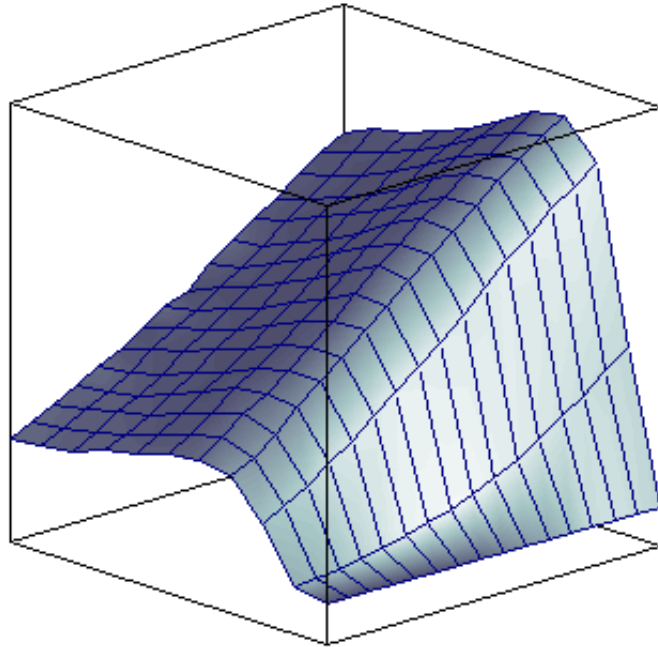


# The risk ridge



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**20 June 2005**

## 1. Introduction

In 1952, Harry Markowitz [1] developed the basic techniques for portfolio optimisation- usually referred to as mean-variance analysis – that is still in widespread use today. In 1964, Bill Sharpe simplified and extended this analysis with the publication of the capital asset pricing model (CAPM), a linear model of financial assets [2]. According to the CAPM differences in premium of certain assets are due to differences in the riskiness of the returns of the assets. The model asserts that correct measure of riskiness is the beta [3]. In CAPM, investment choice is reduced to two “assets” – the entire market and some risk free asset. Portfolio construction consists only of deciding the proportions of a portfolio invested in each, depending only on the investor’s risk appetite.

In 1976 an alternative approach to price financial assets was developed by Ross [4]. In this approach a linear relationship between the asset returns and a given factor model is derived– this model was called the arbitrage pricing theory (APT). In an economy, for example, the returns of the assets are generated by factors such as inflation, oil price, interest rate, etc. The APT attempts to relate the return as a linear combination of these factors.

Some of these theories have also been used to measure investment performances- a direct measure of reward-to-risk. The Sharpe Ratio is calculated by taking a fund’s annualised rate of return minus the risk-free rate divided by the fund’s standard deviation. The difference in returns represents the fund’s excess returns beyond that of a risk free investment, also known as “risk premium Jensen’s alpha [5] is a risk-adjusted performance measure that represents the average return on a portfolio over and above that predicted by the CAPM, given the portfolio's beta and the average market return.

There have been plenty of criticisms of these models over the intervening years, but it should be recognised that they offered profound insights, albeit at the cost of great abstraction from reality. The main criticisms being that these theories use only the first two moments of the distribution of the returns – which in effect assumes that the distribution of the returns are normal. This makes the Sharpe ratio open to ‘gaming’. Lowering the skewness by mechanically writing ordinary calls, for example will raise the Sharpe ratio substantially.

A number of extensions to these theories already exist. In 1994 Sharpe revisited this measure to provide more generality and to cover a broader range of applications [6]. In [7], a generalization of the traditional Sharpe ratio is developed to evaluate non-normal return distributions. They use this measure in benchmarking and evaluating funds. A number of optimisation routines have also been developed to account for transaction costs, taxes, or complicated trading strategies or horizons. Some of these routines will also draw an entire efficient frontier. See [8] for a review. In [9] a number of omega functions are described. The omega function allows for comparing funds with different risk profiles by using all the information about the distribution of the returns. At its simplest it can be expressed as a ratio of the sum of the amounts considered as wins, relative to a threshold multiplied by their corresponding probabilities divided by the sum of the amounts considered as losses, multiplied by their corresponding probabilities. A behavioural CAPM was proposed in [10] and in [11] a robust mathematical approach is used for evaluating and compensating traders.

In this paper we look a new paradigm for developing strategies for building a portfolio. We look at dynamic strategies at different time horizons and based on stochastic simulation we show that there exists a risk frontier that is independent of the investor's apriori risk or return performances. This paradigm can be used to explain some of the anomalies of the financial markets, which up to now have been the domain of behavioural finance [12]. This theory can be used in portfolio construction [12,13] We also explore some extensions of the idea to wider disciplines such as insurance company management, game theory, genetic mutation and humour [16].

The remainder of the paper is organised as follows: in section two we explain the risk ridge by means of an example. In section three we analyse the factors which lead to its emergence. In section four we consider areas where the results may be applied. Section five concludes.

## 2. The Risk Ridge

### 2.1 The problem

The risk ridge paradigm is first introduced by means an example [13,14,15].

The pensions fund management industry in the UK is organised in the following way: trustees of a pension fund give a mandate to a manager to manage the pension fund's assets for a certain mid-range time period, say three years. The fund manager is given a benchmark; say the FTSE All share, which he is expected to match.

At the end of the three-year period, if the cumulative return of the pension fund is within a predetermined range of the corresponding return of the benchmark, the trustees might extend the manager's mandate for a further three years. The purpose of handing out these three year mandates is putatively to avoid managers focusing on the short term, and thus taking sub-optimal investment decisions for very long term funds.

However, an examination of trustees' behaviour will show that if their fund manager underperforms excessively in the short term, they will end the mandate and find a new manager.

In this example we assume that the trustees put in place a mandate with a fund manager as follows:

- Achieve a cumulative return of 3% above the benchmark over 3 years
- Avoid 4% underperformance return relative to the benchmark in any rolling year
- Performance is monitored quarterly

Of course, we are examining a stylised situation here, but the setup will be realistic to those involved in the pensions industry.

## 2.2 The dynamics

Let us examine the dynamics of the system. The fund manager has the goal of maximising his management fees by maximising the return on the fund, and winning a mandate for a further three years. To achieve this goal he has to take on risk – i.e. diversify away from the benchmark (remember, if he only tracks the benchmark he still loses the mandate because he doesn't outperform).

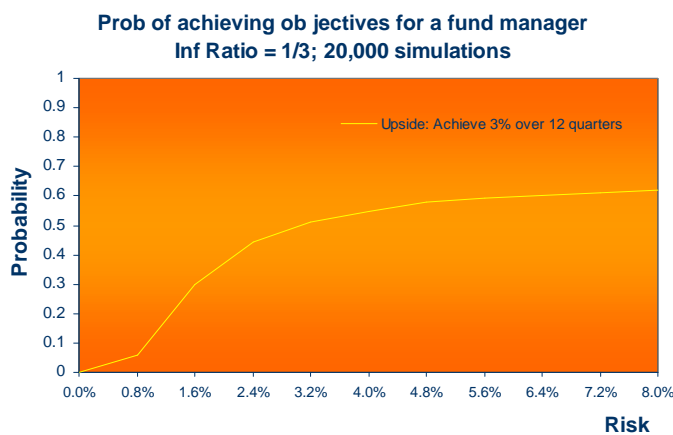
First we need to consider a measure of skill for a fund manager, say the ability to turn risk into return, commonly referred to as the information ratio. For this we use  $I$ , where  $I = \frac{\mu}{\sigma}$ , so given an information ratio of 1/3, you would expect the manager to transform a third of their risk into return.

Secondly we will assume that returns are normally distributed. According to our assumptions, if returns are normally distributed with variance  $\sigma^2$ , then a fund manager with an information ratio of  $I$  will produce returns with mean  $I\sigma$ , i.e. returns are distributed  $\sim N(I\sigma, \sigma^2)$ . Quarterly risk is the annual risk multiplied by

$$\sqrt{\frac{1}{4}}.$$

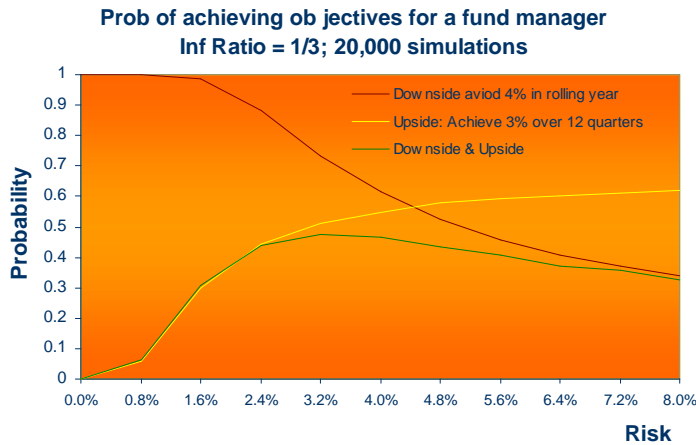
Simulating 20,000 paths allows us to calculate the probability of achieving the objectives as the number of successful paths divided by 20,000. Using this methodology we determined the probability of achieving the upside for different levels of risk and plotted the results below. We can see that the more risk the manager takes on, the higher his chances of outperforming, shown in yellow in Figure 1 below.

Fig 1. A plot of the probability of upside for different levels of risk.



However, he must also, in the interim, avoid getting sacked by underperforming during the three-year period. Clearly the further away from the benchmark he is, the higher the variance of returns, and the lower the chances of escaping short term downside, shown in Figure 2 below.

Figure 2. The probability of upside and downside for different levels of risk.



The optimal level of risk is clearly the one which minimises the chances of being sacked at any point and maximises the chances of outperforming— the highest point of the curve shown in green above.

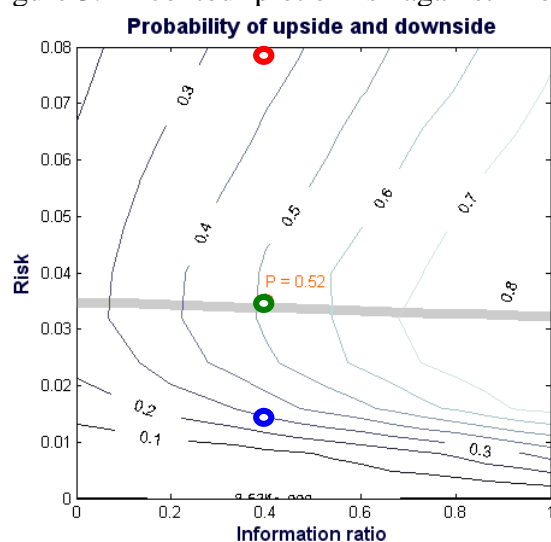
One of the results of the Capital Asset Pricing Model states that investor choice can be reduced to creating a portfolio invested partly in the risk free asset and partly in the market portfolio – the capital market line. We can now see the application to CAPM – rather than considering utility indifference curves, or other arcane methods of choosing a point on the capital markets line to target, the fund manager simply diversifies away from the benchmark until he achieves the optimal level of risk.

## 2.3 The Risk Ridge itself

We now consider another influence on the system: variation in the investment manager's ability to turn risk into return.

To examine this we calculated the probability of achieving the objectives for 16 different levels of risk and 12 different levels of information ratio. We used 20,000 paths to calculate the probabilities at each risk and information ratio combination. This gave us a grid of probabilities, which we then converted into a contour plot shown below.

Figure 3. A contour plot of risk against information ratio



The contours on Figure 3 join all the combinations of risk and information ratio that give the same probability of achieving the objectives. For example the contour labelled 0.5 indicates all combinations of risk and information ratio that give a 50% chance of achieving the objectives. An investment manager would want their risk/ information ratio combination to give them a greater than 50% chance of achieving their objectives, and to be in the region to the right of the 0.5 contour.

The green circle on the chart indicates a point estimate of risk and information ratio. In this case an investment manager with an information ratio of 0.4 and a risk of 3.5%. The manager has approximately a 52% chance of achieving the objectives (calculated here from 40,000 simulations).

If the manager increased their risk to 8% holding their information ratio constant (red circle), then the probability of achieving objectives falls to around 35%. The manager is far more likely to hit the downside than the upside.

If the manager reduced their risk to 1.5% holding their information ratio constant (blue circle) then the probability of achieving their objectives falls to around 30%. However the manager is more likely to fail to outperform than hit the downside restrictions.

So the green circle where a manager with an information ratio of 0.4 is running at a risk level of around 3.5% seems to be an optimal balance between upside and downside.

The optimum level of risk for a given level of skill can be described mathematically as:

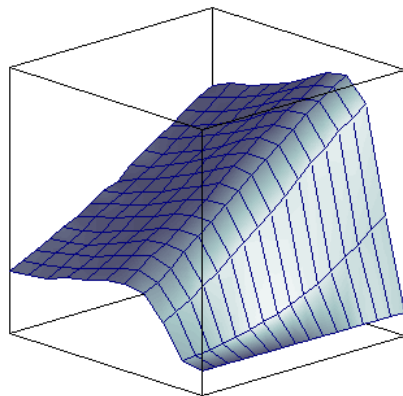
$$OptimumRisk(Skill) = \arg \max(f_{Happiness}(Skill, Risk)).$$

Where ‘Happiness’ is the probability of the manager achieving the objectives. Using the chart we can do this by drawing a vertical line on the chart where the information ratio is 0.4, and reading off the highest probability of happiness for a given level of skill. The optimum level of risk given an information ratio of 0.4 is around 3.5%.

The thick grey line is a simple straight-line fit of the optimum risk level for any given information ratio. Obviously the line will vary slightly from simulation to simulation. Nevertheless on left hand side of the contour chart at an information ratio of 0 the optimum risk to achieve the objectives is around 3.5%. To the right of the chart where the information ratio is 1, the optimum risk to achieve the objectives is a little over 3%. This illustrates that the optimum level of risk lies in a narrow band for a wide range of information ratio. It follows from these observations that the risk level to adopt is *broadly* independent of manager skill. This is the risk ridge.

For completeness we show a plot of this function below, the ridge can be seen as the “spine” of the three-dimensional plot.

Figure 4. A surface plot of the probability of achieving objectives.





## 2.4 Consequences and evidence

The optimal level of risk for a fund manager may not be the optimal level of risk for the client. However, if the benchmark chosen is appropriate, the mismatch may not be too bad.

It is interesting to note that in the UK institutional fund management industry large investment houses tend to centre on a level of active risk from the FTSE All Share index that is consistent with the risk ridge theory. Risk ridge theory would dictate that those running at the incorrect level of risk would shed clients (and funds under management), a kind of Darwinian selection on fund managers.

We are aware of at least one large, highly skilled manager who was taking low levels of risk, and had difficulty in outperforming. This proved to be a contributory factor to the manager shedding clients.

In addition there is the example of one of the largest investment houses in the mid 90's who adopted a strong value investment style. As the market's growth stocks outperformed during the late 90's, the value manager's tracking error from the benchmark kept on increasing. Eventually clients could not stomach the short-term underperformance and sold the fund manager. When the bull market finally turned, and quality of earning returned to the fore, there were few clients left in the fund to benefit from the long-term positive returns.

## 2.5 Summary

To summarise the system above – a participant has to achieve a goal over one time horizon, whilst avoiding a pitfall on another. These goals conflict, and lead to an optimal strategy, which is largely independent of the participant's skill.

While some of our assumptions are clearly violated (e.g. normal distribution of asset returns, etc), we believe that the framework is robust.

### **3. An examination of the fundamentals**

We now consider what aspects of the system described above lead to the emergence of the risk ridge, and so generalise the result. We frame the following in terminology which is more suited to game theory or agent based modelling than financial economics, indeed, we believe that the risk ridge sits most easily within these discipline.

#### **3.1 Requirements**

##### **3.1.1 Participants**

Clearly you need at least one participant. In the case of the fund, it is the fund manager and the market (or other fund managers). A card player needs an opponent and a comedian is nothing without his audience competing with him. However there may be circumstances when you have a participant in isolation. To model the game you need to identify some key objectives of the participant. You don't need to understand all the participant's objectives; just two conflicting objectives are enough to reveal a truth about the participant's behaviour.

##### **3.1.2 Uncertainty or risk**

There are uncertain outcomes in the game. Sometimes uncertainty or financial risk can be specified by a diffusion process, whereas an asset's risk is defined by its standard deviation. Otherwise, and in most cases, uncertainty cannot be defined succinctly. In this case it can only be described in wider terms; we might describe uncertainty as a 'situation' in human relations, or use a phrase like 'uncharted water' to describe a space. We shall assume that the uncertainty is described by the distribution of the outcomes, which is specified *a priori*.

For the risk ridge to be of use as a decision tool a participant will need to exert some control over their levels of uncertainty or risk.

### 3.1.3 Goal

We need to be able to describe a measurable quantifiable goal for the participant, where the goal is made up of two conflicting parts:

#### **An upside**

An outcome, over one or more time periods. The upside needs to increase with uncertainty. A zero risk strategy will not result in achievement of the upside!

#### **A downside**

An outcome, which is measured over one or more time periods.

The downside increases with increasing uncertainty. A zero risk strategy may lead to avoiding the downside.

In our manager example the upside and downside were binary events – outcomes are either achieved or not. In general we may consider a situation where degrees of success are possible.

The upside and downside objectives do not need to be symmetric, for example, the upside could be over a longer period than the shorter-term downside.

We could describe the goal through a utility function which is then used to determine the probability of a participant being happy at the end of the game.

A rational participant will always prefer a higher utility to a lower utility.

This will automatically lead to the following behaviour:

- Where the participant perceives no downside and only upside, they will maximise risk, ie reckless.
- Where the participant perceives no upside and only downside participants will minimise risk, ie ultra conservative.

### 3.1.4 Skill

The participant has skill. This is the ability of the participant to transform uncertainty into an objective.

In our manager example, skill could be described as the ability to transform risk into return, such as the information ratio measure.

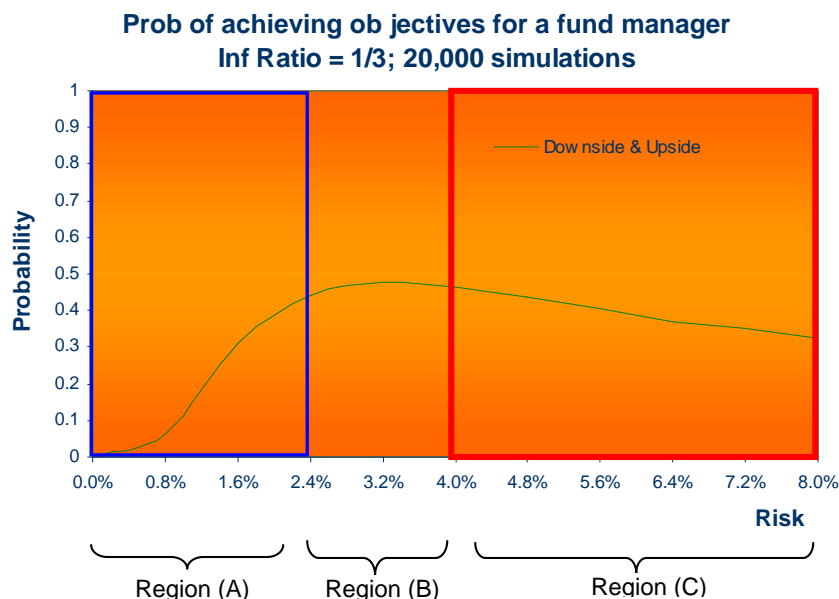
## 3.2 How the Risk Ridge emerges

### 3.2.1 Happiness (Utility)

We will define happiness as ‘achieving upside while avoiding downside’. Arguably happiness is a somewhat woolly term, however it’s quite easy to communicate to a layperson or a participant, you simply ask “are you happy, yes or no?” The answer is a binary outcome, and it gets around the problem of generating difficult to communicate utility functions. Generally the probability of a participant being happy will vary as uncertainty changes, where the setup in section 3.1 holds.

To illustrate this we consider the “manager problem” in more detail below.

Figure 5. The probability of achieving upside and downside



**Region (A):** With low uncertainty the participant can avoid the downside objective, however because there is insufficient risk to achieve an upside, the probability of happiness is low.

From here if we increase uncertainty, we raise the probability of achieving the upside.

$$\frac{\partial \text{Happiness}}{\partial \text{Uncertainty}} > 0.$$

**Region (B):** In a region of medium uncertainty the probability of achieving upside becomes balanced by the probability of the participant incurring downside. The happiness function is at an optimum at this point.

$$\frac{\partial \text{Happiness}}{\partial \text{Uncertainty}} = 0,$$
$$\frac{\partial^2 \text{Happiness}}{\partial^2 \text{Uncertainty}} < 0.$$

**Region (C):** Increasing uncertainty from this point leads to a reduction in happiness because the downside objective bites. So,

$$\frac{\partial \text{Happiness}}{\partial \text{Uncertainty}} < 0,$$

### 3.2.2 Holding uncertainty constant

So far we have ignored a participant's level of skill. However, for a given level of uncertainty we would expect that a participant with more skill is more likely to achieve their objectives and be happy.

$$\frac{\partial \text{Happiness}}{\partial \text{Skill}} \geq 0,$$

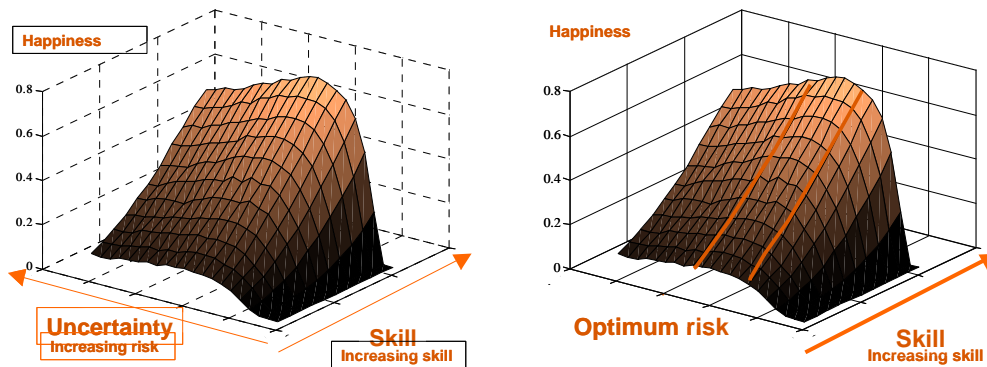
This relation holds up to the point when you have a 100% chance of achieving your objectives, at which point there is no additional benefit from additional skill.

### 3.2.3 The ridge

The two charts below show the shape of this function in uncertainty, risk, happiness space. On the left hand chart I have labelled the axes, and on the right hand chart I have highlighted the ridge using tramlines. Between the tramlines this definition holds:

$$\frac{\partial \text{Happiness}}{\partial \text{Uncertainty}} \leq \frac{\partial \text{Happiness}}{\partial \text{Skill}} .$$

Fig 5.



The first thing that we can draw out is that the optimal level of risk is broadly independent of skill. Whether you have lots of skill or none at all you should be taking similar levels of risk. We add a note of caution here because the optimum does move with changing skill, just very slowly, and it is of second order to the impact of upside and downside tolerances.

### 3.3 Markowitz portfolio optimisation

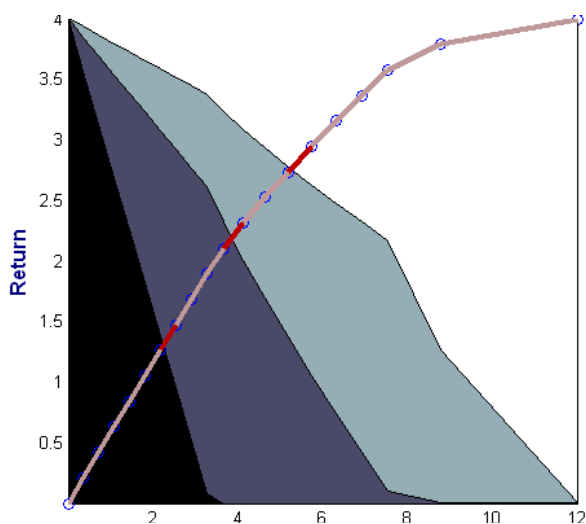
The risk ridge points towards an optimum level of risk that is broadly independent of skill. However is it possible to produce an integrated model that draws together Markowitz portfolio optimisation and the risk ridge?

Harry Markowitz defined portfolio optimisation as a process that would lead to an efficient set of portfolios. By efficient we mean a portfolio with the highest return for a given level of risk. The set of efficient portfolios is referred to as an efficient frontier.

The problem that the investment community face is how to pick off a point on this efficient frontier. Utility functions are one approach which would allow a point to be chosen, however, rarely do you come across practitioners using utility functions. This is largely because utility functions are an unnatural fit to general human thought (if not behaviour). How many people can describe their marginal utility to an additional unit of risk?

However the risk ridge approach gives a far more communicable way of picking a point on an efficient frontier. It considers the probability of achieving your objectives as a third dimension over the mean variance framework.

Fig 6. Efficient frontier overlaid over an optimal portfolio allocation

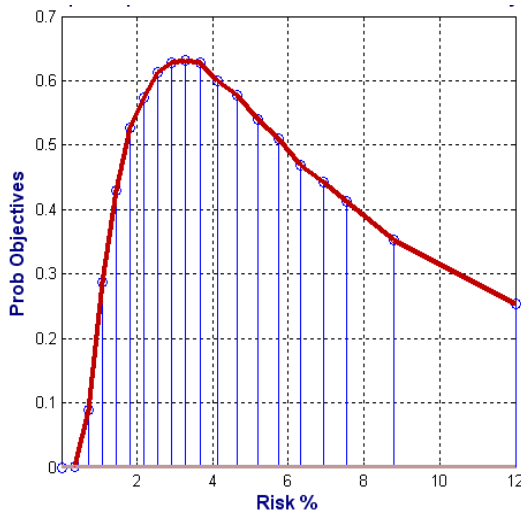


The red line on Fig 6 illustrates a traditional efficient frontier for a mix of 4 notional assets. The blue dots represent 20 portfolios that have been generated along this frontier. Behind the frontier we show an area chart, this illustrates the asset allocation at each point on the frontier. The lowest risk/return (0% & 0%) point is associated with 100% in the black asset, and the highest risk/return portfolio (12% & 4%) is 100% invested in the white asset.



Using the same upside and downside objectives as in the first case study we can take the risk and information ratio for each of the 20 portfolios, and calculate the probability of achieving objectives.

Fig 7. Probability of achieving objectives at each point on the efficient frontier.



We can see from this chart that picking a portfolio from the efficient frontier where the risk is between 3%-3.5% will lead to the highest probability of achieving the objectives, in this case a probability of over 60%.

The efficient frontier on its own could be criticised for being a little fatalistic, it assumes that the risks and returns will be borne out. By contrast, adding the risk ridge dimension includes a probabilistic dimension communicating the uncertainty of the outcomes.

In terms of sensitivity a different set of asset assumptions will shift the curve up or down, and keep the optimum broadly in the same place. However a different set of objectives will shift the position of the optimum left or right. For example if you had more headroom on the downside the optimum shifts to a higher risk on the right, and if you had less headroom on the downside the optimum shifts to the left into a lower risk position.

Fig 8. Optimal portfolio allocation and efficient frontier together

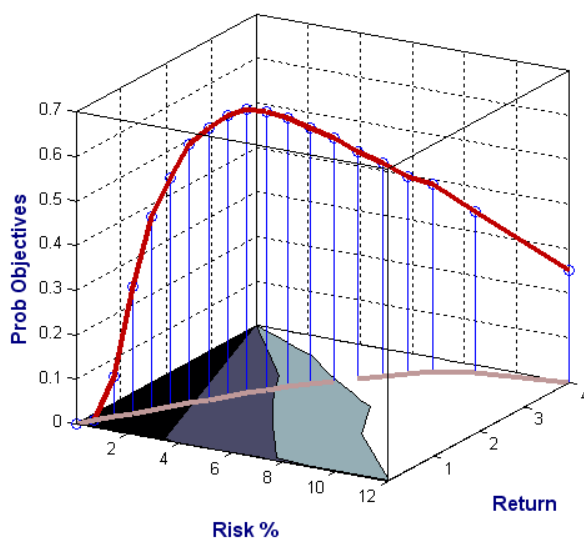


Fig 8 shows the risk ridge as a third dimension to the portfolio optimisation problem.

### 3.4 Poker

It is with care that the casual observer writes anything regarding the game of poker. A quick search on Google will reveal a large volume of research on the subject, some of it involving high-powered probability, game theory and psychology – much like the game itself.

With this in mind we consider an interesting result from the game. We see several similarities between the game of poker and the investment fund industry. Both involve the redistribution of wealth, with players ranging from the keen amateur to the hardened professional. Both involve the weighing up of probabilities, the use of judgement, and above all, the taking of risks in order to make profit.

First, a brief recap for those who have never played the game, or who played it last at university. All players are dealt a number of cards (five or seven, depending on flavour), some of which may be revealed to all players, but at least some of which are only seen by the player to whom they are dealt. Each player in turn makes the decision to “call”, “raise” or “fold”. These decisions revolve around how much money to place in the central pot – essentially each player makes a bet on that they hold the strongest hand.

A player who calls simply matches the last bet in the pot, and challenges the other players to reveal their hands. He may instead decide to raise, which means he matches the previous bet, and increases the stakes. A player who folds retires, and has no further financial interest in the round (beyond anything he has already put into the pot).

Clearly the decision to call, raise or fold depends on the strength of one’s hand, and the amount of money in the pot. If you calculate that the amount of money you might win, as a return on the amount you must stake, is greater than the probability that you hold the highest hand (i.e. you make an expected positive return), you should raise or call as appropriate, otherwise you should fold. The situation is complicated by the fact that players after you may raise or call, changing the odds.

Finally, when all players have called or folded, the hands are revealed, with the highest ranking hand winning the pot. If all but one player folds, that player wins without having to reveal his hand.

The key point is this: since the other players do not know your full hand, it is possible to bluff – backing a weak hand which has no chance of winning, in the hope that all other players will fold. A risky strategy, yet one which is essential to success in the game. For consider this: if a player is playing against an opponent who never bluffs, he will fold on a marginal hand if the opponent bets strongly, knowing he can’t win. Conversely, against a frequent bluffer he will call frequently, even with a marginal hand, knowing that on average, he will make money.

When presented with situations of conflict between players we may turn to game theory, and it is here that we find the result of interest to us: for the reasons above, there is an optimal frequency of bluffing, and this is independent of the skill of the players involved. Repeating the full argument would require more game theory than the average reader may have, the result can be found in, for example, [19].

We note (as does Sklansky) that game theory assumes that both players will pursue an optimal strategy, and success in poker can largely be attributed to identifying your opponents' sub-optimality, and exploiting them. So the theoretical result may not hold in your monthly college reunion game (but probably does in the world championships!).

To conclude – if we consider bluffing in poker analogous to taking of risk in investment, the optimal level of risk to take is independent of player (or investment manager) skill. It is not immediately clear to us how the poker result could be derived from a risk ridge framework, but it may be a fruitful line of enquiry.

### 3.5 Humour

In [17] John Vorhaus describes the need for individuals to be prepared to take the ‘will to risk’ and attempt a comic revelation to a listener. In addition he expresses the point that in identifying a comic revelation the comic will not be successful every time. Indeed the levity hit rate for a novice comic is very low. Vorhaus defines levity as the following:

$$(Truth) + (Pain) = Comedy$$

In other words a comic revelation involves both truth, and pain.

Truth is that a statement must be true, and that the listener shares that truth. No truth, no upside. The downside is that it must hurt a listener at the same time; too much pain and the listener will be offended. Take an old example:

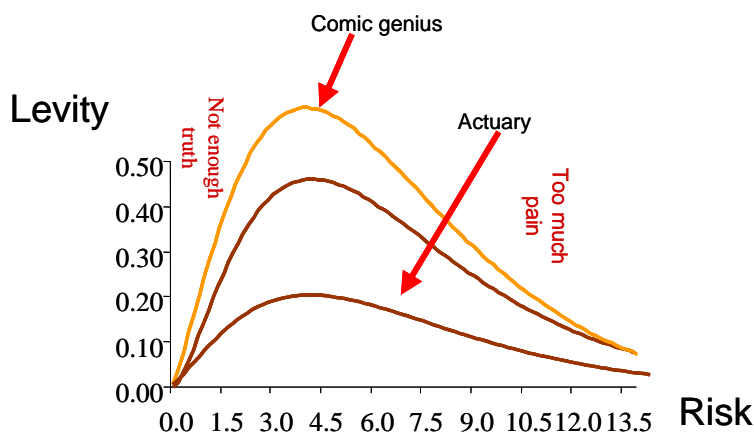
“How do you know you are talking to an extrovert actuary”

“He looks at your shoes when he talks to you”

Actuaries who share the truth that they have a grey reputation, and that they are therefore tarnished with this image would, find it amusing.

The conflict between upside and downside will form a risk ridge [19], where the skill dimension is in the delivery of the truth & pain, or revealing a new comic truth.

Figure 9: Suggested graph of the probability of achieving levity for different participants



Based on the risk ridge principle we would expect that a comic genius and an actuary should take similar levels of ‘will to risk’ in order to maximise the probability of connecting with people as seen in Figure 9. However the comic genius has a higher probability of connecting with his listeners.

## 4. Applications in other fields

As stated above, we believe the risk ridge to be an application of game theory rather than financial economics, and hence expect to find similar results, or see applications, in far wider fields. Some examples may include:

- A gene may mutate by a stable optimum amount from generation to generation. An organism needs to survive long enough to produce progeny without dying in the short term. Too much or too little mutation will reduce the chances of survival to the next generation due to internal or external influences. The risk ridge suggests that the mutation rate should be independent of an organism's efficiency in a particular environment.
- Could the accident hump in mortality tables be driven by an inherited urge in individuals to take sufficient risk with their life— and some individuals take too much?
- Neurons in a brain transform inputs from other neurons into a useful output [18]. In addition, neuron firings are noisy. Too much noise leads to confusion, and too little noise restricts the ability of neurons to make wide associations in thought. Is there some optimum level of noise for a neuron to take, regardless of the neurons processing of its neighbours inputs?

## 5. Conclusion

In this paper we have presented what we believe to be a novel and important concept. By considering a system where a participant has conflicting objectives, we uncover a remarkable result linking risk, skill and reward. Our result can be seen to extend the framework provided by classical finance paradigms such as the Capital Asset Pricing Model, as well as having links with game theory and behavioural finance.

Moving forward we see room for further research – perhaps replicating the risk ridge using option pricing techniques, and other analytical or algorithmic approaches such as Nash equilibria.

Have you ever undertaken actions that led to you feeling pain? Were those actions in pursuit of happiness, perhaps some long harboured desire or hope? If you answered yes to both these questions then you already know what the risk ridge feels like. We hope this paper will add illumination to what you've already experienced first hand. Perhaps you will be inspired to have another go at achieving happiness where previously you failed.

Long may you pursue your dreams at your own private, optimum, level of risk.

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## Bibliography

- [1] Markowitz, H. 1952 Portfolio Selection, *Journal of Finance*, 7, 1, 77-91.
- [2] Sharpe, William F. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19 (September): 425–42
- [3] Jagannathan, R. and McGrattan E.R. 1995 The CAPM debate. Federal Reserve Bank of Minneapolis Quarterly Review Vol. 19, No. 4, Fall 1995, pp. 2–17
- [4] Ross, S., "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, (December 1976) 343-362
- [5] Jensen, M. 1968, The Performance of Mutual Funds in the Period 1945-1964, *Journal of Finance*, Vol. 23, pp. 389-416.
- [6] Sharpe, William F. 1994. The Sharpe Ratio. *Journal of Portfolio Management* ???
- [7] Amin, Gaurav; Kat, Harry. "Generalization of the Sharpe Ratio and the Arbitrage-Free Pricing of Higher Moments". Alternative Investment Research Centre, Working paper
- [8] Mehndi P. 2002. Portfolio Optimisation: The rise of Markowitz Mean-Variance models and beyond *Optirisk Newsletter*.
- [9] Keating C. 2005. Last but not least. *Professional Investor*. PP20-23.
- [10] Hersh Shefrin and Meir Statman. Behavioural Capital Asset Pricing Theory. September 1994 *Journal of Financial; and Quantitative Analysis* Vol 29 #3.
- [11] Ahn, Hyungsole; Dewynne, Jeff; Hua, Philip; Penaud, Antony; Wilmott, Paul. "The End of the Year Bonus. How to optimally Reward a Trader." Feb 14, 2000 Working Paper.
- [12] Brodie, Z., Merton, R., "The Design of Financial Systems: Towards a Synthesis of Function and Structure", Harvard Business School Working Paper No. 02-074, 2004
- [13] Dorey, Martyn. (2001): 'The risk ridge' Presentation to Citigroup Quant Conference, Lausanne, Switzerland
- [14] Dorey, M (2002). [Risk budgeting \(slides\)](#). Younger Members' Convention. Institute and Faculty of Actuaries Conference.
- [15] Dorey, M (2003). [Kylie Minogue asset models and who cares anyway \(slides\)](#). Younger Members' Convention. Institute and Faculty of Actuaries Conference.
- [16] Dorey, M (2004). [The importance of being illogical \(slides\)](#). Younger Members' Convention. 29 Nov 04. Institute and Faculty of Actuaries Conference.
- [17] Vorhaus, J. *The Comic Toolbox*. , ISBN:1-879505-21-5 Silman-James Press
- [18] Bridgeland, Sally; Dorey, Martyn; Palin, Jon [Neural networks : Statistics for professionals?](#). SIAS Paper December 1997

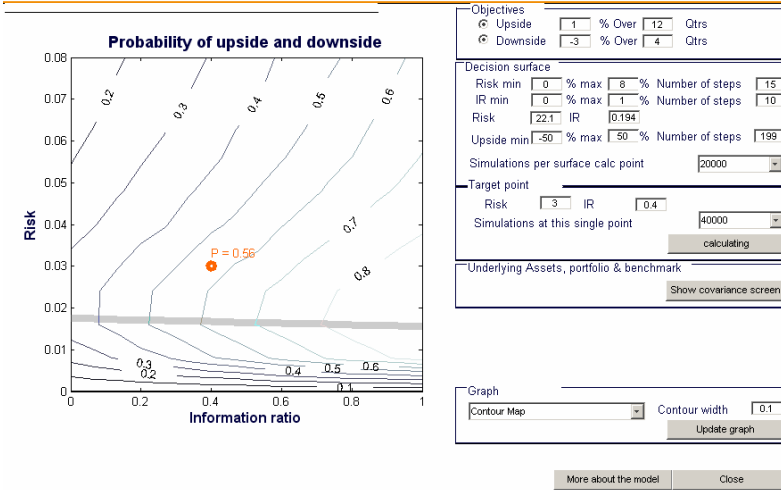


## **Appendix A.**

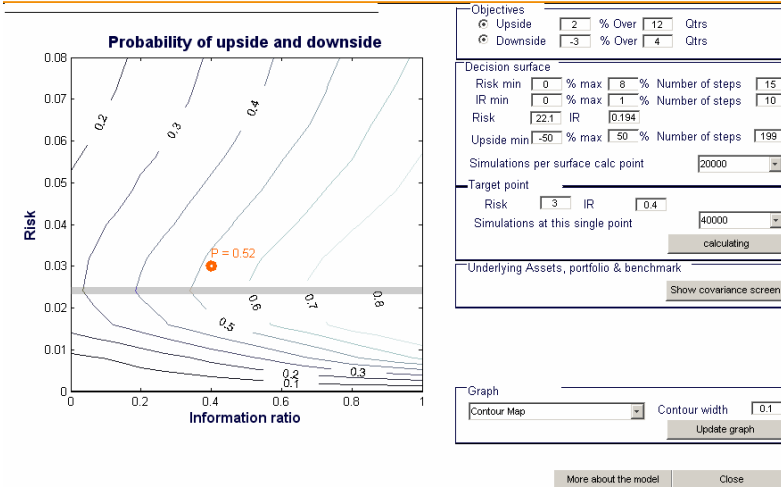
In the following pages we include screen shots of risk ridge models with varying objectives using the assumptions from section 2. Practitioners in the field of portfolio construction may find these a useful input to discussions on risk.

# Sample contour charts of the risk ridge: Downside 3% in rolling year

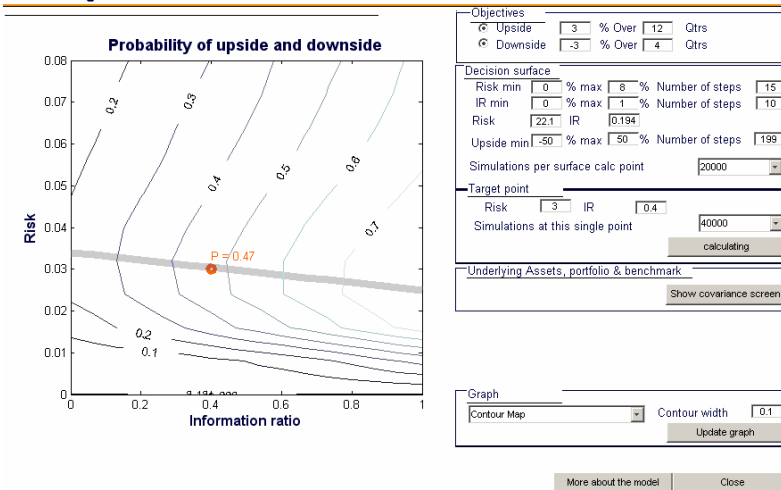
Risk Ridge 1.2



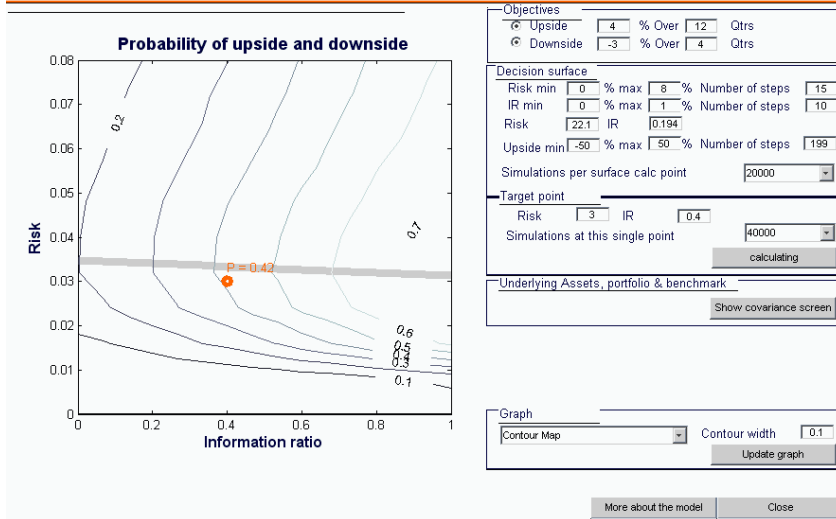
Risk Ridge 1.2



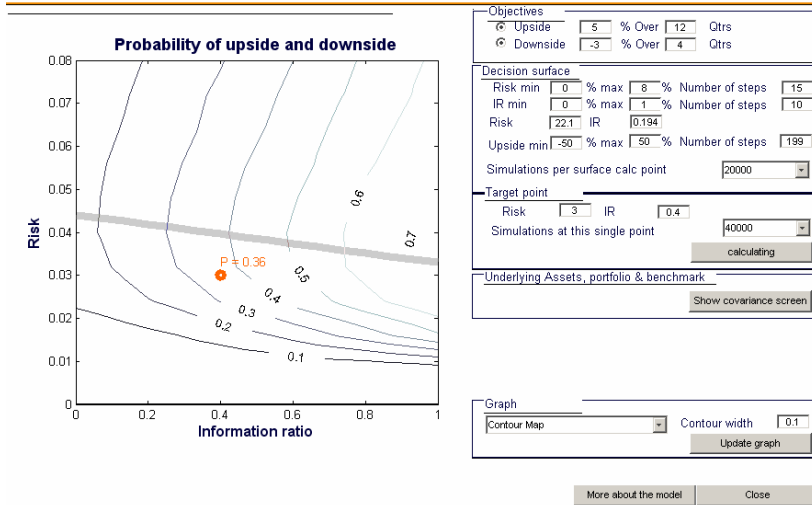
Risk Ridge 1.2



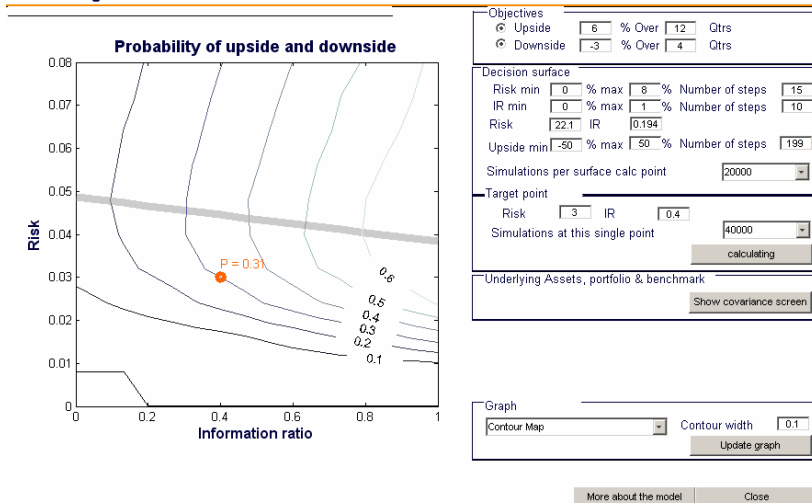
## Risk Ridge 1.2



## Risk Ridge 1.2

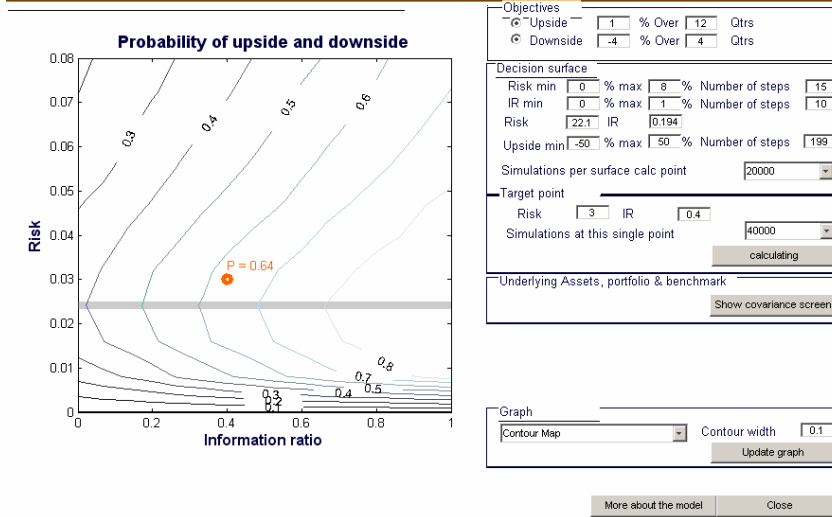


## Risk Ridge 1.2

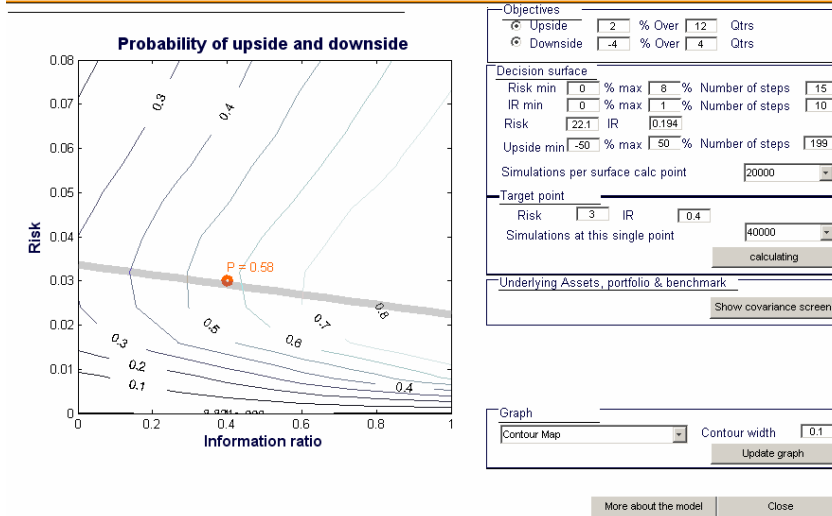


# Sampled contour charts of the risk ridge: Downside 4% in rolling year

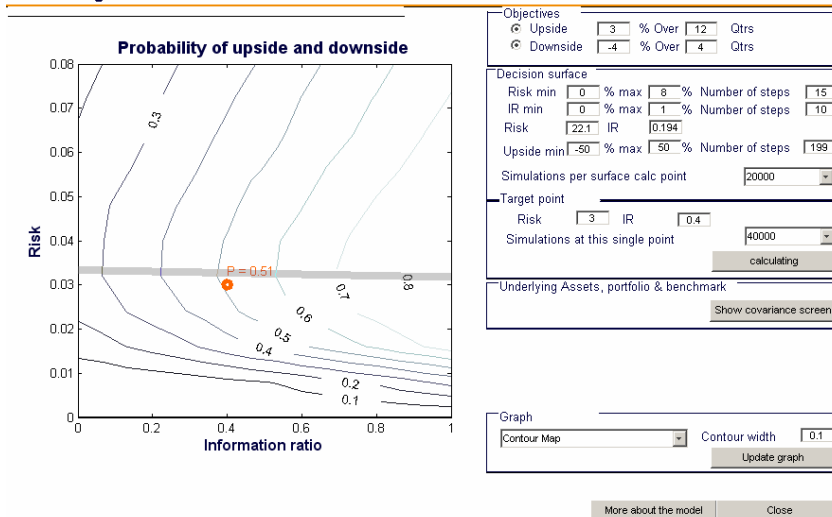
Risk Ridge 1.2



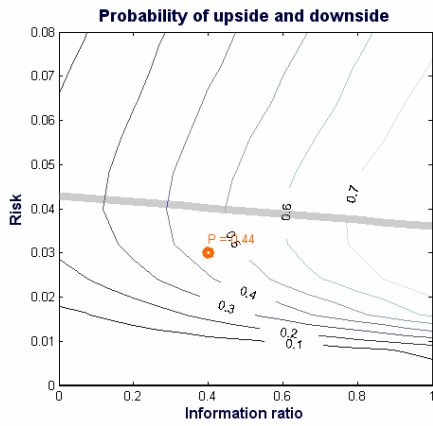
Risk Ridge 1.2



Risk Ridge 1.2



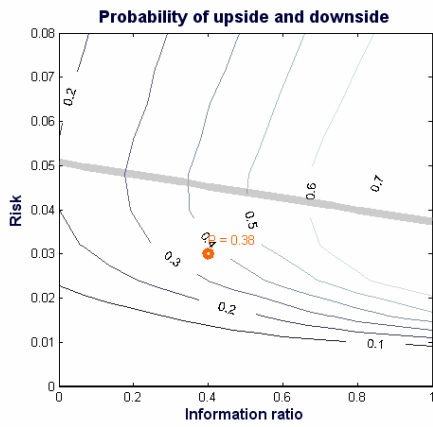
### Risk Ridge 1.2



Objectives	
<input checked="" type="radio"/> Upside	4 % Over 12 Qtrs
<input type="radio"/> Downside	-4 % Over 4 Qtrs
Decision surface	
Risk min	0 % max 8 % Number of steps 15
IR min	0 % max 1 % Number of steps 10
Risk	22.1 IR 0.194
Upside min	-50 % max 50 % Number of steps 199
Simulations per surface calc point	20000
Target point	
Risk	3 IR 0.4
Simulations at this single point	40000
calculating	
Underlying Assets, portfolio & benchmark	
Show covariance screen	
Graph	
Contour Map	Contour width 0.1
Update graph	

More about the model Close

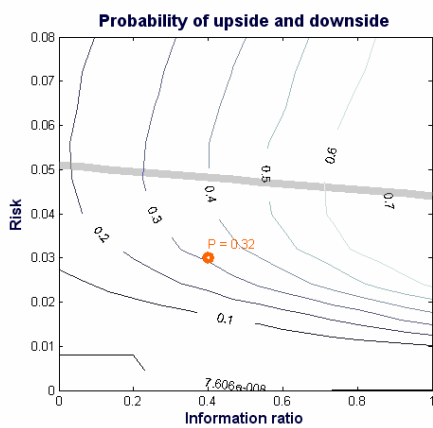
### Risk Ridge 1.2



Objectives	
<input checked="" type="radio"/> Upside	5 % Over 12 Qtrs
<input type="radio"/> Downside	-4 % Over 4 Qtrs
Decision surface	
Risk min	0 % max 8 % Number of steps 15
IR min	0 % max 1 % Number of steps 10
Risk	22.1 IR 0.194
Upside min	-50 % max 50 % Number of steps 199
Simulations per surface calc point	20000
Target point	
Risk	3 IR 0.4
Simulations at this single point	40000
calculating	
Underlying Assets, portfolio & benchmark	
Show covariance screen	
Graph	
Contour Map	Contour width 0.1
Update graph	

More about the model Close

### Risk Ridge 1.2

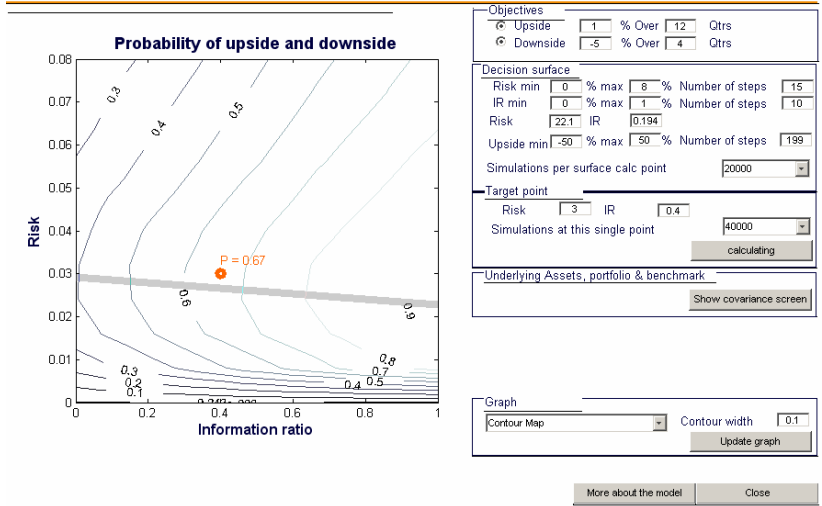


Objectives	
<input checked="" type="radio"/> Upside	6 % Over 12 Qtrs
<input type="radio"/> Downside	-4 % Over 4 Qtrs
Decision surface	
Risk min	0 % max 8 % Number of steps 15
IR min	0 % max 1 % Number of steps 10
Risk	22.1 IR 0.194
Upside min	-50 % max 50 % Number of steps 199
Simulations per surface calc point	20000
Target point	
Risk	3 IR 0.4
Simulations at this single point	40000
calculating	
Underlying Assets, portfolio & benchmark	
Show covariance screen	
Graph	
Contour Map	Contour width 0.1
Update graph	

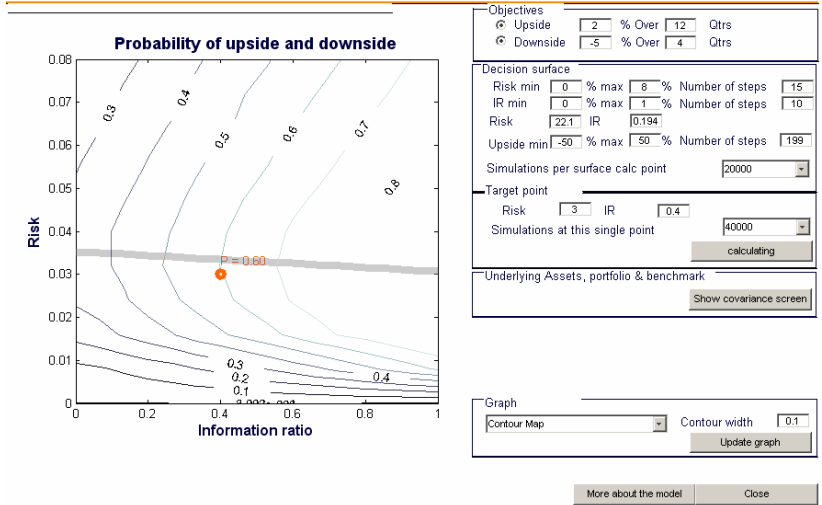
More about the model Close

# Sampled contour charts of the risk ridge: Downside 5% in rolling year

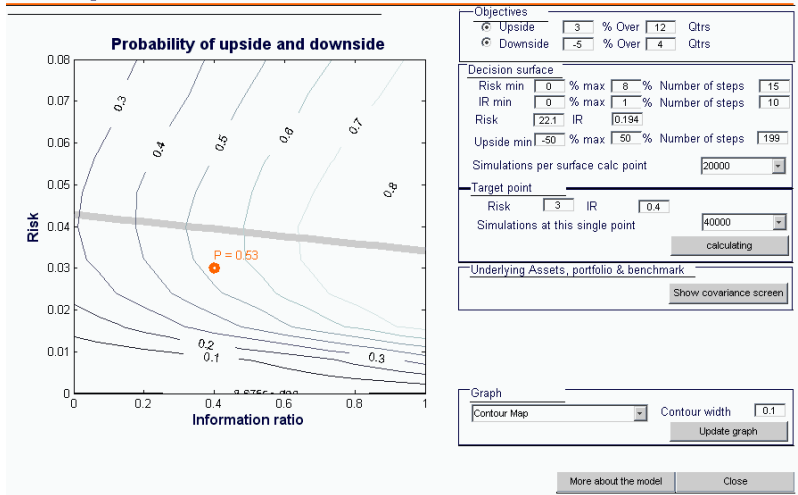
Risk Ridge 1.2



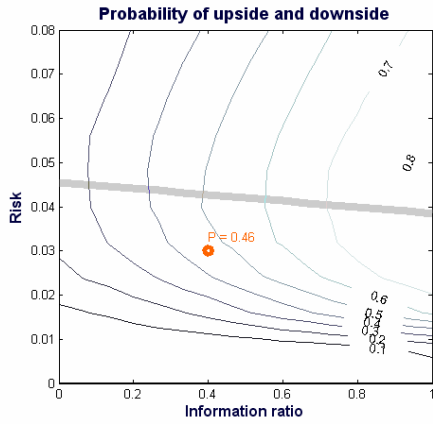
Risk Ridge 1.2



Risk Ridge 1.2



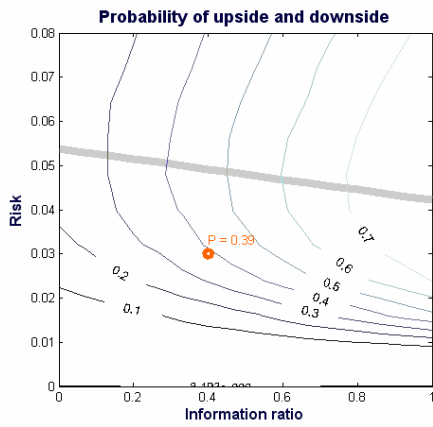
### Risk Ridge 1.2



Objectives			
<input checked="" type="radio"/> Upside	4	% Over	12 Qtrs
<input checked="" type="radio"/> Downside	-5	% Over	4 Qtrs
Decision surface			
Risk min	0	% max	8 % Number of steps 15
IR min	0	% max	1 % Number of steps 10
Risk	22.1	IR	0.194
Upside min	-50	% max	50 % Number of steps 199
Simulations per surface calc point 20000			
Target point			
Risk	3	IR	0.4
Simulations at this single point 40000			
calculating			
Underlying Assets, portfolio & benchmark			
Show covariance screen			
Graph			
Contour Map			Contour width 0.1
Update graph			

More about the model    Close

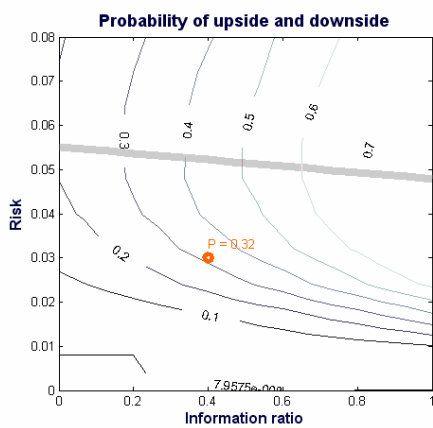
### Risk Ridge 1.2



Objectives			
<input checked="" type="radio"/> Upside	5	% Over	12 Qtrs
<input checked="" type="radio"/> Downside	-5	% Over	4 Qtrs
Decision surface			
Risk min	0	% max	8 % Number of steps 15
IR min	0	% max	1 % Number of steps 10
Risk	22.1	IR	0.194
Upside min	-50	% max	50 % Number of steps 199
Simulations per surface calc point 20000			
Target point			
Risk	3	IR	0.4
Simulations at this single point 40000			
calculating			
Underlying Assets, portfolio & benchmark			
Show covariance screen			
Graph			
Contour Map			Contour width 0.1
Update graph			

More about the model    Close

### Risk Ridge 1.2



Objectives			
<input checked="" type="radio"/> Upside	6	% Over	12 Qtrs
<input checked="" type="radio"/> Downside	-5	% Over	4 Qtrs
Decision surface			
Risk min	0	% max	8 % Number of steps 15
IR min	0	% max	1 % Number of steps 10
Risk	22.1	IR	0.194
Upside min	-50	% max	50 % Number of steps 199
Simulations per surface calc point 20000			
Target point			
Risk	3	IR	0.4
Simulations at this single point 40000			
calculating			
Underlying Assets, portfolio & benchmark			
Show covariance screen			
Graph			
Contour Map			Contour width 0.1
Update graph			

More about the model    Close

