Overview

- Robotic loss reserving
  - What is it?
  - Why is it of interest?
- Requirements of a robot
- Main components of the robot
- Robot supervision
- Future development

Robotic loss reserving – what is it?

- P&C loss reserving
  - Estimation of liabilities for incurred but incomplete claims
  - Central estimate (i.e. mean value)
  - Stochastic properties (statistical distribution of the amount of liability)
- Robotic (or adaptive) loss reserving
  - Software that will produce this output over a sequence of valuation dates
  - Without human intervention
  - With no significant loss of accuracy due to lack of intervention
Robotic loss reserving – why is it of interest?

- Valuation revolving door
  - Many insurers now wish to conduct frequent liability valuations
    - e.g. quarterly
  - They may have 50 or more lines and sub-lines of business recognised for valuation purposes
  - These require separate identification of valuation liabilities because of structurally different models of the claim process
  - These may have many segments
    - State, distribution channel, etc
  - These require separate identification of valuation liabilities for management and/or strategic reasons, e.g. profit measurement
  - These requirements mean that the performance of a quarterly valuation can take about 3 months
    - One valuation ends, another begins

Robotic loss reserving – why is it of interest?

- Valuation revolving door
  - Obvious advantages in automating such quarterly valuations
  - Once an insurer contemplates a move to monthly valuations, conventional actuarial valuation ceases to be feasible at all
  - A robot is the only option

Reserving by means of roll-forwards?

- Rolling a valuation forward
  - Consider the case of full half-yearly valuations
  - Roll these forward to provide intermediate monthly valuations
    - Assume that each half-yearly valuation remains valid over the following 5 months

  Value of liabilities at any one of these months =
  Value at previous half-yearly valuation
  less claims paid since then
  plus allowance for claims incurred since then

- Problem here is that the monthly series of loss reserves tends to run smoothly for 5-month periods with 6-monthly shocks
  - Roll-forwards not reliable
Requirements of a robot

- Must recognise changes in its environment
  - e.g. door is now closed instead of open as it was a minute ago
- Must be able to respond appropriately to these changes
  - e.g. don’t attempt to pass through doorway without first taking action to open the door

Requirements of a loss reserving robot

- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
Requirements of a loss reserving robot

- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
  - Example of Payment per Claim Incurred (PPCI) by accident year
Requirements of a **loss reserving** robot

- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
  - Example of Payment per Claim Incurred (PPCI) by accident year

![Payment per Claim Incurred (PPCI) by accident year](image)

- The tail of the payment pattern tending to extend with increasing accident period
- Case estimates tending to develop more rapidly in more recent accident periods
- Must be able to respond appropriately to these changes
- Model of claim process must evolve over time to reflect these changes

**Robot design**
Main components of the robot

Evolutionary Model 1
Evolutionary Model 2
... Evolutionary Model m

Estimates of loss reserve

Bootstrap of Model 1
Bootstrap of Model 2
... Bootstrap of Model m

Distributions of estimates of loss reserve

Single estimate of loss reserve with distribution
Main components of the robot

Evolutionary Model 1
Evolutionary Model 2
... Evolutionary Model m

Estimates of loss reserve

Bootstrap of Model 1
Bootstrap of Model 2
Bootstrap of Model m

Distributions of estimates of loss reserve

Taylor & McGuire (2007)

Single estimate of loss reserve
with distribution

Evolutionary models

- These are Dynamic Generalised Linear Models (DGLMs)
- Model form is:
  \[ y_t = h^{-1}(X_t \beta_t) + \varepsilon_t \]  
  [GLM for period t]

Data vector function
Link matrix
Design matrix
Parameter vector
Controlled stochastic error

Evolutionary models

- These are **Dynamic Generalised Linear Models (DGLMs)**
- Model form is:
  \[ y_t = h^{-1}(X_t \beta_t) + \varepsilon_t \]  
  [GLM for period t]
  \[ \beta_{t+1} = G_t \beta_t + \eta_t \]  
  [parameter evolution]

Forecasts

- Forecast of \( y_{t+1} \) by means of adaptive filter
  - Hence "adaptive reserving"
- Notation: let
  \[ Y_{1:s} = E(Y_t|\text{data from } 0, 1, \ldots, s) \]
- Estimate
  \[ Y_{1:t} = Y_{1:t-1} + K_t \{ y_t - Y_{1:t-1} \} \]
  or (depending on link function and error terms)
  \[ Y_{1:t} = Y_{1:t-1} + K_t \{ \text{DIAG } Y_{1:t-1}^{-1} y_t - 1 \} \]
Adaptive filter

- This is a second order approximation to Bayesian updating of the parameter vector $\beta_t$ (Taylor, 2008)
- It holds for following cases

<table>
<thead>
<tr>
<th>$h$</th>
<th>$c$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>identity</td>
<td>normal</td>
<td>normal</td>
</tr>
<tr>
<td>log</td>
<td>Poisson</td>
<td>gamma</td>
</tr>
<tr>
<td>log</td>
<td>gamma</td>
<td>gamma</td>
</tr>
</tbody>
</table>

Adaptive filter (cont’d)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t|t-1}$ to $Y_{t+1|t}$
  - Update $Y_{t|t-1} \rightarrow Y_t$, as just illustrated
  - Also update $\text{Var}(Y_{t|t-1}) \rightarrow \text{Var}(Y_t)$
  - Extract updated parameter estimate $\hat{\beta}_t$
Adaptive filter (cont’d)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t+1} \text{ to } Y_{t+12}$
  - Update $Y_{t+1} \rightarrow Y_t$ as just illustrated
  - Also update $\text{Var}[Y_{t+1}] \rightarrow \text{Var}[Y_t]$
  - Extract updated parameter estimate $\beta_t$
  - Update $\text{Var}[Y_{t+1}] \rightarrow \text{Var}[Y_t]$ by means of formula for assumed parameter evolution (in our case $\text{Var}[Y_{t+1}] = \text{Var}[Y_t]$)
  - Also update $\text{Var}[$SomeExpression$] \rightarrow \text{Var}[$SomeExpression$]$

- Also update $Y_{t+1} \rightarrow Y_{t+1}$ using $\beta_{t+1}$

- Also update $\text{Var}[Y_{t+1}] \rightarrow \text{Var}[Y_{t+1}]$

- Iterate
Adaptive filter - examples

Data from earlier examples

PPCI (actual and filtered)

Accident year 1991

PPCI (constant dollars)

Developm ent quarter

Accident year 1993

PPCI (constant dollars)

Accident year 1995

PPCI (constant dollars)
Adaptive filter - examples

Accident year 1997

PPCI (actual and filtered)

$0

$1,000

$2,000

$3,000

$4,000

$5,000

$6,000

$7,000

$8,000

$9,000

1 3 5 7 9 1 11 31 51 71 92 12 32 52 72 93 13 33 53 73 94 14 34 54 74 95 15 3

Development quarter

Accident year 1999

PPCI (constant dollars)

Accident year 1999

N.B. filter has been applied to accident periods
Bootstrapping

Recall standard form of bootstrap for regression model

Data $Y$

Residuals $R_t = Y_t - X_t \beta_t$

$R_t^* =$ Randomised residuals

Pseudo-data $Y_t^* = X_t \beta_t^* + R_t^*$

Pseudo-estimate $\beta_t^*$

Iterate

Distribution of $\beta_t$

Bootstrapping a filter

Recall standard form of bootstrap for regression model (adapted to filter)

Data $Y$

Residuals $R_{t|t-1} = Y_t - X_t \beta_{t|t-1}$

$R_{t|t-1}^* =$ Randomised residuals

Pseudo-data $Y_{t|t-1}^* = X_t \beta_{t|t-1} + R_{t|t-1}^*$

Pseudo-estimate $\beta_{t|t-1}^*$, $t=1,2,\ldots$

Iterate

Distribution of $\beta_{t|t-1}$
Bootstrapping a filter

Recall standard form of bootstrap for regression model (adapted to filter)

Problem here is that bootstrap assumes components of residual vector \( R \) are independent

For a filter

\[ R_{t|t-1} = Y_t - X_t \beta_{t|t-1} \]

The components of \( R_{t|t-1} \) are NOT independent because \( \beta_{t|t-1} \) has been calculated from \( R_{t-1|t-2}, R_{t-2|t-3}, \ldots \)

---

Data Y
Residuals \( R_{t|t-1} = Y_t - X_t \beta_{t|t-1} \)
Randomised residuals
Pseudo-data
Y* = X* \( \beta_{t|t-1} \) + \( R_{t|t-1} \)*
Pseudo-estimate \( \beta_{t|t-1}^*, t=1,2,\ldots \)

---

Bootstrapping a filter – correct version

Procedure provided by Stoffer & Wall (1991)

Let

\[ L_{t|t-1} = \text{Var} [R_{t|t-1}] \]

Standardised innovations are i.i.d.

\[ \epsilon_{t|t-1} = L_{t|t-1}^{-1} R_{t|t-1} \]

\( \epsilon_{t|t-1}^* = \) randomised standardised innovations

---

Data Y
“Innovations” \( R_{t|t-1} = Y_t - X_t \beta_{t|t-1} \)
Standardised innovations \( \epsilon_{t|t-1} = L_{t|t-1}^{-1} R_{t|t-1} \)
\( \epsilon_{t|t-1}^* = \) randomised standardised innovations

---

Bootstrapping a filter – correct version

Procedure provided by Stoffer & Wall (1991)

Let

\[ L_{t|t-1} = \text{Var} [R_{t|t-1}] \]

Standardised innovations are i.i.d.

Note there is no need for pseudo-data

Filter is updated from t-1 to t using innovations only

---

Data Y
“Innovations” \( R_{t|t-1} = Y_t - X_t \beta_{t|t-1} \)
Standardised innovations \( \epsilon_{t|t-1} = L_{t|t-1}^{-1} R_{t|t-1} \)
\( \epsilon_{t|t-1}^* = \) randomised standardised innovations

---

One complete pass of filter

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Distribution of \( R_{t|t-1} \)
Example results of bootstrapping

Using 3 forms of model
- **PPCI**
  - Payments per claim incurred
  - Payment based
- **PPCF**
  - Payments per claim finalised
  - Sensitive to the rate of settlement of claims
- **PCE**
  - Projected case estimates
  - Sensitive to case estimates

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Mean</th>
<th>CV</th>
<th>Mean</th>
<th>CV</th>
<th>Mean</th>
<th>CV</th>
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<td>1</td>
<td>8</td>
<td>240%</td>
<td>132</td>
<td>55%</td>
<td>22</td>
<td>189%</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>132%</td>
<td>202</td>
<td>57%</td>
<td>54</td>
<td>130%</td>
</tr>
<tr>
<td>3</td>
<td>58</td>
<td>60%</td>
<td>369</td>
<td>47%</td>
<td>23</td>
<td>180%</td>
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<tr>
<td>4</td>
<td>345</td>
<td>9%</td>
<td>396</td>
<td>9%</td>
<td>76</td>
<td>30%</td>
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<tr>
<td>5</td>
<td>232</td>
<td>137%</td>
<td>840</td>
<td>56%</td>
<td>351</td>
<td>62%</td>
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<td>292</td>
<td>13%</td>
<td>2.215</td>
<td>7%</td>
<td>671</td>
<td>54%</td>
</tr>
<tr>
<td>7</td>
<td>107</td>
<td>97%</td>
<td>1.257</td>
<td>7%</td>
<td>194</td>
<td>71%</td>
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<tr>
<td>8</td>
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<td>7%</td>
<td>3,514</td>
<td>60%</td>
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<td>9</td>
<td>2,339</td>
<td>40%</td>
<td>3,346</td>
<td>53%</td>
<td>2,058</td>
<td>30%</td>
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<tr>
<td>10</td>
<td>3,166</td>
<td>56%</td>
<td>7,133</td>
<td>23%</td>
<td>2,555</td>
<td>31%</td>
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<tr>
<td>11</td>
<td>5,112</td>
<td>60%</td>
<td>6,765</td>
<td>33%</td>
<td>4,700</td>
<td>31%</td>
</tr>
<tr>
<td>12</td>
<td>8,544</td>
<td>60%</td>
<td>10,816</td>
<td>23%</td>
<td>7,240</td>
<td>31%</td>
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<tr>
<td>13</td>
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<td>23%</td>
<td>9,709</td>
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<tr>
<td>14</td>
<td>13,544</td>
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<td>15</td>
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<td>48%</td>
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<td>5,480</td>
<td>31%</td>
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<tr>
<td>16</td>
<td>7,182</td>
<td>44%</td>
<td>6,742</td>
<td>20%</td>
<td>6,700</td>
<td>31%</td>
</tr>
<tr>
<td>17</td>
<td>8,544</td>
<td>43%</td>
<td>8,664</td>
<td>21%</td>
<td>7,234</td>
<td>33%</td>
</tr>
<tr>
<td>18</td>
<td>9,001</td>
<td>43%</td>
<td>9,015</td>
<td>21%</td>
<td>3,749</td>
<td>98%</td>
</tr>
</tbody>
</table>

Total ex 14 | 30,119| 30% | 41,721| 18%| 29,366| 22% |

Model blending
Model blending – inputs

- Results after filtering and bootstrapping $m$ models consist of:
  - $m$ sets of estimates of liability by accident year
  - $m$ associated sets of standard errors of prediction
  - Case estimates by accident year

Model blending

- Let $L_i^{(j)}$ = estimated liability for accident year $i$ from model $j$
- Take final estimates as $L_i = \sum_{j=1}^{m} w_i^{(j)} L_i^{(j)}$
  with $w_i^{(j)} \geq 0$
  $\sum_{j=1}^{m} w_i^{(j)} = 1$

Model blending - criteria

- We would like
  - $\text{MSEP}[L]$ to be small where $L = \sum_i L_i$
  - $\sum_i \Delta^2 w_i^{(j)}$ to be small for each $j$
    - Smooth weights for each model
  - $\sum_i \Delta^2 \log L/C_i$ to be small where $C_i$ denotes case estimates for accident year $i$
    - Smooth relation of final estimates to case estimates over accident years
Model blending – objective function

- Problem addressed by Taylor (1985, 2000)
- Minimise the objective function

\[ Q = \text{MSEP} + k_1 \sum \Delta^2 w_i(j) + k_2 \sum \left[ \log \frac{L_i}{C_i} \right] \]

with respect to the \( w_i(j) \), where \( k_1, k_2 \) are pre-determined constants that assign weight to the smoothness criteria.

Model blending – example of results

<table>
<thead>
<tr>
<th>Accident year</th>
<th>PPCI</th>
<th>PPCF</th>
<th>PCE</th>
<th>Blended</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>128</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>213</td>
<td>204</td>
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<td>3</td>
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<td>150</td>
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<tr>
<td>4</td>
<td>110</td>
<td>150</td>
<td>150</td>
<td>150</td>
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<tr>
<td>5</td>
<td>202</td>
<td>202</td>
<td>202</td>
<td>202</td>
</tr>
<tr>
<td>6</td>
<td>291</td>
<td>291</td>
<td>291</td>
<td>291</td>
</tr>
<tr>
<td>7</td>
<td>670</td>
<td>704</td>
<td>702</td>
<td>702</td>
</tr>
<tr>
<td>8</td>
<td>817</td>
<td>817</td>
<td>817</td>
<td>817</td>
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<tr>
<td>9</td>
<td>2,259</td>
<td>2,259</td>
<td>2,259</td>
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<td>10</td>
<td>3,544</td>
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<td>7,182</td>
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<td>8,544</td>
</tr>
<tr>
<td>14</td>
<td>9,001</td>
<td>9,001</td>
<td>9,001</td>
<td>9,001</td>
</tr>
</tbody>
</table>

Total 14 38,189 14,136 27,985 37,929 99%

N.B. smaller than for any individual model
Robot supervision

Need for supervision

- Robots affect business bottom line
- Need for strict supervision
- This takes the form of exception reporting
  - Using a range of diagnostics to test whether experience is deviating too far from model predictions

### Example of supervision diagnostics

<table>
<thead>
<tr>
<th>Accident year</th>
<th>Actual ($M)</th>
<th>Forecast ($M)</th>
<th>Actual: forecast</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>0.7</td>
<td>1.6</td>
<td>44%</td>
<td>(10%)</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>2007</td>
<td>25.4</td>
<td>21.0</td>
<td>121%</td>
<td>** (0%)</td>
</tr>
<tr>
<td>Total</td>
<td>78.7</td>
<td>74.9</td>
<td>105%</td>
<td>* (5-10%)</td>
</tr>
</tbody>
</table>
Further development

Ample scope for further development

- Filter has been applied to accident periods (rows)
- Could investigate application to diagonals
- This could filter superimposed inflation parameters
- Project currently under way
- Appears more difficult

Further development (cont’d)

- Test performance of GLM filter against obvious alternatives
  - MCMC
    - Project currently under way
  - Particle filters
  - Neural nets
    - See e.g. Mulquiney (2006)
Further development (cont’d)

- Filter applied here to aggregate claim models
- Try application to micro-models (individual claims)
  - Excluding case estimate information (Taylor & McGuire, 2004)
  - Including case estimate information (Taylor, McGuire & Sullivan, 2007)

Questions?

References (1)

References (2)


References (3)