Risk Measures: Beyond Coherence?

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Agenda

- Risk measures: What are they good for?
- Risk measurement and risk aversion
- Coherent (and other) risk measures
- Beyond coherence:
  - Liquidity and aggregation
  - Background risk
  - A regulatory perspective
- Discussion (hopefully)
Risk Measures

- Risk measures are functions
  - You put in a (loss) distribution
  - You get out a number
- They can be used for
  - Comparing risks
  - Pricing
  - Capital allocation (as in portfolio management)
  - Capital allocation (as in regulatory requirements)
Some risk measures

- Expected loss: $(1+\lambda) \cdot E[X]$
- Standard deviation: $\sigma(X)$
- Percentile: $\Pr(X \leq \text{VaR}_a(X)) = a$
- Tail Conditional Expectation: $E[X|X>\text{VaR}_a(X)]$
- Lloyd’s RBC: $E[\max\{X-(\text{NP}+\text{RBC}),0\}] = \text{NP} \cdot \text{ELC}$
- …
Risk aversion

- A risk measure adds a margin to expected loss.
- Hence it forms a representation of risk aversion:
  - How risk averse are we? How much is the margin?
  - In which way are we risk averse? How do we calculate the margin?

- Ways in which to model risk aversion
  - Exaggerate the probabilities of adverse scenarios
  - Exaggerate the consequences of adverse scenarios
Distorting probabilities (1)

- If working with a probability distribution, the method consists of “blowing up” its tail.
- For a tail function $S(x) = \Pr(X>x)$, apply the non-linear transform $S^*(x) = g(\Pr(X>x))$.
- The risk measure equals the expected loss under the transformed probability distribution:

$$\rho(X) = \int_0^\infty S^*(x)dx \quad \text{(recall that } \mu = \int_0^\infty S(x)dx \text{ )}$$
Transformed Probability Distributions

\[ P(X > x) \]

Probability Distribution

Transformed Probability (PH)

Margin (Loading)

Expected Loss

\[ x \]

0

1

2

3

4

5

6

7

8

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Transformed Probability Distributions

- Probability Distribution
- Transformed Probability (TailVaR)
- Margin (Loading)
- Expected Loss

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Transformed Probability Distributions

- Probability Distribution
- Transformed Probability (TailVaR)

Pr(X>x)

Margin (Loading)

Expected Loss

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Distorting probabilities (2)

- If working with samples (e.g. from a DFA model), re-weight them according to (an increasing function of) their ranks.
- After re-weighting, take the average.
- If the CDF is $F(x) = \Pr(X \leq x)$, this corresponds to:
  $$\rho(X) = \mathbb{E}[X \cdot h(F(X))]$$
  for appropriate increasing function $h$. 
Weighting of samples in the distortion approach

Tail-VaR

Distortion function

Samples

Weights
Distorting losses

- An alternative way of modelling risk aversion relies on the transformation of potential losses.
- Method: transform each sample $x_i$ by $v(x_i)$.
- $v$ is a convex and increasing “disutility” function.
- Take the average of the transformed samples.
- Apply inverse $v^{-1}$ to recover original scale.

$$\rho(X) = v^{-1}(\mathbb{E}[v(X)])$$
Utility-like transformation of losses
Properties of risk measures

- Different risk measures are characterised by alternative sets of properties
- Different sets of properties correspond to alternative notions of risk aversion
- Let’s have a look…
Coherence (Artzner et al., 1999)

- $\rho(X+Y) \leq \rho(X) + \rho(Y)$, meaning that pooling risks is always beneficial.
- $\rho(a\cdot X) = a\cdot \rho(X)$, $a \geq 0$ meaning that the scale of loss does not matter.
- Can be constructed by distorting probabilities: $\rho(X) = E[X\cdot h(F(X))]$
- TailVaR is coherent, VaR isn’t
Additivity (Gerber, 1974)

- $\rho(X+Y) = \rho(X) + \rho(Y)$ for independent $(X,Y)$
- $\rho(X+Y) \leq \rho(X) + \rho(Y)$ for negative correlation
- $\rho(X+Y) \geq \rho(X) + \rho(Y)$ for positive correlation
- Can be constructed by distorting the losses:
  \[ v(x) = \exp(\beta X) \Rightarrow \rho(x) = \frac{1}{\beta} \ln \mathbb{E}[\exp(\beta X)] \]
- Such risk measures are quite sensitive to scale.
Convexity (Föllmer and Schied, 2002)

- $\rho(\lambda X + (1-\lambda)Y) \leq \lambda \rho(X) + (1-\lambda) \rho(Y)$
- “Must diversify in order to pool, not pool in order to diversify”
- The two classes described previously are special cases
- Can construct using a combination of distortion and utility approaches (Tsanakas and Desli, 2003 BAJ)
Departures from coherence

- Coherence nowadays forms the most widely accepted set of properties for risk measures.
- However, there are situations where the properties of coherent risk measures may not be appropriate.
- Typically this happens if the scale of potential losses is an issue.
- Three such situations are now described.
Liquidity risk

- In the case that a highly adverse scenario takes place, additional capital will have to be raised.
- Trying to raise £1m and £100m are two very different things.
- Coherent risk measures are scale invariant: \( \rho(aX) \leq a \cdot \rho(X) \) and do not address this issue.
Aggregation risk

- Some people would also argue that you should never accumulate highly correlated risks.
- This is intricately linked with liquidity – aggregating many highly correlated positions is like investing in one large risk.
- Coherent risk measures are subadditive: $\rho(X+Y) \leq \rho(X) + \rho(Y)$ and again do not take account of this issue.
Convex risk measures to the rescue

- If liquidity and aggregation are concerns, we could use a convex risk measure instead, e.g., “exponential TailVaR”:

\[ \rho(X) = \frac{1}{\beta} \ln E[ e^{\beta X} | X > Q_X(a) ] \]

- For small losses behaves approximately like a coherent risk measure.
- For larger losses it becomes more and more sensitive to liquidity and aggregation.
Sensitivity of risk measures to portfolio size

- **Tail-VaR**
- **Exponential Tail-VaR**

![Graph showing the sensitivity of risk measures to portfolio size]
Background risk

- You hold a risk Y that you cannot get rid of.
- You add some new (say independent) exposure X to that.
- Small amounts of the new risk X diversify your initial exposure.
- The more of X you take on though, the more X dominates your portfolio, hence the less diversification benefit it contributes.
Background risk (cont’d)

- Capital allocation techniques can determine the benefit from taking on exposure in new risk X.
- Consider the portfolio $Y + \lambda \cdot X$
- Calculate the aggregate risk, using e.g. TailVaR
- Determine the contribution of the new exposure $\lambda \cdot X$ to the aggregate risk.
- Plot that against exposure $\lambda$. 
Risk contribution in the presence of background risk

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Regulation

- Coherent risk measures encourage the pooling of portfolios.
- It is desirable that such pooling does not increase the shortfall risk.
- Consider two loss portfolios $X, Y$.
- A regulator suggests a coherent risk measure $\rho$ for determining respective capital $\rho(X), \rho(Y)$. 
The loss from each portfolio, in excess of capital, is borne by “society”.
These losses are respectively:
\[
\max\{X - \rho(X), 0\}, \quad \max\{Y - \rho(Y), 0\}
\]
Suppose now that the holders of risks X and Y decide to merge them.
New risk to “society” is:
\[
\max\{X+Y- \rho(X+Y), 0\}
\]
It was shown by Dhaene et al. (2004), that, if $\rho$ is subadditive, the shortfall risk to “society” after the merger can be higher than before.

Hence pooling can be good for insurers, but bad for policyholders, due to increased shortfall risk!

A bit controversial but there is something in it.
Conclusion

- There are different ways of constructing risk measures, depending on how our risk aversion is manifested.
- Coherent risk measures are the leading paradigm, but sometimes do not adequately capture risk.
- They can be enriched by introducing some sensitivity to the scale of potential shortfall.