REVIEWS

An Introduction to Mathematical Risk Theory. By Hans U. Gerber

As its title implies, this monograph is a textbook on mathematical risk theory. The material has been adapted from a lecture course given by the author at the University of Michigan.

Risk theory is defined in the foreword to be "a collection of related ideas for designing, managing and regulating a risk enterprise". The fundamental attribute of the operation of a risk enterprise is considered to be the possibility that "expenditures may exceed receipts during some accounting periods in the normal course of operation".

The presentation of the author differs from that of the standard texts in the subjects (Risk Theory by R. E. Beard, T. Pentikäinen, and E. Pesonen; Mathematical Models in Risk Theory by H. Bühlmann; Stochastic Theory of a Risk Business by H. L. Seal) which appeared around 1970. In particular the author takes the opportunity of including some of the more recent developments.

The theoretical discussion in the text is interspersed with well-chosen examples which both illustrate and extend the concepts mentioned. At several points the author indicates analogies with other branches of actuarial science, for example the use of martingales in life and other contingencies, decision theory (and in particular Pareto-optimality) in graduation and statistical inference and renewal equations (as from ruin theory) in demography. The text is concise, with the mathematical discussions given in a rigorous and often elegant way, avoiding long and intricate derivations. The author devotes little space to supplementing his main ideas with giving the reader an intuitive feel for what is happening and it is here that the author may be criticized. Further the reviewer found the numbering of paragraphs, equations and examples both confusing and frustrating especially when following up the cross references demanded by the rigorous, progressive discussion.

At the end of each chapter the author employs the useful device of giving short historical notes and listing references worth pursuing both within the field of actuarial science and elsewhere.

The material is presented in ten chapters which may be conveniently divided into four sections. The first section (Chapters 1-3) covers the mathematical tools needed later on, namely distribution theory, stochastic processes and martingales.

The second section (Chapters 4, 5 and 7) covers static models. A knowledge of chapter 1 is a prerequisite. Chapter 4 discusses the distribution of aggregate claims in one year. The individual model and collective model (based on the Compound Poisson distribution), are described with a numerical illustration. The numerical evaluation of the distribution of aggregate claims is less of a problem now that computers can perform complex calculations like convolutions easily and exactly. However, approximation methods (described at length for example in Beard et al.'s book) are mentioned. Bowers' Gamma function and the Gram–Charlier approximations are regarded as particular cases of expansions in terms of orthogonal polynomials. The Edgworth and Esscher approximations are also mentioned, and there is some comparison of the performance of the four methods. In Chapter 5 different principles of premium calculation are discussed and compared with respect to five desirable properties. The exponential principle, based on the concept of zero utility, has the most attractive properties and establishes a neat connection with the adjustment coefficient of ruin theory (Chapter 8). These principles are applied to the reduction of premiums through pooling and the choice of retention limits. Chapter 7 discusses decision-making under conflicting viewpoints and the concept of Pareto-optimal sets with reference to risk exchange between insurance companies, and also deals with the calculation of stop-loss premiums on a computer-oriented basis.

The third section deals with dynamic models (Chapters 6 and 8) and assumes a knowledge of distribution theory and stochastic processes. Chapter 6 provides a rather brief introduction to credibility and experience rating. Chapter 8 deals with the probability of ruin. Following the approach of H. L. Seal, the author relates the probability of ruin in a finite time interval, a function $\psi(x,t)$ of the initial reserve $x$ and time horizon $t$, to the distribution of aggregate claims which is assumed to be compound Poisson. The author then concentrates on the probability of ultimate ruin
$\psi(x)$, deriving a renewal equation which he solves explicitly in the cases of exponential and mixed exponential claim amounts. The result that $\psi(x) \leq e^{-Rx}$ for a suitably defined $R$, the adjustment coefficient, is proved elegantly by induction. An asymptotic formula for $\psi(x)$ is also obtained, by a heuristic argument. The Laplace transform of the maximal aggregate loss is derived, leading to the Beekman-Bowers approximation. The author develops formulae for estimating upper and lower bounds for $R$. The chapter ends with a discussion of excess of loss coverage as an optimal form of reinsurance.

The final section (Chapters 9 and 10) utilizes martingales firstly to derive additional results in ruin theory and then to solve some decision theory problems. Presentation is hindered by the gap between these sections and Chapter 3 where the required theory is introduced. General results for $\psi(x)$ are obtained which are identical to those of Chapter 8. A method to obtain moments and the Laplace transform of the time of ruin are presented, as is a more general approach which produces upper bounds for $\psi(x,r)$. In Chapter 10, decision theory is used to consider problems concerned with maximizing dividend payments and choosing the most suitable time to offer a contract and to 'fire an agent' (assuming the absence of employment protection legislation!). The reader who finds martingales a difficult idea to come to terms with could omit Chapters 3, 9 and 10 with little loss. In fact the author covered the material in this monograph, with the exception of these three chapters, in his lecture course at the University of Michigan.

The book can be recommended as an interesting, stimulating and challenging one. The presence of the word, 'mathematical' in its title should be taken as a warning that the presentation is theoretical and rigorous. The advice in the foreword that 'this monograph is not for the general reader' is not given lightly.

S. Haberman.

Computational Probability. Edited by P. M. Kahn

[Academic Press, 1980]

This volume is a collection of nineteen papers presented at the Actuarial Research Conference on Computational Probability held at Brown University, Rhode Island in August 1975. The conference was sponsored jointly by the Committee on Research of the Society of Actuaries and the Department of Applied Mathematics of Brown University. David Halmstad was a chairman of the conference and secretary to the Committee on Research of the Society of Actuaries. He died subsequently and the volume is dedicated to him. By this chain of events some interesting papers have come in a single volume within reach of British actuaries. None of the contributors came from the British Isles, though several came from Europe. Partly for this reason, the papers included do not seem to be already familiar to British actuaries, except for historical demographers.

Perhaps the most novel item of methodology is presented in the paper on 'Backward Population Projection by a Generalized Inverse' by T. N. E. Greville and Nathan Keyfitz. Generalized inverses have attracted most interest in connection with linear models where the design matrix can be singular. Here is a new and practical application to population projections. Enthusiasts for generalized inverses will be encouraged, but the sceptics won't be convinced as they can retort that backward projection is not of great practical importance.

Some papers augment the literature on specialist actuarial topics. In the field of Workmen's Compensation is the paper by Lena Chang on 'Reversionary Annuities as Applied to the Evaluation of Law Amendment Factors'. Another specialist topic is covered by Gottfried Berger in a paper on 'Accounting Principles for Life Insurance: Reflections on Language and Notation'. The topic of Maturity Guarantees, much debated among British actuaries, is approached by Phelim Boyle in a paper on calculating the value of a maturity guarantee. With $r_t$ as the rate of return in year $t$, he assumes that $\log(1 + r_t)$ is normally distributed and returns in successive years are independent. Boyle adopted a different approach in his subsequent paper (J.I.A. 105, 177). There is also a paper by Hans Gerber and Donald Jones on 'Calculation of Stop-Loss Premiums' in which they illustrate their methods with a simple portfolio of five policies and produce fourteen pages of tables.
Several papers relate directly to computing and data processing. Among these are 'Efficient Sorting by Computer: An Introduction' by Robert Sedgewick, 'Symbolic Information Processing' by Kenneth Levine and Peter Masters and 'APL for Actuaries' by Donald L. Orth. Numerical Analysis is represented by papers such as 'Numerical Fourier Inversion' by Harald Bohman. Nils Wikstad recommends Newton–Raphson methods for solving the Laudberg Risk Equation. John Beekman and Clinton Fuelling contribute a paper on 'Simulation of a Multirisk Collective Model'.

The papers on stochastic processes seem rather academic. In this category is one paper on the Galton–Watson Process and another on Markov Population Processes. Of more practical interest to actuaries is a considerable paper by J. R. Brzezinski on 'Correlates of Life Lapsation'. The Life Insurance Marketing and Research Association in Hartford has collected data on life insurance policy lapsation on all policies in force in a number of insurance companies and investigated the results. Tables were constructed for four classes of business: Pension Business, High Early Cash Value Business, Permanent Insurance and Term Insurance. A table of Graduated Select Lapse Rates is given for Permanent Life Insurance both by Number of Policies and by Amount of Insurance over Policy Years 1–15 with five year age intervals. The highest lapse rate in the first year calculated from Amount of Insurance is .2606 for ages 20–24 and from Number of Policies is .2718 for the same age range. LIMRA has also made considerable use of regression analysis in an attempt to explain lapse rates by considering additional characteristics of the business. The 'dummy variables' chosen were: Male, Female, Medical, Non-medical, Medical and Policy Year 1, Non-medical and Policy Year 1, Permanent Insurance, Pension Insurance, and High Early Cash Value Insurance. One finding was that "women do indeed have better persistency than their male counterparts considering all the variables in the regression". It is anticipated that following trends in lapsation over time will be valuable.

Where this volume appears to be out of date is in the topics missing from it. Techniques such as the EM algorithm, 'Jack-knife' and 'Bootstrap' methods which one would expect to feature in a similar conference today are not mentioned. There is also a noticeable lack of papers on Time Series and Forecasting Methods such as Box-Jenkins which require expertise in computing. In the opening paper Ulf Grenander draws attention to the possibilities of using computers interactively, but other contributors do not follow up this theme. Facilities available now, such as the GLIM interactive system, have progressed far beyond what was being done in 1975. Still the volume remains a desirable acquisition, at £11.80, for an actuary interested in methodology. The appearance and binding are attractive; it would enhance any actuarial library.

P. S. Carroll

Reviews of U.K. Statistical Sources: Vol. XIII-23, Wages and Earnings By ANDREW DEAN

Users of the official statistics of wages and earnings will be grateful to have available this comprehensive description of how these statistics are obtained, together with information about non-official surveys. This applies particularly to pension fund advisers because now that the vast majority of scheme liabilities are related to the earnings of members it is often necessary to revalue using a national earnings index. Although earnings indices do not normally collect as much uninformed criticism as those on prices, the book should allay any fears of bias, and from the wide variety of indices available enable the most appropriate to be chosen. Snags such as the effect of back-dated settlements on the indices, or less than complete coverage of some industries or individuals (such as part-time employees) are pointed-out, particularly in the final evaluation chapter. There is no doubt that since 1970 the New Earnings Survey has largely answered the criticisms of the limitations of the official earnings statistics available in the 1960s, but for some purposes, however, incomes data from the Family Expenditure Survey, the General Household Survey, and the Inland Revenue's Survey of Personal Incomes may be more appropriate. Sources of information on these statistics and on the incomes of the self-employed were described in Volume VI of this series of
Reviews of U.K. Statistical Sources, which dealt with personal incomes in general, and was reviewed in *J.I.A.* 106, 339. Pension Fund advisers may also find useful the list of published information on salary scales for a wide range of public-service employments, and for a limited number of groups outside.

J. L. Field

*The Measurement of Portfolio Performance: An Introduction.* By HYMANS & MULLIGAN

[Kluwer Publishing Limited]

The most important quote in this book is the last exhortation in the last chapter 'If you can't beat 'em don't join 'em but try a bit harder instead'.

This advice is given to fund managers despite evidence that as a group (or herd?) they have tended to underperform market indices over the years. The technical disadvantages they face before they start are clearly and sympathetically set out.

The subject of portfolio performance is becoming increasingly important as insurance companies, merchant banks, stockbrokers, and others offer their investment services to pension fund trustees.

The differences in approach to investment by the various types of investment manager have led to Trustees being presented with reports on their investments in widely differing formats. Most Trustees would like to be able to look at a few simple numbers and be sure that their advisors are acting prudently and successfully. To this end the book sets out the main summary statistics and explains the advantages and disadvantages of each. An illustration is given of the dangers involved in assessing performance by just looking at the overall return after a period of 5 years, even if this is calculated on a time weighted rate of return, or against a notional fund.

The book would not therefore appeal to those seeking a simple solution to performance measurement. The appeal is much more to those seeking a simple explanation of the problems of such measurement.

The layout of the book is particularly attractive. The twenty-two chapters are designed to more or less stand up individually and therefore do not necessarily need to be read sequentially. Whilst on first reading this facility is unlikely to be used, it does mean that the book can easily be consulted later to refresh the memory on a particular point. There are also eight appendices either developing the theoretical framework of some of the statistics or giving background information. The layout could perhaps be criticized for having too many chapters. In particular the 'chapter' on the Trustees report covers just one page.

An area which is not given enough emphasis is the diversification of risk. Evidence is referred to showing that most of the risk of performing in a significantly different manner to the all share index can be diversified away by holding relatively few stocks. The problem arises in the definition of 'significantly different'. Trustees tend to be upset by, say, a 5% underperformance of the index although the statistician might think this 'not significantly different' from index performance.

With these minor reservations the book provides a useful and much needed guide to the new or confused trustee, without a mathematical background, needing to find his way through the jungle of terms which confront him. It also has sufficient content for the more mathematically minded who are likely to become occasionally involved in looking at investment performance.

Lastly, it should appeal to fund managers who can console themselves with the thought that if they were not being criticized for underperforming they would be criticized for taking too many risks.

P. J. Nowell

*Understanding Pension Fund Finance.* By D. DON EIZRA.

[Pagurian Press Ltd, Toronto]

This book achieves its aims of explaining the rôles of actuaries and investment managers in pension
fund finance, for the benefit of managers and administrators, in as simple terms as possible. The contents will be mostly familiar to actuaries, but nevertheless the book is of interest, perhaps especially to students and young actuaries, for the good examples of how to explain actuarial techniques in non-technical terms. The book starts by explaining pension fund characteristics and actuarial techniques. There is an interesting section on the characteristics of assets and liabilities and on investment policy, some of which may require concentrated reading by lay readers although the presentation is clear. The final part of the book explains the common measures of investment performance and gives advice on how to choose an investment manager. Being aimed at readers in Canada and the U.S.A. it is also a convenient source of the definition of some of the pension fund actuarial terms in use in those countries.

R. C. GILDER