

# Actuarial History

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In this talk I propose to outline the fascinating story of how the insurance principle developed from primitive beginnings and the long history of pensions. I shall describe how compound interest was in use for everyday business transactions from the Middle Ages onwards. It was not until the 17<sup>th</sup> century, however, that publications started to appear on the theory of probability. There was also some pioneering statistical analysis on the London Bills of Mortality, which showed for the first time how real data could be obtained and applied to practical uses.

All of these developments came together in 1693, when Edmond Halley published a crucially important paper, using real life data from Breslaw to construct a life table and describing for the first time how to use such a table to work out the purchase price of a life annuity.

Halley's paper had been preceded by one or two pieces of similar work by others, but his was the influential work which established actuarial science. His paper was widely read and led, half a century or more later, to the establishment of pension funds and life assurance companies funded on actuarial principles.

I shall describe how all this came about, taking each of the main strands of development in turn:

- Insurance and pensions
- Compound interest
- Probability theory
- Mortality data.

Finally I shall outline some subsequent developments, including the establishment of The Equitable Assurance Society in 1762 and the subsequent formation of the actuarial profession.

# Insurance

- Needs – individual risk
- Marine insurance – ancient Greeks
- Separation of risk and finance by 1350
- Office of Assurances, London, 1575

The basic need for insurance and pensions stems from individual risk and uncertainty. If you go on a journey or voyage, there is the risk of losing any goods entrusted to you, or your possessions, or even your life. Your house may catch fire and leave you and your family without a roof over your heads. If you are a breadwinner, you run the risk of dying too soon and leaving your dependants destitute. Alternatively you may live too long after retirement and fall into poverty when your savings are exhausted.

These risks existed from the earliest times and the traditional method of relieving poverty was by charity. This was never very satisfactory, however, because it often provided inadequate relief and it had a stigma. It was natural, therefore, to look for some means whereby the necessary sums could be provided as of right at an adequate level, out of funds earmarked for the purpose. Hence insurance was born.

Originally the finance for a sea voyage was commonly advanced by one or more wealthy individuals who agreed not to seek repayment if the cargo was lost. If the ship arrived safely, the money was repaid with a heavy rate of interest, which covered both the risk premium and the true interest on the loan.

This kind of transaction can be traced back as far as the ancient Greeks. In one case quoted by Demosthenes (who was born about 384 B.C.), 3000 drachmas were advanced in respect of a cargo of wine. The rate of interest was to be 22½ per cent, which was to be increased to 30 per cent if the return voyage was delayed until the stormy season.

However, not everyone who was prepared to provide finance would have been happy to take the risk of loss, even if it was compensated by a high rate of interest. It is thought that the separation of the protection element from the financing came during the period 1300-1350 A.D. This probably happened because an expansion of world trade meant that more finance was needed than could be provided from traditional sources.

The first genuine insurance policy we know of was dated 1350. A cargo of wheat from Sicily to Tunis was assured for 300 florins at a premium of 18 per cent. The insurer, Leonardo Cattaneo, undertook to assume all risks from act of God, or of man, and from perils of the sea.



It is not clear how long it took to establish an insurance market in London but it may well have existed by 1500. It seems likely that English merchants at first arranged any insurance they required through their Italian counterparts in London, whom they usually met in Lombard Street, and the risks may well have been borne in Italy. Eventually, however, as English merchants themselves became wealthy, it was probably found more convenient, as well as more profitable, to insure the risks at home.

Marine insurance was now well established. In 1559 Sir Nicholas Bacon said, "Doth not the wise merchant, in every adventure of danger, give part to have the rest assured?"

However, when claims arose the insurers did not always pay up, and this gave rise to numerous disputes. In 1575 the Privy Council acted to establish the Office of Assurances at the Royal Exchange in London. The slide shows the Royal Exchange thronged with merchants.

A copy of every policy had to be copied into the policy register held at the Office of Assurances, by the clerk, Richard Candeler. There was a set of detailed rules governing claims – these rules still survive in the British Library.



## Early life assurance

- One year contracts
- Usually to cover debts or travel
- William Gibbons
- Prescribed rules

The earliest life policy known was issued in 1583. It was taken out on the life of William Gibbons, a citizen of London, for a term of 12 months at a premium of 8 per cent. He died nearly a year after the policy was effected, but the underwriters refused to pay, on the ground that Gibbons had lived for 12 months, taking 28 days to the month. However, the Court ruled that the sum assured must be paid, because it was the established custom to reckon by calendar months.

The Office of Assurances had some rules for life policies (as well as for marine insurance). No life policies were to be issued for more than one year at a time and the maximum sum assured was £1,000. When one person insured his own life there had to be an insurable interest, for example if he was due to receive a lump sum provided he survived to a certain day, which would be lost in the event of his prior death. When insuring someone else's life there also had to be an insurable interest and you could only insure up to 80 per cent of the sum you would lose in the event of the death of the person whose life was being insured.

# Pensions

- Jehoiachin, King of Judah, 562 BC
- Lysias, 400 BC
- Alice Buch, 1259
- Philip de Harwodelme, 1286
- Nicholas Thorne, 1294
- Corrodies

The idea of a pension for life is very old indeed, starting with Jehoiachin, King of Judah in 582 B.C. We read in the Bible (2 Kings 25, 29-30) that he was released from prison in his 37<sup>th</sup> year of exile in Babylon, treated kindly, and given a seat at table and “lived as a pensioner of the king for the rest of his life”.

Around 400 B.C. Lysias, a Greek orator, protested vigorously when he found that his State pension was discontinued. He had been granted the pension when he was younger, possibly because of war injuries. The reasons now given for its cessation were that he was able-bodied and not classed as disabled, was skilled in a trade, could mount a horse, and was well-off financially.

In medieval England Alice Buch, the widow of Guy Buch, a king's crossbowman, was awarded a pension of 12 pence a day to support herself and her children. This was the same sum as Guy had been receiving daily.



Philip de Harwodelme, rector of Bigby, near Exeter, was by 1286 worn down by disease and old age, and was awarded a pension of £13 per annum from the parish tythes.

The monks of St. Augustine's Abbey, Canterbury, were very grateful to Nicholas Thorne, their abbot when he won some benefits for them at Rome. However, stricken with conscience about the practices he had used there, he retired to an abbey at Selby, Yorkshire, and lived there as an ordinary monk. When the monks of St Augustine's heard 11 years later that he was experiencing infirmity and weakness, they awarded him in 1294 a pension of £7 per annum.

Despite the absence of insurance companies it was sometimes possible to buy yourself a pension in the Middle Ages. You paid a lump sum or granted some land to a monastery or other religious institution and in return they would let you live there with board and lodging, and often a small regular cash payment as well. The income and other benefits were called a corrody and the recipient was a corrodian. A payment of £50 to £100 might well secure a man and his wife a good corrody for the rest of their lives (worth perhaps £3 per annum each), though the purchase prices seem have varied quite a bit. Not enough is known about how the purchase prices of corrodies were calculated and there may be scope for further research in this area. In effect a corrody protected its recipients against inflation and gave them security in their old age.

## Pensions after 1500

- Monks displaced by Henry VIII
- The Chatham Chest
- Retirement pensions
- Pensions from one's successor, e.g. Samuel Pepys
- Superannuation scheme for naval officers, 1672

The monks displaced by Henry VIII in the Reformation of the 1530s were typically awarded pensions for life of £6 per annum, though abbots and abbesses could be awarded much more – perhaps up to £100 per annum.

In 1590 the Chatham Chest was established to provide pensions to wounded seamen of the Royal Navy. This was probably the first funded occupational pension fund in the world.

By the 17<sup>th</sup> century retirement pensions were becoming quite common, for example from local authorities to their former officials.

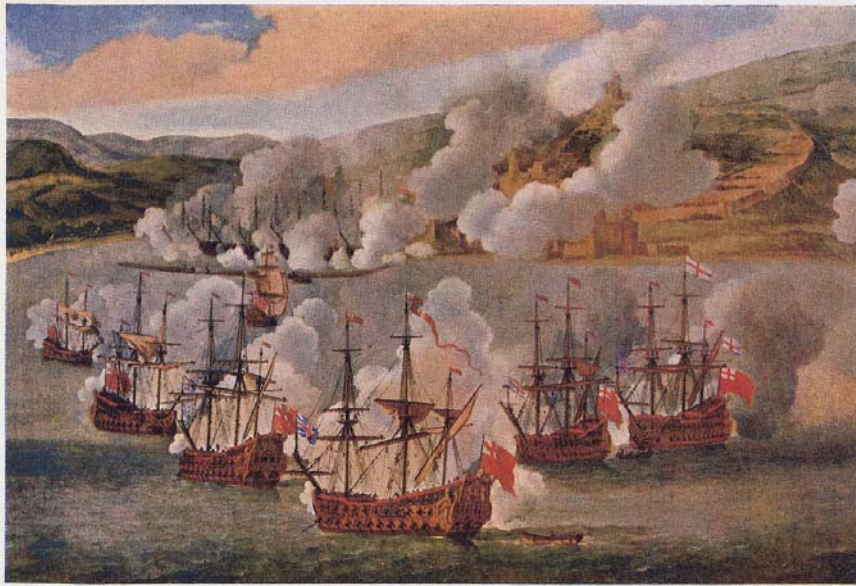
Civil servants sometimes had to pay pensions to their predecessors. Samuel Pepys, for example, became Clerk of the Acts of the Navy Board in 1660, after the Restoration of Charles II. Unfortunately, however, the post was then claimed by Frank Barlow, who had held it before the Civil War. Pepys had to buy him off with a pension of £100 per year, which was a significant sum out of Pepys's salary of £350 per annum.

The system of paying one's predecessor was unfair, because some predecessors lived longer than others, so the introduction of a State-financed superannuation scheme for retired naval officers in 1672 was very significant. This is believed to be the first occupational pension scheme in the world to provide lifetime pensions on retirement due to old age. There was no fixed retirement age. The pension, which was 100 per cent of salary and allowances, became payable to any officer who became unfit for performing his duties because of age, provided he had completed at least 15 years' service..



This is the actual Chatham Chest, which was in use at Chatham in the 17<sup>th</sup> century and still survives at Chatham, where it is on display. Money was paid into the Chest and the benefits were paid out of it. Contributions of sixpence per month were deducted from seamen's wages of 10s.0d per month, i.e. 5% (though the sixpence did not increase when wages were increased, and this was one of the flaws in the financing of the Chest). Pensions were payable on a fixed scale according to the degree of the injury – for example, £6.13s.4d per annum for the loss of a limb or £12 per annum for total blindness. In addition each claimant received a lump sum (called “smart money”) equal to one year's pension. The pensions were payable for life but were reviewed subsequently and could be reduced or terminated if the pensioner was found to have recovered sufficiently to be capable of employment.

At first the finances of the Chest were fairly healthy (despite some problems in ensuring that the deducted contributions actually reached Chatham) and accumulating moneys were invested in property. However, the naval wars of the 1650s produced many claimants and when the wars ceased and the size of the navy was reduced, the contributions dwindled but the pensions did not. From about 1670 onwards the State took over the payment of pensions on a “pay as you go” basis. The Chest continued for many years, eventually being merged into Greenwich Hospital in 1814.



SEA-FIGHT IN A HARBOUR, BY VANDEVELDE.

(Hampton Court Palace.)

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This is the kind of battle which gave rise to claimants on the Chatham Chest. After they were injured they had to attend at Chatham, where their wounds were assessed and the pension was awarded.

# Compound interest

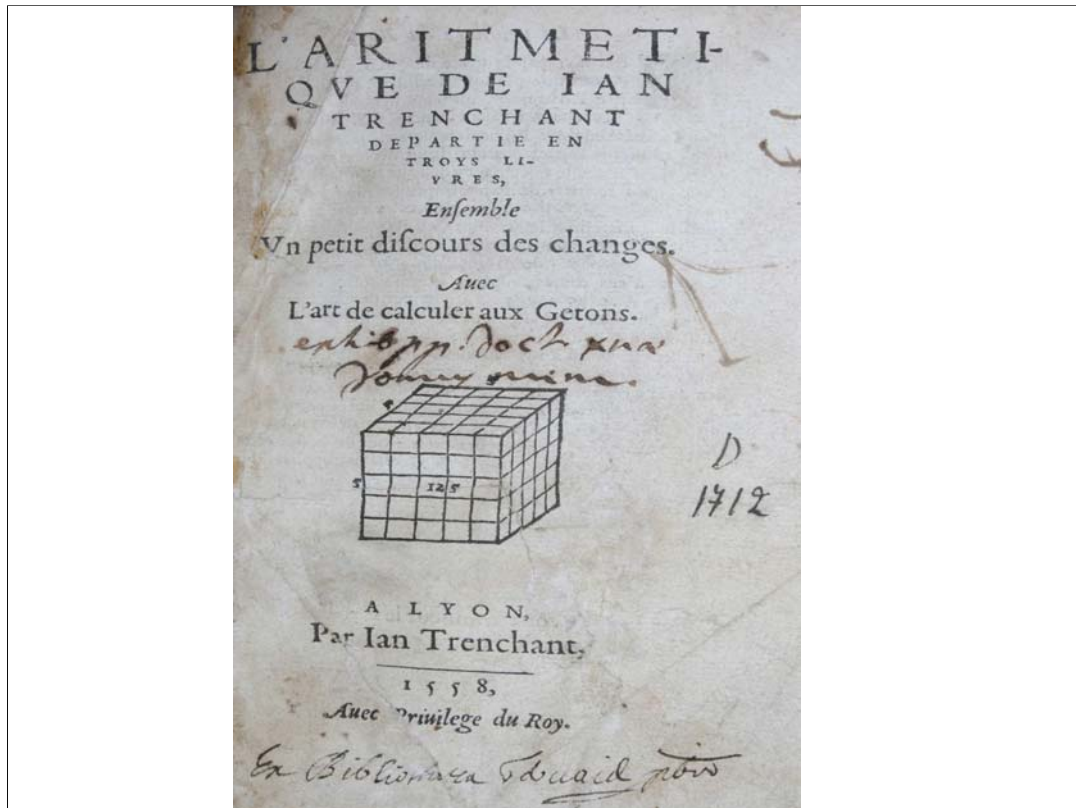
- Usury and chevisance
- Religious arguments
- Trenchant, 1558
- Stevin, 1585
- The 1571 Act
- Richard Witt, 1613

Jews were allowed to practise money-lending at high rates of interest (sometimes as much as 86 per cent per annum) until they were expelled from England in 1290. Thereafter usury was generally forbidden, for religious reasons, so many loans were dressed up as the sale and repurchase of goods to escape detection. This was known as “chevisance” and also became prohibited.

Compound interest was in use in Italy in medieval times. In the 16<sup>th</sup> century two Continental mathematicians published elementary compound interest tables – Jean Trenchant and Simon Stevin.

It was not until 1571 that an English Act of Parliament finally established that money-lending was permissible, provided that the rate of interest charged did not exceed 10 per cent per annum.

The first English book devoted to compound interest was published in 1613 by Richard Witt, a London mathematical practitioner. Even though all his payments were certainties, he thought like a modern actuary and his book is a notable landmark.

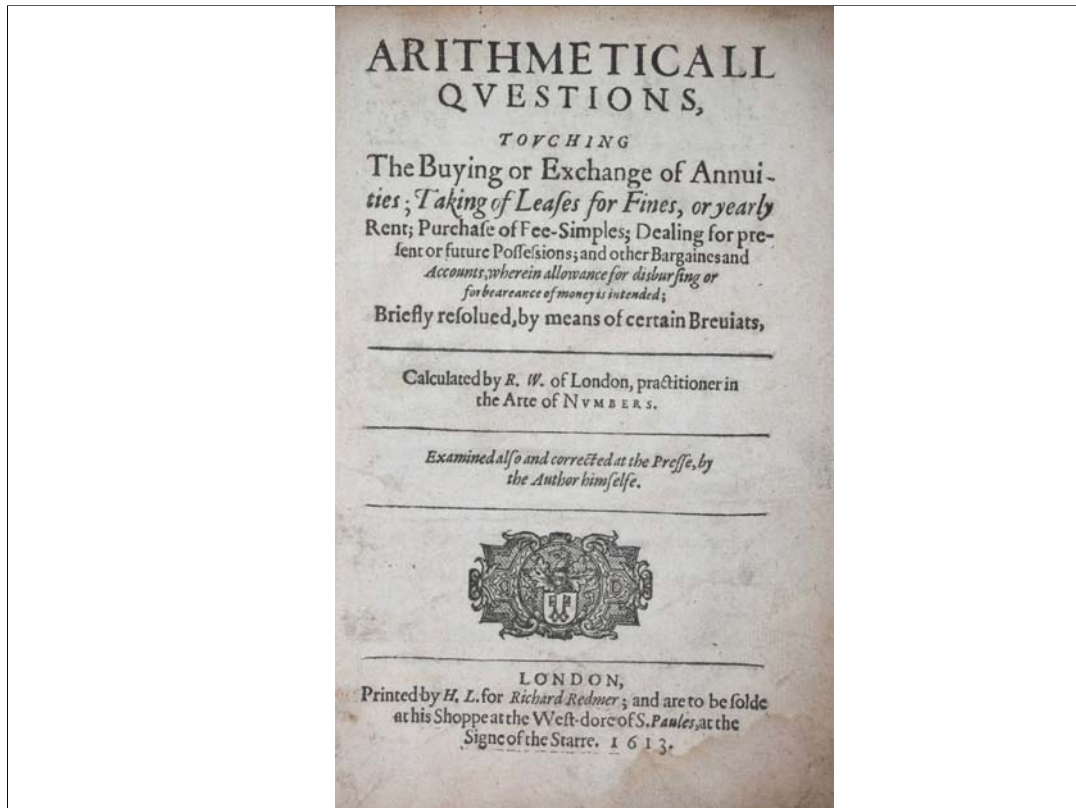


This is the title page of Jean Trenchant's book, first published at Lyons in 1558. It included a chapter on simple and compound interest. Only a few tables were given. One showed the accumulation over a number of periods of an initial investment, calculated at 4 per cent per period, and another showed the total accumulated result of investing a series of payments.





This is Richard Witt's pre-Fire London, showing some of the prominent landmarks in the heart of the City, with the river Thames in the foreground. The spire of the Royal Exchange can be seen at the back, to the left of centre.



The title page of Richard Witt's book. The care he took over it is apparent from the fact that he actually attended while it was being printed and corrected mistakes himself.

In his preface Witt commented that many businessmen thought that they could get by without a knowledge of arithmetic, but then they often made losses – “it is as true that the blind drink many a fly”.



*Arithmetical Questions*

**The Breuiat of the Table  
of 10. per Cent.**

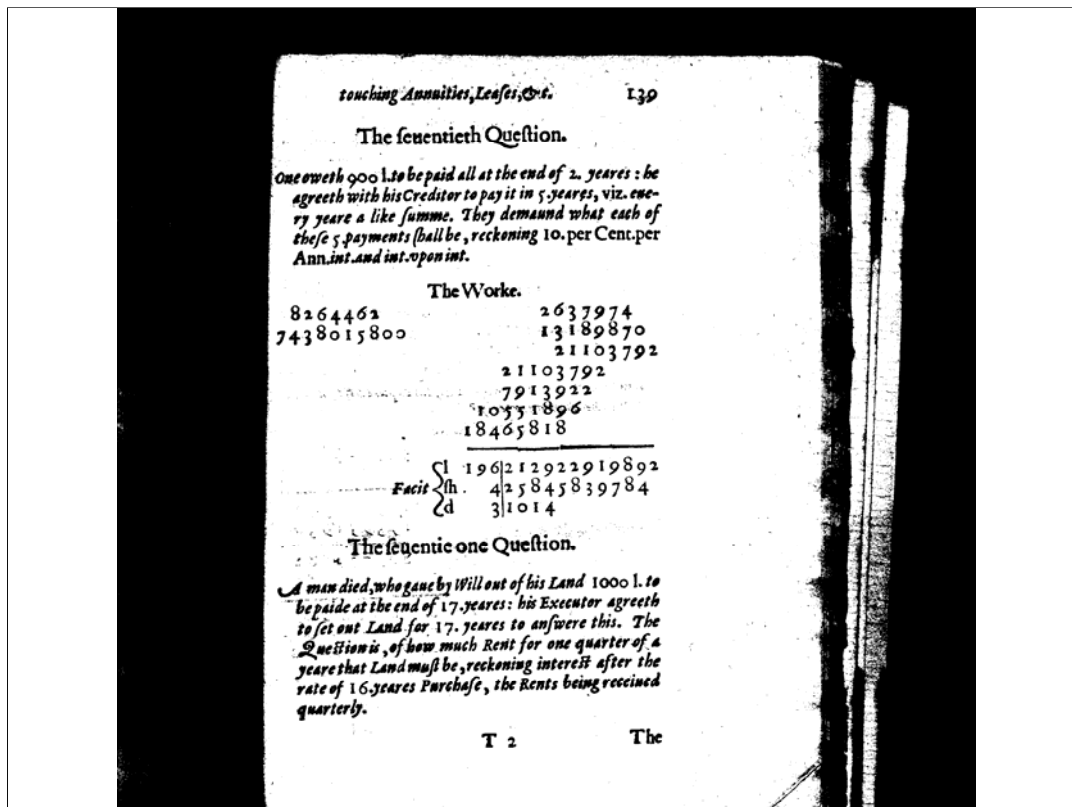
Year	Value	Year	Value
1	11000000	16	45949729
2	12100000	17	50544702
3	13310000	18	55599173
4	14641000	19	61159090
5	16105100	20	67274999
6	17715610	21	74002499
7	19487171	22	81402749
8	21435888	23	89543024
9	23572476	24	98497326
10	25937424	25	108347059
11	28531167	26	119181765
12	31384283	27	131099941
13	34522712	28	144209936
14	37974983	29	158630929
15	41772481	30	174494022

These Numbers in this Breuiat must also be esteemed Numerators, each of them having for Denominator 10000000.

Now follow the Questions: In working whereof, we will use this Breuiat.

The

This is one of the tables in Richard Witt's book and shows the accumulation of £1 for a number of years at the maximum legal rate of interest of 10 per cent per annum.



One of the many illustrative problems given in Witt's book: "One oweth £900 to be paid all at the end of 2 years: he agreeth with his creditor to pay it in 5 years, viz. every year a like sum. They demand what each of these 5 payments shall be, reckoning 10 per cent per annum interest, and interest upon interest." The solution is obtained by first working out the present value of £900 to be paid in 2 years' time and dividing the result by the present value of a series of payments of 1 per annum for 5 years, to give the solution of £196.4s.3d. The method adopted shows a knowledge of a fundamental actuarial concept, the equation of payments.

# Leases

- Witt's examples
- The study of compound interest
- Downward trend in interest rates – 10% (1571), 8% (1625), 6% (1651), 5% (1714)
- Henry Phillippes, 1654 and later

Many of the examples in Richard Witt's book dealt with alterations to leases and the resulting sums which the parties should pay.

There followed a succession of books by other authors on compound interest, mostly of inferior quality.

The downward trend in interest rates meant that fresh tables had to be produced each time a new maximum rate was fixed by law.

Henry Phillippes was the first writer (1654) to discuss which rate of interest should be used for transactions of various types. Valuing land should be done by capitalising the annual income at a low rate of interest because it was a safe investment. Leases, on the other hand, should be valued at a higher rate and he suggested the use of the current maximum legal rate of interest on loans. He commented that the cost of leases for lives should be increased now that interest rates have come down, and proposed that the lump sum should be the amount which would be paid for a lease of 12 years (for one life), 23 years (for two lives), 33 years (for three years) and so on.

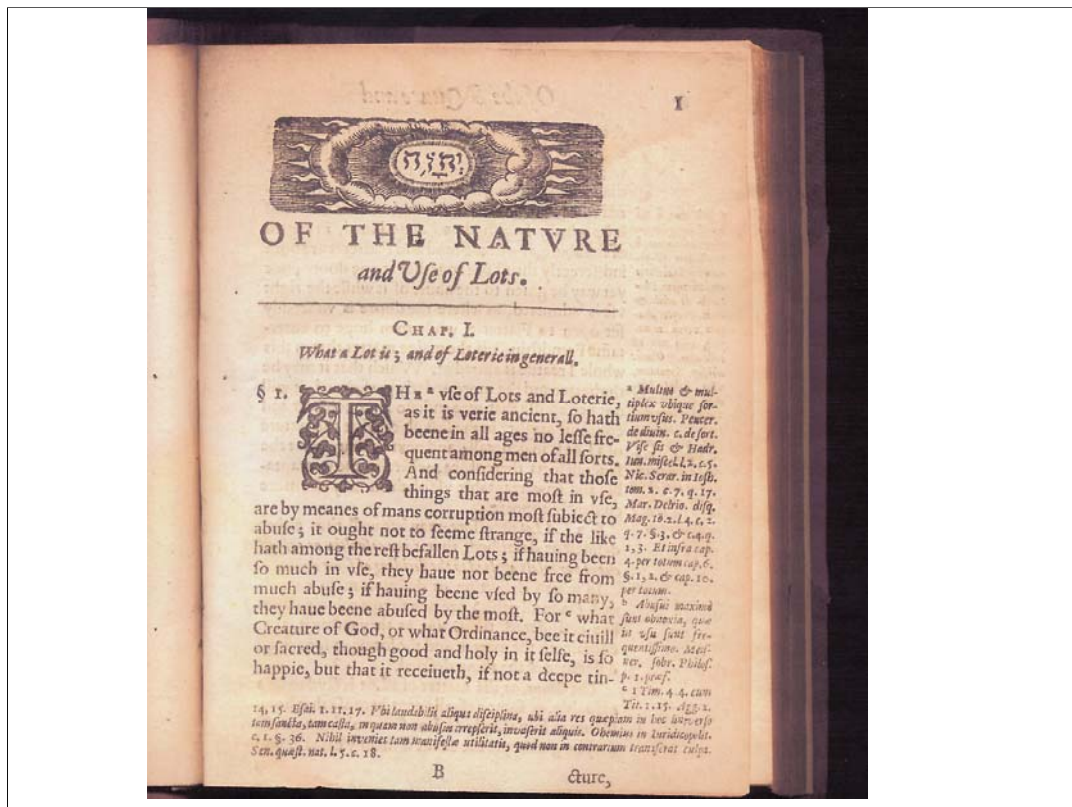
## Probability theory

- Cardan (1501-76) discusses dice games
- Thomas Gataker removes religious barriers, 1619
- Fermat-Pascal private correspondence, 1654
- Christian Huygens publishes first theory, 1657

Turning now to probability theory, Cardan wrote in the 16<sup>th</sup> century about the probabilities involved in dice games – he was an inveterate gambler – but his work remained unpublished for many years and had no effect on the development of the subject.

Thomas Gataker wrote a book in 1619 which removed some religious barriers to the study of probability (see next slide).

The two mathematicians Fermat and Pascal corresponded about some probability problems in 1654, but their work did not see the light of day at that time, and it was Christian Huygens who was the first writer to have his work on probability published, in 1657. His third proposition was of fundamental importance: to have **p** chances of winning **a** and **q** chances of winning **b**, the chances being equal, is worth( **pa+qb**) **divided by (p+q)**.



One of the reasons why probability theory was not developed earlier was almost certainly the widespread religious conviction, based on the Bible, that every event occurred as a result of God's providence. It would have been regarded as blasphemous to develop a theory about the probability of occurrence. Moreover, the study of how often particular outcomes occurred in the past might well not be a reliable guide to God's decisions in future. However, Thomas Gataker, a clergyman of the Church of England, wrote a book on lots (above) in 1619, in which he propounded the important doctrine that it was only on exceptional occasions that God's providence manifested itself, and that most events were natural occurrences which were not influenced by God at all. He also controversially argued that games of chance were permissible, provided they were played for recreation and not for profit. This book made an impact at the time and can be seen as paving the way for the objective study of chance events.

## Insurance in 17<sup>th</sup> century

- Marine insurance
- Ryley and Mabb's fire insurance scheme 1638
- Great Fire of London 1666
- Nicholas Barbon's Fire Office, 1667?
- Life assurance banned in Europe
- One year contracts only in England

We saw earlier how marine insurance became established in England, and it was commonplace in the 17<sup>th</sup> century to insure ships and cargoes at the Royal Exchange.

At the start of the century fire insurance did not yet exist, however. In 1638 a proposed fire insurance scheme was put forward by William Ryley, a civil servant, and Edward Mabb, gentleman, but it did not get off the ground. They proposed a night-time fire watch and a fire brigade, with reserves of water to be set up in convenient places. If their scheme had been adopted, the history of London might have been very different.

However, the scheme was not adopted, and most of London was destroyed in the Great Fire of 1666. This disaster stimulated the formation of fire insurance companies, starting with Nicholas Barbon's Fire Office. There was a very interesting debate about the amount of capital that this office would need to keep in reserve, given that the future claims experience might be very variable from one year to another.

Life assurance was still available at the Royal Exchange in one-year contracts, unlike the rest of Europe where it was banned for fear that it would give rise to murder.

## Mortality data

- James Howell uses Bills of Mortality 1645
- Poul Klingenberg's "tontine" life-table 1653
- John Graunt analyses London Bills of Mortality 1662
- John Hudde analyses Amsterdam annuity cohort 1671
- Sir Richard Corbet analyses Shrewsbury mortality, 1683

Large cities used to publish weekly Bills of Mortality which warned people when to leave the city because the plague was increasing.

In 1645 James Howell published a letter which suggested that the population of Amsterdam was much less than the population of London, because the numbers of deaths shown in the weekly Bills of Mortality were much less. This may be the first use of the Bills for a demographic purpose.

Paul Klingenberg published a life table in 1653 to illustrate the progress of a tontine which was being launched, but the table is thought to be purely illustrative and not based on real life data.

John Graunt analysed the London Bills of Mortality for many years past and published the results in 1662. He was able to draw many interesting conclusions from the data, though it was a handicap that the Bills did not record ages at death. He produced a very inaccurate life table, which was largely a mathematical conception and based to only a very limited extent on the data.

In the Netherlands John Hudde analysed the experience of a cohort of people of known ages who had bought annuities from the City of Amsterdam between 1586 and 1590.

In England Sir Richard Corbet analysed the mortality by age in his home parish in Shrewsbury, but his results remained unpublished.



# The Diseases and Casualties this Week.

<b>A</b> Bortive	6
Aged	50
Ague	1
Apoplexie	2
Childbed	42
Cholera	11
Cold	1
Consumption	99
Convulsion	63
Cough	1
Droptic	22
Drownd at St. Martin in the Fields	1
Feaver	268
Fistula	2
Flux and Small-pox	4
Flux	1
Found dead in the Fields at St. Mary Illington	1

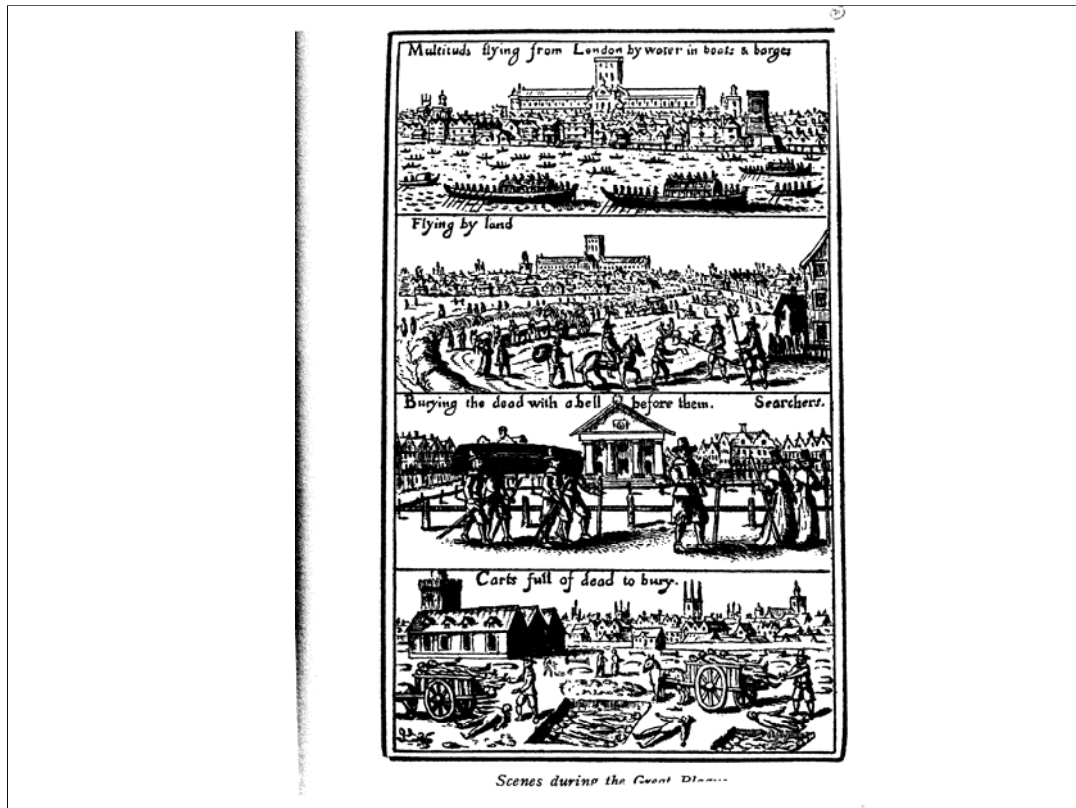
Frighted	1
Gowt	1
Grief	3
Griping in the Guts	35
Jaundies	2
Impoisthume	8
Infants	9
Kingsevil	2
Meagrome	2
Plague	5533
Purples	2
Rickets	10
Rifling of the Lights	13
Rupture	1
Scurvy	5
Spotted Feaver	65
Stilborn	10
Stone	3
Stopping of the Stomach	6
Suddenly	1
Surfeit	36
Teeth	112
Thrush	3
Tiffick	5
Vomiting	4
Winde	1
Wormes	12

Christened	{ Males — 68 }	Buried	{ Males — 3212 }	Plague	5533
	{ Females — 78 }		{ Females — 3248 }		
	{ In all — 146 }		{ In all — 6460 }		
Decreased in the Burials this Week				1837	
Parishes clear of the Plague				7	Parishes Infected 123

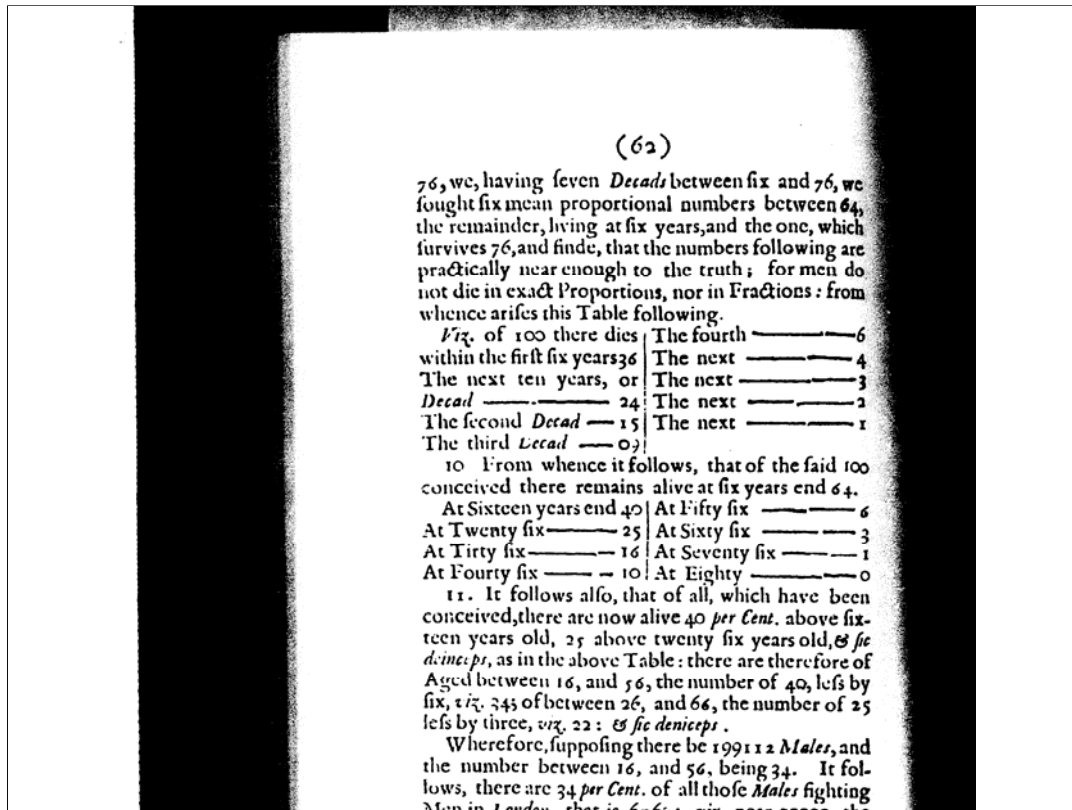
The Assize of Bread set forth by Order of the Lord Mayor and Counc of Aldermen,  
A penny Wheaten Loaf to contain Nine Ounces and a half, and three  
half-penny White Loaves the like weight.

This is part of a weekly Bill of Mortality for London, from which it will be seen that no less than 5533 deaths from plague are recorded in just this one week.





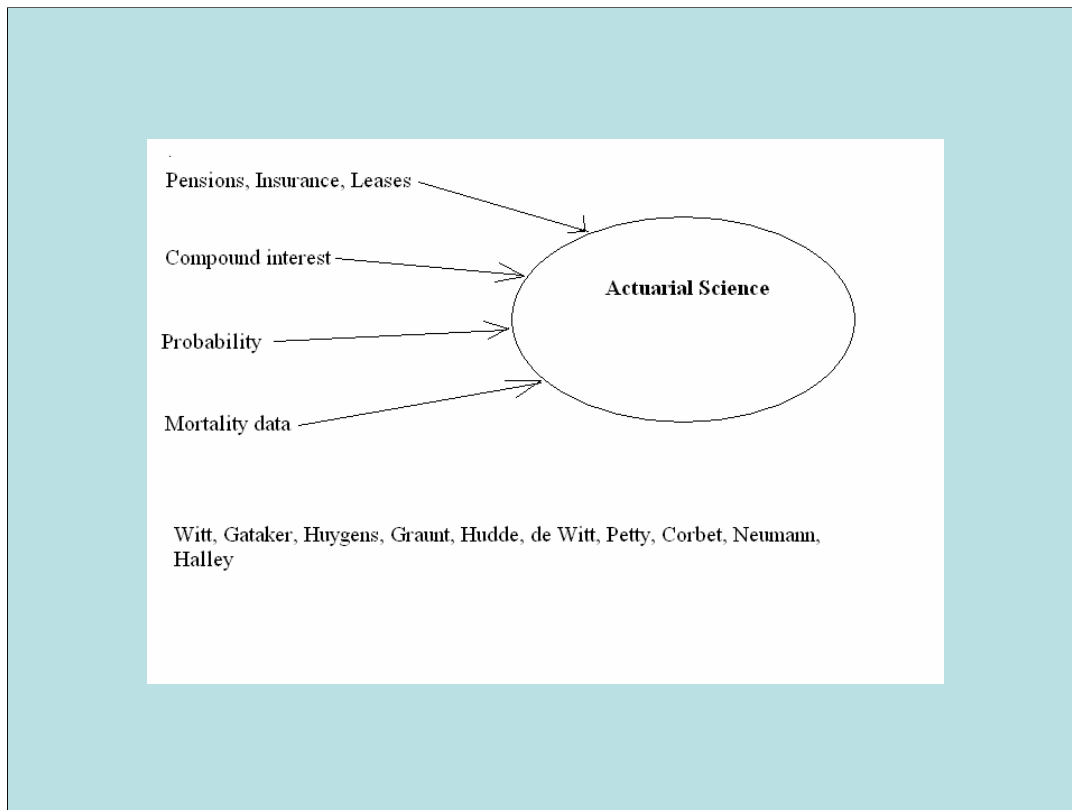
These is a contemporary representation of some of the scenes in London during the Great Plague of 1665



The slide reproduces John Graunt's famous life table of 1662, showing the numbers of people surviving to various ages out of every hundred conceived. Although the number surviving to age 6 is based to some extent on the data he analysed, each of the figures for the other ages is about five-eighths of the previous figure. This is equivalent to an annual mortality rate of 4.6 per cent, independent of age.

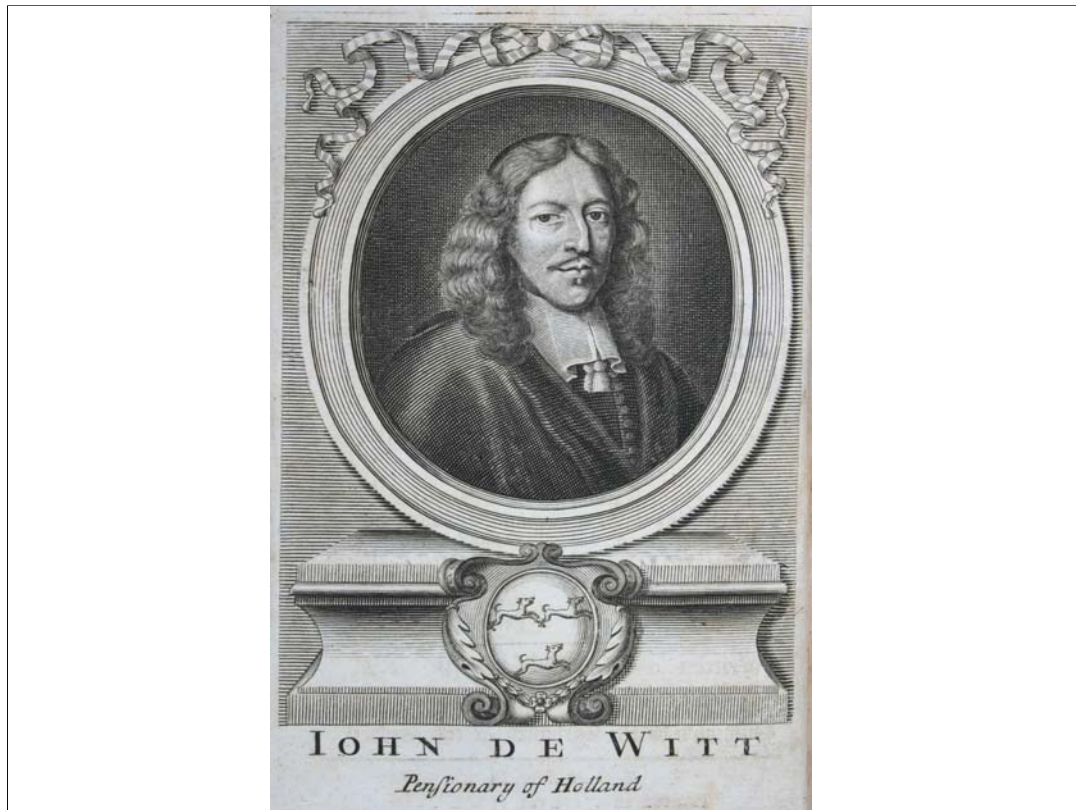
Perhaps Graunt's biggest achievement was to draw attention to the regularity of the patterns of life and death. Among other things he estimated the size of London's population (121 parishes) at 384,000 in 1661, which is thought nowadays to be reasonably close to the truth.

Graunt's book was widely circulated in England and on the Continent, running into several editions.



The time was now ripe for the creation and dissemination of actuarial science. The four main preconditions were in place: a well-established framework of pensions, insurance and leases; a good knowledge of compound interest; the beginnings of probability theory; and a realisation that real-life mortality data could be found and analysed.

The slide shows the names of some of the people who helped to prepare the way. Richard Witt had thoroughly studied compound interest. Thomas Gataker had removed the religious obstacles to the study of probability. Nearly 40 years later Christian Huygens published the first work on probability theory. John Graunt showed how the Bills of Mortality could be used for statistical purposes and produced a life table. John Hudde and Jan de Witt, working in the Netherlands, made the first analysis of mortality data by age and used the results to value life annuities, but their work was not widely published. Sir William Petty, who may well have helped Graunt, published several booklets of a demographic nature. In 1683 Sir Richard Corbet did some pioneering (but unpublished) mortality analysis by age in Shrewsbury. As we shall see, the decisive breakthrough was made when Caspar Neumann in Breslau collected some mortality data there which he sent to Edmond Halley in London for analysis.



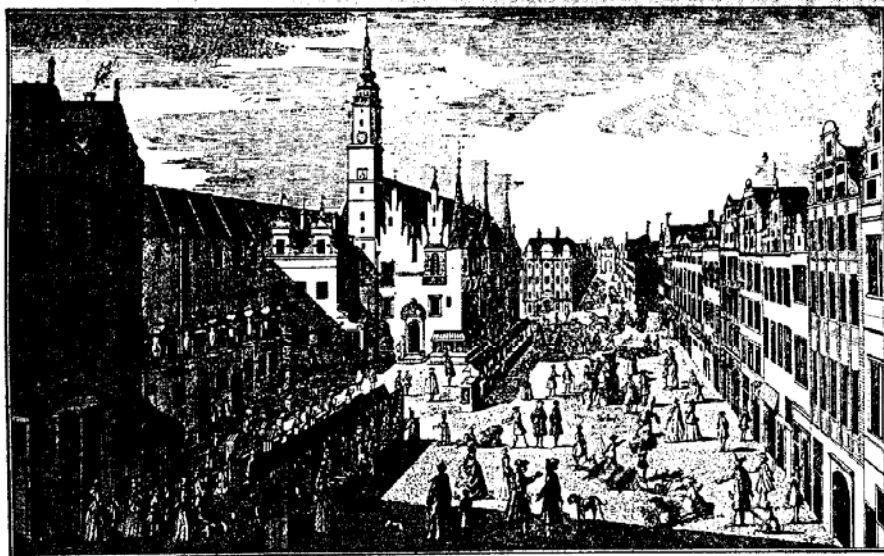
The Dutch Prime Minister, Jan de Witt, produced some pioneering calculations in 1671 to work out the price which should be paid for Dutch government annuities. This work was probably based partly on the mortality investigations which had been carried out by John Hudde at about that time. However, de Witt's treatise, though it was of excellent quality and can be regarded as the first flowering of actuarial science, remained in obscurity and did not influence the mainstream development of the subject.

## Creation of actuarial science

- Caspar Neumann's Breslau data, 1687-92
- Edmond Halley's life table, 1693
- Halley evaluates life annuities, including those on two lives

The creation of actuarial science, as far as the wider world was concerned, dates to 1693, when Edmond Halley, the famous mathematician, published a paper on the subject in the Royal Society's *Philosophical Transactions*. The trigger for Halley's work was supplied by Caspar Neumann, who collected some data from the Bills of Mortality for Breslaw, a town far from the sea and devoted to linen manufacture, and sent it to the Royal Society in London.

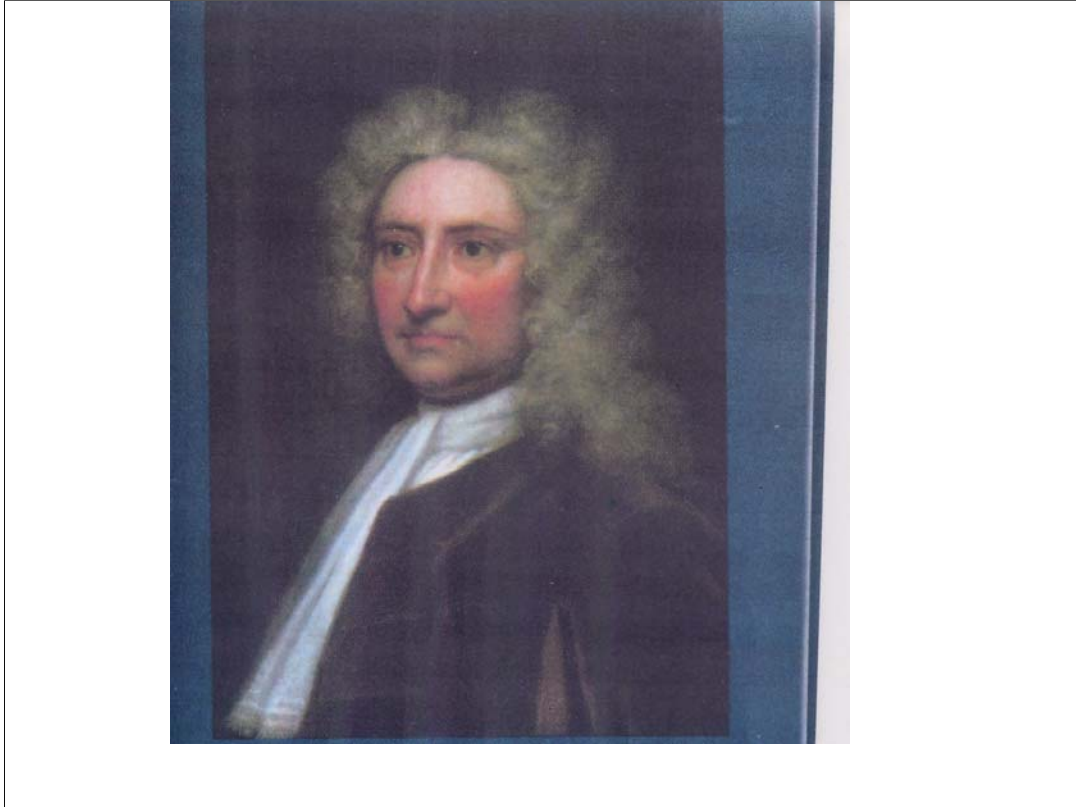
Considering that Halley's main expertise was in quite a different field – astronomy- it is remarkable that he was able to produce such an excellent paper, which laid the foundation for actuarial science.



BRESLAU : TOWN HALL AND MARKET-PLACE, SEEN FROM THE HAY-MARKET.

From an engraving after a drawing by Friedrich Bernhard Werner (1730).

This old print of Breslaw suggests that it was a typical market town. Caspar Neumann was the 44-year old pastor of the imposing church of St. Mary Magdalen, the foremost Protestant church of the region, where he preached in a simple and natural style. He had a lively interest in scientific matters, including meteorology, astronomy and botany. He decided to collect birth and death statistics from the church registers. The Royal Society heard about this and asked him to send his figures over to London. Neumann responded by sending tables for 1687-91 from four parishes only (and excluding Catholics), though Halley wrongly assumed that they related to the whole town. For the whole 5-year period there were 6,193 births and 5,869 deaths, and the deaths were subdivided by age.



Edmond Halley (above) was aged 36 at this time and editor of the *Philosophical Transactions*. His paper started by reviewing the work of Graunt and Petty and criticised the validity of the deductions they had made, because their data did not take age into account.

Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.	Age.	Per- sons.
Curt.		Curt.		Curt.		Curt.		Curt.		Curt.		Curt.		Curt.	
1	1000	8	680	15	628	22	586	29	559	36	481	43	401	50	344
2	854	9	670	16	620	23	578	30	551	37	474	44	407	51	335
3	798	10	661	17	614	24	573	31	543	38	463	45	397	52	324
4	760	11	653	18	610	25	567	32	515	39	454	46	389	53	313
5	732	12	646	19	604	26	560	33	507	40	440	47	377	54	302
6	710	13	640	20	598	27	553	34	499	41	430	48	367	55	292
7	692	14	634	21	592	28	546	35	490	42	427	49	357	56	282
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72. Halley's life table, 1693, was based on the bills of mortality for Breslau, which had the advantage that the deaths were classified by age. The table shows how many of 1,000 babies who have not yet reached their first birthday will survive to successive ages. Halley used the table to show how much should be paid to purchase a life annuity. *Philosophical Transactions*, 1693.

Halley produced his famous life table (above) from Neumann's data, and used it to estimate the total number of people in Breslau as 34,000, with a subdivision into 13 age groups. (This estimate was much too small because the data did not relate to the whole town).

He had no actual census of the population of Breslau, so he was obliged to assume that there was a stationary population. In fact the population of the town may well have been distorted by a plague epidemic in 1633, when about 30% of the town's population died. If, as is likely, these people were mainly replaced by an influx of young adults from the surrounding countryside, this would have resulted in the town having a "population bulge" of people born around 1605-15, which may have meant that Halley's life table overstated the number of people normally surviving into old age.

Having produced his life table, Halley used it in the same way as an actuary would do today, to estimate the present value of a payment in a future year which depended upon survival to that year. Thus for an annuitant aged 27, the value of the payment in one year's time would be evaluated, as if it were a certainty, from a compound interest table, and the result was then multiplied by 546/553 to reflect the chance of surviving to receive it. A similar process would be followed for the payment in two years' time, multiplying the value from the compound interest table by 539/553, and so on. Summing these results for all future payments of a life annuity would give the price which should be paid for the annuity at the outset.

Based on his results, Halley pointed out that the British Government was selling life annuities much too cheaply at only 7 years' purchase, whereas the true value for a young life was over 13 years' purchase. The Government ignored his advice, however.

Not content with this, Halley went on to consider the value of an annuity dependent on the continuation of two lives.



## Application of actuarial science

- Progress slow
- Bubble insurances, 1710-20
- Techniques developed by Abraham de Moivre and Thomas Simpson
- Applications to leases on lives – John Richards, 1730

Although Halley had shown the way, in a paper which was widely circulated, the commercial world was very slow to take advantage of actuarial science and the money-making opportunities it offered. A large number of insurance companies were formed in the first 20 years of the 18<sup>th</sup> century, in the period leading up to the South Sea Bubble, but they were nearly all unsound financially and died away. One notable exception was the Amicable Society, formed by John Hartley, a London bookseller, in 1706, which offered long-term life assurance to the public. Part of the aggregate premium income was used to meet death claims in the early years and the remainder was set aside to build up a fund. Hartley does not seem to have used Halley's work in any way, and claims proved heavier than expected as the members aged, but the Society had a long and honourable life, being finally absorbed into the Norwich Union in 1866.

The mathematicians Abraham de Moivre and Thomas Simpson developed techniques based on Halley's approach. One of their main aims was to cut down on the amount of labour involved in the calculations, and de Moivre developed an ingenious approximation which achieved this without too much loss of accuracy. His method assumed that the numbers of those living, according to the life table, decreased in arithmetical progression with increasing age.

Perhaps the earliest practical application of Halley's method was in the valuation of leases based on lives. John Richards of Exeter published a useful book in 1730, to help stewards and tenants to value their interests in leases and estates, where these depended on the survival of named people.

ANNUITIES  
UPON  
LIVES:

OR,

The VALUATION of  
ANNUITIES upon any  
Number of LIVES ; as  
also, of REVERSIONS.

To which is added,  
An APPENDIX concerning the  
EXPECTATIONS of LIFE, and  
Probabilities of SURVIVORSHIP.

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By A. DE MOIVRE. F. R. S.

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L O N D O N,  
Printed by WILLIAM PEARSON,  
over-against Wright's-Coffee-House in  
Aldersgate-Street. MDCCXXV.

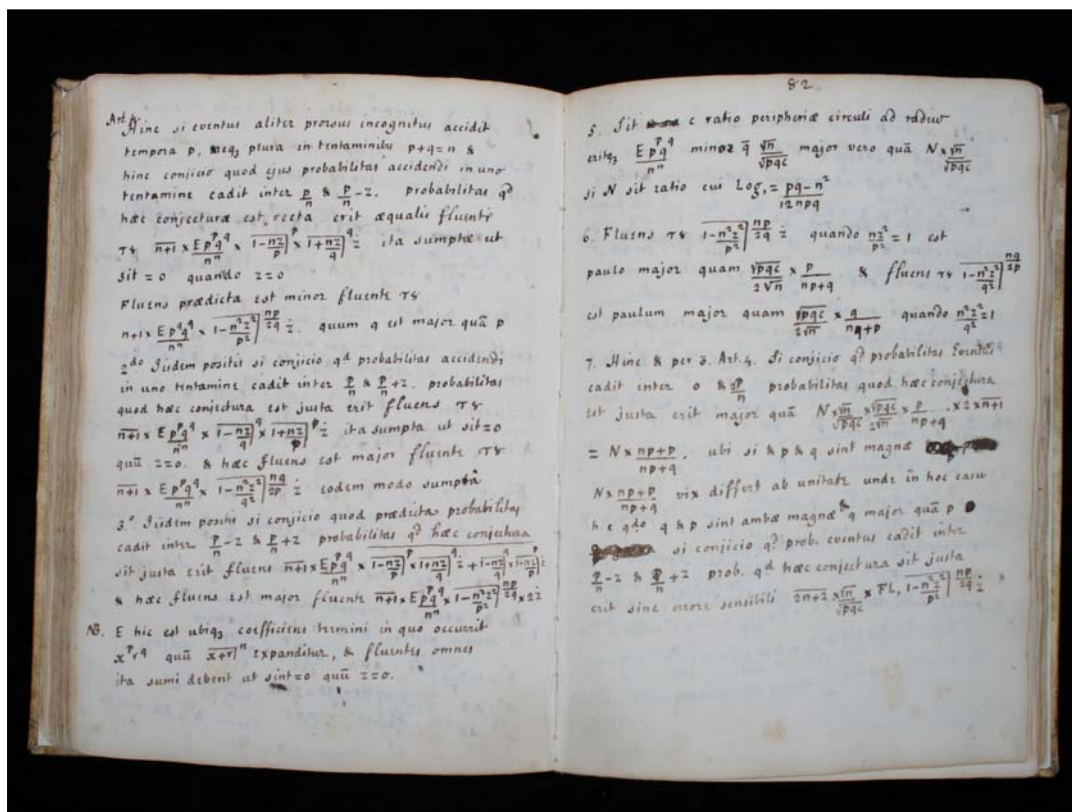
This is the title page of de Moivre's book. Among other problems the book described a method for valuing reversion, where a benefit was receivable after someone's death.

		One Life 32 Years old.				
32 Years old.	2 <sup>d</sup> Life	4 <sup>per</sup> C.	5 <sup>per</sup> C.	6 <sup>per</sup> C.	7 <sup>per</sup> C.	8 <sup>per</sup> C.
	3 <sup>d</sup> Life					
32		7.55	7.35	7.09	6.77	6.25
42		6.96	6.76	6.52	6.25	5.87
52		6.27	6.12	5.93	5.71	5.42
62		5.26	5.16	5.03	4.88	4.60
72		3.83	3.77	3.70	3.61	3.48
82		1.23	1.22	1.21	1.20	1.19
42 Years old.	42	6.49	6.27	6.03	5.79	5.53
	52	5.87	5.70	5.51	5.33	5.11
	62	4.96	4.85	4.72	4.58	4.42
	72	3.65	3.60	3.53	3.42	3.29
	82	1.20	1.19	1.18	1.17	1.16
52 Years old.	52	5.34	5.24	5.12	4.95	4.74
	62	4.58	4.50	4.40	4.28	4.14
	72	3.42	3.38	3.33	3.26	3.16
	82	1.17	1.16	1.15	1.14	1.13
62	62	3.98	3.92	3.85	3.77	3.66
	72	3.06	3.03	2.99	2.94	2.86
	82	1.11	1.10	1.09	1.08	1.07
72	72	2.46	2.44	2.41	2.38	2.33
	82	0.98	0.97	0.96	0.95	0.94
	82	0.55	0.54	0.54	0.53	0.52

Some pioneering tables by John Richards (above) showed the value of annuities on three lives, ceasing on first death. If the three lives were aged 32, 42 and 52, for example, and money earned 5 per cent per annum, the cost of purchase would be £5.70.

Richards also used his tables to find the value of a lease which terminated on the *last survivor* of two or three lives, using a result derived by de Moivre, namely that the value of a last survivor annuity on two lives of given ages is obtained by adding the single-life annuity values for each life and then deducting the value of an annuity during their joint continuance. A slightly more complex variant of the same technique is used if there are three lives.

Richards discussed the risks facing the purchaser of an estate and said that the method he had used to allow for such risks was to allow the purchaser a greater rate of interest on the money laid out.



Probability theory continued to be developed during the 18<sup>th</sup> century. This Latin manuscript by Thomas Bayes, probably dating from 1747-49, includes some of his work on probability as shown above. The result for which he is mainly remembered – Bayes's Theorem – is not included here and was published after his death by Richard Price, who is credited nowadays with being the first to attempt a general Bayesian account of inductive reasoning.

## The mathematics of insurance

Corbyn Morris looks at ruin, 1747 –

If risk is 33% and insurance is spread over 12 voyages:

more than 8 succeed - 39% chance

exactly 8 succeed - 24% “

less than 8 succeed - 37% “

Probability of losing all is 2 in a million,  
compared with 33% if no spread

An important advance was made in the theory of insurance in 1747 by Corbyn Morris, who demonstrated mathematically that the insurer's probability of ruin decreases as he spreads his available wealth between a larger number of risks. He postulated a number of similar voyages, in each of which there was a one-third risk of loss, and calculated the possible outcomes. If the insurer spread his risks over twelve voyages rather than one, the risk of making some loss was only marginally increased, from 33% to 37%, but the probability of losing everything was reduced from 33% to only 2 in a million.

# The Equitable

- James Dodson's first lecture 1756
- Foundation of the Society 1762
- Richard Price acts as consultant
- William Morgan becomes actuary 1775
- First actuarial valuation 1776
- Distribution of surplus
- Mortality investigations

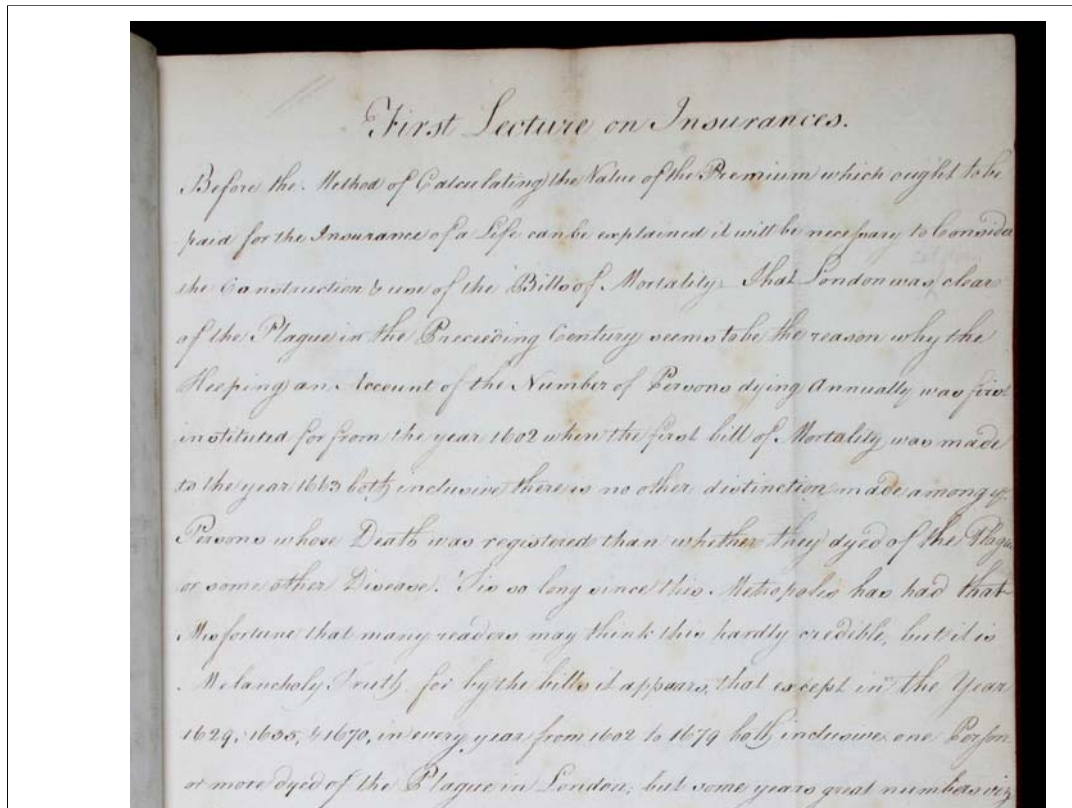
James Dodson, the Master of the Royal Mathematical School (which taught mathematics and navigation for the Royal Navy) wrote his *First Lecture on Insurances* in 1756. He started by constructing a life table based on the London Bills of Mortality from 1728 (when age at death first started to be recorded) to 1750. He then looked at how a life assurance company based on scientific principles would operate. He explored the worst case facing such a company, assuming that it experienced deaths in the early years as heavy as the actual experience of 1741, a year of high mortality. In the light of this he suggested that a large sum should be borrowed at the outset as a contingency reserve.

Dodson advertised a meeting of those who would be interested in forming such a company, but he died in 1757 before the project came to fruition. Eventually The Equitable was formed in 1762.

From about 1770 onwards Richard Price, a non-conformist minister, acted as a consultant to the Society. He was an expert in actuarial matters, on which he wrote extensively. In 1774 he wrote a paper for the Directors, which still survives among the Equitable's archives, setting out his recommendations on a method for ascertaining the true financial state of the Society. The first actuarial valuation was carried out in 1776 under his guidance.

Price's nephew, William Morgan, was appointed as actuary to the Society in 1775 and he remained in that post for more than fifty years. The Society's periodic valuations revealed surpluses, because the premiums turned out to be higher than proved necessary for the professional classes who insured with the Society, and various methods were used for distributing surplus funds to policyholders, including a reversionary bonus system. The Society analysed its own mortality experience from time to time, to check that the premiums it was charging were sufficient.





Dodson's *First Lecture on Insurances* (above) commences: "Before the method of calculating the value of the premium which ought to be paid for the insurance of life can be explained it will be necessary to consider the construction and use of the Bills of Mortality. That London was clear of the Plague in the preceding century seems to be the reason why the keeping an account of the number of persons dying annually was first instituted, for from the year 1602 when the first Bill of Mortality was made [this is incorrect – there were earlier Bills] to the year 1663 both inclusive there is no other distinction made among the persons whose death was registered than whether they died of the Plague or some other disease. 'Tis so long since this metropolis has had that misfortune that many readers may think this hardly credible, but it is melancholy truth....." This is a very important document in actuarial history, and it still exists only in manuscript form.

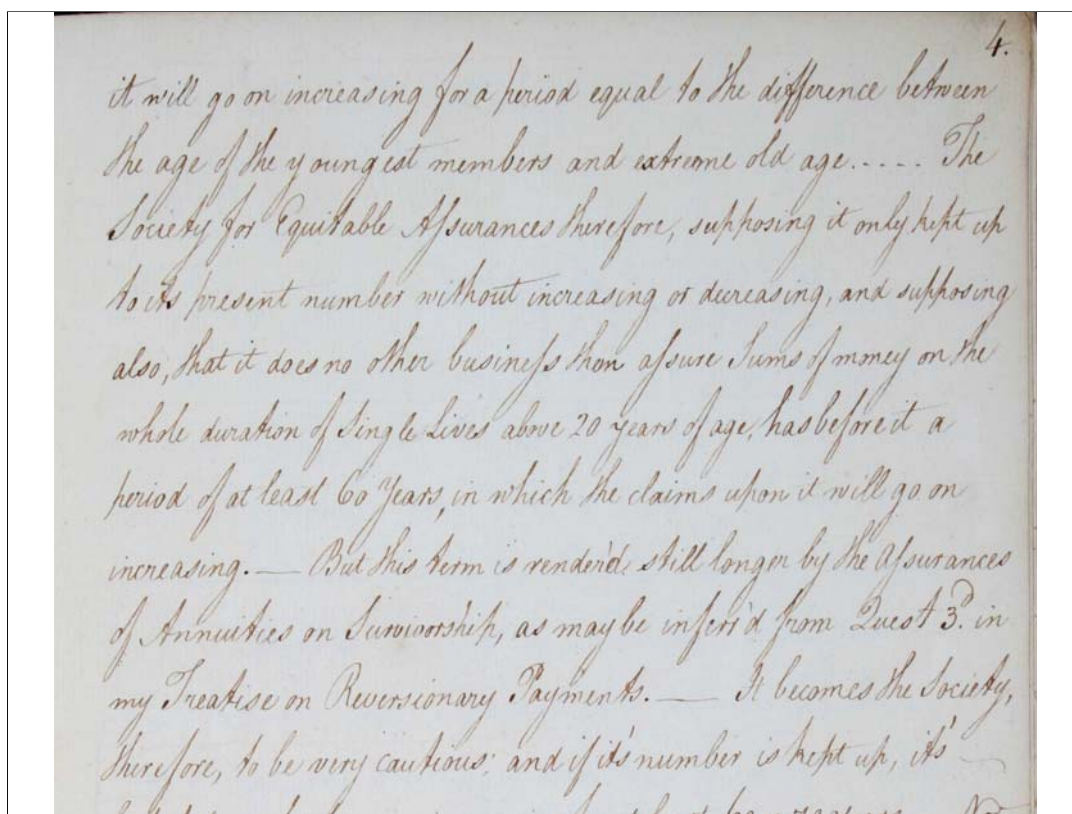
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*A Table of the Annual Premiums payable for insur-  
ing 100 £ during the whole continuance of a single Life of  
any of the following Ages according to the foregoing principles*

<i>Age</i>	<i>Annual Premium</i>	<i>Age</i>	<i>Annual Premium</i>	<i>Age</i>	<i>Annual Premium</i>	<i>Age</i>	<i>Annual Premium</i>
<i>14</i> <i>or</i> <i>Under</i>	2,358	28	3,404	42	4,858	56	7,438
15	2,427	29	3,501	43	4,988	57	7,295
16	2,500	30	3,612	44	5,122	58	7,556
17	2,578	31	3,698	45	5,226	59	7,901
18	2,630	32	3,801	46	5,332	60	8,272
19	2,712	33	3,880	47	5,466	61	8,673
20	2,775	34	3,976	48	5,599	62	9,108
21	2,846	35	4,076	49	5,735	63	9,581
22	2,921	36	4,180	50	5,864	64	10,094

The first table of premium rates used by The Equitable. The cost of a life policy varied according to the age of the life to be assured, and once fixed it would not be varied as the person grew older.





Here is an extract from Richard Price's important report to the Directors of The Equitable in 1774 about how to establish the Society's financial state. "The Society for Equitable Assurances therefore, supposing it only kept up to its present number without increasing or decreasing, and supposing also, that it does no other business than assure sums of money on the whole duration of single lives above 20 years of age, has before it a period of at least 60 years, in which the claims upon it will go on increasing... It becomes the Society, therefore, to be very cautious..."

This plea for caution was often echoed by William Morgan, the Society's actuary, during the next half a century, when he faced repeated calls for the distribution of surplus assets to policyholders.

In 1776 each of the Society's 922 policies was valued during a four month period. The 89 deaths which had occurred during the previous ten years was slightly less than expected under Halley's life table (99) and much less than under Dodson's and Simpson's tables (136 and 139 respectively). The premiums had, of course, been based on Dodson's table.

		Midsummer Quarter 177A	529 11
July 10	1	To Oliver Taylor paid 1/2 Salary due to J <sup>n</sup> Dugood deceased at Mich <sup>l</sup> last.....	10
27	1	To William Miller paid on Account for Furniture & Expenses.....	50
Aug. 21	1	To John Pocock, Actuary, paid 1/2 Salary due at Mich <sup>l</sup> last.....	56 5
21	1	To Wills..... paid him for his Clock.....	15
21	1	To William Morgan, Assistant Actuary, paid 1/2 Salary due at Mich <sup>l</sup> last.....	27 10
Sept. 1	1	To John Lewis, Printer paid his Bill.....	7 11 6
7	1	To William Miller, paid him more on Account.....	25
28	1	To the Assignees of Marmaduke Storr, paid for his care of the Clock for half a year.....	10 6
29	1	To Wills Disbursements paid since the 2A <sup>th</sup> June last.....	27 5 9
		Michaelmas Quarter 177A	240 15 0
Oct. 5	2	To John Pocock, Actuary 1/2 Salary due at Mich <sup>l</sup> last.....	27 10
5	2	To William Morgan, Assistant Actuary, 1/2 Salary due at Michaelmas last.....	25
5	2	To John Pocock, paid him 1/2 Salary for his Clock due at Mich <sup>l</sup> last.....	15
Nov. 21	2	To William Miller, paid him on Account for Work, Furniture & Expenses.....	50
28	2	To John Pocock, Actuary, paid 1/2 Salary due at Xmas last.....	27 10
28	2	To William Morgan, Assistant Actuary, paid 1/2 Salary due at Xmas last.....	25
28	2	To John Pocock, paid him 1/2 Board & Wages for his Clock due at Xmas last.....	15
28	2	To Mess <sup>rs</sup> Gunttenden & Finch, paid for their Bill of Costs in the Suit with the Executors of Francis Milne.....	29 9 6
29	2	To Wills Disbursements paid since the 29 <sup>th</sup> of September last.....	16 17 2
			221 6 8
		177A L. 298. q. 11	

In this extract from the account book of The Equitable, we see some entries for the quarterly salaries paid to the Society's first actuaries. William Morgan, at this stage only the assistant actuary, was paid a salary of £100 per annum, while John Pocock, the actuary (who died in 1775) was paid £150 per annum.

## 36] T A B L E S.

## T A B L E VI.

Shewing the Probabilities of the Duration  
of Human Life at all Ages, formed from  
the Register of Mortality at *Northampton*,  
for 46 Years from 1735 to 1780.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decrem.
0	11650	1340	31	4310	75	65	1632	80
3 months	10310	554	32	4235	75	66	1552	80
6 months	9750	553	33	4100	75	67	1472	80
9 months	9203	553	34	4085	75	68	1392	80
1 year	8650	1367	35	4010	75	69	1312	80
2 years	7283	502	36	3935	75	70	1232	80
3	6781	335	37	3860	75	71	1152	80
4	6446	197	38	3785	75	72	1072	80
5	6249	184	39	3710	75	73	992	80
6	6065	140	40	3635	76	74	912	80
7	5925	110	41	3559	77	75	832	80
8	5815	80	42	3482	78	76	752	77
9	5735	60	43	3404	78	77	675	73
10	5675	52	44	3326	78	78	602	68
11	5623	50	45	3248	78	79	534	65
12	5573	50	46	3170	78	80	469	63
13	5523	50	47	3092	78	81	406	60
14	5473	50	48	3014	78	82	346	57
15	5423	50	49	2936	79	83	289	55
16	5373	53	50	2857	81	84	234	48

One of Richard Price's many achievements was the production of the Northampton life table (above), which was used for many years by The Equitable and other early life assurance offices, to calculate their premium rates. Price analysed the christenings and burials in the parish of All Saints in that town from 1735 to 1780 and made some adjustments (later thought to be unsound) to achieve consistency with other data.

## Some later actuarial landmarks

- John Finlaison's mortality investigation 1829
- Joint Offices mortality tables 1843
- Formation of Institute 1848
- Manly and Thomas on pension fund valuations 1901
- Redington on immunisation 1952

The 19<sup>th</sup> century saw an immense growth in life assurance and the twentieth century a proliferation and growth of pension schemes, based on the solid actuarial foundations which had been established earlier. There were many technical improvements during this period, and only some of them can be touched on here.

John Finlaison can be regarded as the first Government Actuary (though that was not his official title). In 1829 he conducted a comprehensive investigation into the mortality experiences of various groups of lives. Like Halley over a century before, he concluded that the British Government was still selling life annuities too cheaply.

Benjamin Gompertz attempted in 1825 to fit a simple mathematical curve to observed mortality experiences in order to find a "law of mortality". Although obsolete, his method is still occasionally used as an approximation for some practical purposes. Interest has continued and in 1953 M.E. Ogborn submitted a paper for discussion at a meeting of the Institute of Actuaries, wholly devoted to the question of whether there is a mathematical formula which will express the way in which mortality changes age by age.

In 1843 the various life offices which were now in existence published the results of a collaborative mortality investigation in which they pooled their policy data. The success of this venture was probably one of the factors which led to the formation of the Institute of Actuaries in 1848, with John Finlaison as the First President.

Later in the 19<sup>th</sup> century various technical improvements were introduced in the methods used to graduate mortality tables, so that the crude rates produced from the data were adjusted so as to run smoothly from age to age. W.S.B. Woolhouse was the first to explain graduation using differences, in 1866.

In 1901 two actuaries (H.W. Manly and E.C. Thomas) wrote the first comprehensive paper on the valuation of staff pension funds, in which they produced a multiple decrement table. Whereas an ordinary life table had only one cause of decrement (death), the new table allowed also for withdrawals and retirements.

In 1952 Frank Redington produced his classic paper on immunisation theory, which discussed the matching of the durations of the assets with the durations of the liabilities, so as to reduce the possibility of a loss arising from a change in interest rates.

Immense changes have come from the introduction of modern electronic computing later in the twentieth century, which have enabled much more complex financial products to be introduced and much more complex modelling carried out. Various difficulties have arisen in recent years, partly as a result of all this complexity. However, the basic human needs for which insurance and pensions were introduced remain unchanged, as has the need for proper unbiased scientific advice on the terms on which financial products should be offered. The actuarial profession is responding to the new challenges and, in looking forward, is also conscious of the lessons which can be learnt from the past.

Looking back will help us to look  
forward

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