Introduction

- Often actuaries are required to estimate the "mean of possible outcomes"
- This presentation discusses the impact of both process and parameter uncertainty upon the mean estimate not just the spread of potential outcomes
- The presentation considers this impact in a “Bayesian world”
- The implications of both GRIT and the ICA work => more consideration of process and parameter uncertainty within actuarial estimates
- We have put forward a “devil’s advocate” case that the impact of considering uncertainties surrounding actuarial modeling should result in actuaries increasing their mean selections
- We have shown both some practical examples and theory supporting these examples
Reserving Risk and the Impact on the Mean

Parameter Uncertainty - Introduction

- Most of the distributions we deal with are right skewed.
- These distributions do not necessarily behave intuitively.
- The relationship between the parameters and the mean of these distributions is highly nonlinear.
- Small changes in these parameters can result in large movements of the resultant mean.
- By their very nature there is more room for upwards (adverse) movement than downwards movement.

Parameter Uncertainty – Example 1

Example 1 – Gamma Distribution

- If we assume $X \sim \text{Gamma}(\alpha, \beta)$ with both fitted mean and standard deviation of 1,000.
- $E(X) = \frac{\beta}{\alpha}$
- This has the parameters $\alpha = 0.001, \beta = 0.00001$.
- Let us assume that the parameters are equally likely to be 5% higher, 5% lower than the fitted parameters.

Parameter Uncertainty - Example 1

Example 1 cont. – Gamma Distribution

- Gamma Distribution Means with different parameters.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>-5%</th>
<th>0%</th>
<th>+5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Mean</td>
<td>1.003</td>
<td>1.003</td>
<td>1.003</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.050</td>
<td>0.050</td>
<td>0.050</td>
</tr>
</tbody>
</table>

- This has an overall mean of 1,002 > 1,000.
Example 2 – Lognormal Distribution

- If we assume $X \sim \text{LogNormal}(\mu, \sigma)$ with both fitted mean and standard deviation of 1,000.
- $\mathbb{E}(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)^2$
- This has the parameters $\mu = 6.561, \sigma = 0.833$.
- Let us assume that the parameters are equally likely to be 5% higher, 5% lower and the fitted parameters.

Lognormal Distribution Means with different parameters.

<table>
<thead>
<tr>
<th>%</th>
<th>5%</th>
<th>0%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mu</td>
<td>996</td>
<td>1000</td>
<td>1004</td>
</tr>
<tr>
<td>Sigma</td>
<td>176</td>
<td>178</td>
<td>170</td>
</tr>
<tr>
<td>Sample</td>
<td>758</td>
<td>1353</td>
<td>720</td>
</tr>
</tbody>
</table>

This has an overall mean of 1,037 > 1,000.

Let us now look at this on a more technically robust basis.

Taking example 2 from above (LogNormal) with a fitted mean and standard deviation of 1,000 we shall now examine the effect on the mean of a specific fit.

Firstly we simulate 18 independent values from the LogNormal with the fitted parameters previously given.

The mean of our sample happens to be 996.
Bayes Theorem states:
\[ f(\mu, \sigma | x) = \frac{L(x | \mu, \sigma)g(\mu, \sigma)}{P(x)} \]

Where \( f \) is the posterior distribution, \( L \) is the likelihood and \( g \) is the prior distribution.

The log of our observations is Normally distributed, so assuming the Jeffreys prior we get:
\[ f(\mu | x) \propto \left( \frac{1}{\sqrt{2\pi}\sigma} \right)^n \exp \left( -\frac{1}{2\sigma^2} \sum (x_i - \mu)^2 \right) \]

Using the following hierarchy it is easy to use Monte Carlo simulation to look at the effect of this parameter uncertainty on the mean:
\[
\begin{aligned}
&\text{Normal} \left( \mu, \frac{1}{\tau^2} \right) \\
&\Gamma \left( \frac{2\alpha}{\beta}, \frac{2\alpha}{\beta} \right) \\
&\sigma \sim \text{Normal} \left( \frac{1}{\tau^2} \sum (x_i - \mu)^2, \frac{2}{\tau^2} \right)
\end{aligned}
\]

The effect on the mean in this case is to give us a mean of 1,066.

If we fit the parameters by method of moments, (ie unbiased sd) we obtain a mean of 1,016.

If we fit the parameters by ML, (ie biased sd for Normal) we obtain a mean of 980.

By modelling parameter uncertainty we obtain a mean greater than that obtained through merely fitting the parameters.
Impact on Reserve Risk

- For simplicity let us consider a single development period of a simple triangle:

<table>
<thead>
<tr>
<th>Year</th>
<th>Dev 1</th>
<th>Dev 2</th>
<th>Development Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>10,000</td>
<td>12,000</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>10,000</td>
<td>12,000</td>
<td>1.20</td>
</tr>
<tr>
<td>6</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>10,000</td>
<td>8,000</td>
<td>0.80</td>
</tr>
<tr>
<td>9</td>
<td>10,000</td>
<td>8,000</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>10,000</td>
<td>10,000</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Excess Weighted 0.10</td>
</tr>
</tbody>
</table>

Impact on Reserve Mean

- If we start by making assumptions akin to that for the ODP model, namely:
  \[ E(C_i) = (1+\lambda) \delta_i \]
  \[ Var(C_i) = \lambda (1+\lambda) \delta_i^2 \]
- We will now assume that the model is LogNormal with these as the means and variances.
- The log of the incremental claims over the cumulative claims in the previous development period are thus distributed as a normal distribution with parameters:
  \[ \mu = \ln(1+\lambda) - \frac{\lambda}{2}(1 + \frac{\delta_i^2}{\lambda}) \]
  \[ \sigma^2 = \frac{\lambda}{2}(1 + \frac{\delta_i^2}{\lambda}) \]

Impact on Reserve Mean

- Due to our careful choice of cumulative claims in the first development period, these are i.i.d.
- Our claims distribution assuming a Jeffreys prior as before is:
  \[ \Pi(x_1, \ldots, x_n) = \text{Gamma} \left( \frac{n-1}{2}, \frac{1}{2} \left( \sum \frac{x_i}{\delta_i} \right) \right) \]
  \[ \Pi(x_1, \ldots, x_n) = \text{Normal} \left( \frac{1}{\sqrt{2 \pi}} \frac{1}{\delta_i} \left( \sum \frac{x_i}{\delta_i} \right), \frac{1}{\delta_i^2} \right) \]
Impact on Reserve Mean

- The bootstrap approach assumes that the selected mean is the mean of the distribution post parameter uncertainty.
- In our example, the lack of volatility from the ODP Bootstrap is derived from the fact that the expected incremental claims for the historic are backward looking. They are calculated by differencing the cumulative claims in period 2 and the cumulative claims in period 2 divided by the volume weighted development factor. This is clearly a limitation of the ODP bootstrap approach.
- Of more interest to us is the difference in the means of both approaches.
- Strictly the mean should increase when we factor in parameter uncertainty.

Process Uncertainty

- To illustrate the point with regards to process uncertainty I shall slightly modify the example data, so as to increase both the parameter and process uncertainties.
- This table highlights the differences.
Reserving Risk and the Impact on the Mean

Process Uncertainty

The table below compares the results for the example with greater uncertainty with that for less certainty.

<table>
<thead>
<tr>
<th>%</th>
<th>Mean</th>
<th>SD</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Other Issues

The BF method does not hold up to scrutiny in the context of mean claims reserving. Clearly the expected value of the inverse of the factor to ultimate is not the same as the inverse of the expected value of the factor to ultimate.

If the consideration of risk really should affect the mean then diversification credit will mitigate some of this.

Model Uncertainty.

In our examples we have assumed that the loss distributions are infinite in range, which is not true. Allowing for the curtailing of these distributions would offset some of the downside risk and reduce the increase in the mean.

Summary

From the above Parameter and Process uncertainty lead to a potential increase in the mean estimate.

Some of these results are often already allowed for:

- Actuaries have traditionally loaded reserves where either the parameters are more uncertain or the class is known to be more unstable than the data suggests.
- It would appear that this practice is justified on a statistical level.
Conclusion and Comment

- In order to illustrate the points we have made in this presentation we have taken liberties with both the data we have used and the techniques.
- Most actuaries, including us, are using some form of, mean invariant, bootstrapping to quantify the reserve uncertainty. However, although this is a generally accepted actuarial technique there are limitations with the approach.

Conclusion and Comment

- There don’t appear to be many approaches which allow for effects as we have described.
- There appears to be some way to go before we have settled these issues.
- This presentation implies that the most important factor in terms of claims reserving and understanding the volatilities is the historic triangles. This is rarely the case.
- However, we hope that this presentation has been food for thought.

Questions and Discussion