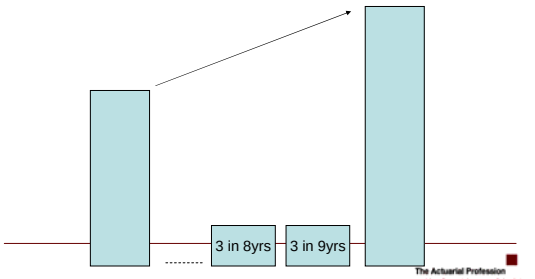
Hedge cash-flows certain – MCEV
We can use this to value guaranteed element 3, 3, ……103



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Present value of profit sharing

- A Series of caplets
  - Nominal 80% of Projected mathematical reserves, strike 3%
  - Or
  - Nominal 100% of projected mathematical reserves, strike 3.5%
- Or just take out a swap (remove the volatility)!
- Not many Companies do this with participating business. The theory is that for SP business they should have purchased bonds when interest rates were higher and amortised. Some have realised too much gain for commercial reasons exposing the guarantees.

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Complications to valuation

- Replicating portfolio difficult to find:
  - Management decisions e.g. complex investment strategy
  - Assets not invested in cash
  - Accounting not always “marked to market” e.g. amortisation schedules, URG on property and equity
  - Policyholder discretion
    - Lapse
    - Bonus as cash or re-invested
    - Decrements.

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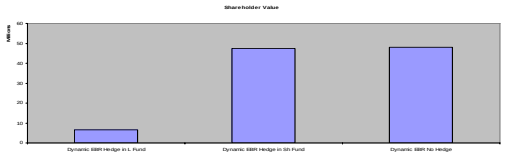
Swaptions – impact on value / Capital efficiency



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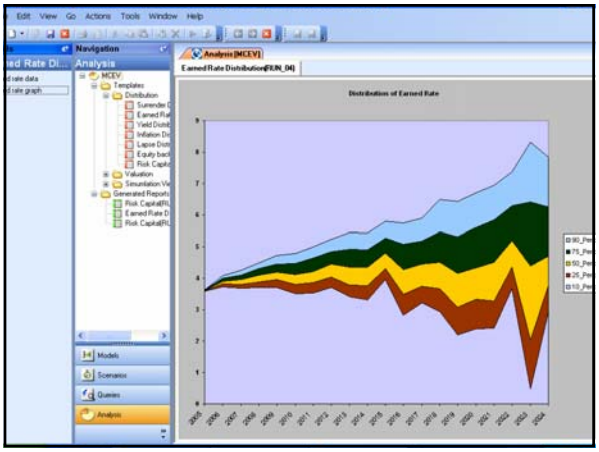

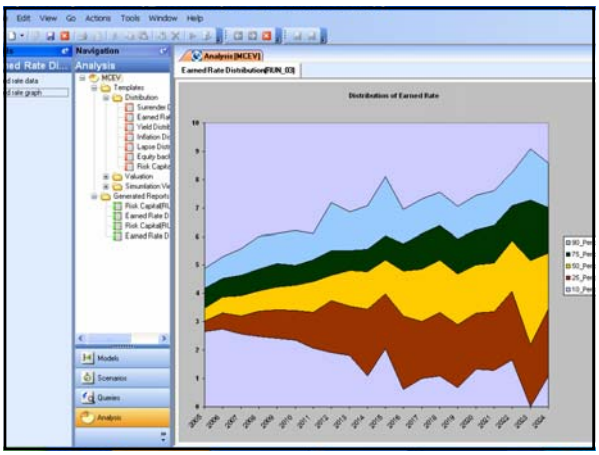

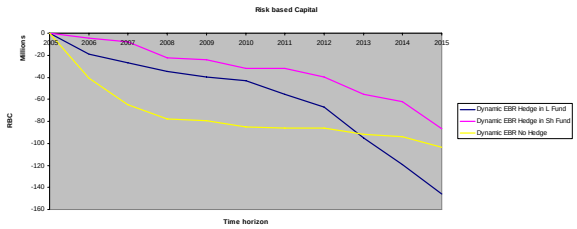

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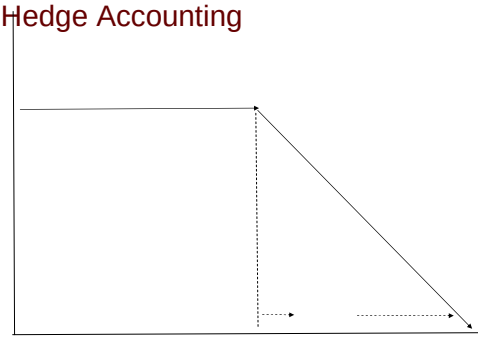
Hedge Accounting



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Path dependency!!!!!!!!!!!!

- Bond portfolios – credit risk exposure
- Swaption hedges
  - Rise in interest rates
  - Fall in interest rates
- Duration mismatching
- M to M tendency
  - Denmark, Swiss solvency test, FTK in Holland

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Complications

- U-rate business – Holland
- Fund RFB – Germany
- Fund PB - France

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